Perturbative and non-perturbative aspects of renormalization and power counting in nucleon-nucleon interaction within chiral effective field theory

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in collaboration with E. Epelbaum

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Outline

- LO interaction contains long- and short-range parts
  LO interaction is (close to) non-perturbative
- Power counting at LO: perturbative case
- NLO: perturbative renormalization
- Cutoff dependence
- Power counting at LO: non-perturbative case
- NLO: non-perturbative renormalization
- Summary

Chiral EFT

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

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"Perturbative" calculation of the S-matrix, spectrum, etc.

Expansion parameter: (soft scale)/(hard scale) $Q = \dfrac{q}{\Lambda_b}$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$$

Weinberg power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)

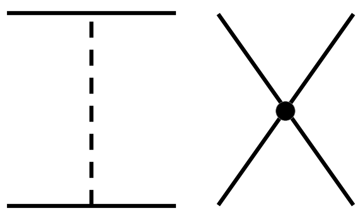
For potential (2N-irreducible) contributions:

$$D = 2L + \sum_{i=\text{vertices}} \left(d_i + \frac{n_i}{2} - 2 \right)$$

d_i – number of derivatives and quark masses

n_i – number of nucleon fields , L – number of loops

$\mathcal{O}(Q^0)$



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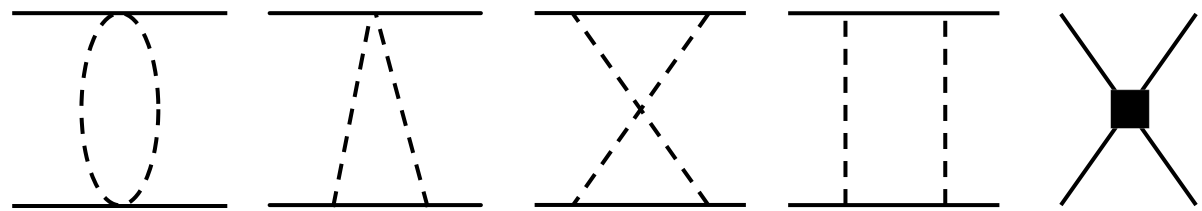

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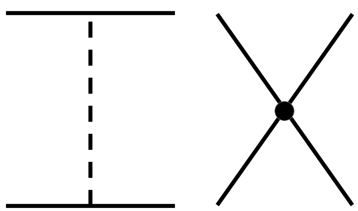

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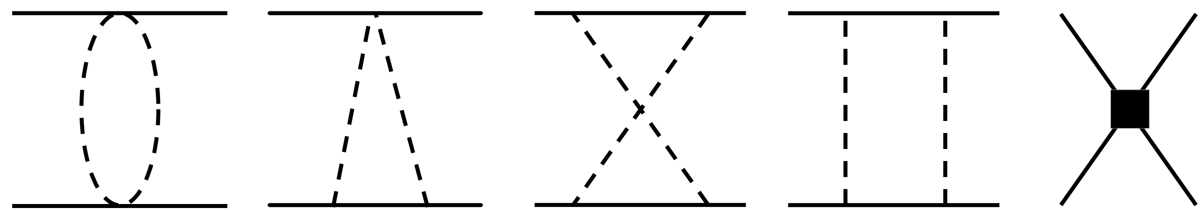

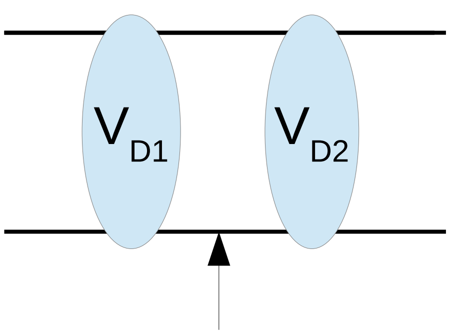

$$\sim \frac{m_N q}{\Lambda_b^2} \sim 1$$

Enhancement due to the infrared singularity: V_0 must be iterated

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

Regularization

Divergent:

$$T_0 = V_0 + V_0GV_0 + V_0GV_0GV_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0G)^m\, V_2\, (GV_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

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$\longrightarrow$ Regulator: cutoff Λ

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→ Regulator: cutoff Λ

Infinite number of counter terms to absorb positive powers of Λ

Infinite cutoff ($\Lambda >> \Lambda_b$)

All positive powers of Λ cancel

$$T \approx 1 + \Lambda + \Lambda^2 + \cdots = \frac{1}{1-\Lambda}$$

A. Nogga, R. Timmermans,
U. van Kolck, **PRC72**, 054006 (2005)
B. Long, C. Yang, **PRC85**, 034002 (2012)
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No justification beyond simplest applications

E. Epelbaum, J. Gegelia, **EPJA41**, 341 (2009)
E. Epelbaum, AM, J. Gegelia,
U.-G. Meißner, **EPJA54**, 186 (2018)

Finite cutoff

$$\Lambda \approx \Lambda_b$$

Phenomenological success (NN): ≥N^{4}LO

P. Reinert, H. Krebs, and E. Epelbaum, **EPJA54**, 86 (2018)
D. R. Entem, R. Machleidt, and Y. Nosyk, **PRC96**, 024004 (2017)

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Consistent with power counting?

Power counting. Leading order.

$$T_0^{[n]} = V_0(GV_0)^n \sim \mathcal{O}(Q^0)$$

$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1}dp}{(\Lambda_{\mathrm{V}})^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$$

G. P. Lepage, nucl-th/9706029
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Rigorously proved
under rather general conditions on V_0
if T_0 is perturbative
(P-waves and higher except for $^3P_0$):

$$\mathrm{T}_0 = \sum_{n=0}^{\infty} T_0^{[n]}$$

$$T_0^{[n]} \leq \mathcal{M}_1 \left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^n \qquad \mathcal{M}_1\,,\mathcal{M}_2 \sim 1$$

AG, E.Epelbaum,
PRC 105, 024001 (2022)

Renormalization. NLO

Renormalization: power counting in terms of renormalized quantities

$$T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from momenta: $p \sim \Lambda \, , p' \sim \Lambda$ in $V_2(p', p)$

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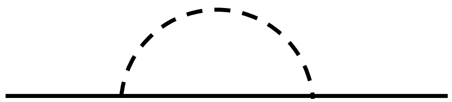
Can be absorbed by LO contact interactions? $\mathbb{R}\left(T_2^{[m,n]}\right) \sim \frac{q^2}{\Lambda_b^2}\left(\frac{\Lambda}{\Lambda_V}\right)^{m+n}$?

Power counting violation. Examples in χPT

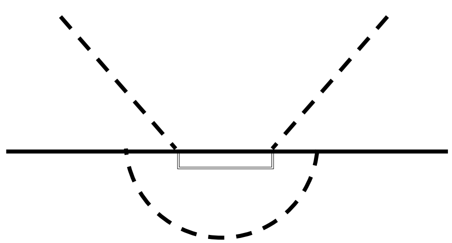
Covariant χPT, new scale: m_N

T. Becher, H. Leutwyler, **EPJC9**, 643 (1999)
T. Fuchs, et al., **PRD68**, 056005 (2003)



$\sim m_N^3 \neq Q^3$

Δ-ful HBχPT, new scale: $\Delta = m_\Delta - m_N$



$\sim \Delta \neq 1/\Delta$ Decoupling

H. Krebs, AG, E. Epelbaum
PRC98(1), 014003 (2018)

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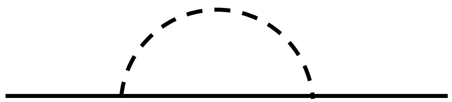

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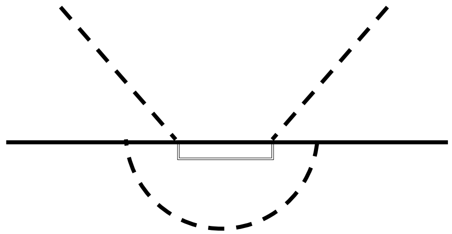

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Power counting and decoupling can be restored
by renormalizing lower order LECs

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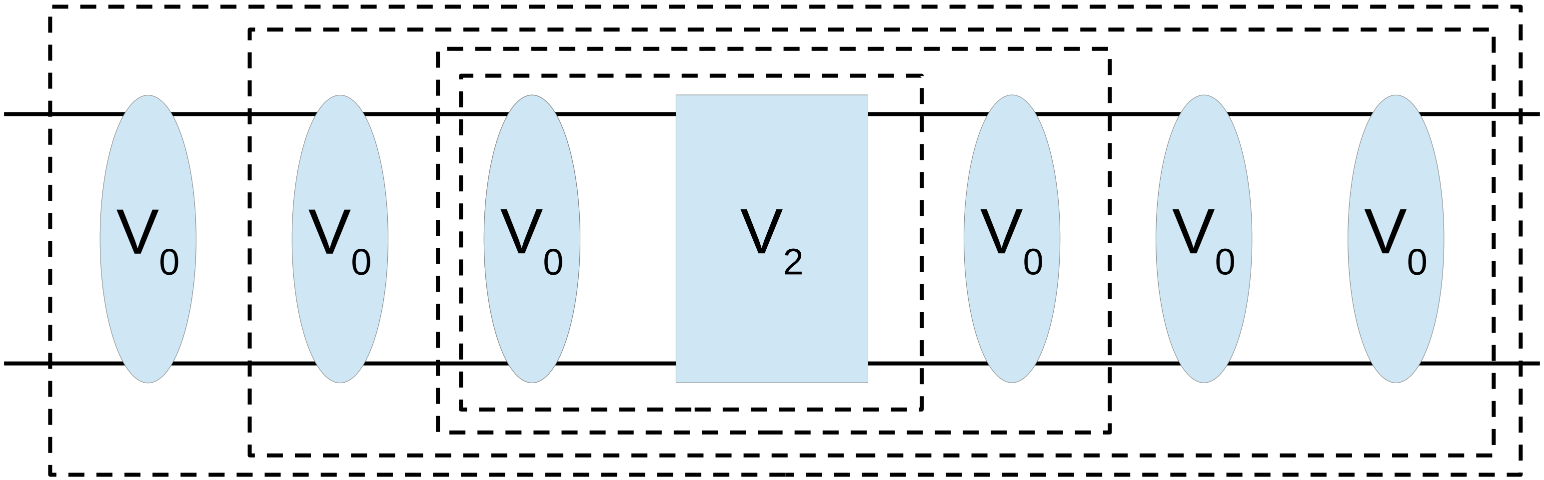
Non-perturbative effects?

Renormalization of NLO amplitude to arbitrary order in V_0. BPHZ subtraction scheme

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



Subtraction operation: $\mathbb{T}(X)(p', p, p_{\rm on}) = X(p'=0, p=0, p_{\rm on}=0)$

Renormallized amplitude (forest formula): $\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}$

$$U_k = ((m_{k,1}, n_{k,1}), (m_{k,2}, n_{k,2}), \ldots), \quad m \geq m_{k,i+1} \geq m_{k,i} \geq 0, \; n \geq n_{k,i+1} \geq n_{k,i} \geq 0 .$$

Power counting in the perturbative case

AG, E.Epelbaum, PRC 105, 024001 (2022)

Convergent series in V_0:

$$\mathbb{R}\left(T_2\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$$

$$\boxed{\left|\mathbb{R}(T_2^{[m,n]})(p)\right| \le \mathcal{M}_1 \left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^{m+n} \frac{p^2}{\Lambda_b^2} \log \Lambda/M_\pi}$$

$$\mathcal{M}_1\,,\mathcal{M}_2 \sim 1$$

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☺

Cutoff dependence. Systematic study.

Regulated potential: $V_0 \equiv V_\Lambda = V_{\Lambda=\infty} + \delta V_\Lambda$

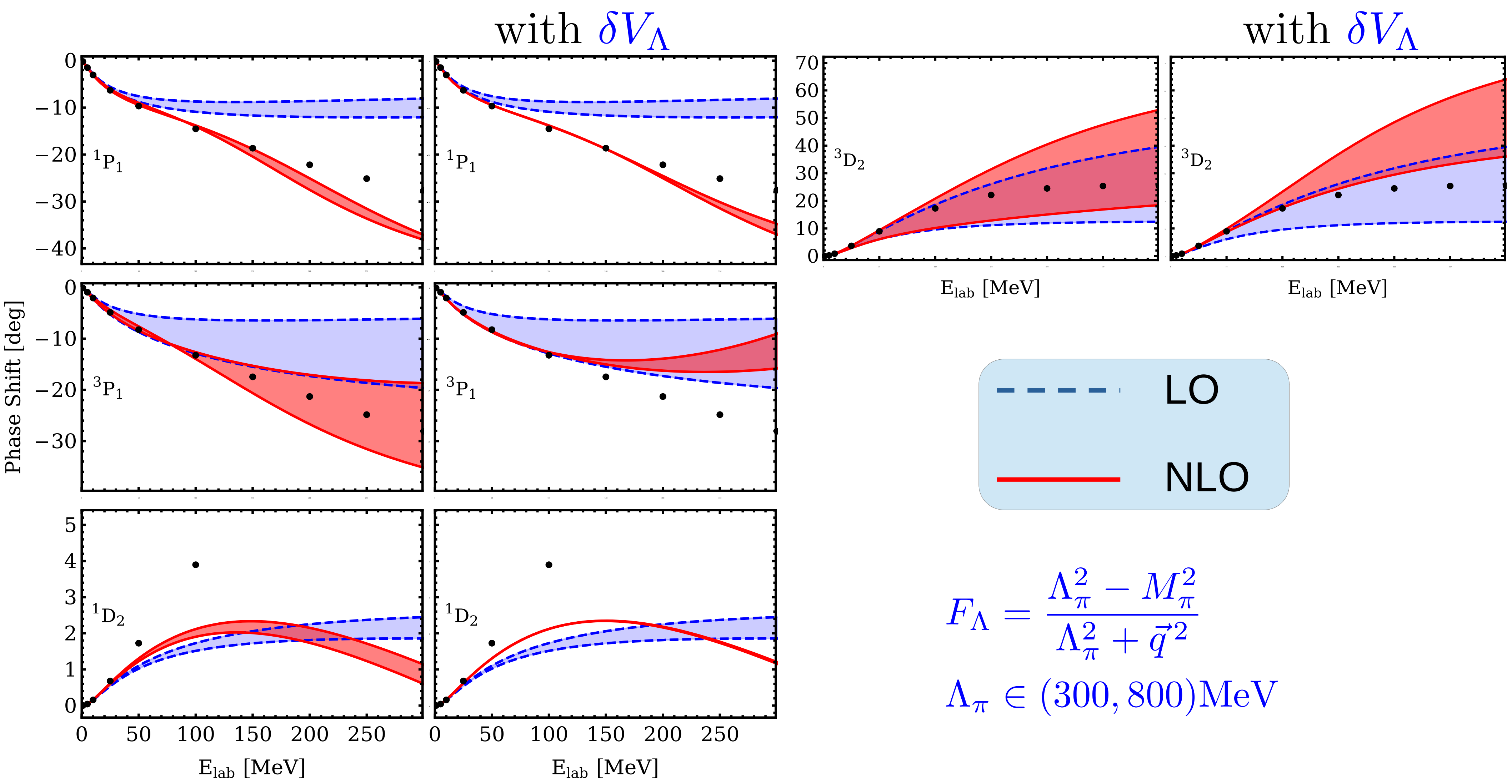
Perturbative inclusion of δV_Λ: $\delta T_2^\Lambda = (1 + T_0 G)\delta V_0^\Lambda (1 + G T_0) \sim \mathcal{O}(Q^0)$

After renormalization: $\mathbb{R}\left(\delta T_2^\Lambda\right) \sim \mathcal{O}(Q^2)$

Removing Λ-dependence perturbatively

Cutoff dependence: P and D-waves. Uncoupled perturbative channels

AG, E.Epelbaum, PRC 105, 024001 (2022)



$$F_\Lambda = \frac{\Lambda_\pi^2 - M_\pi^2}{\Lambda_\pi^2 + \vec{q}^{\,2}}$$

$$\Lambda_\pi \in (300, 800)\mathrm{MeV}$$

Cutoff dependence with δV_Λ is weaker

S-waves. Non-perturbative LO. Fredholm formula

$$T_0 = V_0 R = \bar{R} V_0 \qquad R = \frac{1}{\mathbb{1} - G V_0} = \frac{N}{D}\,, \quad \bar{R} = \frac{1}{\mathbb{1} - V_0 G} = \frac{\bar{N}}{D}$$

Convergent series in V_0 :

$$N = \sum_{i=0}^{\infty} N_i\,, \quad D = \sum_{i=0}^{\infty} D_i$$

(Quasi-) bound state:

$$D(p) \sim \frac{p}{M_\pi}$$

Enhancement at threshold:

$$T_0(p) = \frac{N_0(p)}{D(p)} \sim \mathcal{O}(M_\pi/p)$$

$$\text{For } p > M_\pi : \; T_0(p) \sim \mathcal{O}(Q^0)$$

NLO. Using Fredholm formula.

$$T_2(p) = (1 + T_0 G) V_2 (1 + G T_0) = \frac{N_2(p)}{D(p)^2}$$

Convergent series in V_0 :

$$N = \sum_{i=0}^{\infty} N_i \, , \;\; D = \sum_{i=0}^{\infty} D_i$$

The same for the counter terms:

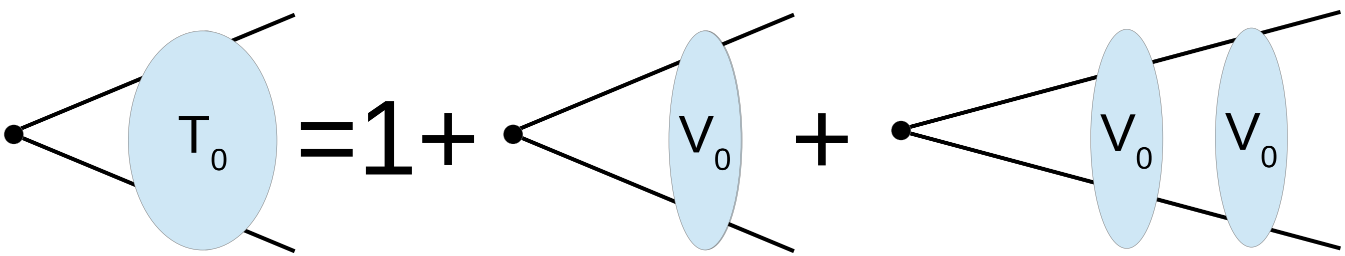
$$\delta T_2 = (1 + T_0 G) \delta V_0^{ct} (1 + G T_0)$$

S-waves. NLO.
Subtractions in the non-perturbative case

The series for ***R(T₂***

The series for $R(T_2^{[m,n]})$ can be summed explicitly

$$\mathbb{R}\left(T_2\right)(p) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p) = T_2(p) - T_2(p=0)\left[\frac{\psi_p(0)}{\psi_{p=0}(0)}\right]^2$$

$\psi_p(0) = 1 +$ 

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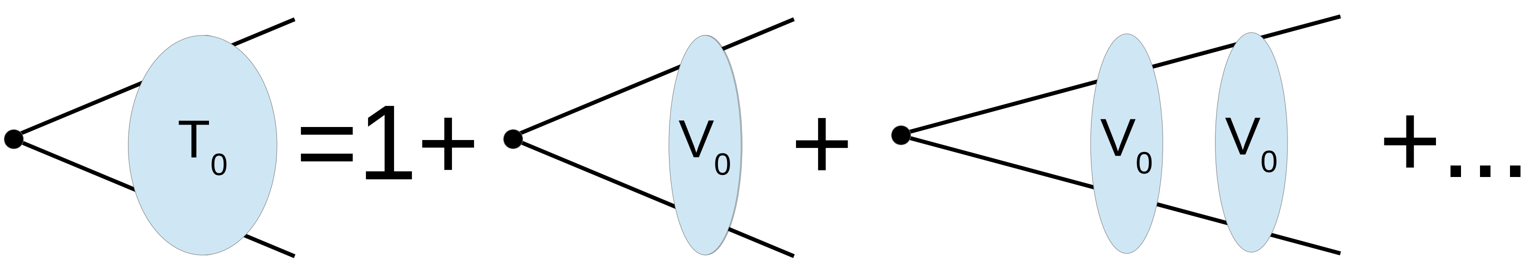

$$\mathbb{R}\left(T_2\right)(p=0) = 0$$

Non-perturbative NLO. Renormalizability. Constraints on V_0

$$\mathbb{R}\left(T_2\right)(p) = \frac{1}{D(p)^2}\left[N_2(p) - N_2(0)\frac{N_\psi(p)}{N_\psi(0)}\right]$$

$$\psi_p(0) = N_\psi(p)/D(p)$$

Convergent series in V_0:

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\ , \ \ N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

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Inside the region: $V_0 \to \lambda V_0\,,\ |\lambda| < 1$

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even if

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More constraints at higher orders!

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More constraints at higher orders!

The same constraints apply to the cutoff dependence

Summary

- ✔ We considered NN Chiral EFT (LO is nonperturbative and contains long- and short-range contributions) with a finite cutoff at NLO
- ✔ Power-counting breaking contributions at NLO can be absorbed by the renormalization of the LO contact interactions for perturbative LO under rather general conditions
- ✔ In the case of non-perturbative LO, the requirement of renormalizability imposes certain constraints on the LO potential