北京航空航天大學
BEIHANG UNIVERSITY

高精度相对论手征核力：现状与未来

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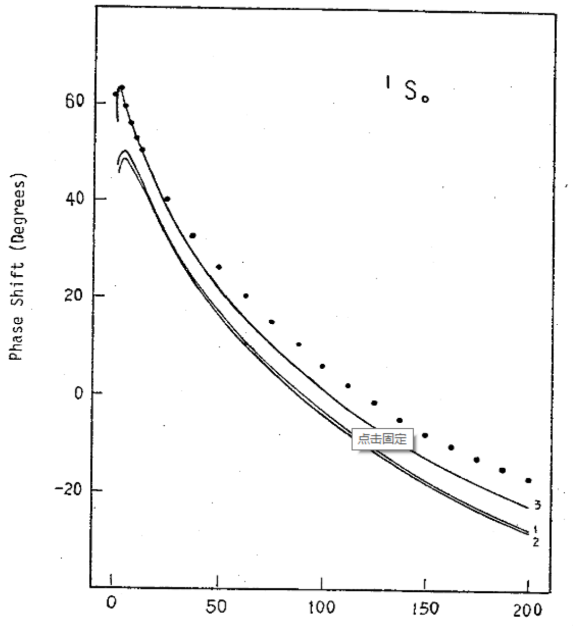
2111.07766: Jun-Xu Lu, Yang Xiao, Chun-Xuan Wang, Li-Sheng, Geng, Jie Meng, and Peter Ring

向前辈科学家致敬

1988	Quark model	CPL 5 (1988) 297
1991	Quark-antiquark pair creation model	NPA 528 (1991) 513
1993	Quark potential model	NPA 561 (1993) 595
2003	Extended chiral SU(3) quark model	NPA 727 (2003) 321



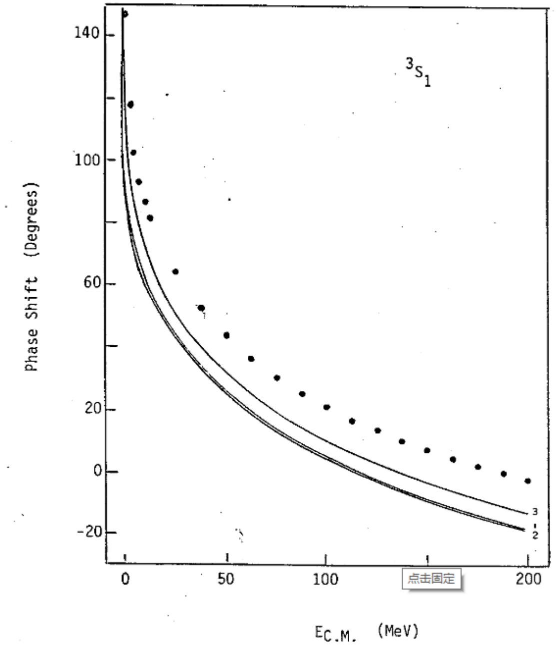


Fig.4

contents

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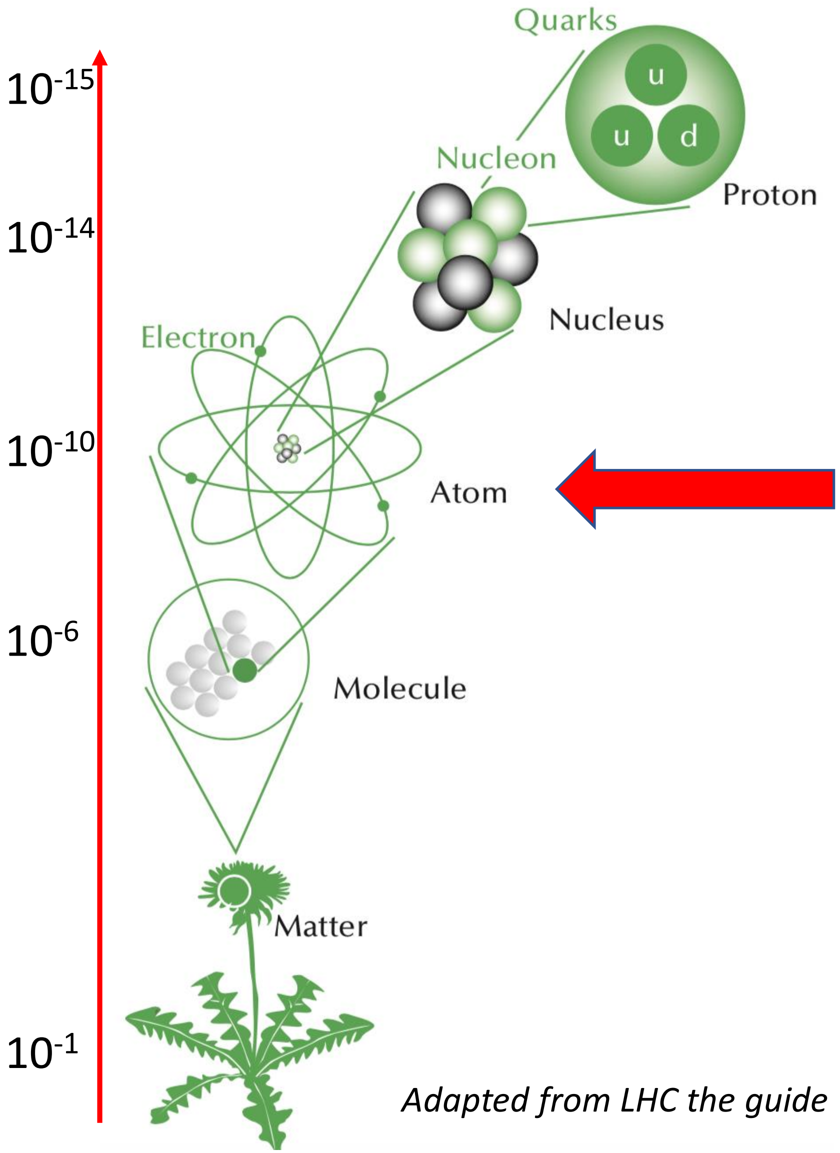
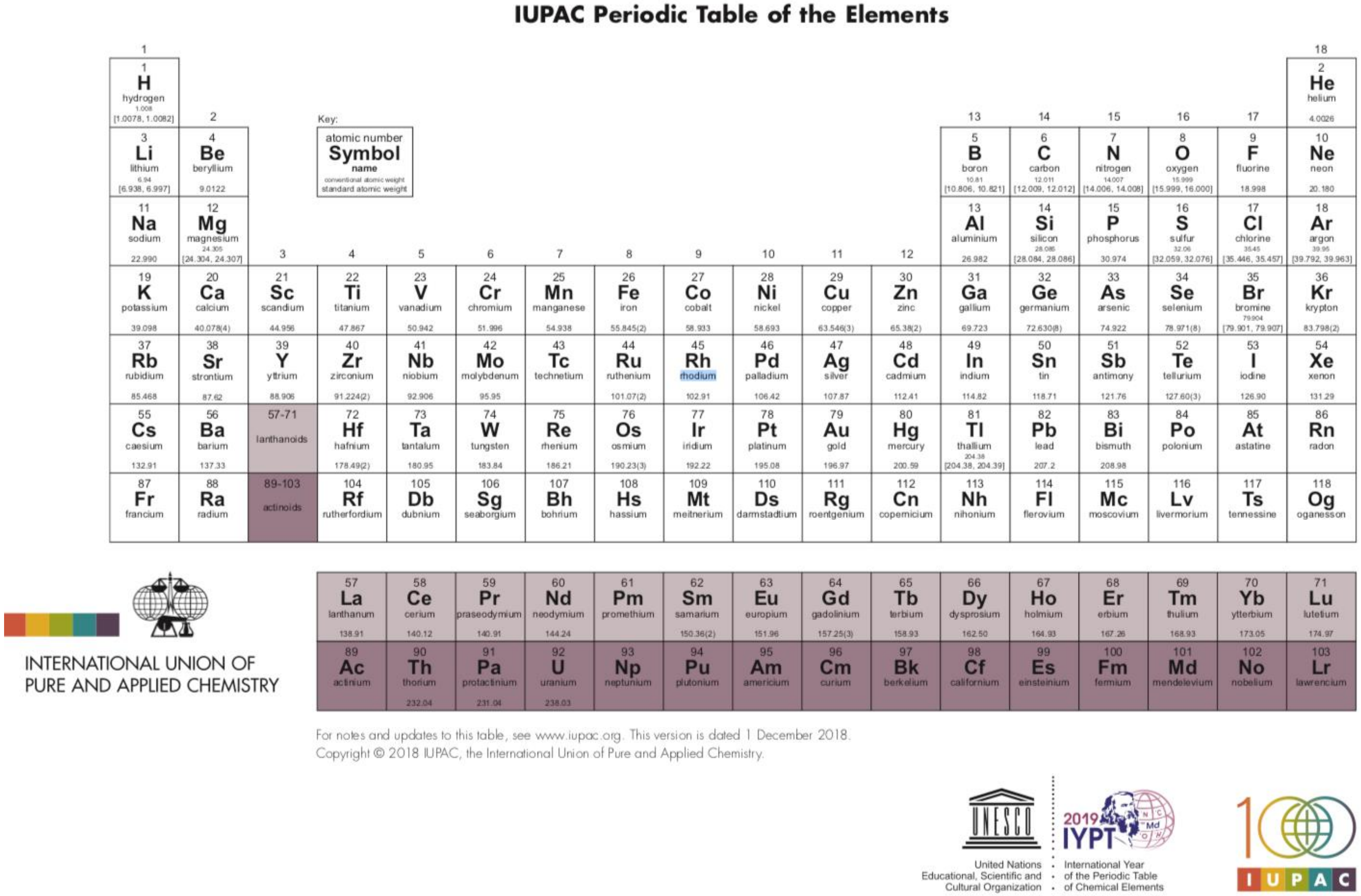
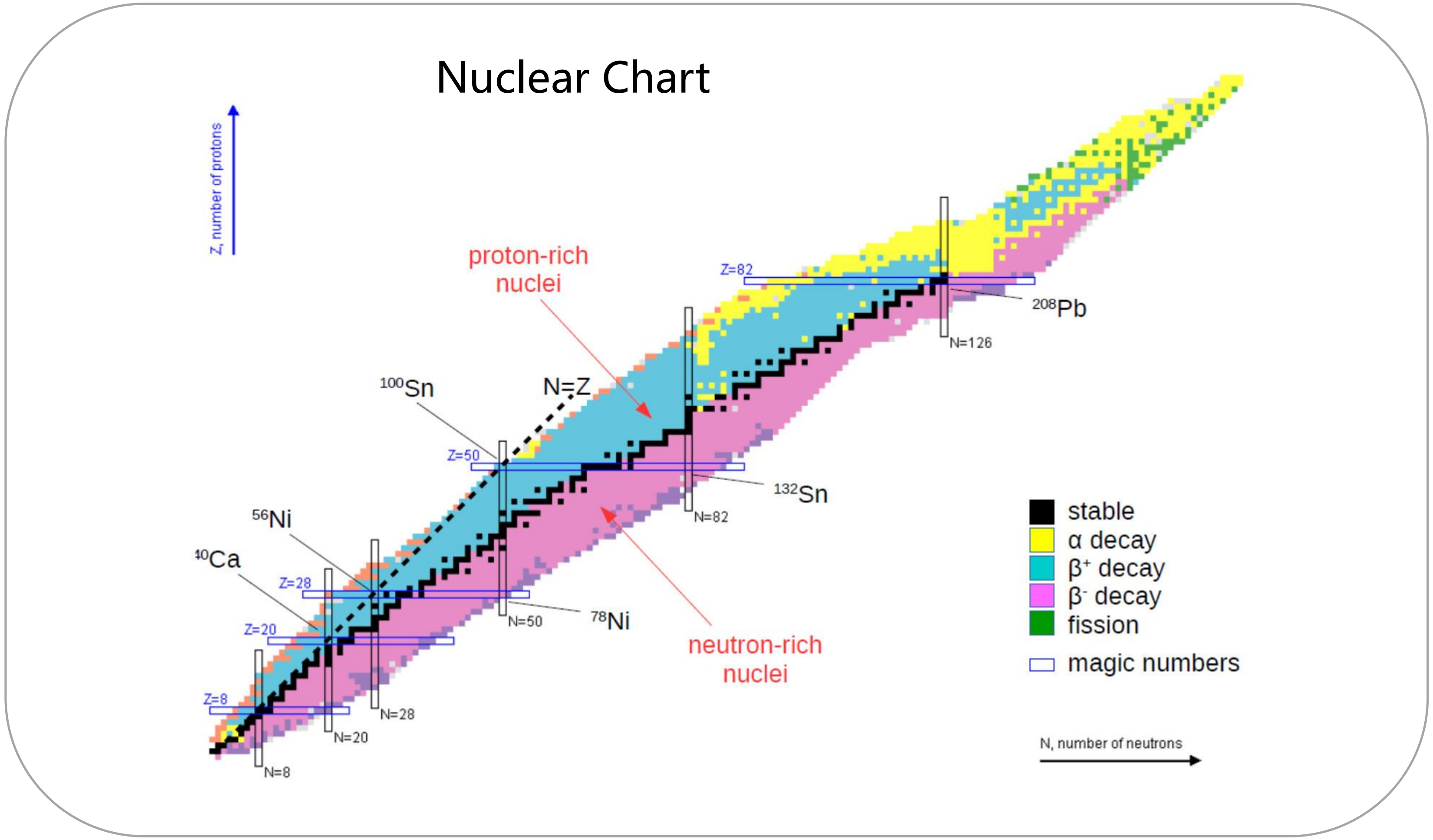
Nucleons are the essential building blocks of Matter!



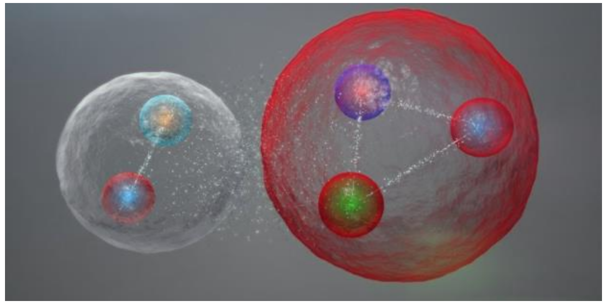
Adapted from LHC the guide

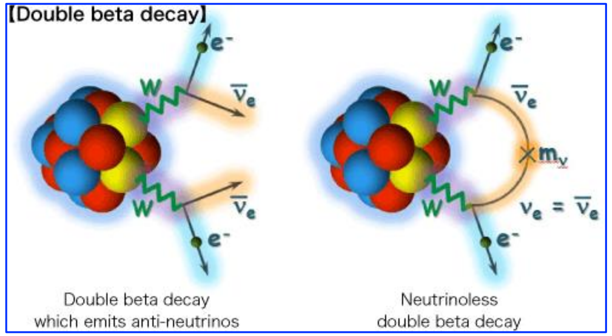


NN: most important input for microscopic understanding of nuclei



- Nuclear structure
- Nuclear reaction
- Nuclear astrophysics
- Exotic hadrons
- Searches for BSM physics
- ……

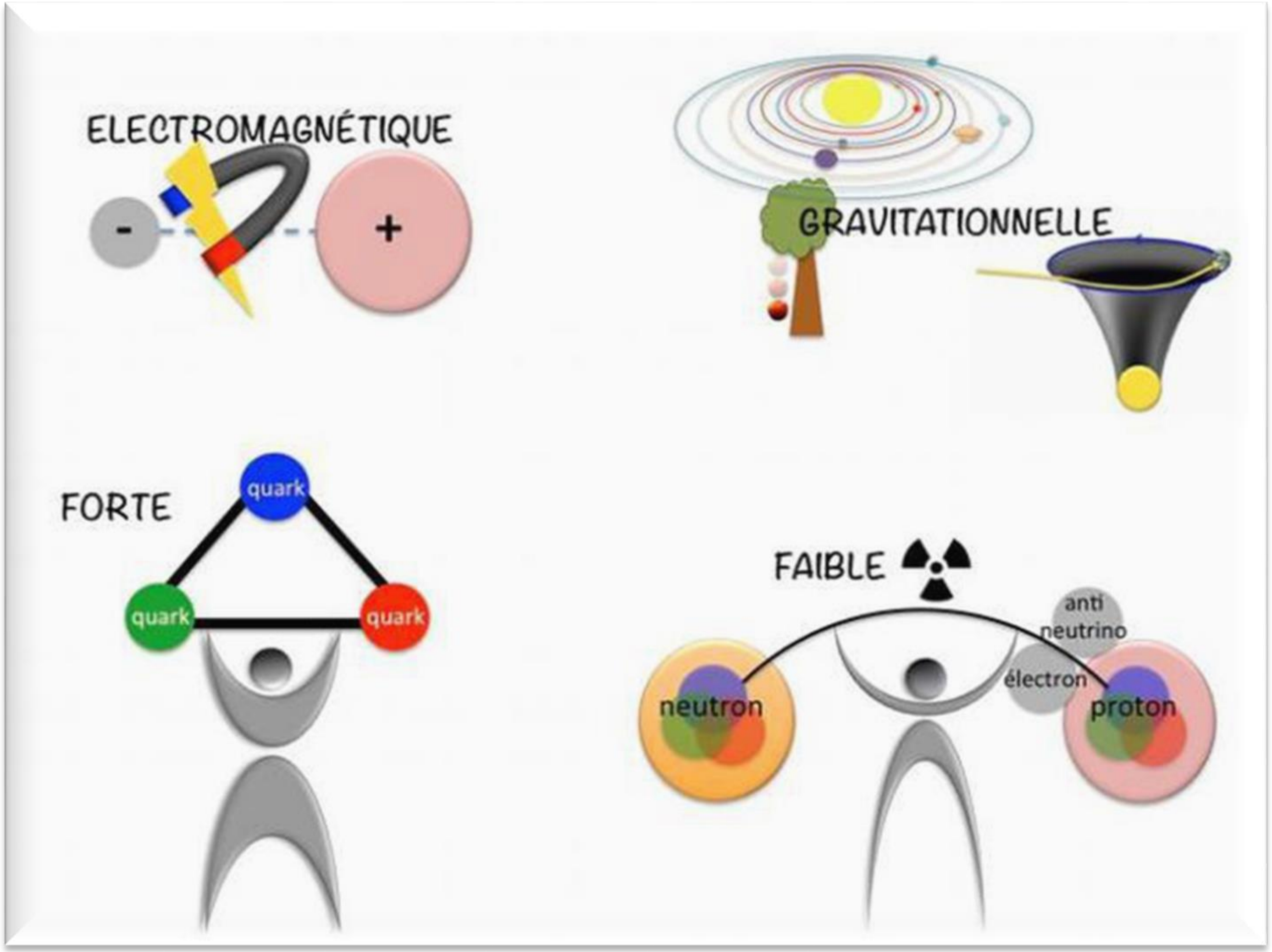




Four interactions in Nature

F= $\frac{kq_1q_2}{r^2}$

F=$q\vec{v} \times \vec{B}$



F= $\frac{Gm_1m_2}{r^2}$

F=..............
....

F=G_F

One of the most difficult problems

- Hans Bethe
- Nobel Prize in Physics 1967

SCIENTIFIC AMERICAN, September 1953

What Holds

the Nucleus Together?

by Hans A. Bethe

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind.

A brief account of the long history

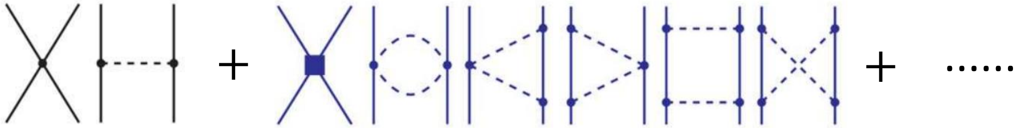
1935 ➢ Yukawa, Meson theory

1950'
1960' ➢ One pion exchange, One boson exchange

QCD

1980' ➢ QCD inspired quark models

1990' ➢ Weinberg, **Chiral effective field theory** S. Weinberg, PLB1990

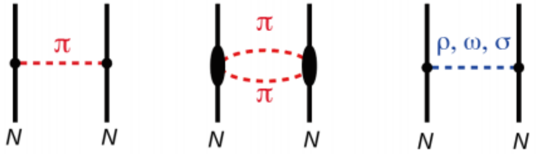


1994 ➢ **High precision** pheno. Models: **AV18** and **Reid93** with operator parameterization

$$V_{NN} = V_c(r)\hat{1} + V_\sigma(r)\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2} + V_{LS}(r)\,\boldsymbol{L}\cdot\boldsymbol{S} + V_T(r)\,\boldsymbol{\sigma_1}\cdot\boldsymbol{q}\boldsymbol{\sigma_2}\cdot\boldsymbol{q} + \cdots\cdots$$

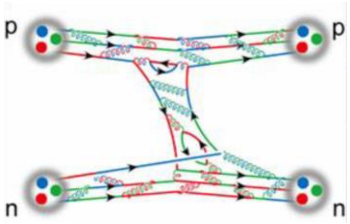
V. Stoks, PRC1994
R. Wiringa,PRC1994

2001 ➢ **High precision** pheno. Models: **CD-Bonn** based on meson-exchange R. Machleidt, PRC2001



2006 ➢ Lattice QCD with full dynamics S.R.Beane PRL2006

Complexity: as residue of QCD / similar to van der Waals force



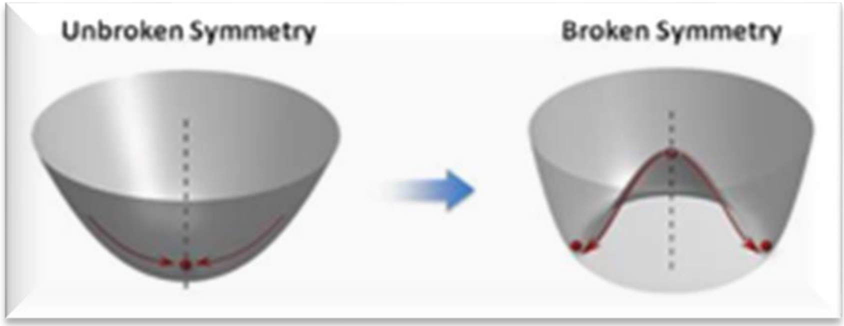

1935 Yukawa


1990 Weinberg


1974 Wilson

Why chiral (effective field theory)

- **Chiral perturbation theory—low energy EFT of QCD**
  - ✓ Because of **quark confinement and asymptotic freedom**, low energy QCD can not be solved perturbatively
  - ✓ **Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons**
  - ✓ **Allows a perturbative formulation of low energy QCD in powers of external momenta and light quark masses, by utilizing chiral symmetry and its breaking pattern (the third feature of QCD)**

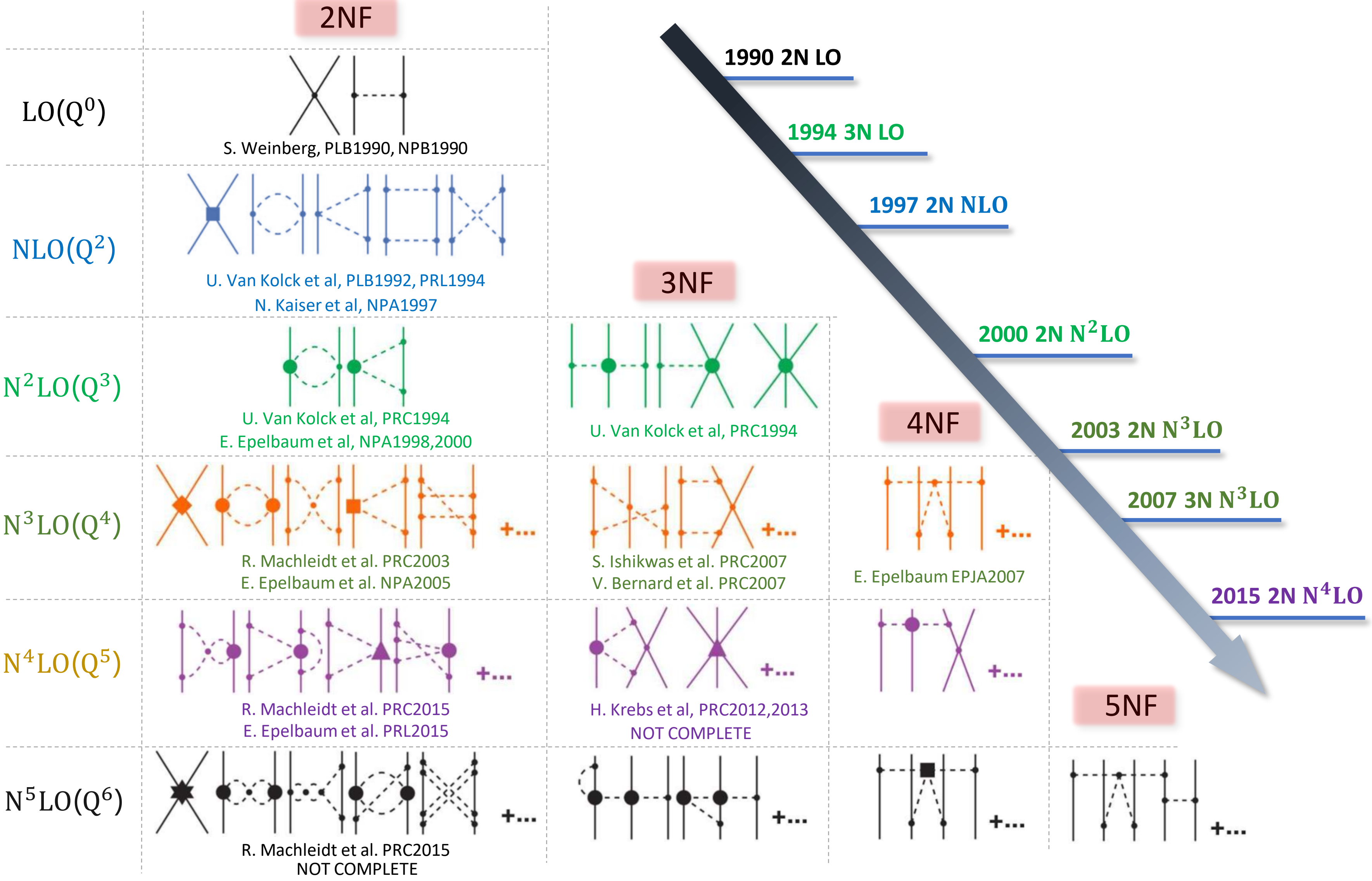


- **Development—Trilogy**
  - ✓ 1979, pion-pion, Weinberg
  - ✓ 1989, to the one-baryon sector, Gasser, Sainio, Svarc
  - ✓ 1990/91/92, to NN/NNN, Weinberg—very successful

Steven Weinberg
Nobel Prize in Physics in 1979

Reviews of Modern Physics 81(2009)1773； Physics Reports 503(2011)1; 1906.12122

Many scientists contributed



Adopted from Front.in Phys. 8 (2020) 57

van Kolck

Kaiser

Epelbaum

Machleidt

Kaplan

...

Why chiral nuclear forces

□ Fewer parameters, similar precision

	AV-18	N3LO	NNLO	NLO
No. of parameters		24	9	9
Description of 2402 np data	1.04	1.10	10.1	36.2

□ More importantly, they are derived from EFTs

- ✓ Closer link with QCD
- ✓ Systematic/order-by-order improvements
- ✓ Consistent descriptions of two/three/four body interactions on the same footing

Why relativistic/covariant

- **Lorentz invariance is one of the most important symmetries in Nature**.
- Both kinematical and dynamical relativistic corrections self-consistently included
- **Relativistic approaches successful in explaining fine structures**
  - ✓ Atomic and molecular systems: why gold is yellow
  - ✓ Nuclear system： **spin-orbit splitting, pseudospin symmetry， covariant DFT**
  - ✓ One-baryon sector： magnetic moments、 masses、 sigma terms

ELSEVIER
Progress in Particle and Nuclear Physics
Volume 109, November 2019, 103713

Review

Towards an *ab initio* covariant density functional theory for nuclear structure

Shihang Shen [a, b, c], Haozhao Liang [d, e], Wen Hui Long [f, g], Jie Meng [a, h, i], Peter Ring [a, j]

Einstein Dirac Mayer Jensen Arima

Why covariant/relativistic

- Lorentz invariance or relativistic corrections might play an important role in speeding up the convergence of ChEFT

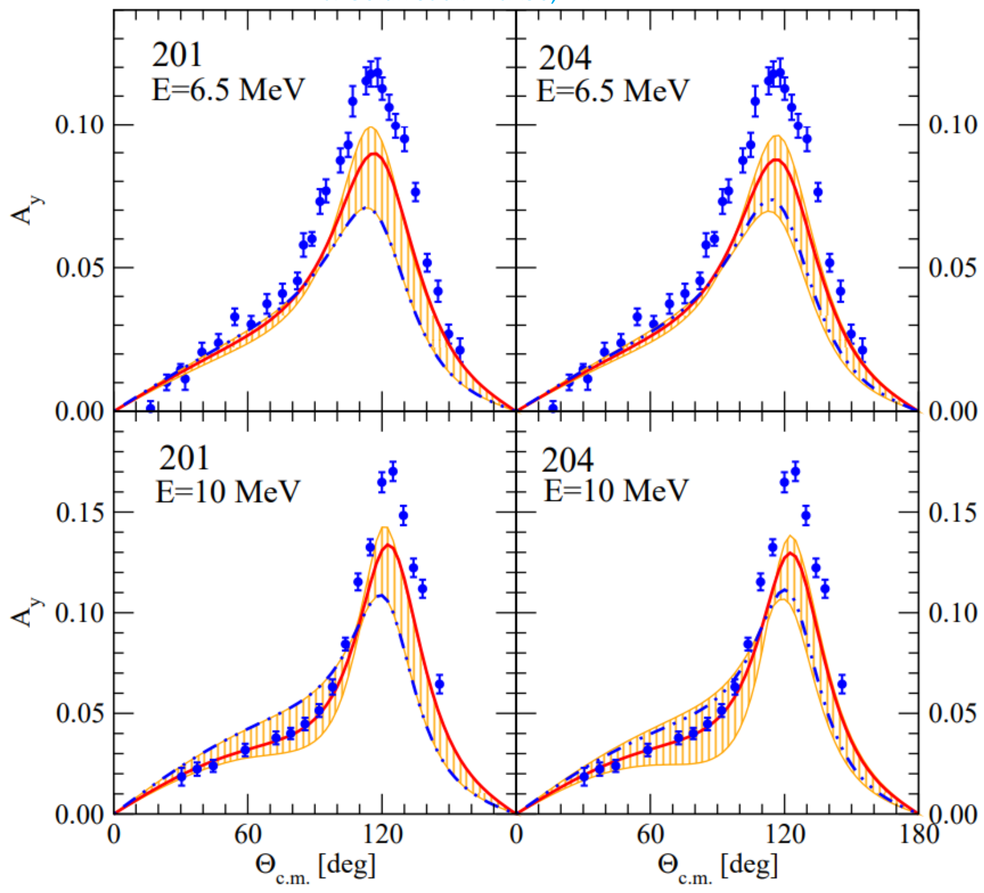
T_{lab} [MeV]	1	50	100	150	200	250	300
P_{cm} [MeV/c]	21.67	153.22	216.68	265.38	306.43	342.60	375.30
P_{cm}/M_N	0.023c	0.16c	0.23c	0.28c	0.33c	0.36c	0.40c

In comparison m_π/m_N=138/939~0.15

Difficulties in current HB chiral nuclear forces

A_y puzzle in N-d scattering

J. Golak et al. EPJA50,177

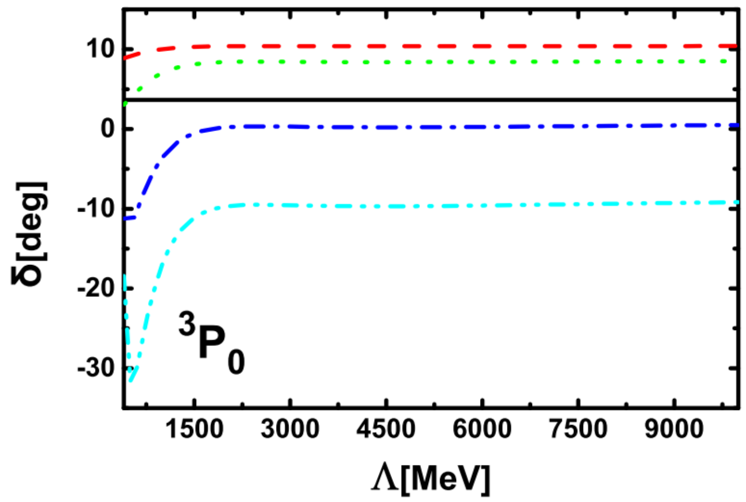


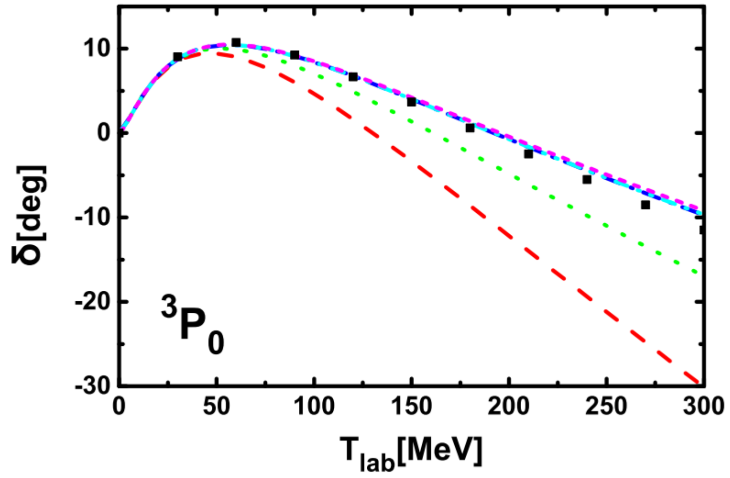
Red: N3LO chiral NN potential

Blue: N3LO chiral NN + full 3N potential

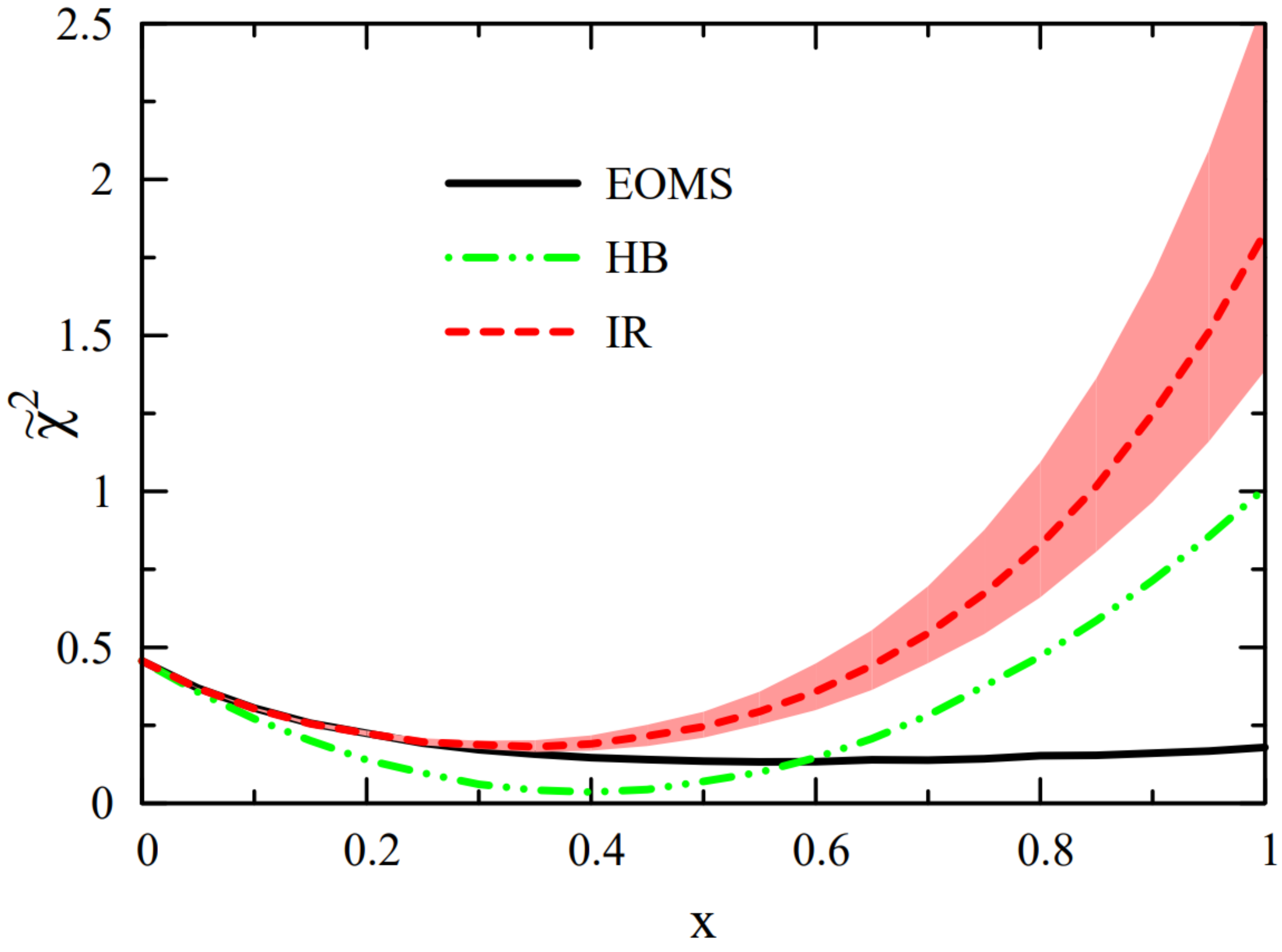
Nenormalization group invariance in 3P0

C.X. Wang et al. CPC45,054101





One-Baryon sector: covariant BChPT is essential



The EOMS covariant ChPT solves a longstanding problem in our understanding of the baryon magnetic moments

*LSG et al., PRL*101 (2008) 222002, *Front.Phys.(Beijing)* 8 (2013) 328

contents

Purpose 1: high precision relativistic chiral nuclear force

- **Provide inputs for ab initio nuclear structure and reaction studies in a covariant setting**

Progress in Particle and Nuclear Physics

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Review

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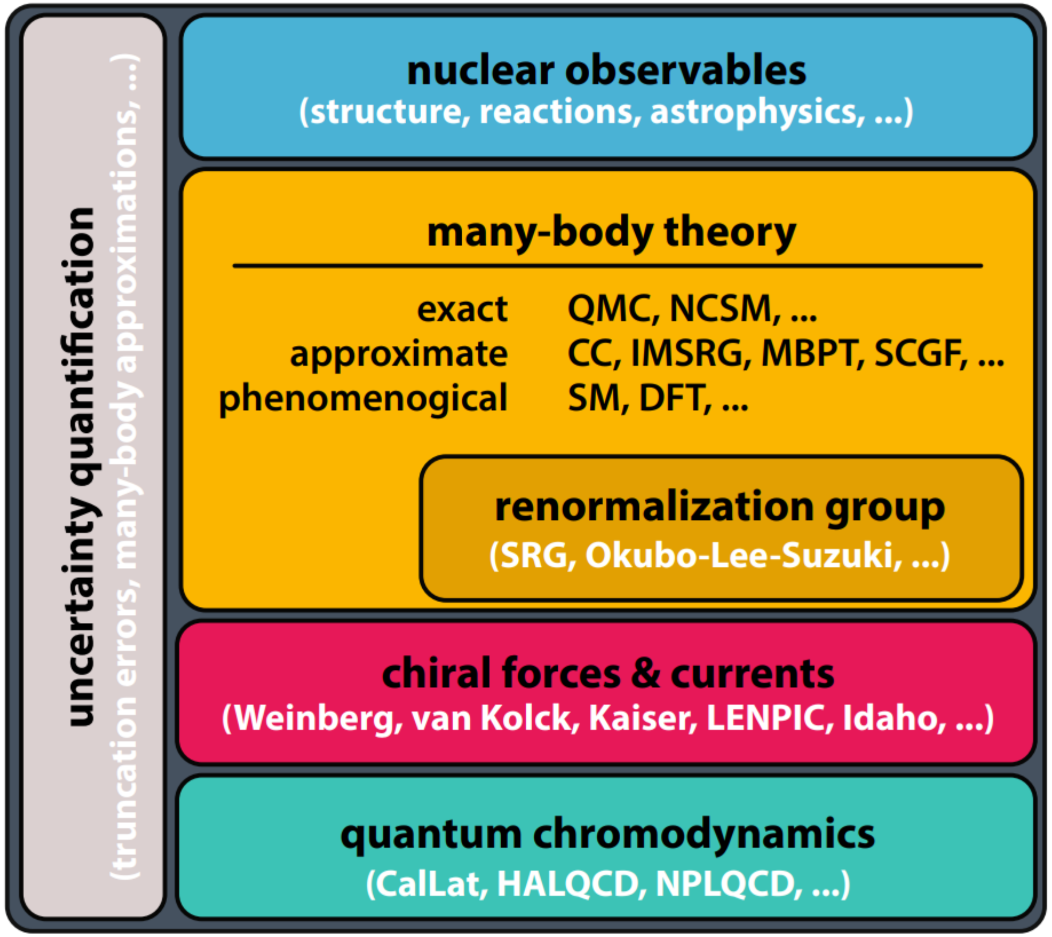
Shihang Shen [a, b, c], Haozhao Liang [d, e], Wen Hui Long [f, g], Jie Meng [a, h, i], Peter Ring [a, j]

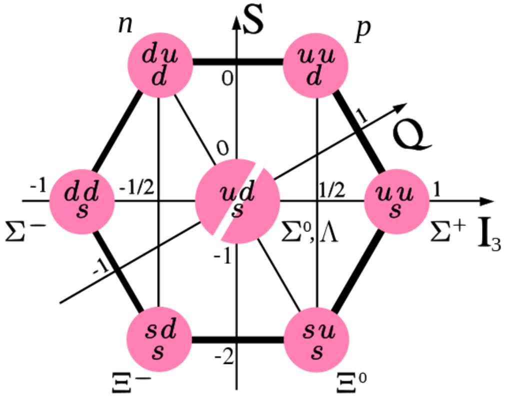
Idealized workflow for ab initio many-body calculations in modern nuclear theory

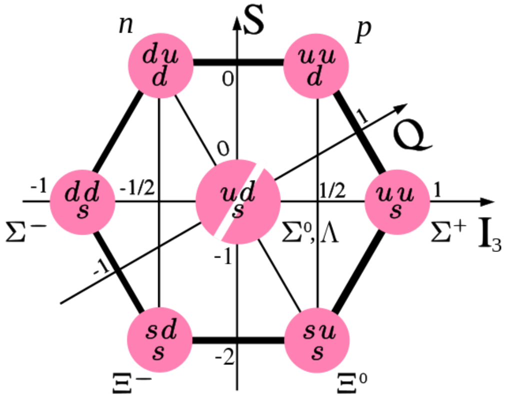


Christian Drischler · Scott K. Bogner, Few-Body Syst (2021) 62:109

Purpose 2: from NN to BB

- ☐ Nucleon-nucleon
- ☐ Hyperon-nucleon
- ☐ Hyperon-hyperon



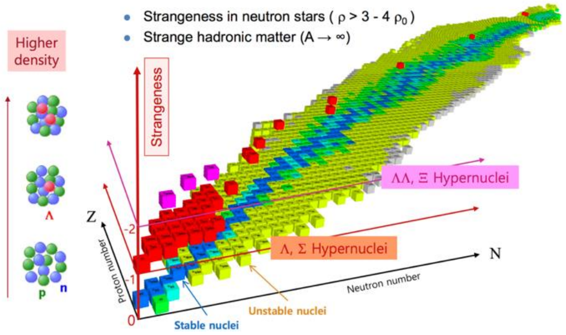


A bound H-dibaryon?

Inoue PRL 106 (2011) 162002

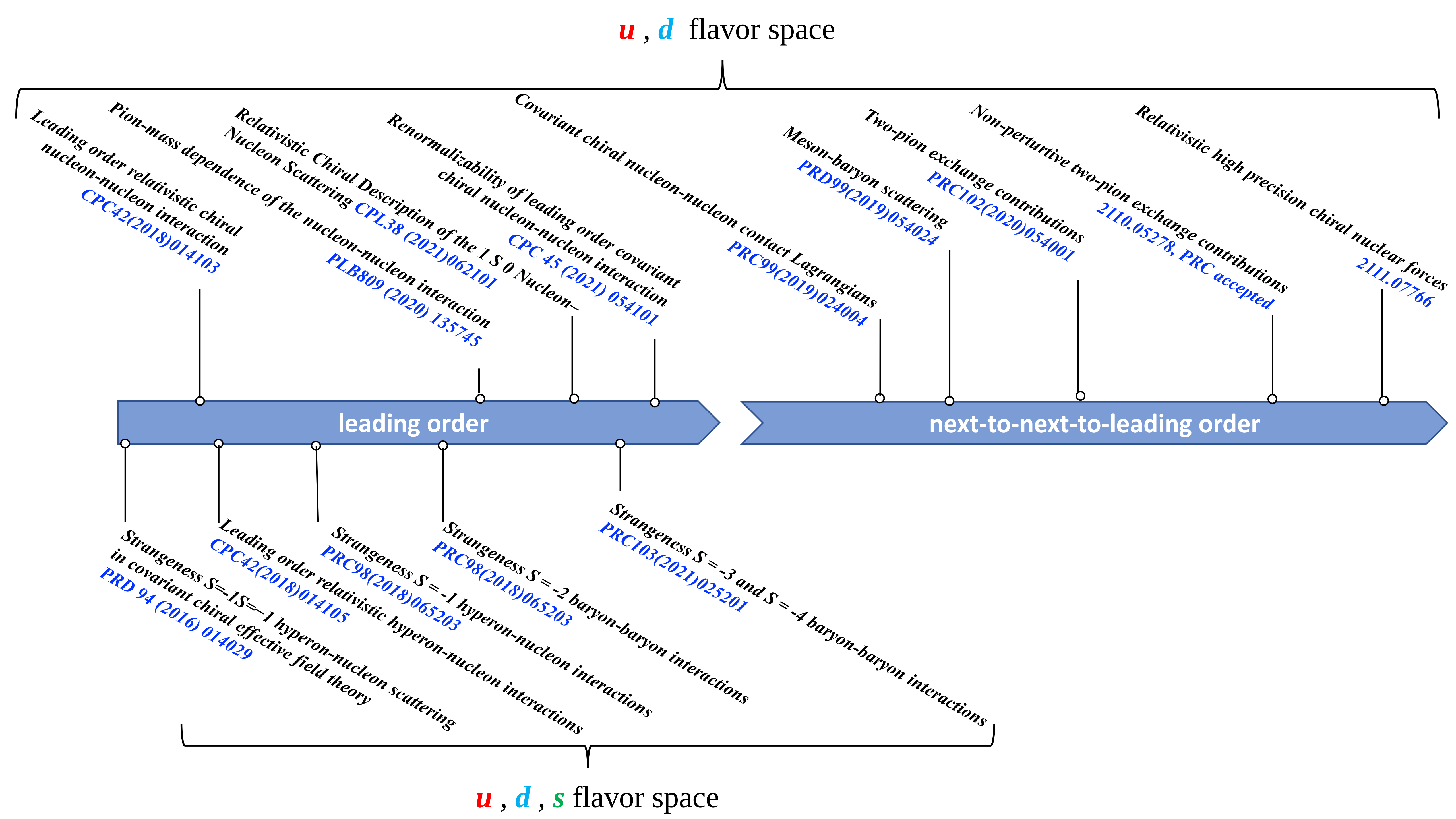
Neutron star

Lonardoni PRL 114 (2015) 092301



Three D nuclear chart

Kaneta M, Tohoku University, Japan)



u , d flavor space
Leading order relativistic chiral nucleon-nucleon interaction CPC42(2018)014103
Pion-mass dependence of the nucleon-nucleon interaction PLB809 (2020) 135745
Relativistic Chiral Description of the 1 S 0 Nucleon–Nucleon Scattering CPL38 (2021)062101
Renormalizability of leading order covariant chiral nucleon-nucleon interaction CPC 45 (2021) 054101
Covariant chiral nucleon-nucleon contact Lagrangians PRC99(2019)024004
Meson-baryon scattering PRD99(2019)054024
Two-pion exchange contributions PRC102(2020)054001
Non-perturtive two-pion exchange contributions 2110.05278, PRC accepted
Relativistic high precision chiral nuclear forces 2111.07766
leading order
next-to-next-to-leading order
Strangeness S=-1S=-1 hyperon-nucleon scattering in covariant chiral effective field theory PRD 94 (2016) 014029
Leading order relativistic hyperon-nucleon interactions CPC42(2018)014105
Strangeness S = -1 hyperon-nucleon interactions PRC98(2018)065203
Strangeness S = -2 baryon-baryon interactions PRC98(2018)065203
Strangeness S = -3 and S = -4 baryon-baryon interactions PRC103(2021)025201
u , d , s flavor space

contents

How to become relativistic/covariant

- □ **Dirac spinors and algebra (instead of non-relativistic wave functions and Pauli matrices)**

$$u(\boldsymbol{p},s) = N_p \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\epsilon_p} \end{pmatrix} \chi_s, N_p = \sqrt{\frac{\epsilon_p}{2M_N}}$$

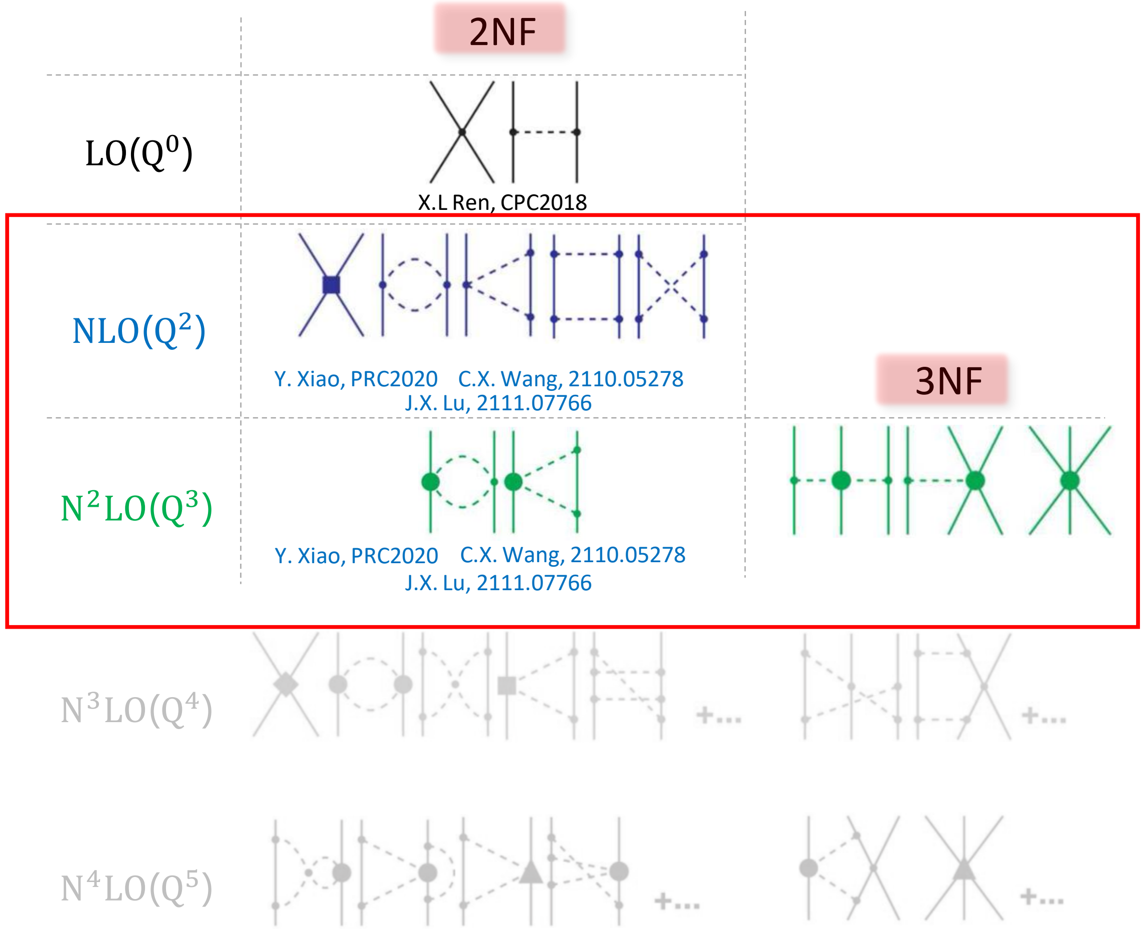
	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+
$\mathcal{O}$	0	1	0	0	0	−	0	1

- □ **Covariant scattering equation (instead of Lippmann-Schwinger equation)**

$$\mathcal{T}(p',p \mid W) = \mathcal{A}(p',p \mid W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p',k \mid W) G(k \mid W) \mathcal{T}(k,p \mid W)$$

$$G(k \mid W) = \frac{i}{[\gamma^\mu (W+k)_\mu - m_N + i\epsilon]^{(1)} [\gamma^\mu (W-k)_\mu - m_N + i\epsilon]^{(2)}}$$

A systematic project from scratch

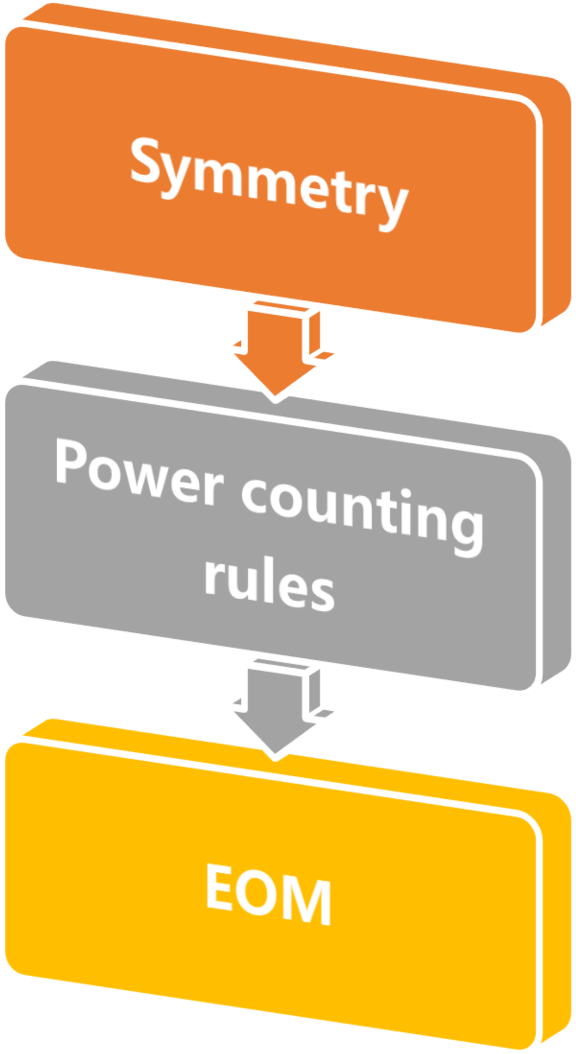


Three key ingredients

1. Four nucleon (baryon) vertices:
   - ✓ 1812.03005
2. Meson-baryon vertices:
   - ✓ 1812.03799
3. Two-meson exchanges
   - ✓ 2007.13675
   - ✓ 2110.05278

How to construct covariant Lagrangians

- **Symmetry Constraints**
  - ✓ Lorentz invariance: α, β, γ … …
  - ✓ Chiral symmetry: matter field $\psi \to K\psi K^{\dagger}$, NGB as usual
  - ✓ Hermitian conjugation: add an appropriate "i".
  - ✓ Parity and Charge conjugation symmetries:
  - ✓ Time reversal symmetry: CPT theory.
- **How to raise chiral order ?**
  - → **Power counting rules**
- **How to deal with redundant terms ?**
  - → **Equation of motion (EOM)**



Symmetry requirements

□Building blocks (Dirac matrices & partial derivatives)

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+
$\mathcal{O}$	0	1	0	0	0	−	0	1

□General form of a Lagrangian term

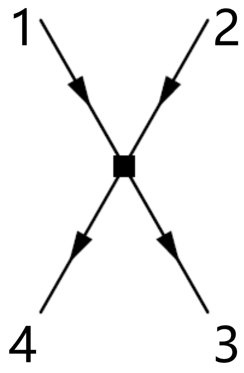
$$\frac{1}{(2m)^{N_d}}\left(\bar{\psi}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}...\Gamma_A\psi\right)\partial^{\lambda}\partial^{\mu}...\left(\bar{\psi}i\overleftrightarrow{\partial}^{\sigma}i\overleftrightarrow{\partial}^{\tau}...\Gamma_B\psi\right),$$

Note $\overleftrightarrow{\partial}^{\alpha} = \bar{\psi}(\overrightarrow{\partial}^{\alpha} - \overleftarrow{\partial}^{\alpha})\psi$ vs. $\partial^{\alpha} = \partial^{\alpha}(\bar{\psi}\Gamma\psi)$

Power counting rules

$$\frac{1}{(2m)^{N_d}}\left(\bar{\psi} i \overleftrightarrow{\partial}^{\alpha} i \overleftrightarrow{\partial}^{\beta} \ldots \Gamma_A \psi\right) \partial^{\lambda} \partial^{\mu} \ldots \left(\bar{\psi} i \overleftrightarrow{\partial}^{\sigma} i \overleftrightarrow{\partial}^{\tau} \ldots \Gamma_B \psi\right)$$

N_d is the number of $\overleftrightarrow{\partial}$, $\overleftrightarrow{\partial} = \vec{\partial} - \overleftarrow{\partial}$



- Nucleon field: $\psi = \binom{p}{n} \sim O(p^0)$, Nucleon mass: $m \sim O(p^0)$,
- Dirac matrices: $\Gamma \in \{1, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim O(p^0), \gamma_5 \sim O(p^1)\}$
- **Covariant derivative:** $\partial(\bar{\psi}\Gamma\psi) \sim O(p^1)$, $\left(\bar{\psi}\overleftrightarrow{\partial}\psi\right) \sim O(p^0)$, **except**

  $(\bar{\psi}\sigma_{\mu\nu}\psi)\left(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi\right) \sim O(p^1)$, $(\bar{\psi}\gamma_5\gamma_\mu\psi)(\bar{\psi}\overleftrightarrow{\partial}^{\mu}\Gamma\psi) \sim O(p^1)$

- **Treatment for covariant derivative:**

$$\tilde{O}^{(n)}_{\Gamma_A\Gamma_B} = \frac{1}{(2m)^{2n}} (\bar{\psi}\, i \overleftrightarrow{\partial}^{\mu_1}\, i \overleftrightarrow{\partial}^{\mu_2} \cdots i \overleftrightarrow{\partial}^{\mu_n}\, \Gamma^{\alpha}_{A}\, \psi)\, (\bar{\psi}\, i \overleftrightarrow{\partial}_{\mu_1}\, i \overleftrightarrow{\partial}_{\mu_2} \cdots i \overleftrightarrow{\partial}_{\mu_n}\, \Gamma_{B\alpha}\, \psi)$$

-Expansion of such structure:

-up to $\boldsymbol{O(q^2)}$: $\boldsymbol{n = 0, 1;}$

-up to $\boldsymbol{O(q^4)}$: $\boldsymbol{n = 0, 1, 2.}$

$$\frac{[(p_1 + p_3)\cdot(p_2 + p_4)]^n}{(2m)^{2n}} \Leftrightarrow \left[1 + \frac{(s - 4m^2) - u}{4m^2}\right]^n$$

Reduction using equation of motion (EOM)

- **Equation of motion :** $\not{D}B = \gamma^\mu D_\mu B = -\mathrm{i}M_0 B + \mathcal{O}(q)$
- Beyond the obvious replacements one can bring terms that do not containing $\not{D}B$ into a form where they do. *Annals Phys., 283:273, (2000)*

$$-2im(\bar{\psi}\Gamma\psi) \approx 2(\bar{\psi}\Gamma \times \gamma_\lambda \partial^\lambda \psi) = (\bar{\psi}\Gamma'_\lambda \overleftrightarrow{\partial}^\lambda \psi) + \partial^\lambda(\bar{\psi}\Gamma''_\lambda \psi),$$

TABLE II. Decomposition of the Dirac matrix products $\Gamma \times \gamma_\lambda$ into charge conjugation even (Γ'_λ) and charge conjugation odd (Γ''_λ) parts [43].

Γ	Γ'_λ	Γ''_λ
$\mathbb{1}$	γ_λ	0
γ_μ	$g_{\mu\lambda}1$	$-i\sigma_{\mu\lambda}$
γ_5	0	$\gamma_5\gamma_\lambda$
$\gamma_5\gamma_\mu$	$\frac{1}{2}\epsilon_{\mu\lambda\rho\tau}\sigma^{\rho\tau}$	$g_{\mu\lambda}\gamma_5$
$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\lambda\tau}\gamma_5\gamma^\tau$	$-i(g_{\mu\lambda}\gamma_\nu - g_{\nu\lambda}\gamma_\mu)$
$\epsilon_{\mu\nu\rho\tau}\gamma^\tau$	$\epsilon_{\mu\nu\rho\lambda}1$	$g_{\mu\lambda}\gamma_5\sigma_{\nu\rho} + g_{\rho\lambda}\gamma_5\sigma_{\mu\nu} + g_{\nu\lambda}\gamma_5\sigma_{\rho\mu}$
$\epsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau$	$g_{\mu\lambda}\sigma_{\nu\rho} + g_{\rho\lambda}\sigma_{\mu\nu} + g_{\nu\lambda}\sigma_{\rho\mu}$	$\epsilon_{\mu\nu\rho\lambda}\gamma_5$
$\epsilon_{\mu\nu\rho\alpha}\sigma^\alpha_\tau$	$\gamma_5\gamma_\rho(g_{\lambda\nu}g_{\mu\tau} - g_{\lambda\mu}g_{\nu\tau}) + \gamma_5\gamma_\nu(g_{\lambda\mu}g_{\rho\tau} - g_{\lambda\rho}g_{\mu\tau}) + \gamma_5\gamma_\mu(g_{\lambda\rho}g_{\nu\tau} - g_{\lambda\nu}g_{\rho\tau})$	$ig_{\lambda\tau}\epsilon_{\mu\nu\rho\alpha}\gamma^\alpha - i\epsilon_{\mu\nu\rho\lambda}\gamma_\tau$
$\frac{i}{2}\epsilon_{\mu\nu\rho\tau}\sigma^{\rho\tau} = \gamma_5\sigma_{\mu\nu}$	$\frac{1}{i}(g_{\mu\lambda}\gamma_5\gamma_\nu - g_{\nu\lambda}\gamma_5\gamma_\mu)$	$\epsilon_{\mu\nu\lambda\rho}\gamma^\rho$

Covariant NN contact Lagrangians (N2LO)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_\mu\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_\nu\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\mu\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_\mu\psi\right)$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_1$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_2$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_3$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^\alpha\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_\alpha\psi\right)-\widetilde{O}_4$

O_S	$(N^\dagger N)(N^\dagger N)$
O_T	$(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\boldsymbol{\sigma}N)$
O_1	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}^2 N)+\text{h.c.}$
O_2	$(N^\dagger N)(N^\dagger\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_3	$i(N^\dagger\boldsymbol{\sigma}N)\cdot(N^\dagger\overrightarrow{\nabla}\times\overleftarrow{\nabla}N)$
O_4	$(N^\dagger\sigma^j N)(N^\dagger\sigma^j\overrightarrow{\nabla}^2 N)+\text{h.c.}$
O_5	$(N^\dagger\sigma^j N)(N^\dagger\sigma^j\overrightarrow{\nabla}\cdot\overleftarrow{\nabla}N)$
O_6	$(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overrightarrow{\nabla}N)+\text{h.c.}$
O_7	$(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)(N^\dagger\boldsymbol{\sigma}\cdot\overleftarrow{\nabla}N)$

Relativistic: 17

vs.

Non-relativistic: 9

Comparison with other works (N2LO)

	Terms	Procedure	Advantage	Disadvantage
L.Girlanda [1]	36	① n=0,1 $(\frac{1}{(2m)^{2n}}(\bar{\psi}\partial^{\mu_1}\dots\partial^{\mu_n}\psi)(\bar{\psi}\partial_{\mu_1}\dots\partial_{\mu_n}\psi))$	A complete set of *NN* contact Lagrangians	**Not minimal**
Stefan Petschuaer [2]	25 (*NN* case)	① n=0,1,2; ② Apply EOM; ③ Ignore Lagrangians with $\partial^\mu(\bar{\psi}\sigma_{\mu\nu}\psi)$ cause they claim it contribute to higher order $O(p^1)$ and can be subsumed in higher order Lagrangians	Contains less terms compared with [1]	**Not complete**
Our work	**17**	① n=0,1; ② Apply EOM; ③ Include Lagrangians with $\partial^\mu(\bar{\psi}\sigma_{\mu\nu}\psi)$ cause it contains unique Lorentz structure	A **complete and minimal** set of *NN* contact Lagrangians	

[1] *PRC81 (2010) 034005* [2] *NPA916 (2013) 1*

Covariant NN contact Lagrangians (N4LO)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)(\bar\psi\gamma_\mu\psi)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha}\psi\right)\partial^2\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)(\bar\psi\gamma_5\gamma_\mu\psi)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^\beta\partial_\nu\left(\bar\psi\sigma_{\alpha\beta} i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)(\bar\psi\sigma_{\mu\nu}\psi)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}\left(\bar\psi\psi\right)\partial^4\left(\bar\psi\psi\right)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu\psi\right)\partial^4\left(\bar\psi\gamma_\mu\psi\right)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu\psi\right)\partial^4\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}\psi\right)\partial^4\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\alpha}\psi\right)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}\left(\bar\psi\psi\right)\partial^2\left(\bar\psi\psi\right)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu\psi\right)\partial^2\left(\bar\psi\gamma_\mu\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu} i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^\alpha\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\psi\right)\partial^2\left(\bar\psi\gamma_5\psi\right)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\nu i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\alpha} i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu} i\overleftrightarrow{\partial}^{\alpha} i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\nu} i\overleftrightarrow{\partial}_{\alpha} i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

Relativistic: 40

vs.

Non-relativistic: 24

Non-relativistic reduction

- **Non-relativistic expansion:** $\psi \rightarrow N$, expand Lagrangians in terms of **1/m**
  - ✓ **Relativistic nucleon field operator:** $$\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tilde{b}_s(\mathbf{p})\, u^{(s)}(\mathbf{p})\, e^{-ip\cdot x},$$
  - ✓ **Non-relativistic nucleon field operator:** $$N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} b_s(\mathbf{p})\, \chi_s\, e^{-ip\cdot x}$$
  - ✓ **Expansion of field operator**
  - ✓ **Dirac matrices expressed in term of pauli matrices**

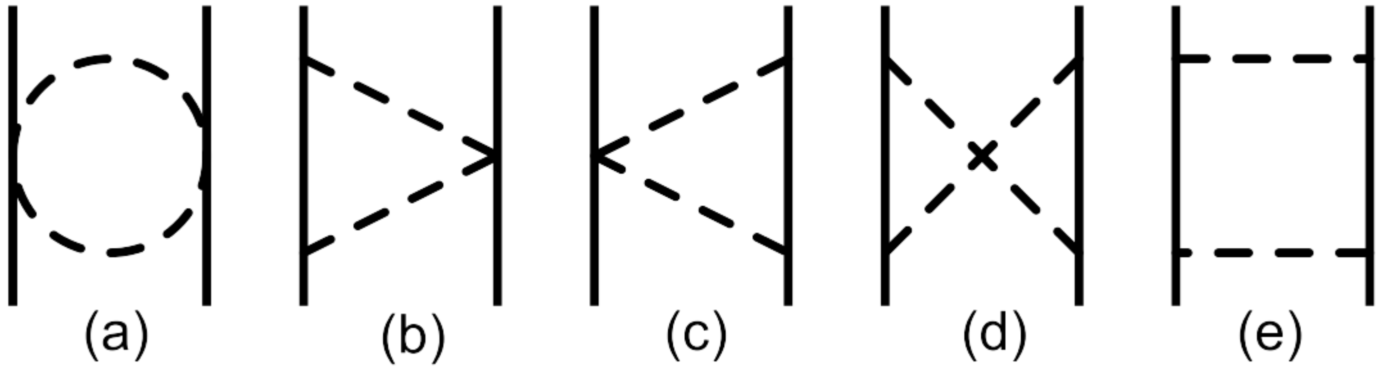
$$\psi(x) = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{2m}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix} \boldsymbol{\nabla}^2 \\ 0 \end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix} 0 \\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2 \end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix} \boldsymbol{\nabla}^4 \\ 0 \end{pmatrix}\right] N(x) + \mathcal{O}(Q^5)\,.$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

  - ✓ **After expansion and keeping only appropriate powers of $1/m_N$, we can reduce the 40 relativistic terms into the 2+7+15 non-relativistic terms**

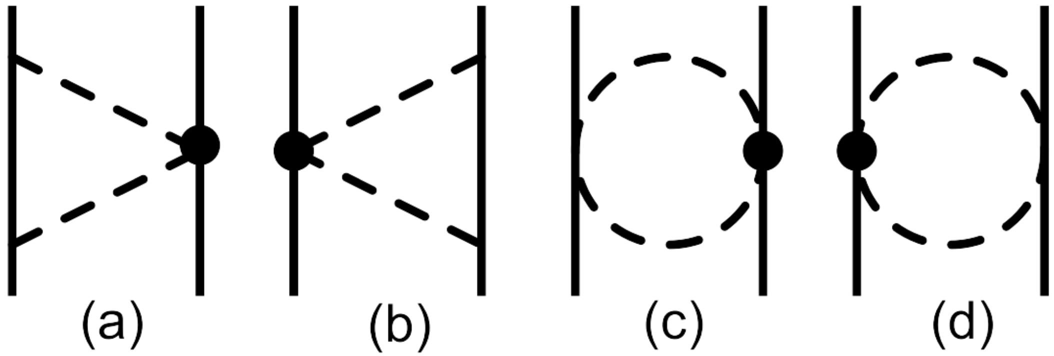
Two-pion exchanges: perturbative and nonpeturbative

LO



g_A
f_π

NLO



c_{1-4}

Highly nontrivial

Kaiser

- There are no unknown LECs
- Contribute to all the partial waves, but almost saturate partial waves of $L \geq 3$
- Perfect candidates to check chiral corrections and the convergence of chiral expansions

Peripheral nucleon-nucleon phase shifts and chiral symmetry #1

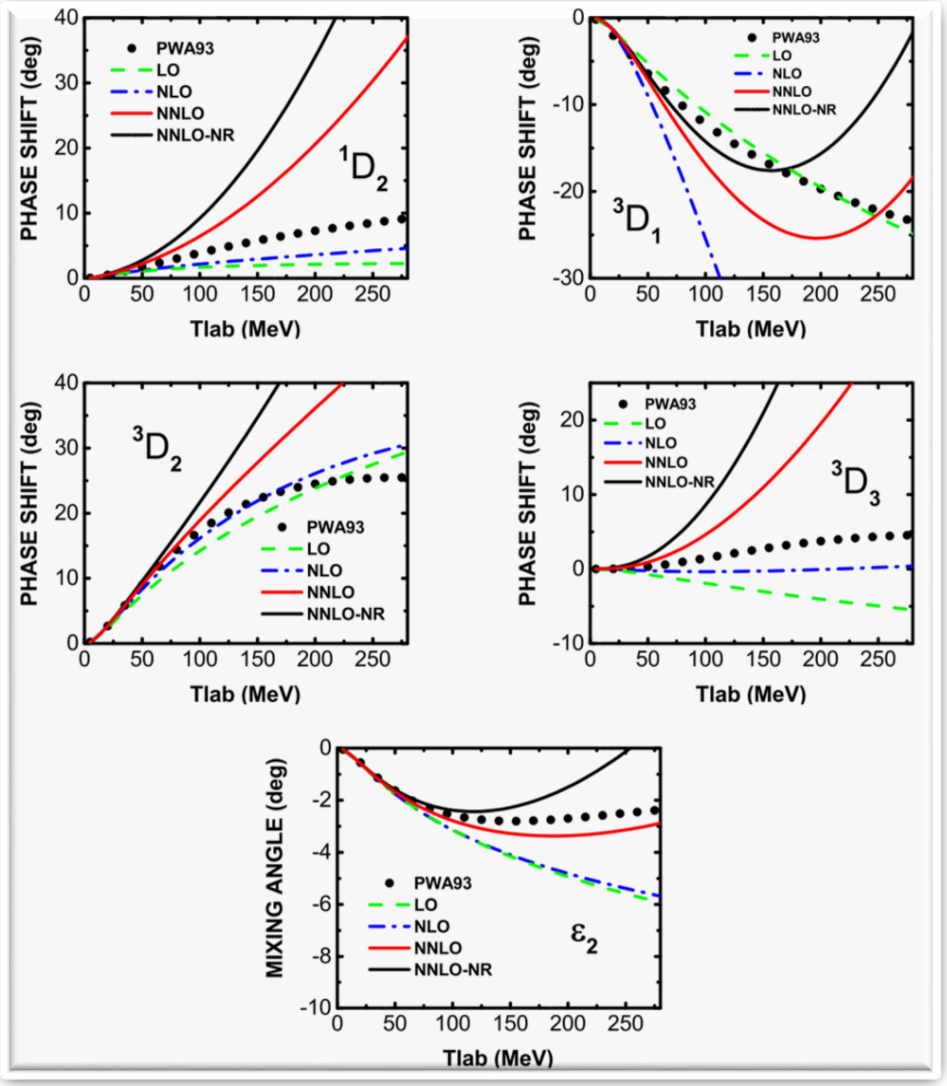
Norbert Kaiser (Munich, Tech. U.), R. Brockmann (Mainz U., Inst. Kernphys.), W. Weise (Munich, Tech. U.)

(Jun 19, 1997)

Published in: *Nucl.Phys.A* 625 (1997) 758-788 • e-Print: nucl-th/9706045 [nucl-th]

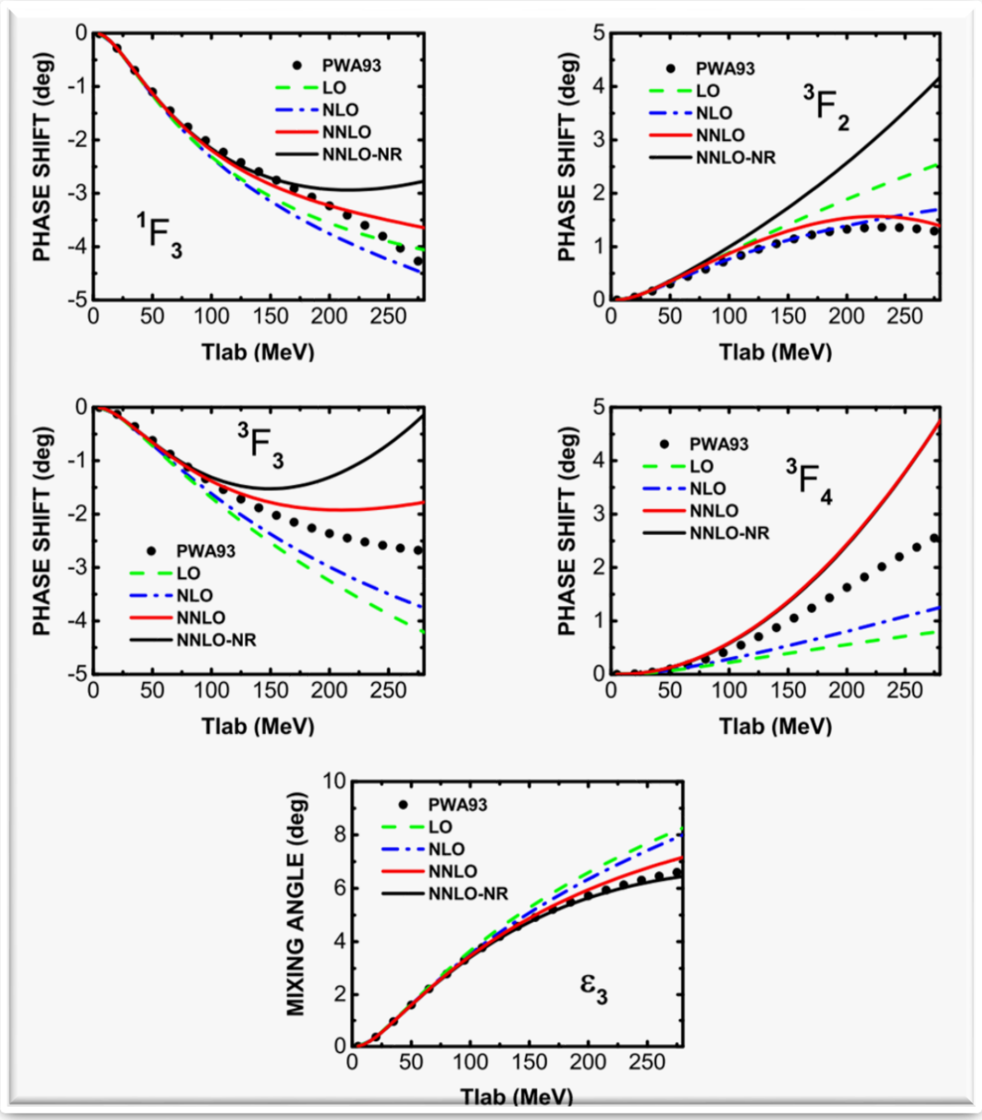
pdf DOI cite

D-waves



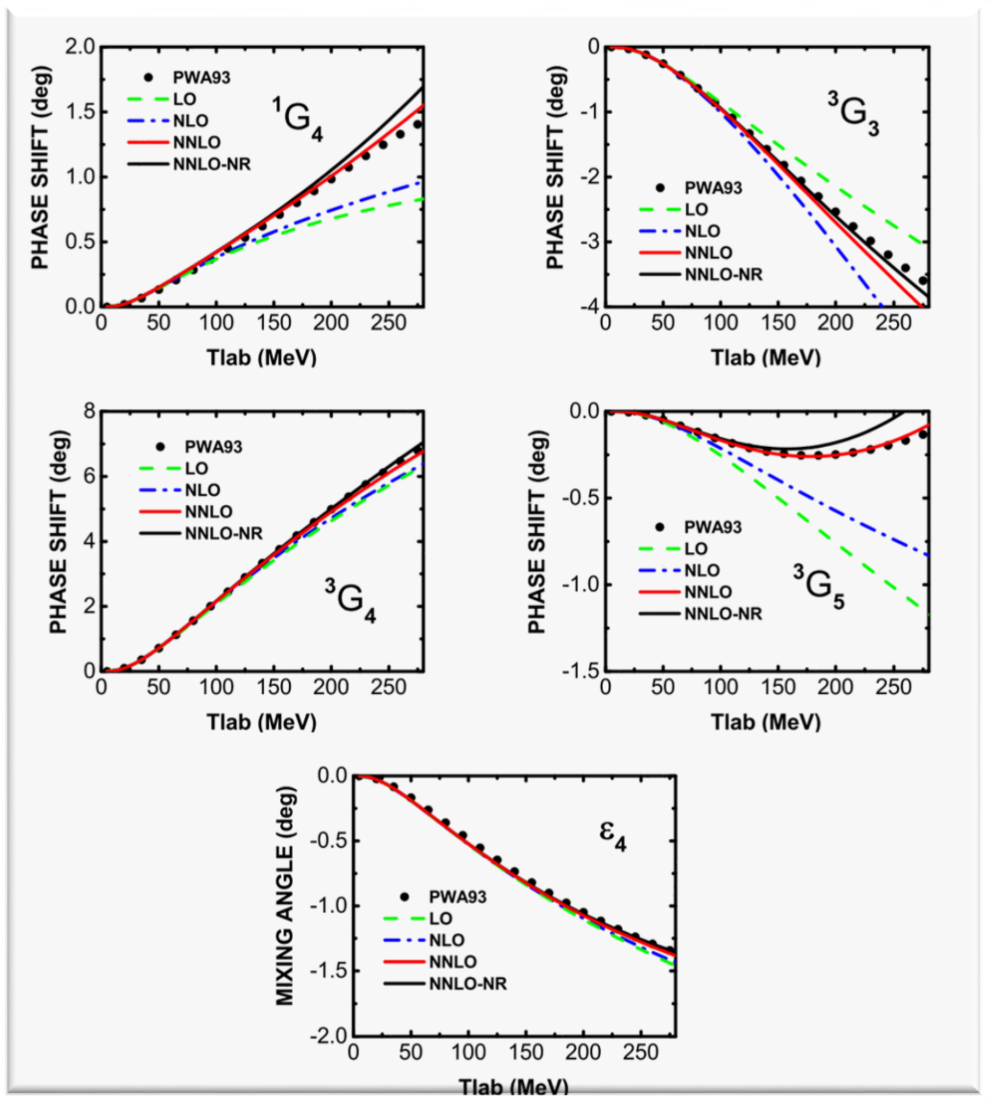
- Up to Tlab=50 MeV, agreement with data is good
- Relativistic TPE more moderate then non-relativistic TPE, and agree better with data
- NLO TPE larger than LO TPE
- Short-range contributions are needed

F-waves



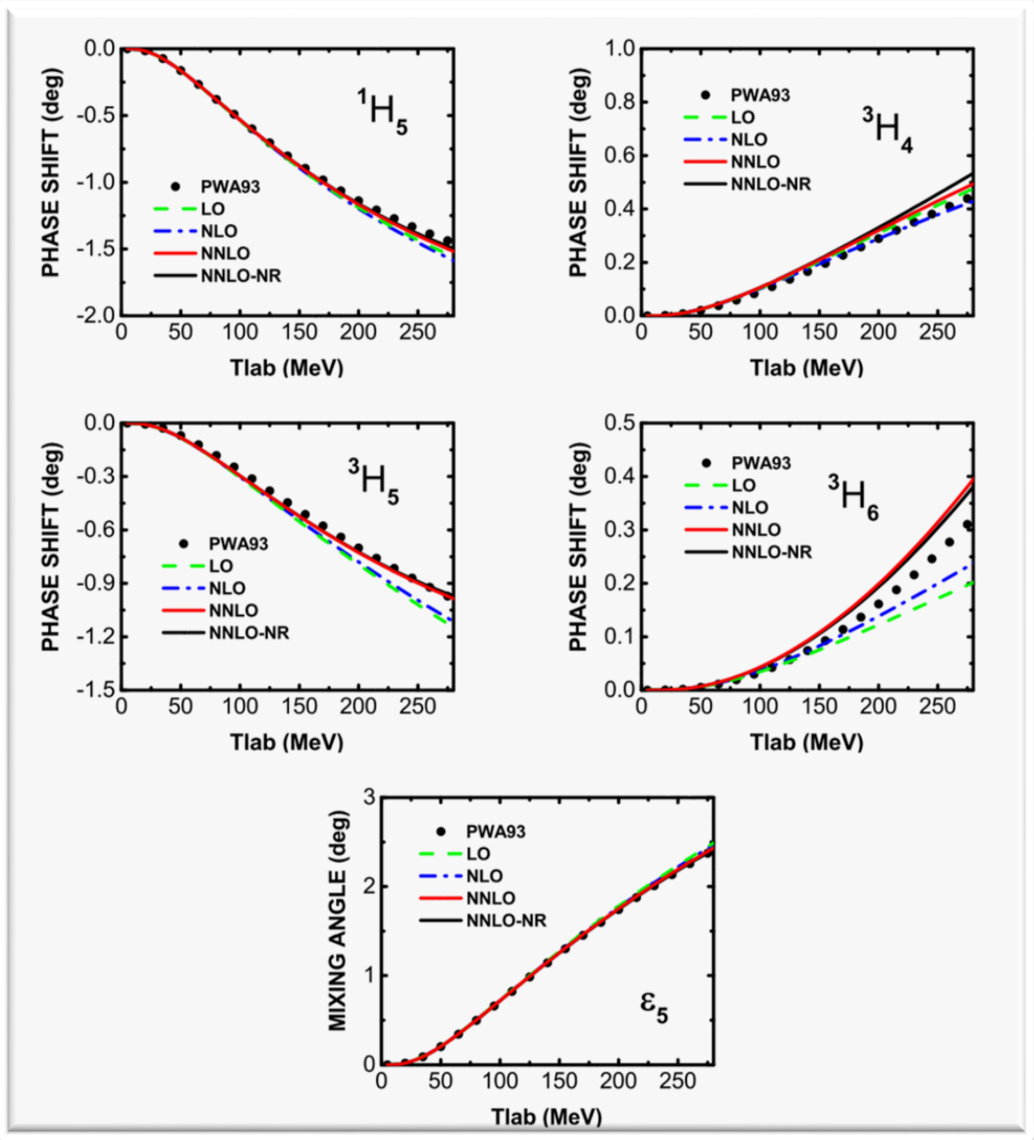
- Agreement with data is better than D-wave
- Relativistic TPE more moderate than non-relativistic TPE, and agree better with data (except 3F4)
- Improvement still needed

G-waves



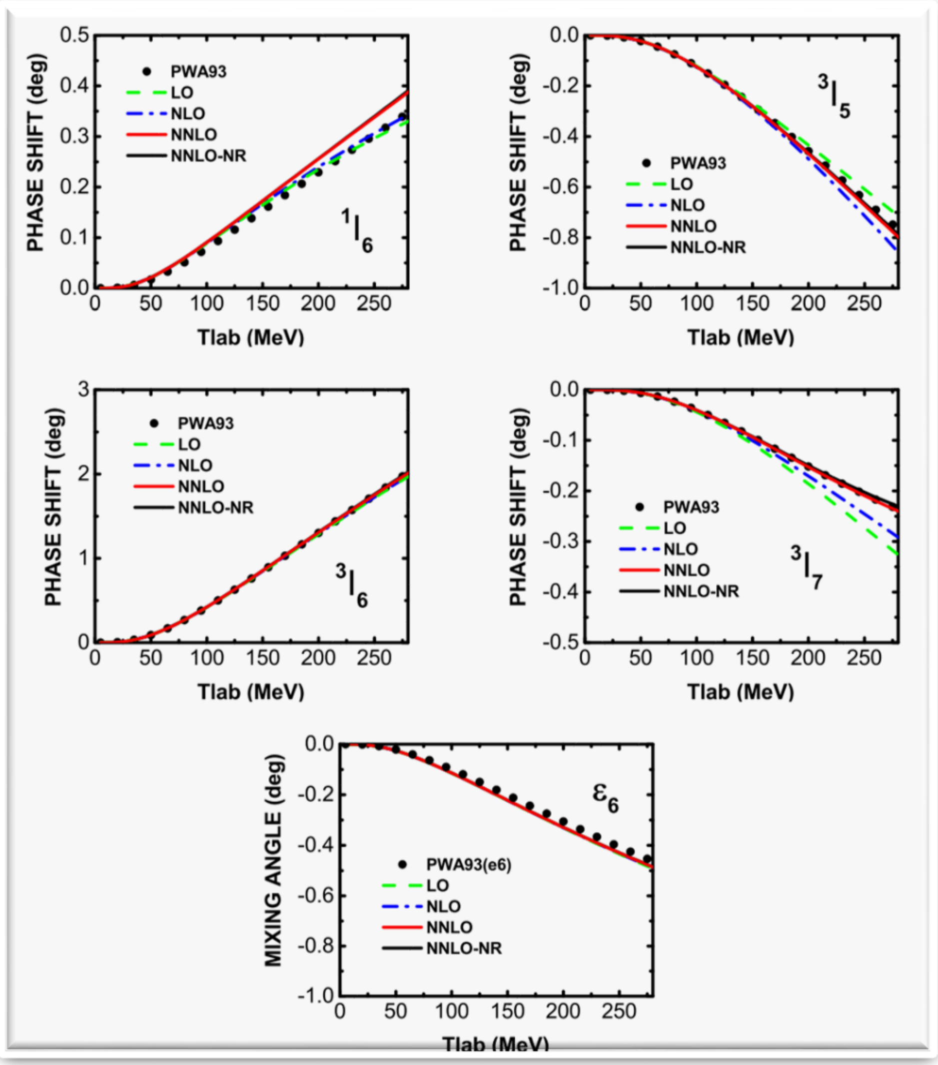
- Agreement with data is better than F&D-waves
- Relativistic TPE more moderate than non-relativistic TPE, and agree better with data (except 3G3)
- Not much Improvement needed

H-waves



- **Small TPE contributions**
- **Agreement with data quite good**
- **Not much Improvement needed**

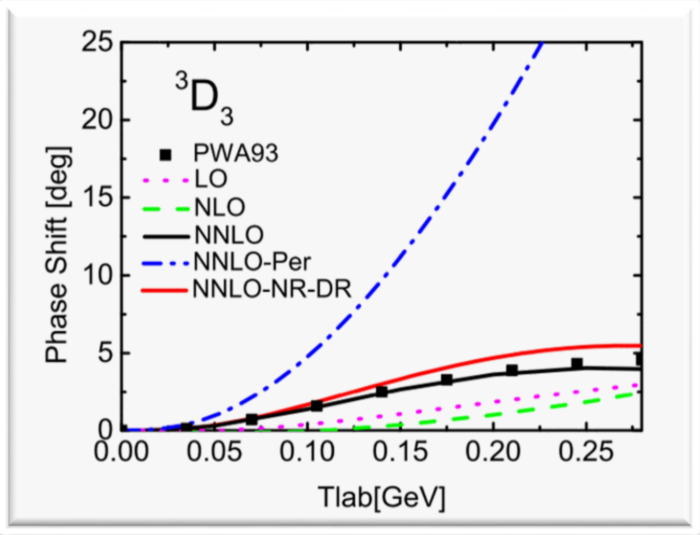
I-waves

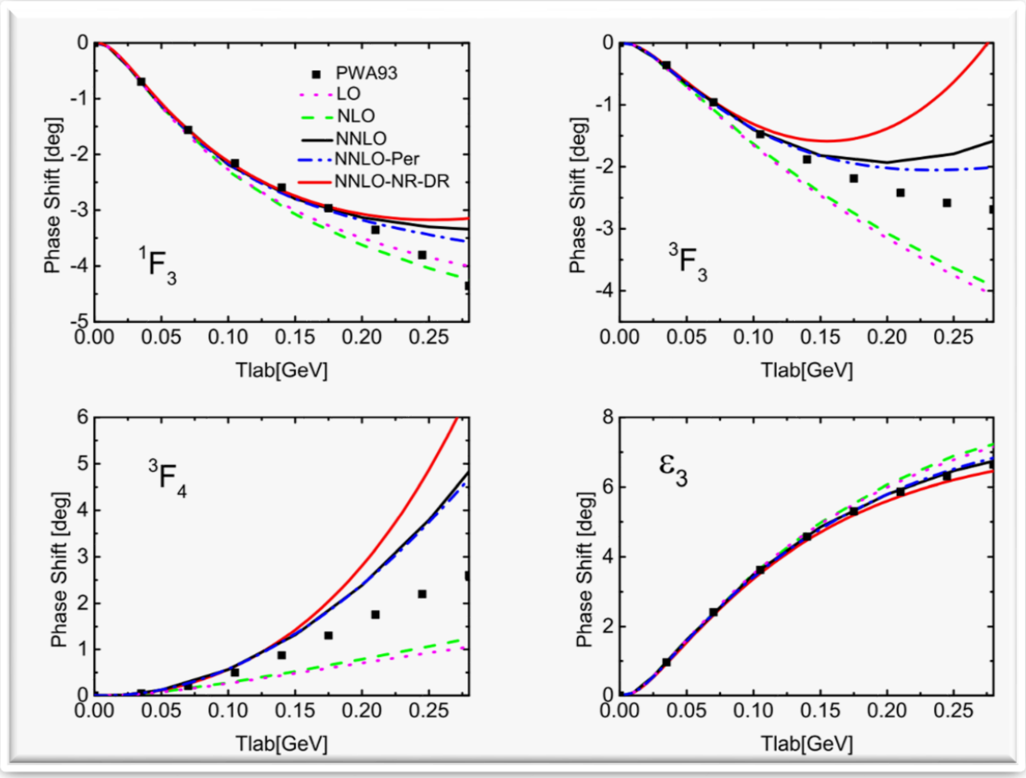


- Small TPE contributions
- Agreement with data quite good
- Not much difference between relativistic TPE and non-relativistic TPE

TPE contributions in a word

- Perturbative relativistic corrections are relatively small
- **Amazingly,** they **improve** the NR results
- Non-perturbative summation improves further the description





NNLO high precision relativistic chiral force

- Fit to the phase shifts of all the partial waves with $J \leq 2$ at $E_{lab} = 1,5,10,25,50,100,150,200\ \mathrm{MeV}$

$$\tilde{\chi}^2 = \sum(\delta^i - \delta^i_{\mathrm{PWA93}})^2,$$

- Inputs

c_1	c_2	c_3	c_4	f_π	g_A
-1.39	4.01	-6.61	3.92	92.4	1.29

- Fit results

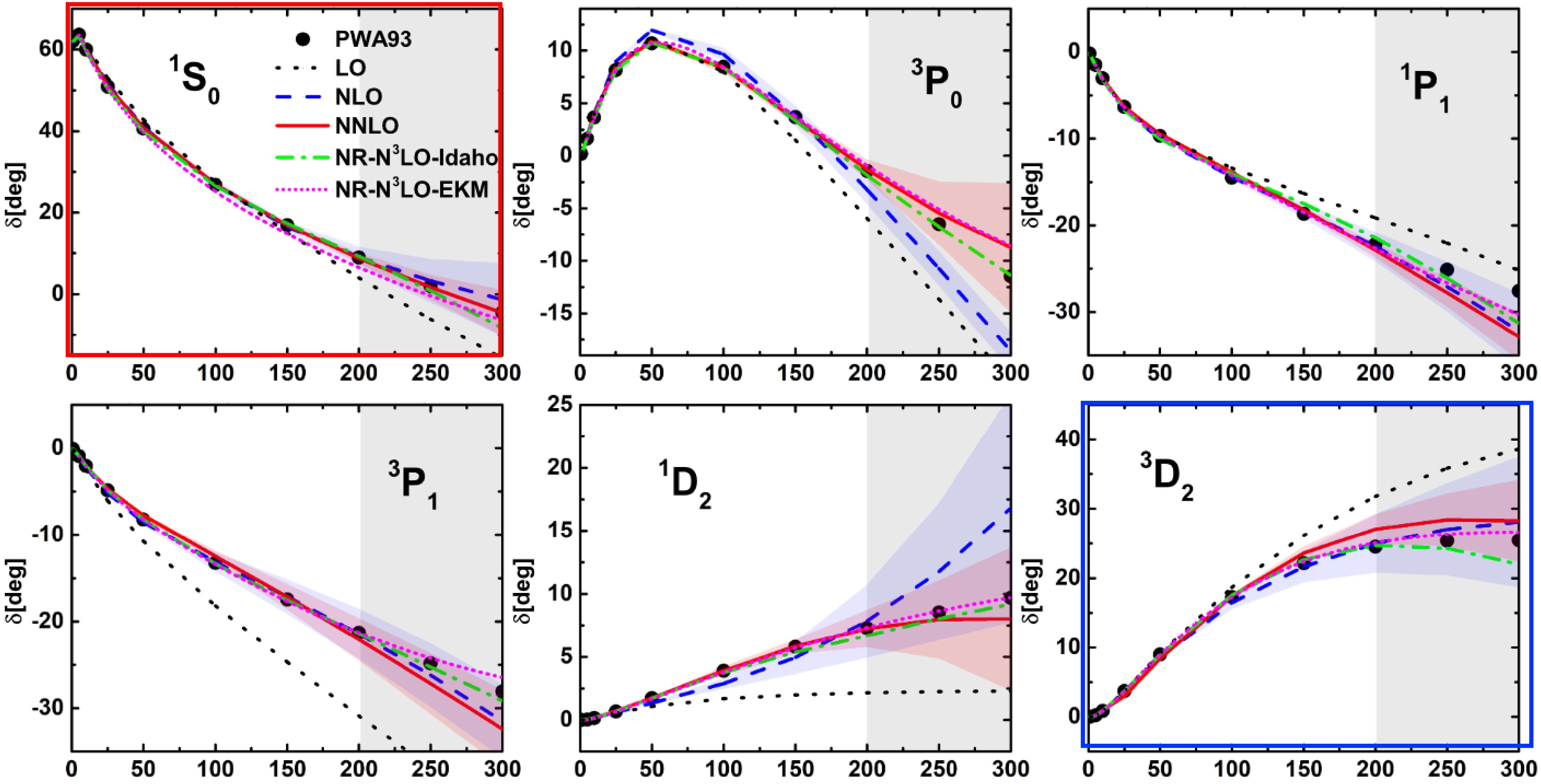
	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9	O_{10}	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}	O_{17}	D_1	D_2
LO	-13.23	-2.06	-9.34	3.14															
NLO	-2.62	9.45	-5.42	-6.05	30.09	9.02	-9.19	8.74	4.74	7.02	3.52	11.42	-6.03	-20.55	-4.99	-12.80	6.30	0.42	0.28
NNLO	-14.83	-2.25	-4.85	6.24	-0.82	1.96	-6.89	7.19	1.44	3.50	-8.10	-9.38	-4.33	-12.89	-12.26	-11.69	3.86	-1.88	-0.63

TABLE III. $\tilde{\chi}^2 = \sum_i(\delta^i - \delta^i_{\mathrm{PWA93}})^2$ of different chiral forces for partial waves up to $J \leq 2$.

	Total	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
NLO	17.02	1.02	7.04	0.46	0.33	1.80	1.69	0.15	2.18	1.35	0.95	0.01	0.04
NNLO	16.61	0.18	0.30	1.07	1.55	3.36	0.26	0.03	0.01	9.56	0.01	0.27	0.01
NR-N^3LO-Idaho	8.84	1.53	0.30	2.41	0.04	2.33	1.00	0.02	0.57	0.42	0.17	0.03	0.02
NR-N^3LO-EKM	16.08	13.45	0.29	0.34	0.06	0.01	0.13	0.01	0.02	0.43	0.12	1.22	0.00

Fit results for $J \leq 2$ partial waves

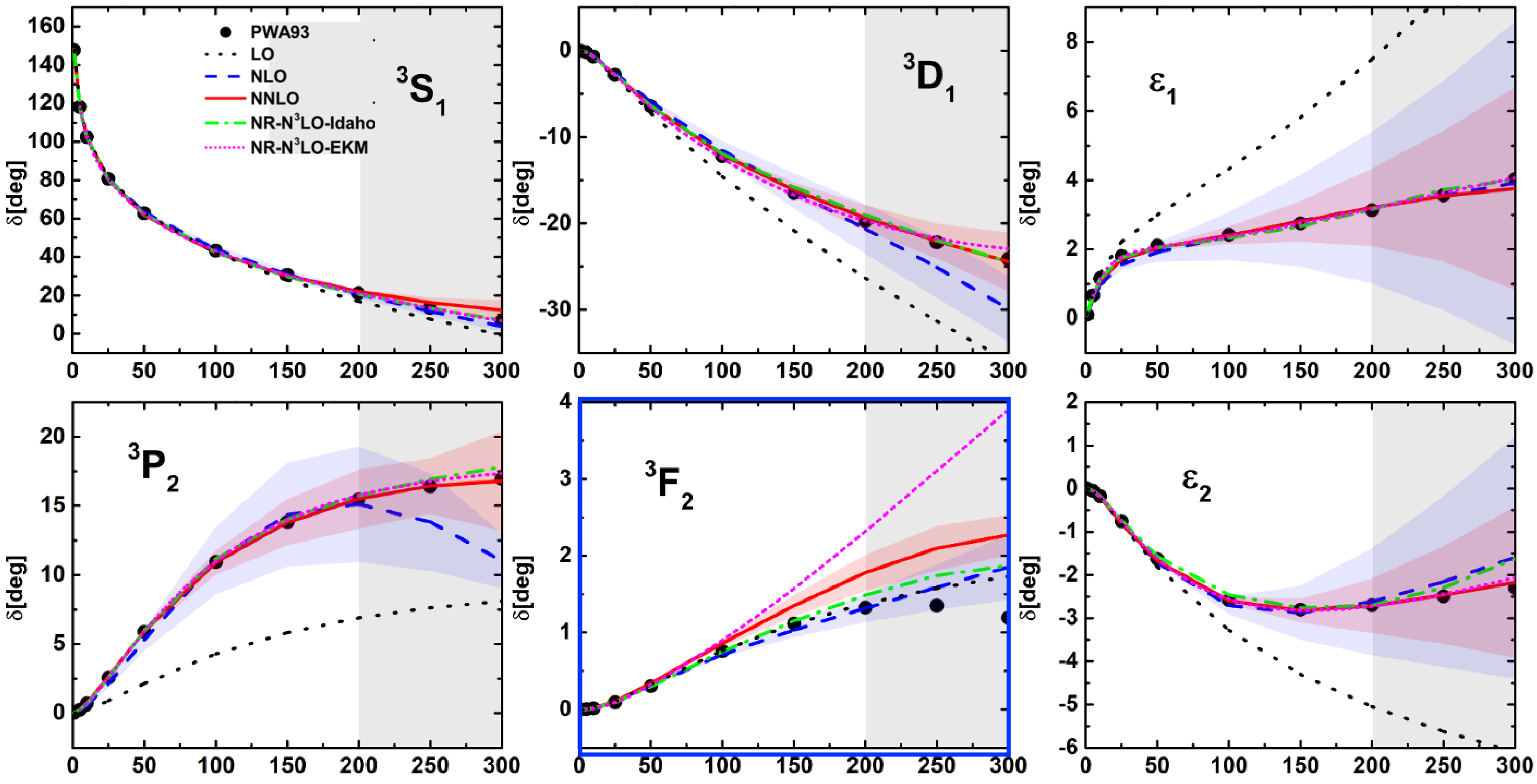
$$\Delta^{\rm NLO} = {\rm Max}\{Q^3 \cdot |\delta^{\rm LO}|, Q \cdot |\delta^{\rm LO} - \delta^{\rm NLO}|\},$$
$$\Delta^{\rm NNLO} = {\rm Max}\{Q^4 \cdot |\delta^{\rm LO}|, Q^2 \cdot |\delta^{\rm LO} - \delta^{\rm NLO}|, Q \cdot |\delta^{\rm NLO} - \delta^{\rm NNLO}|\}.$$



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

Fit results for $J \leq 2$ partial waves

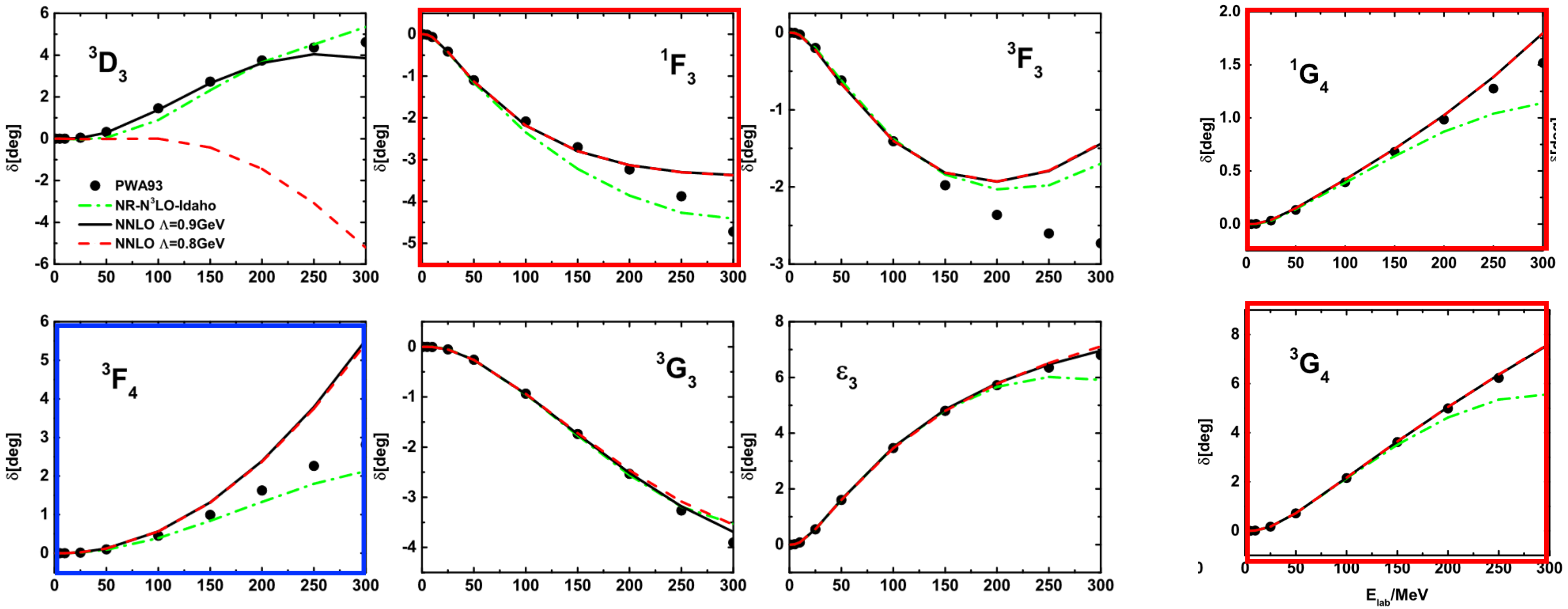
$$\Delta^{\rm NLO} = {\rm Max}\{Q^3 \cdot |\delta^{\rm LO}|, Q \cdot |\delta^{\rm LO} - \delta^{\rm NLO}|\},$$
$$\Delta^{\rm NNLO} = {\rm Max}\{Q^4 \cdot |\delta^{\rm LO}|, Q^2 \cdot |\delta^{\rm LO} - \delta^{\rm NLO}|, Q \cdot |\delta^{\rm NLO} - \delta^{\rm NNLO}|\}.$$



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (201

Predictions for higher partial waves

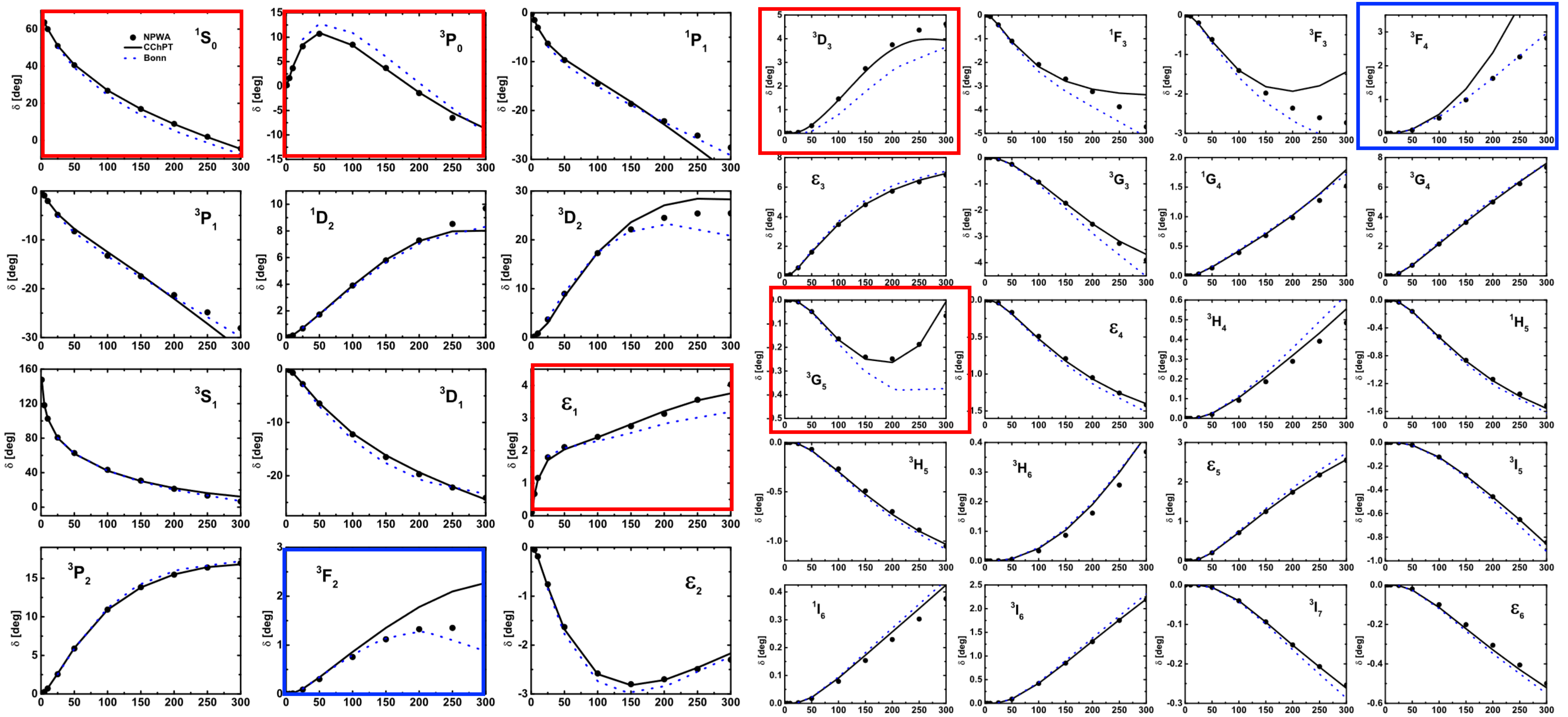
	Total	$^3D_3$	$^1F_3$	$^3F_3$	$^3F_4$	$^3G_3$	ϵ_3	$^1G_4$	$^3G_4$
NNLO	0.98	0.03	0.03	0.21	0.70	0.00	0.01	0.00	0.00
NR-N^{3}LO-Idaho	1.73	0.58	0.73	0.13	0.12	0.00	0.01	0.01	0.15



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011

In comparison with the Bonn potential

R. Machleidt, K. Holinde, C. Elster, Phys.Rept. 149 (1987) 1-89



contents

Summary and outlook

- ☐ Based on the consideration of symmetries and convergence, we proposed to build a high-precision nuclear force in covariant baryon chiral perturbation theory
- ☐ After many years of hard work, we have constructed a high precision covariant chiral nuclear force, which are ready to be used for ab initio nuclear structure and reaction studies

Summary and outlook

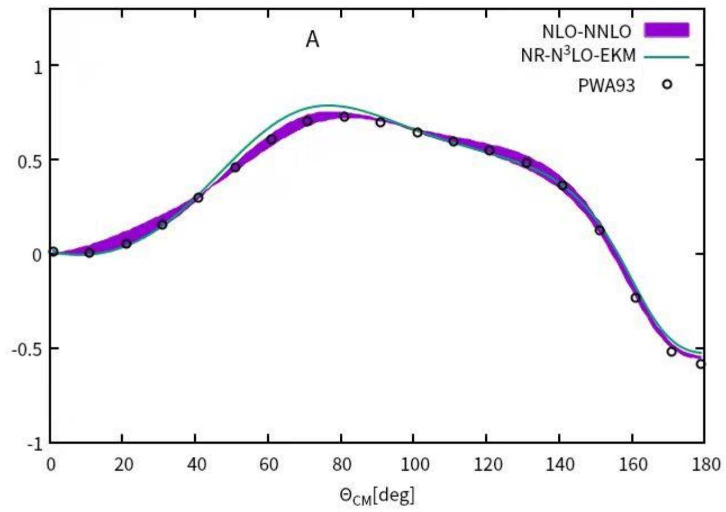
- ☐ Higher order hyperon-nucleon, hyperon-hyperon interactions
- ☐ Correlation functions
- ☐ Three-nucleon forces
- ☐ Ay puzzle in n-d scattering
- ☐ RG invariance up to higher orders
- ☐ Relativistic ab initio nuclear structure and reaction studies
- ☐ ……

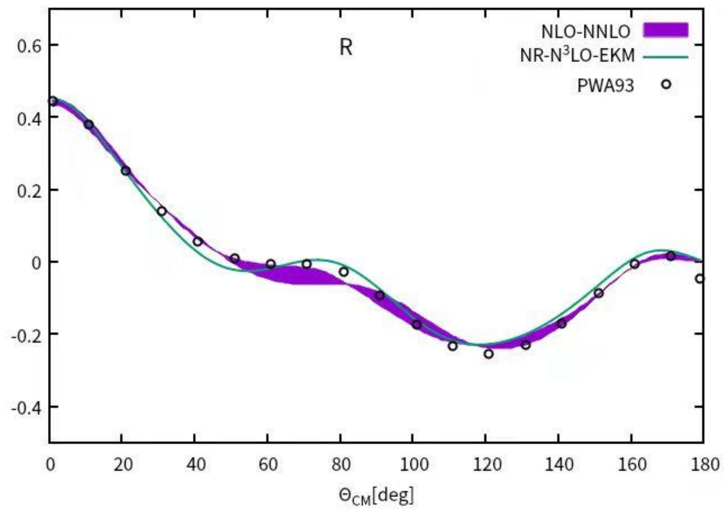
北京航空航天大學
BEIHANG UNIVERSITY

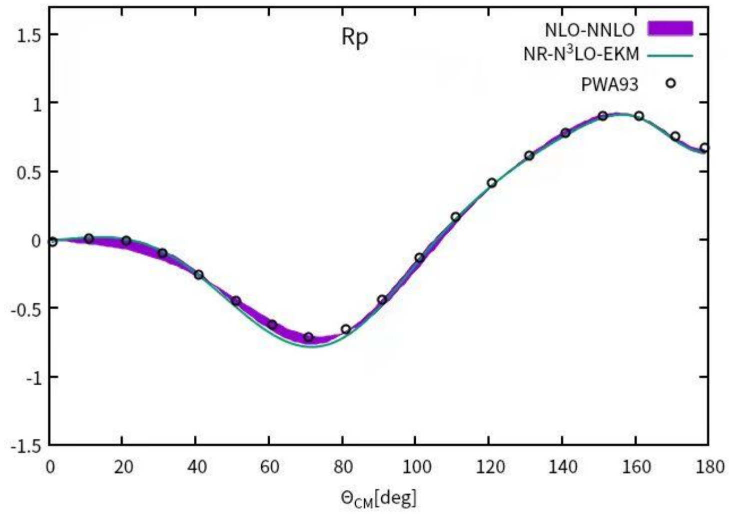
Thanks for your attention！

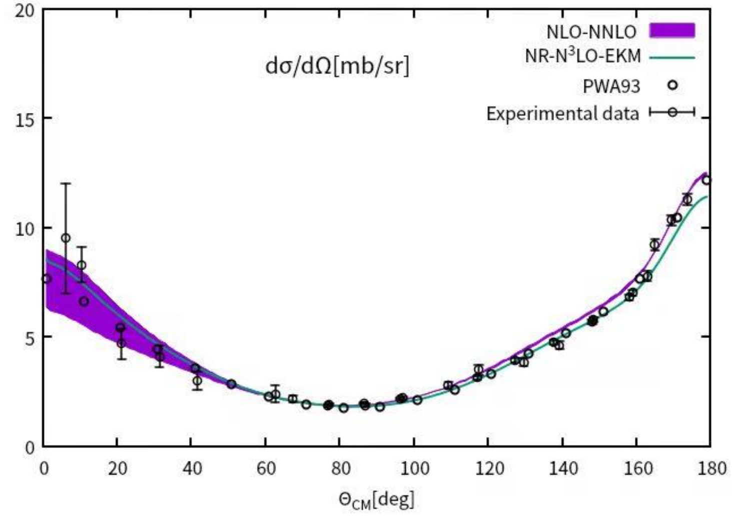
January 12, 2022

Predictions for observables at Tlab=200 MeV









Family of chiral two- plus three-nucleon interactions for accurate nuclear structure studies

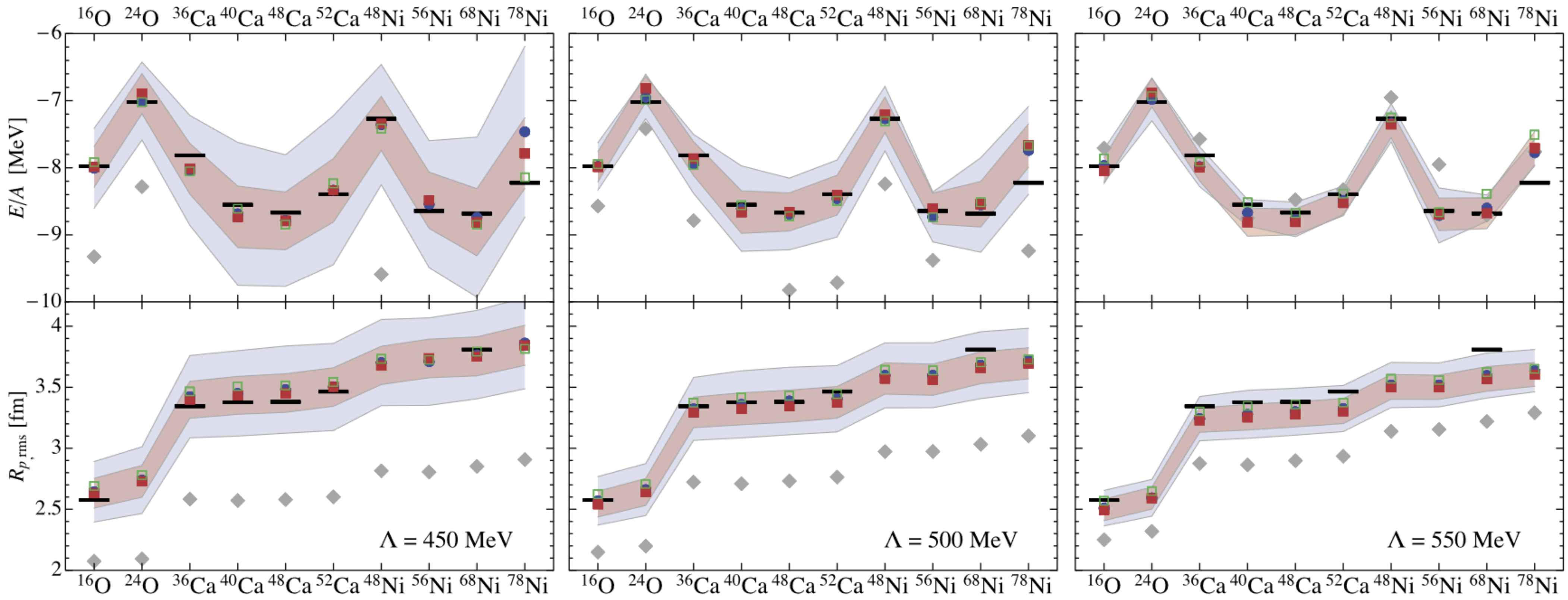


Fig. 4. Ground-state energies (top panels) and point-proton rms radii (bottom panels) obtained in IM-SRG calculations for the NLO (solid gray diamonds), N^2LO (blue circles), N^3LO (red boxes), and N^3LO' (open green boxes) interactions with $\Lambda = 450$ MeV (left), 500 MeV (center), and 550 MeV (right). The error bands for N^2LO (blue) and N^3LO (red) are derived from the order-by-order behavior and include the many-body uncertainties (see text). Experimental data is indicated by black bars [5,37–39].

Thomas Hüther, Klaus Vobig, Kai Hebeler, Ruprecht Machleidt, Robert Roth, Physics Letters B 808 (2020) 13565

Non-perturbative treatment

- **Blankenbecler-Sugar** equation(BbS quation)

$$\mathcal{T}(p',p|W) = \mathcal{A}(p',p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p',k|W)G(k|W)\mathcal{T}(k,p|W),$$

Solution: difficult

  - 3D reduction

$$\mathcal{T} = \mathcal{V} + \mathcal{V}g\mathcal{T}, \qquad \mathcal{V} = \mathcal{A} + \mathcal{A}(G-g)\mathcal{V}$$

$$g = \frac{\pi i \delta(k^0)\Lambda_+^1(\boldsymbol{k})\Lambda_+^2(-\boldsymbol{k})}{2E_k(E_k^2 - s/4 - i\epsilon)}$$

3D propagator

$$T_{l'l}^{sj}(p',p|\sqrt{s}) = V_{l'l}^{sj}(p',p|\sqrt{s}) + \sum_{l''} \int \frac{k^2dk}{(2\pi)^3} V_{l'l''}^{sj}(p',k|\sqrt{s}) \frac{M^2}{E_k} \frac{1}{p^2 - k^2 + i\varepsilon} T_{l''l}^{sj}(k,p|\sqrt{s})$$

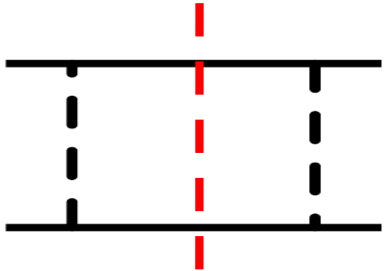
  - Ultraviolet divergence: **Regulator**

$$V_{l'l}^{sj}(p',p|\sqrt{s}) = f_R(p)V(p',p|\sqrt{s})f_R(p')$$

$$f_R(p) = f_R^{\mathrm{sharp}}(p) = \theta(\Lambda^2 - p^2)$$

- **Iterated** OPE

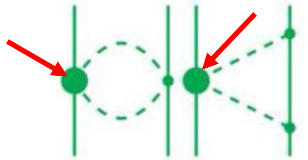
  - Planar box diagram and once-iterated OPE: double counting



$$V_{\mathrm{IOPE}}^{l'l,sj}(p',p|\sqrt{s}) = \sum_{l''} \int \frac{k^2dk}{(2\pi)^3} V_{\mathrm{OPE}}^{l'l'',sj}(p',k|\sqrt{s}) G_{\mathrm{BbS}}(k|\sqrt{s}) V_{\mathrm{OPE}}^{l''l,sj}(k,p|\sqrt{s}),$$

fitting

- n-p phase shifts for partial waves with $J \leq 2$ and $T_{\mathrm{lab}} \in \{1,5,10,25,50,100,150,200\}$ MeV
- Function to be minimized: $\tilde{\chi}^2 = \sum (\delta^i - \delta^i_{\mathrm{PWA93}})^2$ PWA93: V. G. J. Stoks et al, PRC1993
- Fixed input: LECs $c_{1,2,3,4}$ from $\mathcal{L}^{(2)}_{MB}$



NNLO

Covariant NNLO πN scattering   Y.-H. Chen et al. PRD2013

c_1	c_2	c_3	c_4
-1.39	4.01	-6.61	3.92

- Parameters and regulators

CO-NNLO	**19**	**4(LO) + 13(NLO) + 2(promoted)**	**Sharp cutoff**
NR-N^3LO-Idaho	**29**	**2(LO) + 7(NLO) + 15(N^3LO) + 2(Charge) + 3($c_{2,3,4}$ semi-free)**	$e^{-p^{n_1}}$**(S)**, $e^{-p^{n_2}}$**(L)**
NR-N^3LO-EKM	**26**	**2(LO) + 7(NLO) + 15(N^3LO) + 2(Charge)**	$e^{-p^{n_1}}$**(S)**, $(1-e^{-r^2})^{n_2}$**(L)**

- NR-N^3LO-Idaho: R. Machleidt and D. R. Entem, Phys.Rev.C(2003), Phys.Rept.(2011)
- NR-N^3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur.Phys.J.A(2015), Phys.Rev.Lett. (2015).

- **Truncation uncertainties** instead of residue cutoff dependence   E. Epelbaum et al. PRL2015
  - The expansion parameter of chiral EFT

$$Q = \mathrm{Max}\{\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\}$$

  - The NLO and NNLO truncation uncertainties

$$\Delta^{\mathrm{NLO}} = \mathrm{Max}\{Q^3 \cdot |\delta^{\mathrm{LO}}|, Q \cdot |\delta^{\mathrm{LO}} - \delta^{\mathrm{NLO}}|\}$$

$$\Delta^{\mathrm{NNLO}} = \mathrm{Max}\{Q^4 \cdot |\delta^{\mathrm{LO}}|, Q^2 \cdot |\delta^{\mathrm{LO}} - \delta^{\mathrm{NLO}}|, Q \cdot |\delta^{\mathrm{NLO}} - \delta^{\mathrm{NNLO}}|\}$$

TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [38] below 290 MeV by various *np* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

TABLE III. χ^2/datum for the reproduction of the 1999 *pp* database [38] below 290 MeV by various *pp* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	795	1.05	6.66	57.8	0.96
100–190	411	1.50	28.3	62.0	1.31
190–290	851	1.93	66.8	111.6	1.82
0–290	2057	1.50	35.4	80.1	1.38

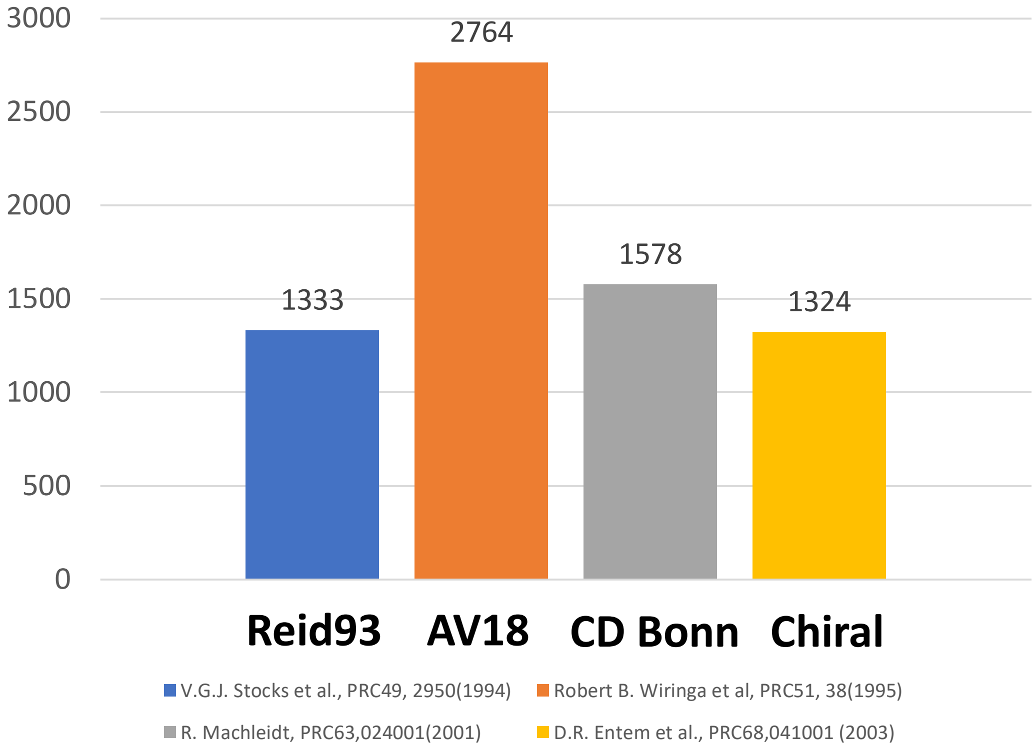
R. Machleidt et al. PRC2003

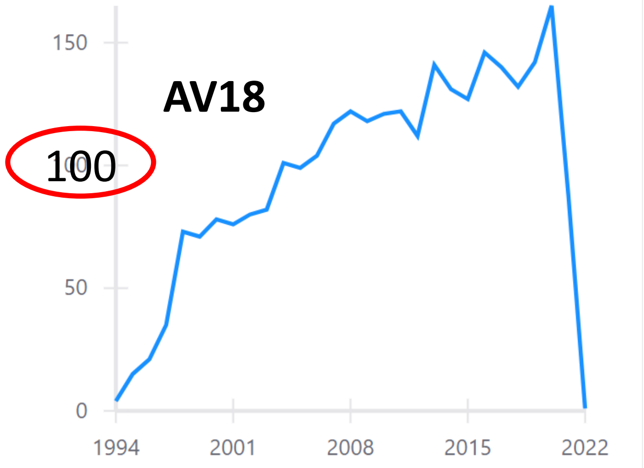
N^3LO: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

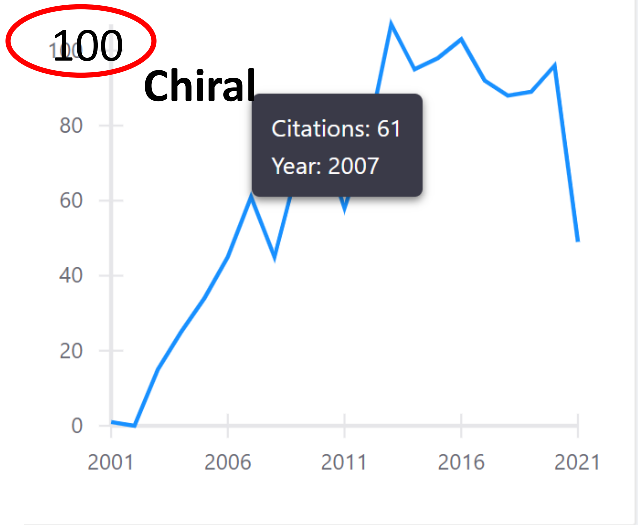
NUMBER OF PARAMETERS
for the *np* potential

	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^3LO Q^4 (N^4LO)	N^5LO Q^6
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$-$^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$-$^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	9	24	50

Why (bare) nuclear forces







Yearly citation about 100 times