Two-point functions from chiral kinetic theory in magnetized plasma

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The 7_{th} Chirality, Vorticity and Magnetic Field in HIC
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Based on LXY, PRD 105, 074039, 2022
S. Lin, LXY, JHEP 06, 054, 2021

Outline

- Motivation: chiral transports in magnetized plasma
- Chiral kinetic theory from Landau level basis
- Response functions as functional derivatives
- Summary and outlook

Anomalous chiral transports in QGP

Talks by Kharzeev, H. Huang, Q. Shou, J. Gao, F. Wang, X. Huang, S. Shi, Mameda, Y. Lin, X. Zhao, W. Wu ...

CME/CSE

$$\mathbf{J} = C\mu_5 e\mathbf{B} \qquad \mathbf{J}_5 = C\mu e\mathbf{B}$$

$$J_5^{\mu} = -\epsilon^{\mu\nu} J_{\nu}$$

CMW: expected in CKT

Vector/Axial CVE

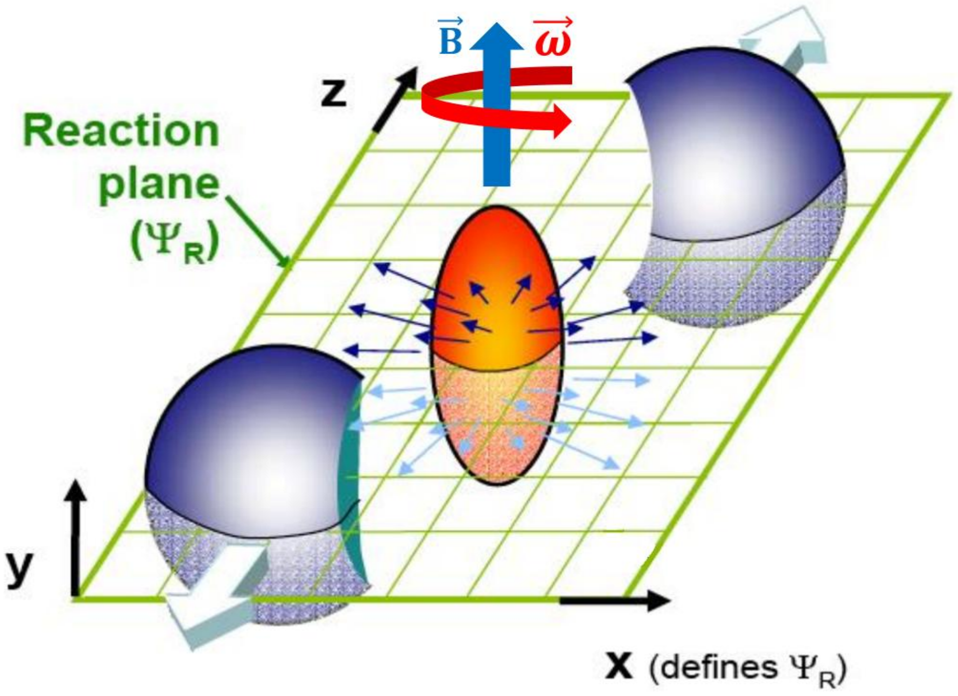
$$\mathbf{J} = C\mu\mu_5\boldsymbol{\omega} \qquad \mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

CESE

$$\mathbf{J}_5 \propto \frac{\mu\mu_5}{T^2}\sigma e\mathbf{E}$$

Ohm/Hall currents(non-anomalous)

$$\mathbf{J} = \sigma e\mathbf{E} + \sigma_H e^2 \mathbf{E}\times\mathbf{B}$$



CKT with Landau level basis

S. Lin, LXY, PRD 2020

H. Gao, S. Lin, Z. Mo, PRD 2020

CKE for chiral fermion in collisionless limit

chirality right-handed

$$p_\mu j^\mu = 0$$
$$\Delta_\mu j^\mu = 0$$
$$p^\mu j^\nu - p^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma$$

$$A_\mu \to A_\mu + a_\mu$$
$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$
$$f_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho \mathcal{B}^\sigma + E_\mu u_\nu - E_\nu u_\mu$$
$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu}(F_{\mu\nu} + f_{\mu\nu})$$

Wigner functions in LL basis → $2W(x,p) = j^\mu \bar{\sigma}_\mu$

Valid to all order of background B & first order $O(a)$ $O(\partial a)$

$B \sim p^2 \sim O(1)$ $a_\mu \sim O(a)$ $\mathcal{B}$ & $E \sim O(\partial a)$

Background LLL

$$j^\mu_{(0)} = (u + b)^\mu \delta(\bar{p}_0 - \bar{p}_3) f(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

1+1D, fluid velocity: u^μ
direction of B^μ: b^μ

Current & stress energy tensor

$$J^\mu = \int d^4p\, j^\mu \qquad T^{\mu\nu} = \int d^4p \frac{1}{2}(p^\mu j^\nu + p^\nu j^\mu)$$

Perturbation: vector/axial gauge field

CKE in collisionless limit

chirality
s=±1

$$p_\mu j_s^\mu = 0$$

$$\Delta_\mu j_s^\mu = 0$$

$$p^\mu j_s^\nu - p^\nu j_s^\mu = -\frac{s}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho^s j_\sigma^s$$

$$A_\mu \to A_\mu + a_\mu^s$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$

$$f_{\mu\nu}^s = \epsilon_{\mu\nu\rho\sigma}u^\rho \mathcal{B}_s^\sigma + E_\mu^s u_\nu - E_\nu^s u_\mu$$

$$\Delta_\mu^s = \partial_\mu - \frac{\partial}{\partial p_\nu}\left(F_{\mu\nu} + f_{\mu\nu}^s\right)$$

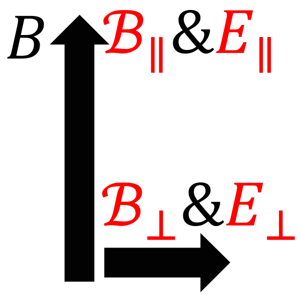
Background LLL $$j_{(0)s}^\mu = (u + sb)^\mu \delta(\bar{p}_0 - s\bar{p}_3) f_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

Perturbation expansion

$$j_s^\mu = j_{(0)s}^\mu + j_{as}^\mu + \sum_{\mathcal{A}s} j_{\mathcal{A}s}^\mu$$

$O(a)$ $\quad$ $O(\partial a)$

$$\mathcal{A}s = \mathcal{B}_\parallel^s, \mathcal{B}_\perp^s, E_\parallel^s, E_\perp^s$$



Perturbative solution: CMW

At $O(a)$ give CMW

$$j^{\mu}_{as} = (u + sb)^{\mu}\Delta\mu_s \delta(\bar{p}_0 - s\bar{p}_3) f'_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

CMW propagating with the speed c in the limit $\boldsymbol{B} \to \infty$

Kharzeev, Yee, PRD 2011

$$j^{\mu}_{(0)s} + j^{\mu}_{as} = j^{\mu}_{(0)s}(\mu_s \to \mu_s + \Delta\mu_s)$$ shifted by $\Delta\mu_s$

$$\Delta\mu_s \equiv \frac{\bar{q}_3\bar{a}_0^s - \bar{q}_0\bar{a}_3^s}{s\bar{q}_0 - \bar{q}_3}$$

redistribution of the chiral plasma

$$\Delta\mu = \frac{\Delta\mu_R + \Delta\mu_L}{2}$$

$$\Delta\mu_5 = \frac{\Delta\mu_R - \Delta\mu_L}{2}$$

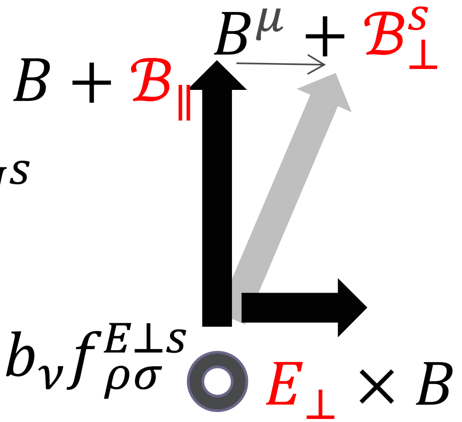
At $O(\partial a)$ collisionless limit $\to$ non-dissipative $\to$ $j^{\mu}_{E\parallel s} = 0$

$$j^{\mu}_{(0)s} + j^{\mu}_{\mathcal{B}\parallel s} = j^{\mu}_{(0)s}(B \to B + \mathcal{B}^s_{\parallel})$$

$$j^{\mu}_{(0)s} + j^{\mu}_{\mathcal{B}\perp s} = j^{\mu}_{(0)s}(b^{\mu} \to b^{\mu} + \mathcal{B}^s_{\perp}/B)$$

$$j^{\mu}_{(0)s} + j^{\mu}_{E\perp s} = j^{\mu}_{(0)s}(u^{\mu} \to u^{\mu} + U^{\mu}_s/B)$$

shifted by $\mathcal{B}^s_{\parallel}, \mathcal{B}^s_{\perp}, U^s$



$$U^{\mu}_s = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} b_{\nu} f^{E\perp s}_{\rho\sigma}$$

Constitutive relation of currents

At $O(1)$

$$J^\mu_{(0)} = \frac{\mu B}{2\pi^2} u^\mu + \underbrace{\frac{\mu_5 B}{2\pi^2} b^\mu}_{\text{CME}}$$

$$J^\mu_{(0)5} = \frac{\mu_5 B}{2\pi^2} u^\mu + \underbrace{\frac{\mu B}{2\pi^2} b^\mu}_{\text{CSE}}$$

At $O(a)$

$$J^\mu_a = \frac{\Delta\mu B}{2\pi^2} u^\mu + \frac{\Delta\mu_5 B}{2\pi^2} b^\mu$$

$$J^\mu_{a5} = \frac{\Delta\mu_5 B}{2\pi^2} u^\mu + \frac{\Delta\mu B}{2\pi^2} b^\mu$$

At $O(\partial a)$

Hall flow

$$J^\mu_{\mathcal{A}} = \frac{\mu\mathcal{B}_\parallel + \mu_5\mathcal{B}^5_\parallel}{2\pi^2} u^\mu + \frac{\mu U^\mu}{2\pi^2} + \frac{\mu_5 U^\mu_5}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^\mu_\perp + \mathcal{B}^\mu_\parallel\right)}{2\pi^2} + \frac{\mu\left(\mathcal{B}^\mu_{\perp 5} + \mathcal{B}^\mu_{\parallel 5}\right)}{2\pi^2}$$

$$J^\mu_{\mathcal{A}\mathbf{5}} = \frac{\mu_5\mathcal{B}_\parallel + \mu\mathcal{B}^5_\parallel}{2\pi^2} u^\mu + \frac{\mu_5 U^\mu}{2\pi^2} + \frac{\mu U^\mu_5}{2\pi^2} + \frac{\mu\left(\mathcal{B}^\mu_\perp + \mathcal{B}^\mu_\parallel\right)}{2\pi^2} + \frac{\mu_5\left(\mathcal{B}^\mu_{\perp 5} + \mathcal{B}^\mu_{\parallel 5}\right)}{2\pi^2}$$

No Ohm current in collisionless limit

Perturbation: longitudinal vorticity

Vorticity from metric $\omega^{\mu}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_{\nu}\partial_{\rho}u_{\sigma}=\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_{\nu}\partial_{\rho}(u^{\alpha}g_{\alpha\sigma})$

chirality
right-handed

$$\Pi_{\mu}j^{\mu}=0$$

$$\Delta_{\mu}j^{\mu}=0$$

$$\Pi^{\mu}j^{\nu}-\Pi^{\nu}j^{\mu}=-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_{\rho}j_{\sigma}$$

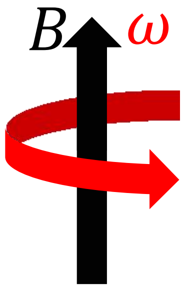
gradient $\partial u^{\rho}\sim O(\partial)$

homogeneous $B^{\sigma}\sim O(1)$

$\Rightarrow \partial F_{\mu\nu}\sim O(\partial) \Rightarrow \Pi_{\mu}=p_{\mu}-\frac{1}{12}\frac{\partial^2}{\partial p_{\nu}\partial p_{\lambda}}\partial_{\lambda}F_{\mu\nu}$

Perturbation expansion $j^{\mu}=j^{\mu}_{(0)}+j^{\mu}_{\omega}$, with j^{μ}_{ω} of order $O(\omega)$

$j^{\mu}_{\omega}\propto\omega$ for $\omega^{\mu}=\omega b^{\mu}$



Response functions as functional derivatives

Response functions from generating functional Γ

$$\mathcal{G}_{VA}^{\mu\nu}(q)\delta^{(4)}(k-q) = \frac{\delta\mathcal{J}^{\mu}(k)}{\delta a_{\nu}^{5}(q)} = \frac{\delta^{2}\Gamma}{\delta a_{\nu}^{5}(q)\delta a_{\mu}(-k)} \qquad \mathcal{G}_{VV}^{\mu\nu} \quad \mathcal{G}_{AA}^{\mu\nu} \quad \mathcal{G}_{AV}^{\mu\nu}$$

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_{\mu}(q)} = \frac{\delta^{2}\Gamma}{\delta a_{\mu}(q)\delta g_{\lambda\nu}(-k)} \qquad \mathcal{G}_{VT}^{\mu,\lambda\nu} \quad \mathcal{G}_{TA}^{\lambda\nu,\mu} \quad \mathcal{G}_{AT}^{\mu,\lambda\nu}$$

Derivative symmetry $\quad \mathcal{G}_{VA}^{\mu\nu}(q) = \mathcal{G}_{AV}^{\nu\mu}(-q)$

General form $\quad \mathcal{G}_{ab}(q) = \mathcal{G}_{ba}(-q) \qquad a, b = \mathcal{J}_{V}^{\mu}, \mathcal{J}_{A}^{\mu}, T^{\mu\nu}\ldots$

CKT solutions satisfy derivative symmetry?

CKT with consistent & covariant anomaly

Consistent $\mathcal{J}$ & covariant J anomaly

$$\partial_\mu \mathcal{J}^\mu = \partial_\mu J^\mu + \frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f^5_{\rho\lambda} = 0 \qquad \partial_\mu \mathcal{J}^\mu_5 = \partial_\mu J^\mu_5 = -\frac{1}{8\pi^2}\epsilon^{\mu\nu\rho\lambda}F_{\mu\nu}f_{\rho\lambda}$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

$$\int_q \Delta_\mu j^\mu$$

CKT with covariant anomaly

Son, Yamamoto, PRL 2012, PRD 2013

Manuel, Torres-Rincon, PRD 2014

Gorbar, Miransky, Shovkovy, Sukhachov, PRL 2017

in contrast to consistent anomaly

Carignano, Manuel, Torres-Rincon, PRD 2018

Relation between consistent & covariant current

$$\mathcal{J}^\mu = J^\mu + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}a^5_\nu F_{\rho\lambda} \qquad \mathcal{J}^\mu_5 = J^\mu_5$$

$G^{\mu\lambda}_{VA,a}(q) \neq G^{\lambda\mu}_{AV,a}(-q)$ → Chern-Simons term → $\mathcal{G}^{\mu\lambda}_{VA,a}(q) = \mathcal{G}^{\lambda\mu}_{AV,a}(-q)$

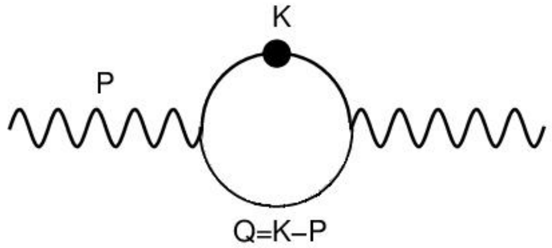
Photon polarization

Structures of response functions

$$\mathcal{G}^{\mu\lambda}_{VV,a}(q) = \mathcal{G}^{\mu\lambda}_{AA,a}(q) = \frac{B}{2\pi^2}\frac{\bar{q}_3^2 u^\mu u^\lambda + \bar{q}_0^2 b^\mu b^\lambda + \bar{q}_0\bar{q}_3 u^{\{\mu}b^{\lambda\}}}{\bar{q}_0^2 - \bar{q}_3^2}$$

LLL photon polarization

Fukushima, PRD 2011



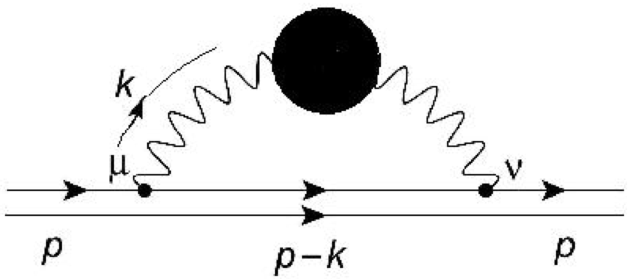
$$\mathcal{G}^{\mu\lambda}_{VV,\mathcal{A}}(q) = \mathcal{G}^{\mu\lambda}_{AA,\mathcal{A}}(q)$$
$$= \frac{i\mu_5}{2\pi^2}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_3 - b^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma - \frac{i\mu}{2\pi^2}\left(\epsilon^{\mu\lambda\rho\sigma}\bar{q}_0 + u^{[\mu}\epsilon^{\lambda]\nu\rho\sigma}q^T_\nu\right)u_\rho b_\sigma$$

CME CSE

Effect on photon splitting and polarization

→ spin polarization of probe fermion

Talk by Lihua Dong



Correlators of consistent currents satisfy Onsager relations

$$\mathcal{G}_{ab}(q_0, \mathbf{q}, \widetilde{\mathbf{B}}) = \gamma_a\gamma_b\mathcal{G}_{ba}(q_0, -\mathbf{q}, -\widetilde{\mathbf{B}})$$

Summary

- CKT from Landau level basis gives covariant anomaly
- Response functions from CKT gives photon polarization in strong B

Outlook

- Finite interactions, collisional terms in strong B
- Effect on photon splitting and polarization of probe fermion

Thanks for your attention!

Consistent & covariant anomaly

Consistent anomaly & covariant anomaly

$$\partial_\mu \mathcal{J}_s^\mu = \pm \frac{1}{96\pi^2} \epsilon^{\mu\nu\rho\lambda} F^s_{\mu\nu} F^s_{\rho\lambda}$$

$$\partial_\mu J_s^\mu = \pm \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} F^s_{\mu\nu} F^s_{\rho\lambda}$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

Bardeen counterterms

↓

Consistent axial anomaly & covariant anomaly

$$\partial_\mu J^\mu + \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f^5_{\rho\lambda} = \partial_\mu \mathcal{J}^\mu = 0$$

$$\partial_\mu J_5^\mu = \partial_\mu \mathcal{J}_5^\mu = -\frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}$$

CKT with free particle basis

$O(1)$: spinless particle $\partial_t f + \mathbf{v}\cdot\nabla_x f + (\boldsymbol{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_p f = 0$

$\delta(p^2)$

$O(\hbar)$: particle with Berry curvature $\boldsymbol{\Omega} = \dfrac{\mathbf{p}}{2|\mathbf{p}|^3}$

$$(1 + \hbar\boldsymbol{\Omega}\cdot\mathbf{B})\partial_t f$$
$$+ [\mathbf{v} + \hbar(\mathbf{E}\times\boldsymbol{\Omega}) + \hbar(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B}]\cdot\nabla_x f$$
$$+ [\mathbf{E} + \mathbf{v}\times\mathbf{B} + \hbar(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega})]\cdot\nabla_p f = 0$$

magnetic moment $\dfrac{\mathbf{p}}{2|\mathbf{p}|^2}$ $\delta(p^2) \to \delta(\tilde{p}^2)$

$$\tilde{p}^2 \equiv p^2 + \hbar\frac{\mathbf{B}\cdot\mathbf{p}}{p_0}$$

valid when $\sqrt{\hbar E}, \sqrt{\hbar B}, \hbar\partial_X \ll p$

$O(\hbar^2)$: particle no longer on-shell, simple picture lost

$$p_\mu G^\mu_{(0)}[f\delta(\tilde{p}^2)] + \frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times[\mathbf{p}f\delta(\tilde{p}^2)]\right\} + \hbar^2 C(f) = 0$$

$\delta(p^2) \nrightarrow \delta(\tilde{p}^2)$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang, PRD 2018

Noncommutable limits

Derivative symmetry in static limit

Shu Lin, LXY, JHEP 2021

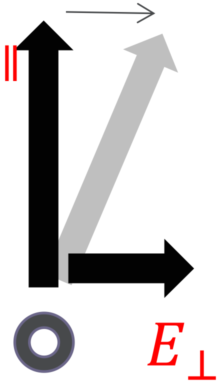
$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_\mu(q)} \qquad \mathcal{G}_{VT}^{\mu,\lambda\nu}(q)\delta^{(4)}(k-q) = \frac{\delta J_\nu^\mu(k)}{\delta g_{\lambda\nu}(q)}$$

With limits $\bar{q}_3 \to 0$ before $\bar{q}_0 \to 0$ Son, Yamamoto, PRD 2013

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q) = \mathcal{G}_{VT}^{\mu,\lambda\nu}(-q)$$

Two limits noncommutable, finite interactions needed Satow, Yee, PRD 2014

Suggestive form: LLL, on-shell



At $O(a)$ shifted by $\Delta\mu_s$

$$j^\mu_{(0)s} + j^\mu_{as} = j^\mu_{(0)s}(\mu_s \to \mu_s + \Delta\mu_s)$$

At $O(\partial a)$ in static limit, shifted by $\mathcal{B}_\parallel^s, \mathcal{B}_\perp^s, U^s$

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\parallel s} = j^\mu_{(0)s}(B \to B + \mathcal{B}_\parallel^s)$$ magnitude enhanced

$$j^\mu_{(0)s} + j^\mu_{\mathcal{B}\perp s} = j^\mu_{(0)s}(b^\mu \to b^\mu + \mathcal{B}_\perp^s/B)$$ tilted

$$j^\mu_{(0)s} + j^\mu_{E\perp s} = j^\mu_{(0)s}(u^\mu \to u^\mu + U_s^\mu/B)$$ boosted

$$U_s^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} b_\nu f_{\rho\sigma}^{E\perp s}$$

Phenomenology of LLL in HIC

Estimate parameters in QGP

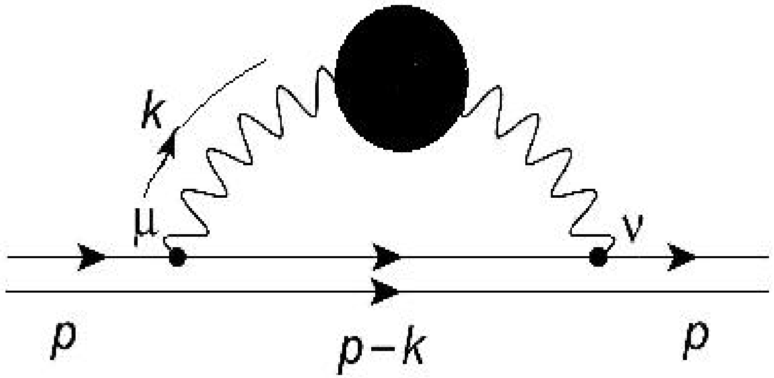
$$eB = m_\pi^2 \sim 10 m_\pi^2$$

$$T = 350MeV$$

$$\frac{eB}{T^2} = 0.2 \sim 1.6$$

higher LLs to be included

UPC? $\frac{eB}{T^2} \gg 1$ lower LLs summation



Effect on photon splitting and polarization → spin polarization of probe fermion

Lihua Dong, Shu Lin, poster at Guangzhou & upcoming works