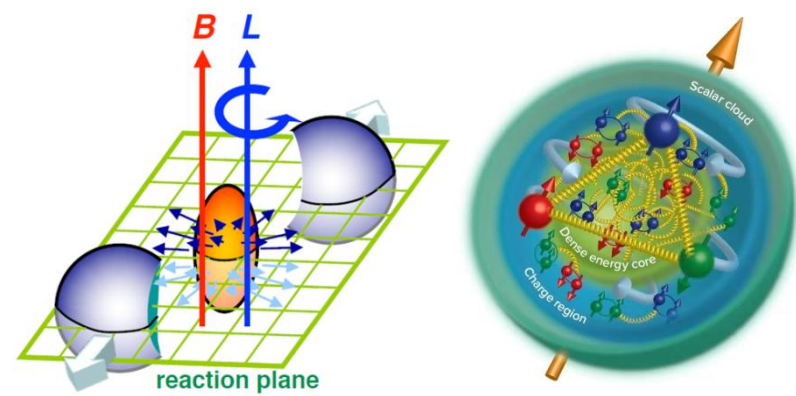




EM fields from the extended Kharzeev-McLerran-Warringa model and nucleon relativistic EM structures

Yi Chen

[YC, X.-L. Sheng & G.-L. Ma, NPA1011(2021)122199]

[YC, C. Lorcé, PRD106(2022)116024]

[YC, C. Lorcé, PRD107(2023)096003]

thanks to:

Xin-Li Sheng, Guo-Liang Ma, Cédric Lorcé, Qun Wang



**Department of Modern Physics,
University of Science and Technology of China**

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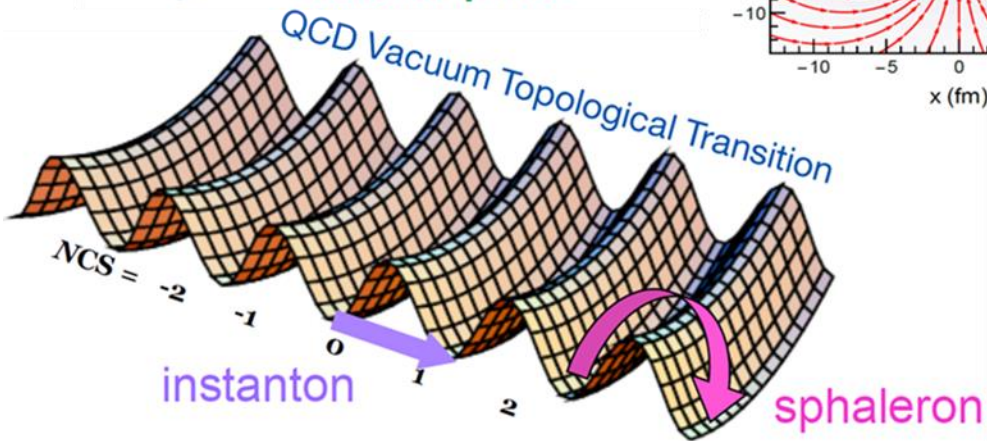
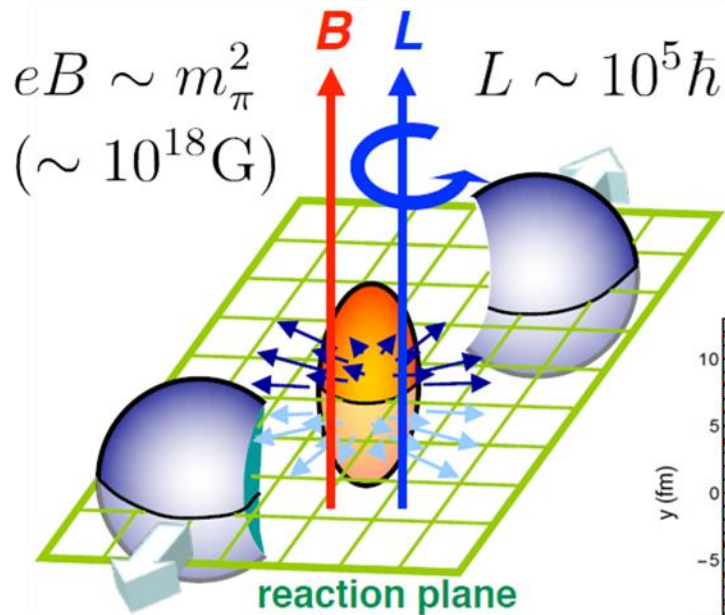
Outline

1. Introduction and motivations for strong EM fields
2. The extended KMW model for EM fields (**part 1**)
3. Nucleon relativistic EM structures (**part 2**)
4. Summary and outlook

†**KMW**: Kharzeev-McLerran-Warringa

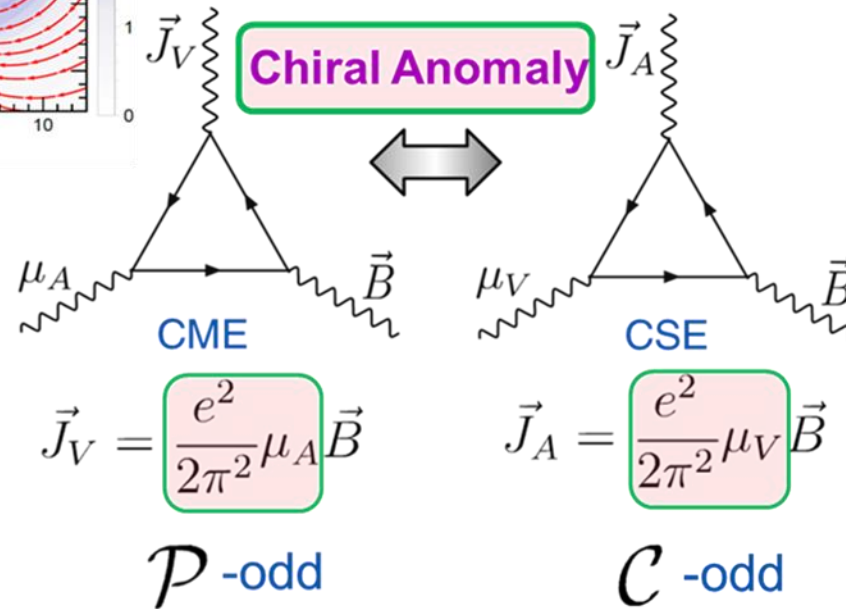
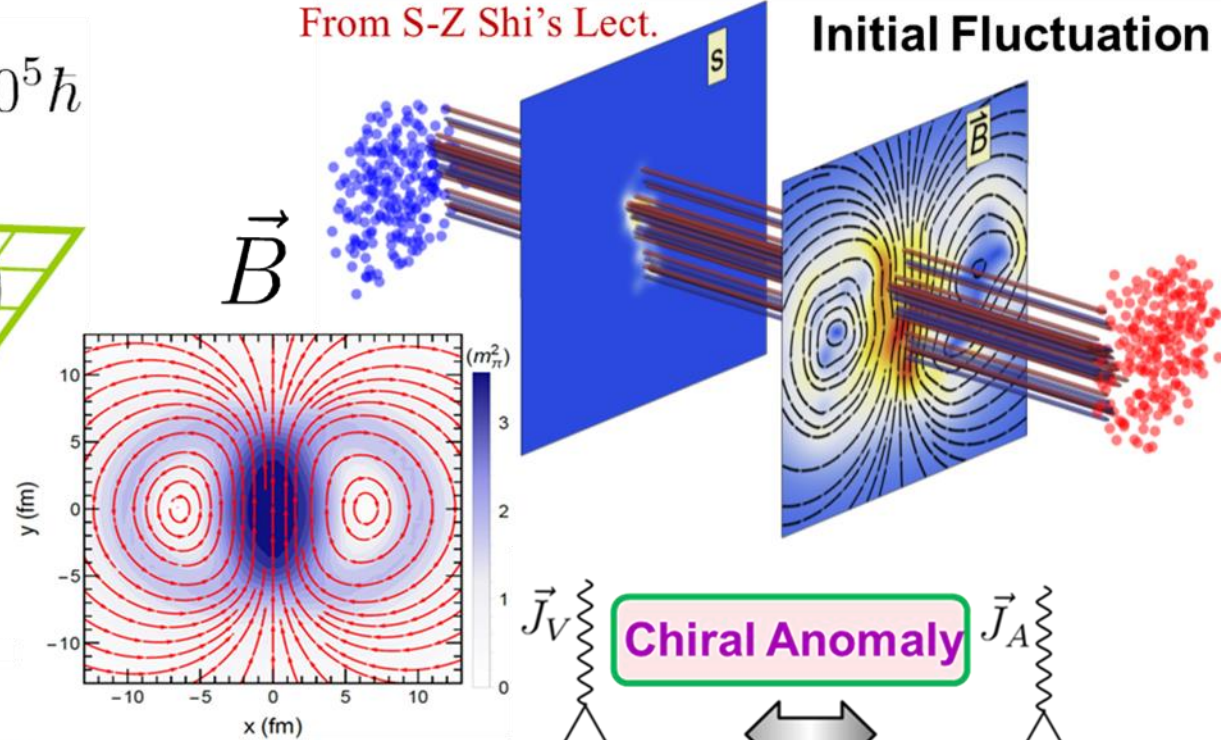
Kharzeev, McLerran & Warringa, NPA 803 (2008) 227-253.

Motivations: Strong EM fields and their effects



From S-Z Shi's Lect.

Initial Fluctuation



Z.-T. Liang & X.-N. Wang, PRL94, 102301 (2005).
 Kharzeev, McLerran & Warringa, NPA803 (2008).

Developments and many calculations on EM fields

Initial work on strong fields in HICs:

From 1974 to 2008 (almost 32 years!)

Rafelski & B. Mueller (1976).

Liénard-Wiechert (L-W) equations of EM fields:

Skokov, Illarionov & Toneev (2009'); S. Voloshin (2010'); Voronyuk, et al. (2011'); L. Ou & B-A. Li (2011'); Bzdak & Skokov (2012'); Deng & Huang (2012'); Toneev et al.(2013,2014'); J. Bloczynski, X-G Hang, X. Zhao & J. Liao (2013'); Deng-Huang (2015'); V. Roy & S. Pu (2016'); X.-L. Zhao, G.-L. Ma & Y.-G. Ma (2018',2019'); Y.-L. Cheng, S. Zhang, Y.-G. Ma, et al.(2019'); J. Hammelmann, et al.(2020'); X.-G. Deng & Y.-G. Ma (2020'); Irfan, Sheng & Wang (2021');

(Semi-) Analytic approaches:

Kharzeev, McLerran & Warringa (2008'); Tuchin (2010', 2013', 2015', 2017', ...); Asakawa (2010'); Liu & Greiner & Ko (2014'); Gürsoy & Kharzeev & Rajagopal (2014'); H. Li, X.-L. Sheng & Q. Wang (2016'); Hattori & Huang (2016); Gürsoy & Kharzeev & Marcus & Rajagopal & Shen (2018'); D. She & Feng & Zhong & Yin (2018'); Chen & Feng (2020'); Chen, Sheng & Ma (2021'); Grayson, Formanek, Rafelski & Mueller (2022);

Numerically solving Maxwell's equations (with Ohm conductivity):

McLerran & Skokov (2014'); Zakharnov (2014'); Z. Wang, Zhao, Greiner, Xu & Zhuang (2022'); A. Huang, She, Shi, Huang & Liao (2023); H. Li, Xia, Huang & Huang (2023);

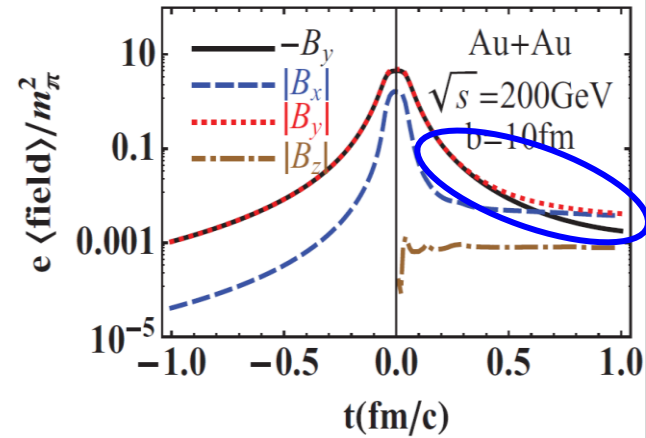
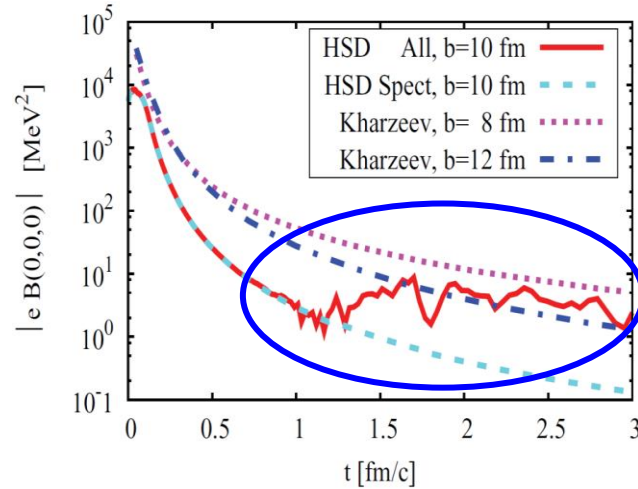
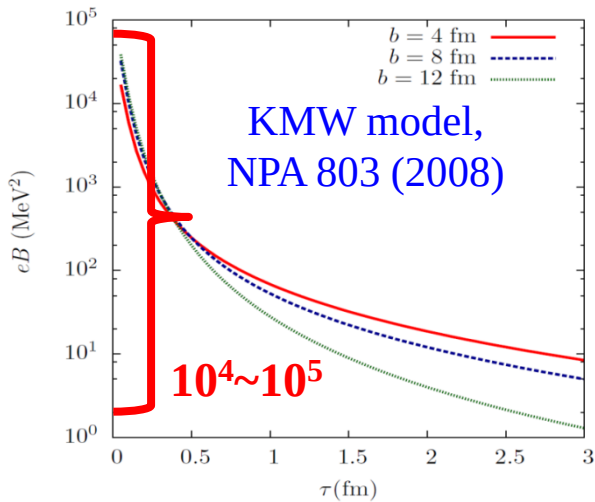
Magneto-hydrodynamics (considering QGP expansion and evolution):

Gürsoy & Kharzeev & Rajagopal (2014'); Roy & Pu & Rezzolla & Rischke (2015'); Inghirami & Zanna, et al. (2016'); Gürsoy & Kharzeev & Marcus & Rajagopal & Shen (2018'); Dash, Shokri, Rezzolla, & Rischke (2023);

See, A. Dash's talk on July 18.

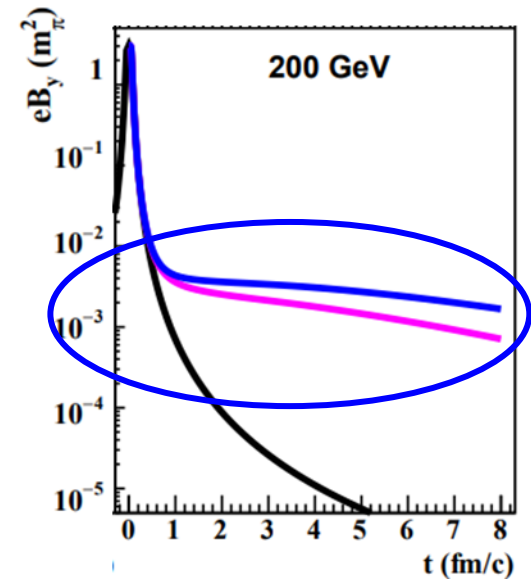
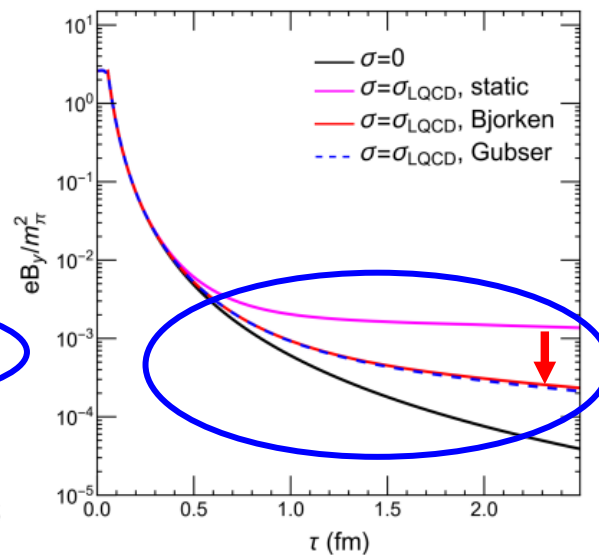
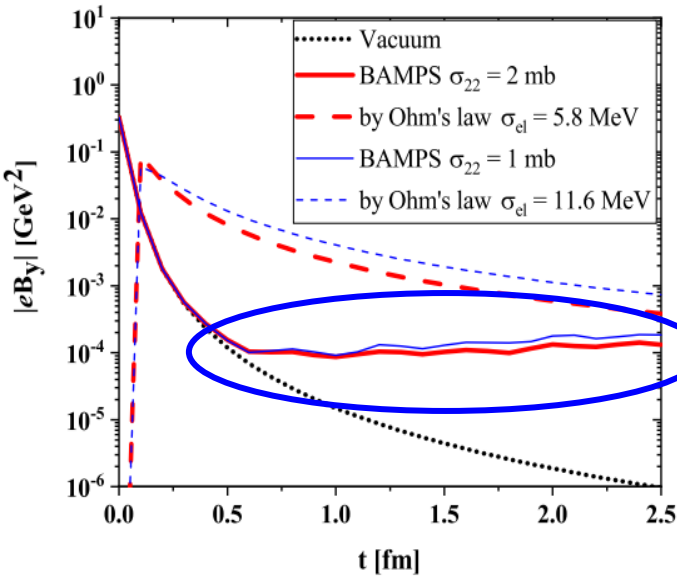
**There are many theoretical efforts in the past 15 years for the calculations of EM fields.
(I sincerely apologize if I may have missed listing your works)**

Time evolution of the strong magnetic field



PHSD: PRC 83, 054911 (2011)

W.-T. Deng & X.-G. Huang (2012)



Wang, Zhao, Greiner, Xu & Zhuang, PRC (2022)

Huang, She, Shi, Huang & Liao PRC (2023)

Li, Xia, Huang & Huang (2023)

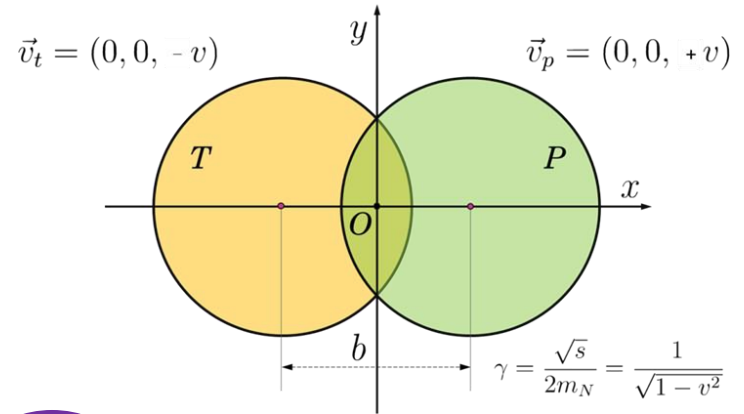
The original KMW model

Decomposition of EM fields:

$$\vec{B} = \vec{B}_s^+ + \vec{B}_s^- + \vec{B}_p^+ + \vec{B}_p^-$$

$$\vec{E} = \vec{E}_s^+ + \vec{E}_s^- + \vec{E}_p^+ + \vec{E}_p^-$$

EM fields in the vacuum:



$$e\vec{B}_s^\pm(\tau, \eta, \vec{x}_\perp) = \pm Z\alpha_{\text{EM}} \sinh(Y_0 \mp \eta) \int d^2\vec{x}'_\perp \rho_\pm(\vec{x}'_\perp) [1 - \theta_\mp(\vec{x}'_\perp)]$$

$$\times \frac{(\vec{x}'_\perp - \vec{x}_\perp) \times \vec{e}_z}{[(\vec{x}'_\perp - \vec{x}_\perp)^2 + \tau^2 \sinh(Y_0 \mp \eta)^2]^{3/2}}$$

$$e\vec{B}_p^\pm(\tau, \eta, \vec{x}_\perp) = \pm Z\alpha_{\text{EM}} \int d^2\vec{x}'_\perp \int_{-Y_0}^{Y_0} dY f(Y) \sinh(Y \mp \eta) \rho_\pm(\vec{x}'_\perp) \theta_\mp(\vec{x}'_\perp)$$

$$\times \frac{(\vec{x}'_\perp - \vec{x}_\perp) \times \vec{e}_z}{[(\vec{x}'_\perp - \vec{x}_\perp)^2 + \tau^2 \sinh(Y \mp \eta)^2]^{3/2}}$$

The original KMW model

Widely used L-W formula:

See, part 2. 

$$e\mathbf{E}(t, \mathbf{r}) = \alpha_{\text{EM}} \sum_{n=\pm} Z_n \frac{\mathbf{R}_n (1 - v_n^2)}{(R_n^2 - [\mathbf{v}_n \times \mathbf{R}_n]^2)^{3/2}},$$

$$e\mathbf{B}(t, \mathbf{r}) = \alpha_{\text{EM}} \sum_{n=\pm} Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n (1 - v_n^2)}{(R_n^2 - [\mathbf{v}_n \times \mathbf{R}_n]^2)^{3/2}},$$



"Singularity" problem

$$\begin{cases} \lim_{R_n \rightarrow 0} e\vec{E} = +\infty \\ \lim_{R_n \rightarrow 0} e\vec{B} = +\infty \end{cases}$$

δ -functions: $\rho_n(t, \mathbf{r}') = Q\delta^{(3)}(\mathbf{r}') = \underline{Ze\delta(x')\delta(y')\delta(z' - z'_n \mp |\mathbf{v}_n|t)}$

In terms of rapidity: Y 

$$e\mathbf{E}(t, \mathbf{r}) = \alpha_{\text{EM}} \sum_{n=\pm} \frac{Z_n \cosh(Y_n) \cdot \mathbf{R}_n}{\left\{ (x - x'_n)^2 + (y - y'_n)^2 + [t \sinh Y_n - (z - \textcircled{z'_n}) \cosh Y_n]^2 \right\}^{3/2}},$$

$$e\mathbf{B}(t, \mathbf{r}) = \alpha_{\text{EM}} \sum_{n=\pm} \frac{Z_n \sinh(Y_n) \cdot \mathbf{e}_z \times \mathbf{R}_n}{\left\{ (x - x'_n)^2 + (y - y'_n)^2 + [t \sinh Y_n - (z - \textcircled{z'_n}) \cosh Y_n]^2 \right\}^{3/2}}.$$

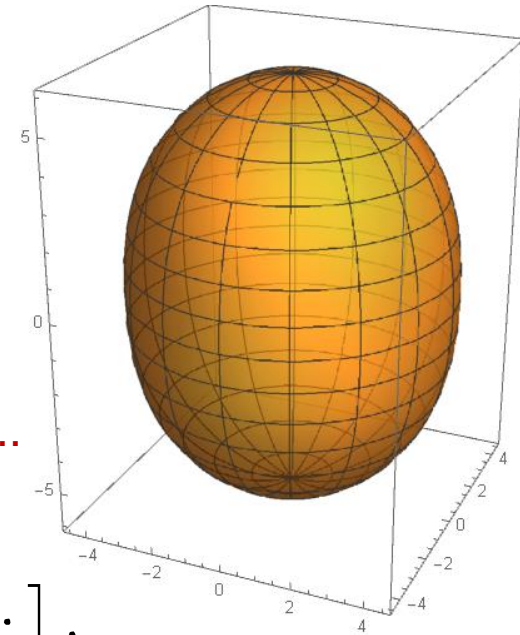
Possible extensions: I

Generalized charge distributions (with Lorentz contraction effect):

$$\rho(\mathbf{r}') = \gamma \rho_0 \frac{1 + \omega [r'/R'(\theta, \phi)]^2}{1 + \exp \{ [r' - R'(\theta, \phi)]/d \}} \Theta(\mathbf{r}'),$$

$$r' = r'(\gamma) \equiv \sqrt{\mathbf{r}'_{\perp}^2 + (\gamma z')^2},$$

E.g., U, Zr,...

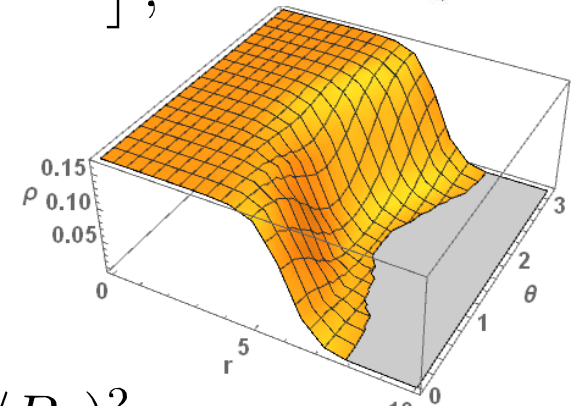


Axial deformation:

$$R' = R'(\theta, \phi) \equiv R_0 [1 + \beta_2 Y_2^0(\theta) + \beta_4 Y_4^0(\theta) + \dots],$$

Normalization:

$$Z = \int_{-R_A}^{+R_A} dx' \int_{-R_A}^{+R_A} dy' \int_{-R_A/\gamma}^{+R_A/\gamma} dz' \rho(\mathbf{r}'),$$



Spherically symmetric:

$$\rho(\mathbf{r}') = \gamma \rho_0 \frac{1 + \omega (r'/R_0)^2}{1 + \exp [(r' - R_0)/d]} \Theta(\mathbf{r}').$$

Possible extensions: II

Baryon-junction stopping effect:

$\alpha_y \sim$ Regge trajectory intercept.

$$f_{\pm}(Y) = \frac{\alpha_y}{2 \sinh(\alpha_y Y_0)} e^{\pm \alpha_y Y}, \quad -Y_0 \leq Y \leq Y_0$$

See, Prof. Zhangbu Xu's talk on July 18.

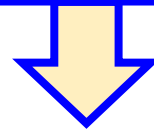


**Refined Baryon-junction stopping effects
with retardation correction (w/ RC)**

$$\Psi_{\pm}(Y) = \theta(t_{\text{ret}} - t_c) \cdot f_{\pm}(Y) + \theta(t_c - t_{\text{ret}}) \cdot \delta(Y \mp Y_0).$$

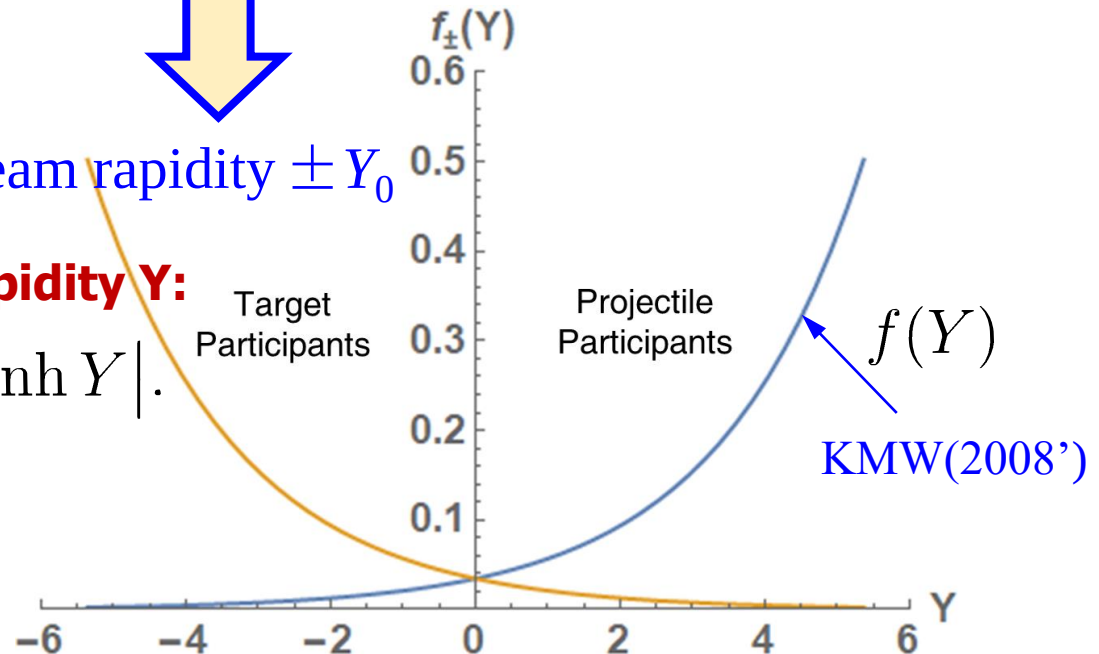


normalized $f_{\pm}(Y)$ beam rapidity $\pm Y_0$



Retardation relation for each rapidity Y :

$$t_{\text{ret}} = t - |\mathbf{r} - \mathbf{r}' - t_{\text{ret}} \cdot \mathbf{e}_z \tanh Y|.$$



KMW, NPA 803 (2008) 227-253.

ALICE, EPJC 73, 2496 (2013);

Gürsoy & Kharzeev & Marcus & Rajagopal & Shen (2018');

The extended KMW model: vacuum case

Contributions of **spectators**:

$$e\mathbf{E}_S^\pm(t, \mathbf{r}) = \alpha_{\text{EM}} \cosh(Y_0) \int_{V_\pm} d^3\mathbf{r}' \frac{\boxed{\rho_\pm(\mathbf{r}')} [1 - \tilde{\Theta}_\mp(\mathbf{r}'_\perp)] \mathbf{R}_\pm}{\left\{ (\mathbf{r}_\perp - \mathbf{r}'_\perp)^2 + [t \sinh(\pm Y_0) - (z - \textcolor{red}{z}') \cosh Y_0]^2 \right\}^{3/2}},$$

$$e\mathbf{B}_S^\pm(t, \mathbf{r}) = \alpha_{\text{EM}} \sinh(\pm Y_0) \int_{V_\pm} d^3\mathbf{r}' \frac{\boxed{\rho_\pm(\mathbf{r}')} [1 - \tilde{\Theta}_\mp(\mathbf{r}'_\perp)] \mathbf{e}_z \times \mathbf{R}_\pm}{\left\{ (\mathbf{r}_\perp - \mathbf{r}'_\perp)^2 + [t \sinh(\pm Y_0) - (z - \textcolor{red}{z}') \cosh Y_0]^2 \right\}^{3/2}},$$

Contributions of **participants**:

$$e\mathbf{E}_P^\pm(t, \mathbf{r}) = \alpha_{\text{EM}} \int_{V_\pm} d^3\mathbf{r}' \int_{-Y_0}^{Y_0} dY \frac{\textcolor{red}{\Psi_\pm(Y)} \cosh Y \cdot \boxed{\rho_\pm(\mathbf{r}')} \tilde{\Theta}_\mp(\mathbf{r}'_\perp) \mathbf{R}_\pm}{\left\{ (\mathbf{r}_\perp - \mathbf{r}'_\perp)^2 + [t \sinh Y - (z - \textcolor{red}{z}') \cosh Y]^2 \right\}^{3/2}},$$

$$e\mathbf{B}_P^\pm(t, \mathbf{r}) = \alpha_{\text{EM}} \int_{V_\pm} d^3\mathbf{r}' \int_{-Y_0}^{Y_0} dY \frac{\textcolor{red}{\Psi_\pm(Y)} \sinh Y \cdot \boxed{\rho_\pm(\mathbf{r}')} \tilde{\Theta}_\mp(\mathbf{r}'_\perp) \mathbf{e}_z \times \mathbf{R}_\pm}{\left\{ (\mathbf{r}_\perp - \mathbf{r}'_\perp)^2 + [t \sinh Y - (z - \textcolor{red}{z}') \cosh Y]^2 \right\}^{3/2}},$$

The extended KMW model: QGP evolution (mixed) case

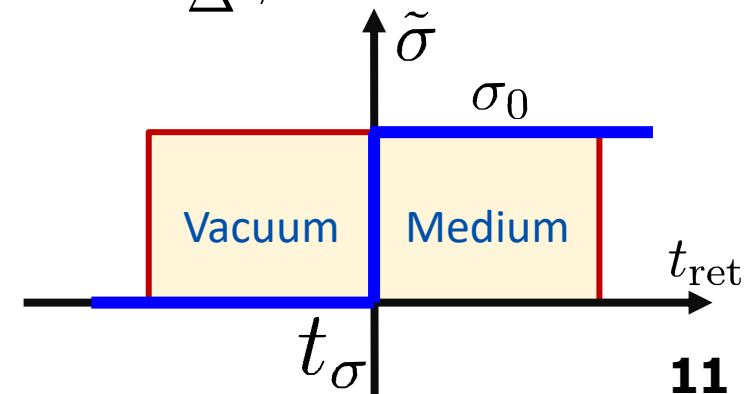
Time-dependent Ohm conductivity during the QGP evolution:

$$\sigma \rightarrow \tilde{\sigma}(t) = \sigma_0 \cdot \theta(t - t_\sigma), \quad \sigma_0 \approx 5.8 \text{ MeV (LQCD)} \\ t_\sigma \sim \text{Medium time}$$

Contributions of spectators & participants:

$$e\mathbf{B}_S^\pm(t, \mathbf{r}) = \lim_{Y \rightarrow \pm Y_0} \alpha_{\text{EM}} \sinh Y \int_{V_\pm} d^3\mathbf{r}' \frac{\rho_\pm(\mathbf{r}') \left[1 - \tilde{\Theta}_\mp(\mathbf{r}'_\perp)\right] \mathbf{e}_z \times \mathbf{R}_\pm}{\Delta^{3/2}} \\ \times \left[1 + \frac{\tilde{\sigma}(t_{\text{ret}}) \sinh |Y|}{2} \sqrt{\Delta}\right] e^{\tilde{A}},$$

$$e\mathbf{B}_P^\pm(t, \mathbf{r}) = \alpha_{\text{EM}} \int_{V_\pm} d^3\mathbf{r}' \int_{-Y_0}^{Y_0} dY \frac{\Psi_\pm(Y) \sinh Y \cdot \rho_\pm(\mathbf{r}') \tilde{\Theta}_\mp(\mathbf{r}'_\perp) \mathbf{e}_z \times \mathbf{R}_\pm}{\Delta^{3/2}} \\ \times \left[1 + \frac{\tilde{\sigma}(t_{\text{ret}}) \sinh |Y|}{2} \sqrt{\Delta}\right] e^{\tilde{A}}.$$

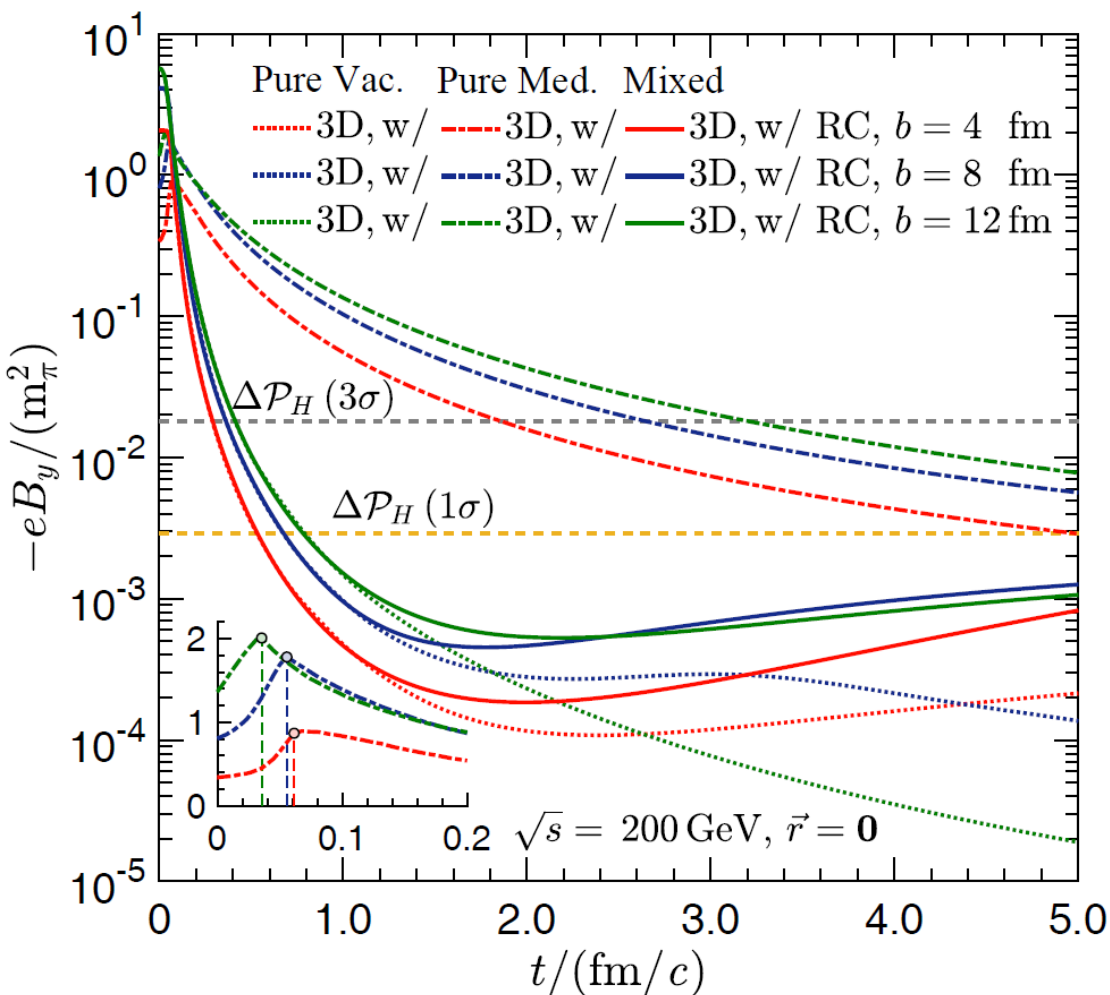


H. Li, X.-L. Sheng & Q. Wang PRC (2017).

YC, X.-L. Sheng & G.-L. Ma, NPA 1011 (2021) 122199.

H. Li, X. Xia, X.-G. Huang & H.-Z. Huang (2023)

Results from the extended KMW model



$$e|B| = \frac{eT_s |\Delta \mathcal{P}_H|}{2|\mu_\Lambda|}$$

$$\Delta \mathcal{P} = \mathcal{P}_\Lambda - \mathcal{P}_{\bar{\Lambda}}$$

$T_s \sim$ Thermal freeze-out temperature (~ 150 MeV)

$$\mu_\Lambda = -\mu_{\bar{\Lambda}} = -0.613\mu_N$$

↓ **STAR Coll.**

$1\sigma, 3\sigma$ constraints
from Λ and $\bar{\Lambda}$ polarizations

$t_s \sim$ hydrodynamic thermal freeze-out time.

$$t_s \simeq 5 \text{ fm}/c \text{ (Hydro)}$$

$$\sigma_0 \approx 5.8 \text{ MeV (LQCD)}$$

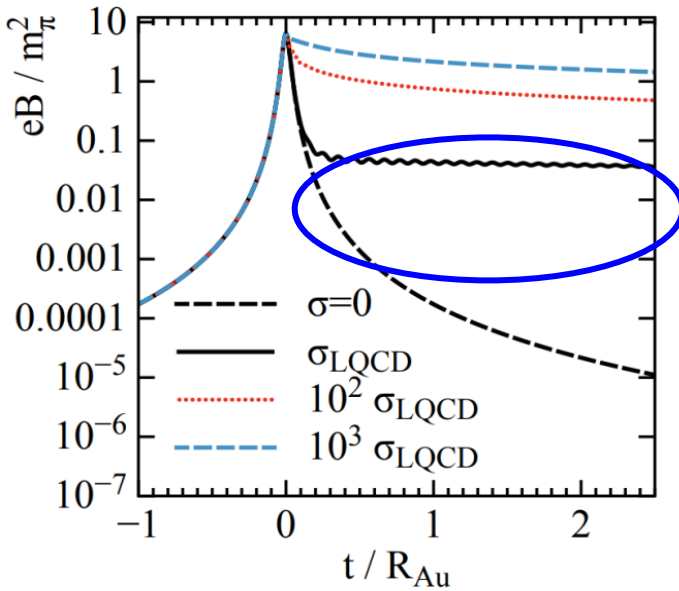
Becattini, Karpenko, Lisa, Upsal, & Voloshin, PRC (2017).

STAR paper, PRC 98, 014910 (2018).

Müller & Schäfer, PRD 98, 071902(R) (2018).

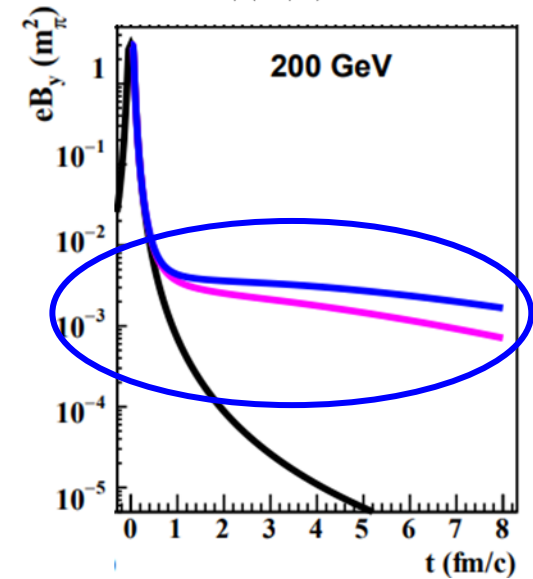
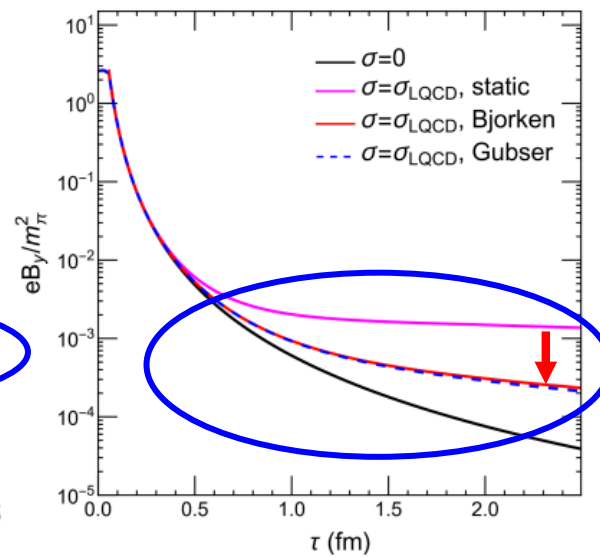
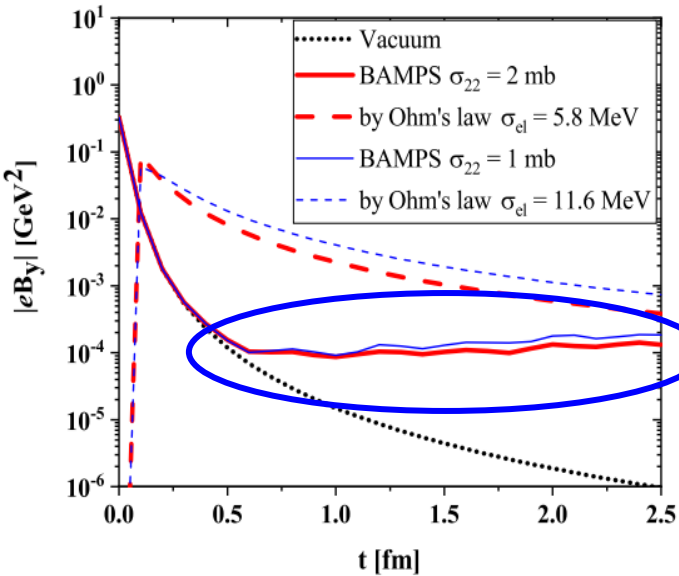
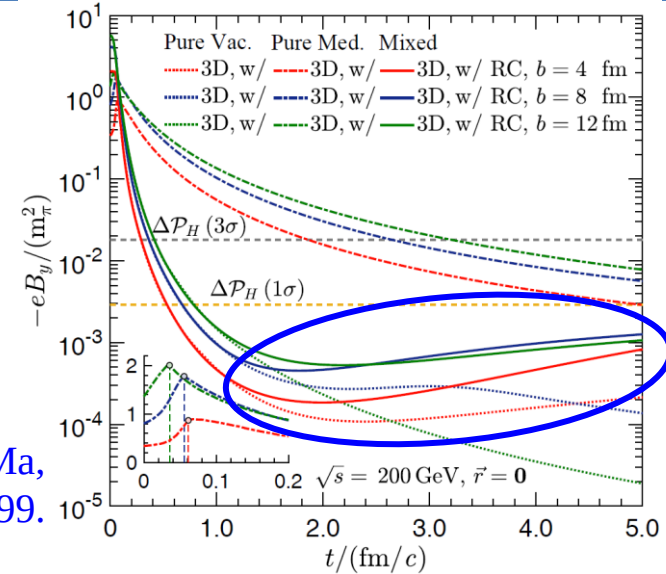
Guo, Shi, Feng & Liao, PLB 798 (2019) 134929.

Time evolution of the strong magnetic field



McLerran-Skokov, NPA
929 (2014) 184-190.

YC, X.-L. Sheng & G.-L. Ma,
NPA 1011 (2021) 122199.

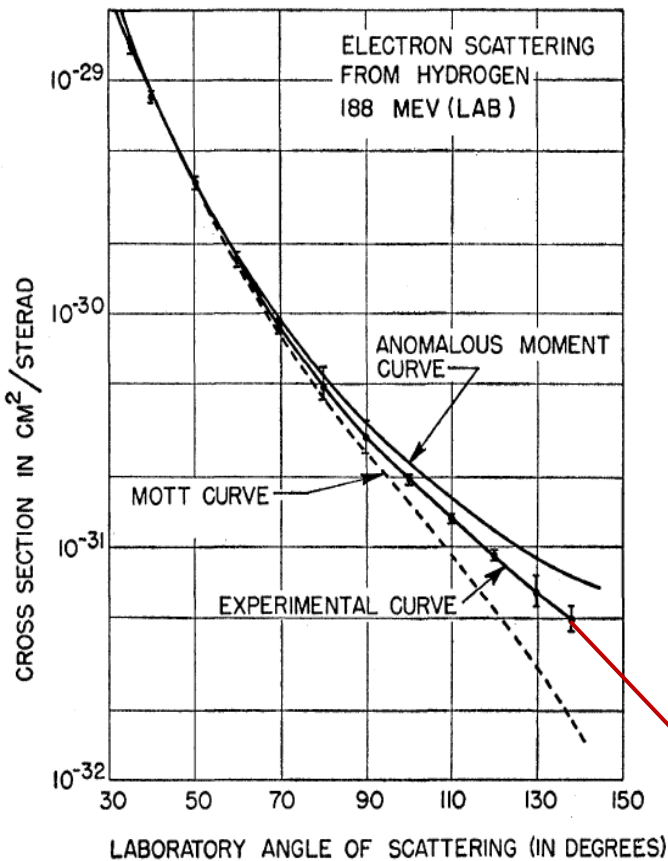


Wang, Zhao, Greiner, Xu & Zhuang,
PRC (2022)

Huang, She, Shi, Huang & Liao
PRC (2023)

Li, Xia, Huang & Huang (2023)

Elastic e-N scattering



Nobel prize in physics (1961)



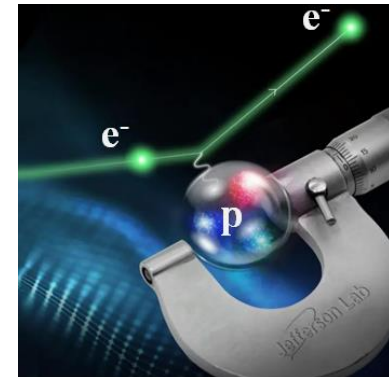
Robert Hofstadter



Rudolf Mössbauer

Nucleon is not a point-like particle! It has clear meaningful structures, known as EM form factors.

$$r_E^p \approx 0.78 \text{ fm}$$



Robert Hofstadter and Robert W. Mcallister. "Electron Scattering from the Proton" Phys. Rev. 98, 217 (1955).

W. Xiong et al. (PRad), Nature (London) 575, 147 (2019).

$$r_E^p \approx 0.831 \text{ fm}$$

Nucleon relativistic EM four-current distributions

* Increasing spin will increase the complexity of a system.

See, Prof. Yang Li's talk on July 19.

● BF distributions (3D):

$$\frac{1}{\sqrt{1+\tau}} \approx 1 - \frac{\tau}{2} + \dots$$

Darwin-Foldy term

Foldy and Wouthuysen, PR 78, 29(1950).

$$J_B^0(\mathbf{r}) = e\delta_{s's} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta\cdot\mathbf{r}} \frac{M}{P_B^0} G_E(\Delta^2),$$

$$\mathbf{J}_B(\mathbf{r}) = e \frac{\nabla \times \boldsymbol{\sigma}_{s's}}{2M} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta\cdot\mathbf{r}} \frac{M}{P_B^0} G_M(\Delta^2)$$

● Poincaré symmetry:

[Durand, Celles & Marr, PR126 (1962) 1882]

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s's'_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu}_{\text{Boost}} \underbrace{\langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle}_{\text{BF amplitude}}$$

* Spin Wigner rotation will play a central role for any a moving spinning system.

Wigner rotation angle:

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

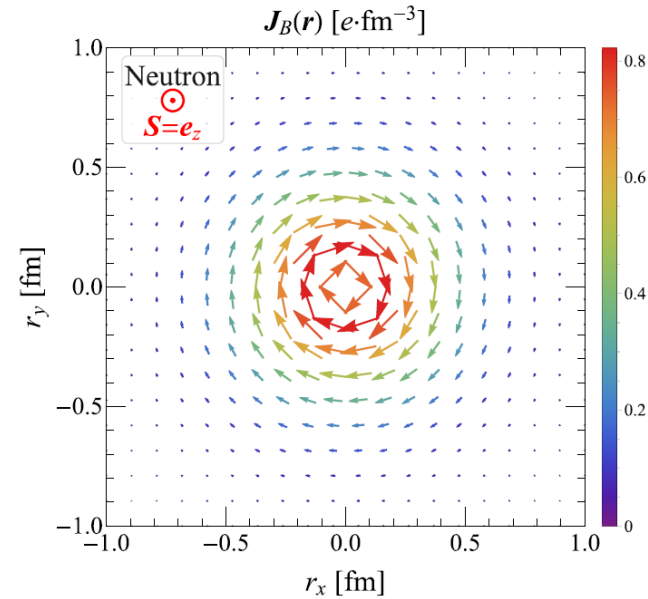
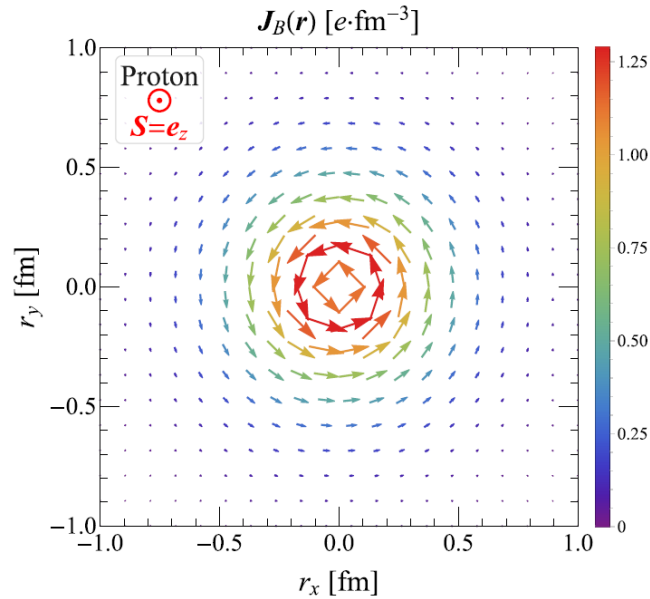
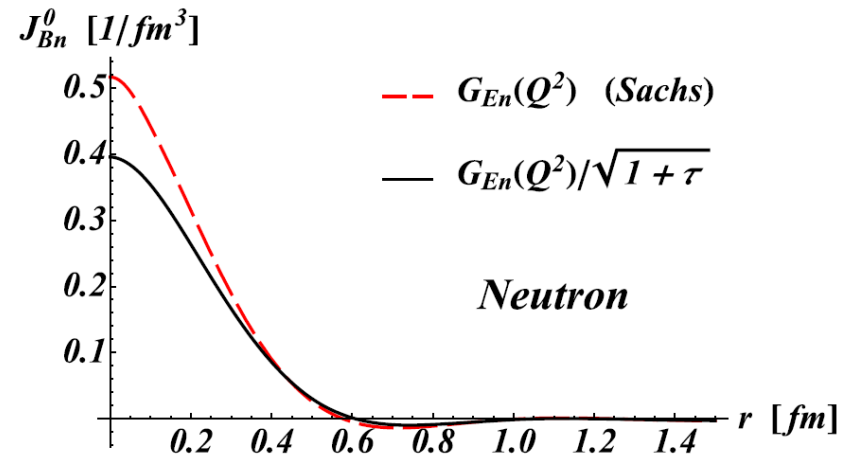
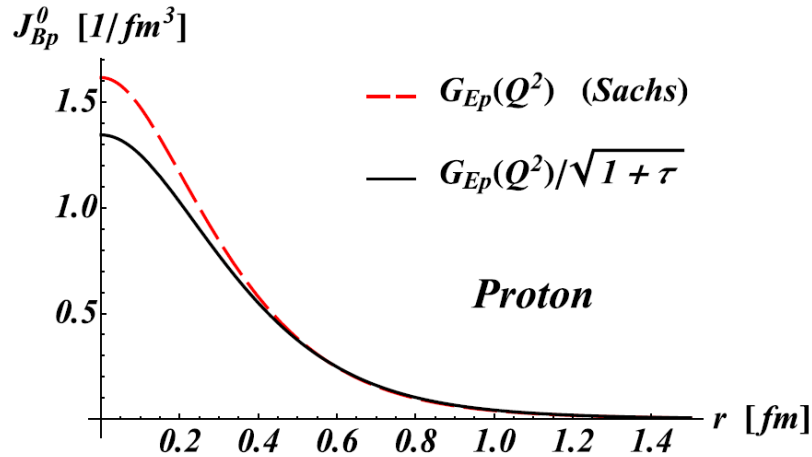
[C. Lorcé PRL125 (2020) 232002]

[YC, C. Lorcé PRD106 (2022) 116024]

[YC, C. Lorcé PRD107 (2023) 096003]

$$p^{(\prime)\mu} = \Lambda^\mu{}_\nu p_B^{(\prime)\nu}$$

Nucleon relativistic 3D EM four-current distributions



N.B.: Our results are very recently confirmed by A. Freese and G. Miller.

Using the tilted LF coordinates

[C. Lorcé PRL125 (2020) 232002]
 [YC, C. Lorcé PRD106 (2022) 116024]
 [A. Freese and G. Miller, PRD 107 (2023) 074036]

Nucleon relativistic EM four-current distributions

● EF distributions (2D):

$$J_{\text{EF}}^0(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\ + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$J_{z,\text{EF}}(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\ + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$\mathbf{J}_{\perp,\text{EF}}(\mathbf{b}_\perp; P_z) = e(\sigma_z)_{s's} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{(\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp}{2P^0} G_M(\Delta_\perp^2)$$

free of spin Wigner rotation

suppressed by Lorentz contraction

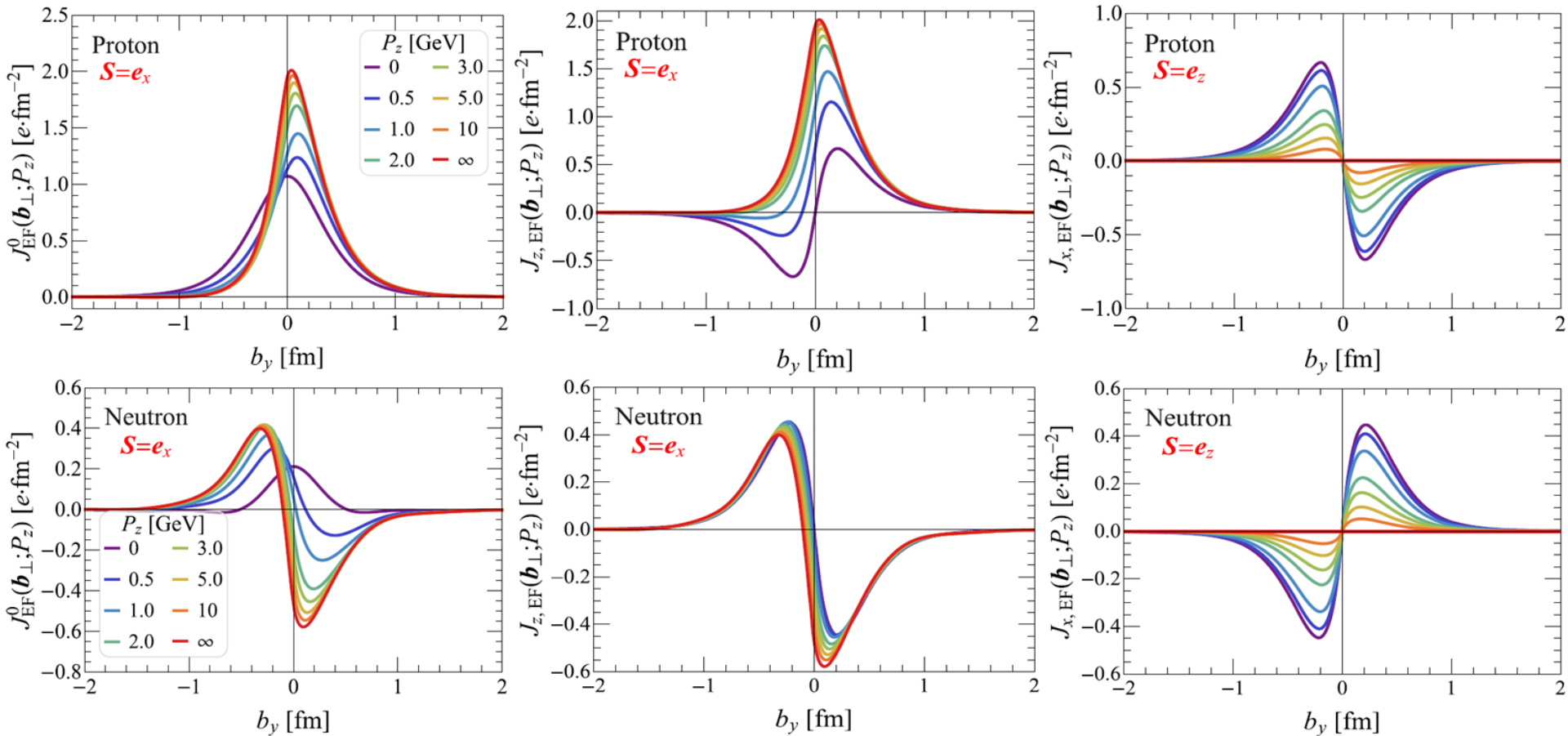
➡ Spin Wigner rotation indeed plays a central role for a moving spinning target.

[C. Lorcé PRL125 (2020) 232002]

[YC, C. Lorcé PRD106 (2022) 116024]

[YC, C. Lorcé PRD107 (2023) 096003]

Nucleon relativistic 2D EM four-current distributions



➡ Relativistic spatial distributions are in general frame-dependent. The deformations from axial symmetry (J^0) or anti-symmetry ($J^3=J_z$) is mainly due to **spin Wigner rotation**.

In particular, the 2D LF charge ("+"-component) distribution is reproduced in the IMF:

$$J_{LF}^+(\mathbf{b}_\perp; P^+) = J_{EF}^0(\mathbf{b}_\perp; \infty) = J_{z,EF}(\mathbf{b}_\perp; \infty)$$

Summary & Outlook

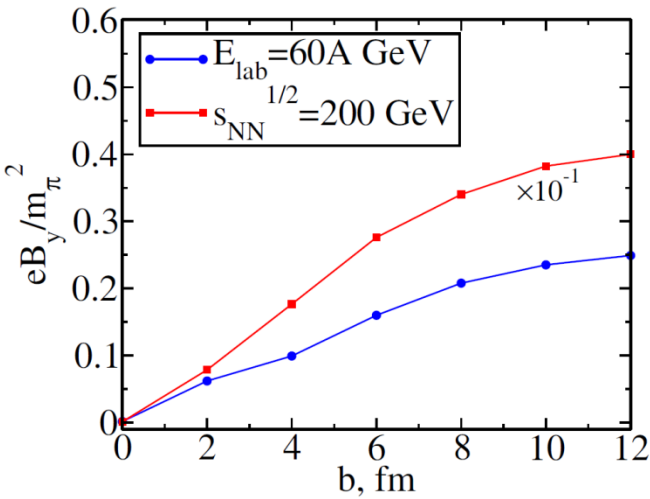
1. Based on the original KMW model (**which has inspired us a lot**), we generalized the formulation for the estimation of strong EM fields → **EKMW model**. Our results on the time evolution of magnetic field are confirmed recently by many research works.
2. Relativistic distortions of EM four-current distribution arising for any a spinning target with non-vanishing average momenta can be understood as a combination of Lorentz boosts and Wigner spin rotations. The latter are very essential and significant!

**Thank you very much
for your attention!**

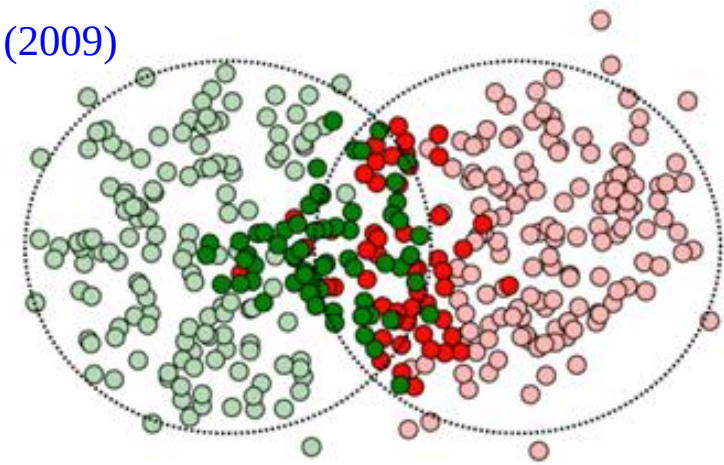
Kharzeev, McLerran & Warringa, NPA 803 (2008) 227-253;
Fukushima, Kharzeev & Warringa, PRD 78, 074033 (2008);
Fukushima, Kharzeev & Warringa, NPA 836, 311 (2010);
Kharzeev, Ann. Phys. 353, 205 (2010);
Kharzeev, Prog.Part. Nucl. Phys. 75, 133 (2014);.....

**Thanks to D. E. Kharzeev
for his brilliant ideas and
very illuminating papers!**

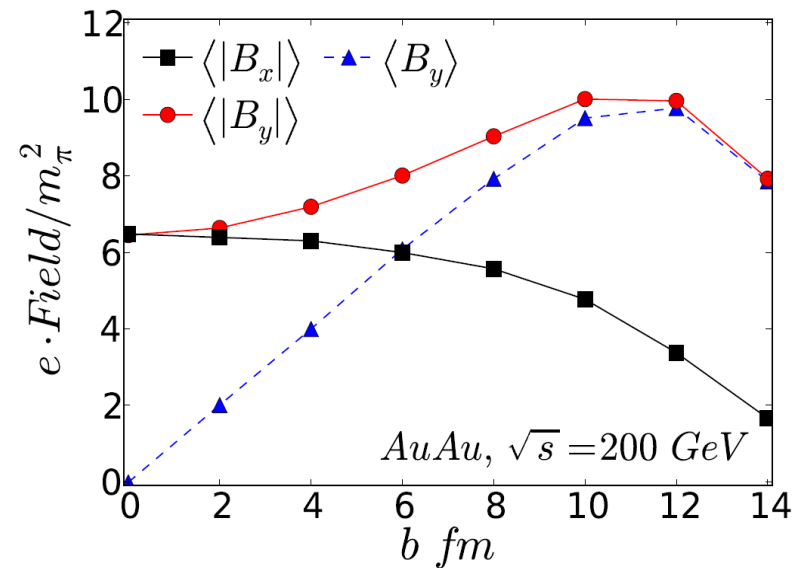
Centrality dependence of the strong magnetic field



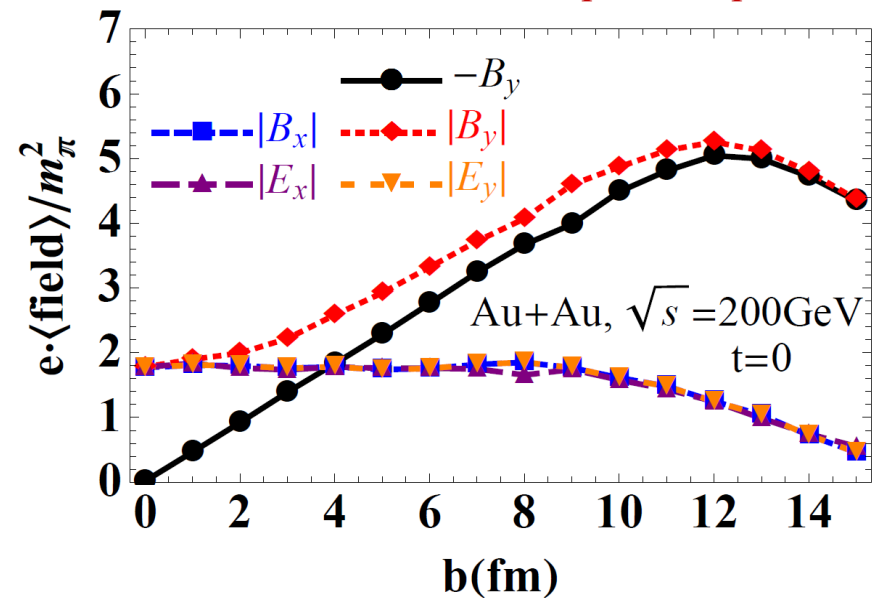
Skokov, Illarionov & Toneev (2009)



Fluctuations of protons' positions



Bzdak & Skokov (2012)



W.-T. Deng & X.-G. Huang (2012)