

立德樹人 求實創新



Holography QCD versus QCD data

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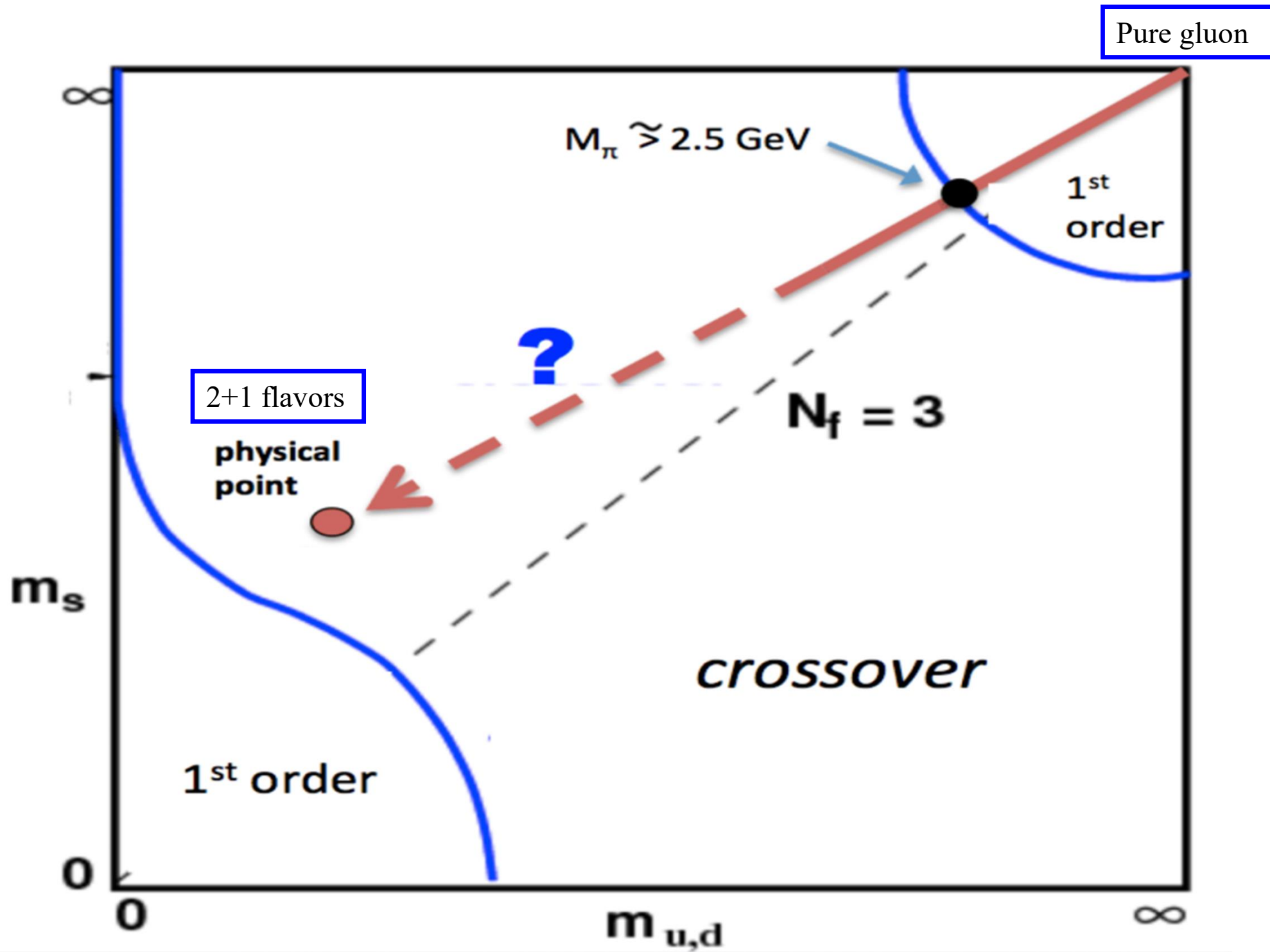
@[The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions](#)

[w/. Rong-Gen Cai, Li Li, Yuan-Xu Wang, Phys.Rev.D 106 \(2022\) 12, L121902,](#)
[w/. Li Li, Zhibin Li, Shao-Jiang Wang, 2210.14094,](#)
[w/. Li Li, Zhibin Li, Jingmin Li, 2305.13874,](#)
[& Working in progress.](#)

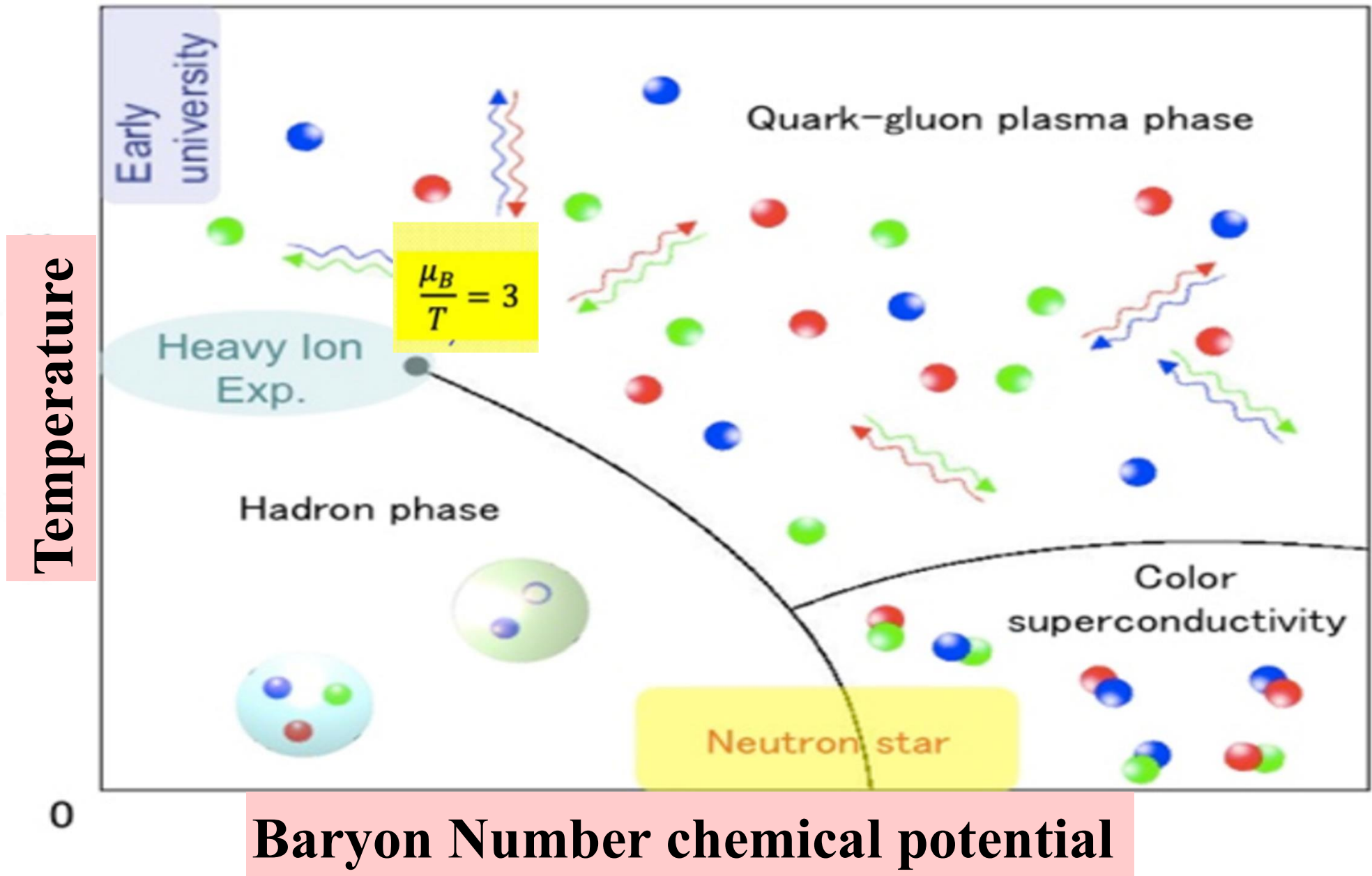
Outline

- I. Motivations & hQCD**
- II. Holographic QCD model (hQCD)**
- III. Confront with QCD Phase diagram**
- IV. Holographic pure gluon model**
- V. Summary**

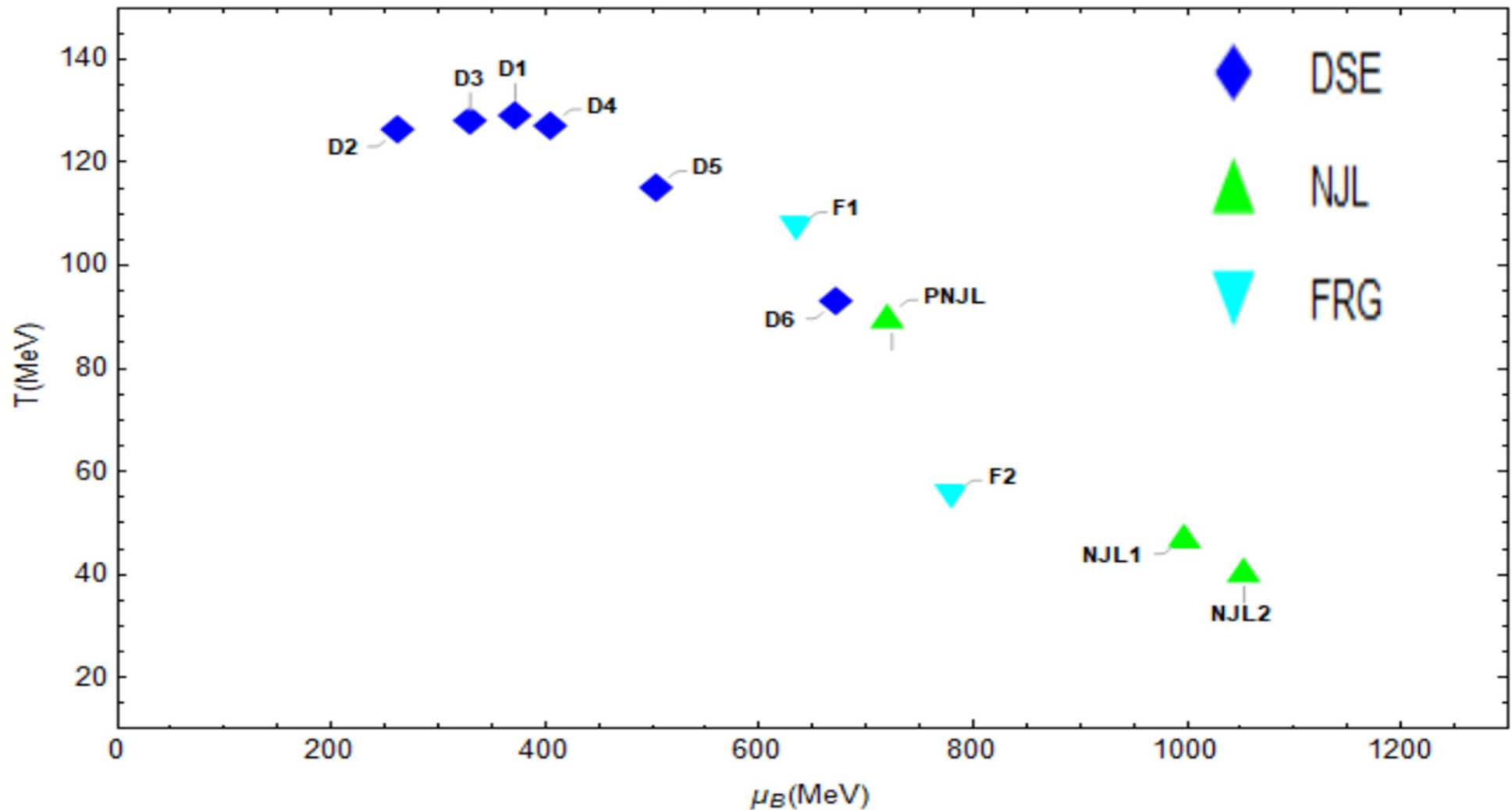
Motivations



A schematic view of QCD Phase diagram



Status of searching CEP



Schwinger–Dyson equation (DSE), 2109.09935 [hep-ph], 1607.01675 [hep-ph], 1011.2876 [nucl-th], 1403.3797 [hep-ph], 1405.4762 [hep-ph], 2002.07500 [hep-ph].

Nambu–Jona-Lasinio models (NJL, PNJL), arXiv:1801.09215 [hep-ph], Nucl. Phys. A 504 (1989), 668-684

Functional renormalization group (FRG). 1909.02991 [hep-ph], 1709.05654 [hep-ph].

Motivations

I. Low energy QCD is strong coupled system

P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. 81 (2009), 1031-1050 [arXiv:0801.4256 [hep-ph]].

S. Gupta, X. Luo, B. Mohanty, H. G. Ritter and N. Xu, Science 332 (2011), 1525-1528 [arXiv:1105.3934[hep-ph]].

II. Finite density QCD v.s. sign Problem

O. Philipsen, Prog. Part. Nucl. Phys. 70 (2013), 55-107 [arXiv:1207.5999 [hep-lat]].

III. AdS/QCD offers a practical approach

O. DeWolfe, S. S. Gubser and C. Rosen, Phys. Rev. D 84 (2011), 126014 [arXiv:1108.2029 [hep-th]].

R. G. Cai, S. He and D. Li, JHEP 03 (2012), 033 [arXiv:1201.0820 [hep-th]].

U. Gursoy, M. Jarvinen and G. Nijs, Phys. Rev. Lett. 120 (2018) no.24,242002 [arXiv:1707.00872 [hep-th]].

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, Phys. Rev. D 104 (2021) no.3, 034002 [arXiv:2102.12042[nucl-th]].

AdS/QCD

AdS/CFT conjecture

$$AdS_5 \times S^5 \longleftrightarrow N = 4 \text{ SYM theory}$$

**If it is true for any gauge theory
(???)**

**String theory,
quantum gravity**



Non-Abelian gauge theory

**Then what is the dual string theory of QCD?
(It is nature to ask the question here)**

?



QCD

Whether there is a holographic QCD model to capture the numerical simulation and phenomenon data in quantitative level?

hQCD model for $2+1-$ flavor QCD

HQCD model for 2+1 flavor system

Einstein-Maxwell-Dilaton system

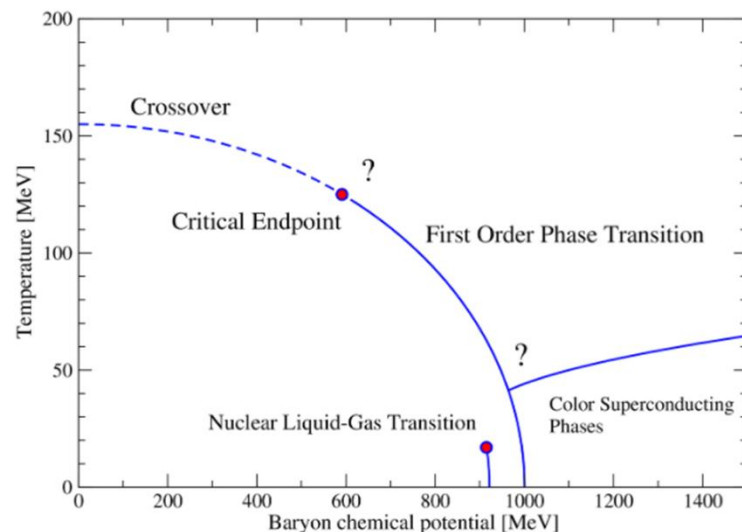
Motivation

To cover the degree of freedom in QCD phase Diagram.
Quarks (chemical potential) & gluons(dilaton potential)

Gravity Action

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \right.$$

$$\left. - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right],$$



Break conformal symmetry

Introduce baryon chemical potential

HQCD model for 2+1 flavor system

Einstein-Maxwell-Dilaton system

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right],$$

Model Parameters

[Rong-Gen Cai](#), [Song He](#), [Li Li](#), [Yuan-Xu Wang](#), [2201.02004](#)

$$V(\phi) = -12 \cosh[c_1 \phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2 \phi^6,$$

$$Z(\phi) = \frac{1}{1+c_3} \text{sech}[c_4 \phi^3] + \frac{c_3}{1+c_3} e^{-c_5 \phi},$$

Effective Newton constant

κ_N^2

Scalar source+Renormalization **b**

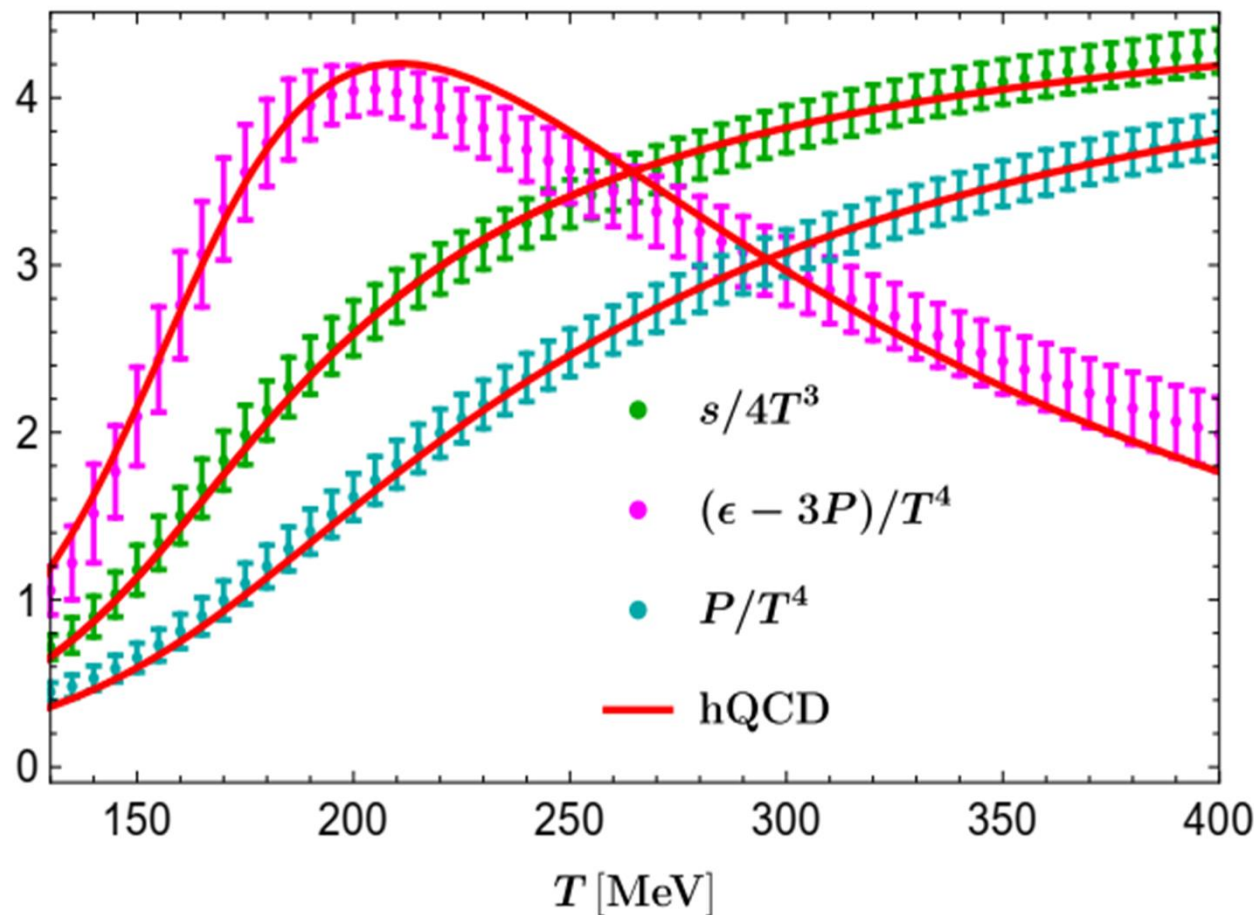
model	c_1	c_2	c_3	c_4	c_5	κ_N^2	$\phi_s(\text{GeV})$	b
pure $SU(3)$	0.735	0				$2\pi(4.88)$	1.523	-0.36458
2+1 flavor	0.710	0.0037	1.935	0.085	30	$2\pi(1.68)$	1.085	-0.27341

To fix model parameters by thermal dynamics

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

c_1, c_2, c_3, c_4, c_5 Effective Newton constant κ_N^2 Scalar source ϕ_s + Renormalization b

Equations of state at **vanishing chemical potential**, s , trace anomaly, pressure



$$T = \frac{1}{4\pi} f'(r_h) e^{-\eta(r_h)/2},$$

$$s = \frac{2\pi}{\kappa_N^2} r_h^3$$

$$\epsilon := T_{tt}$$

$$P := T_{xx}$$

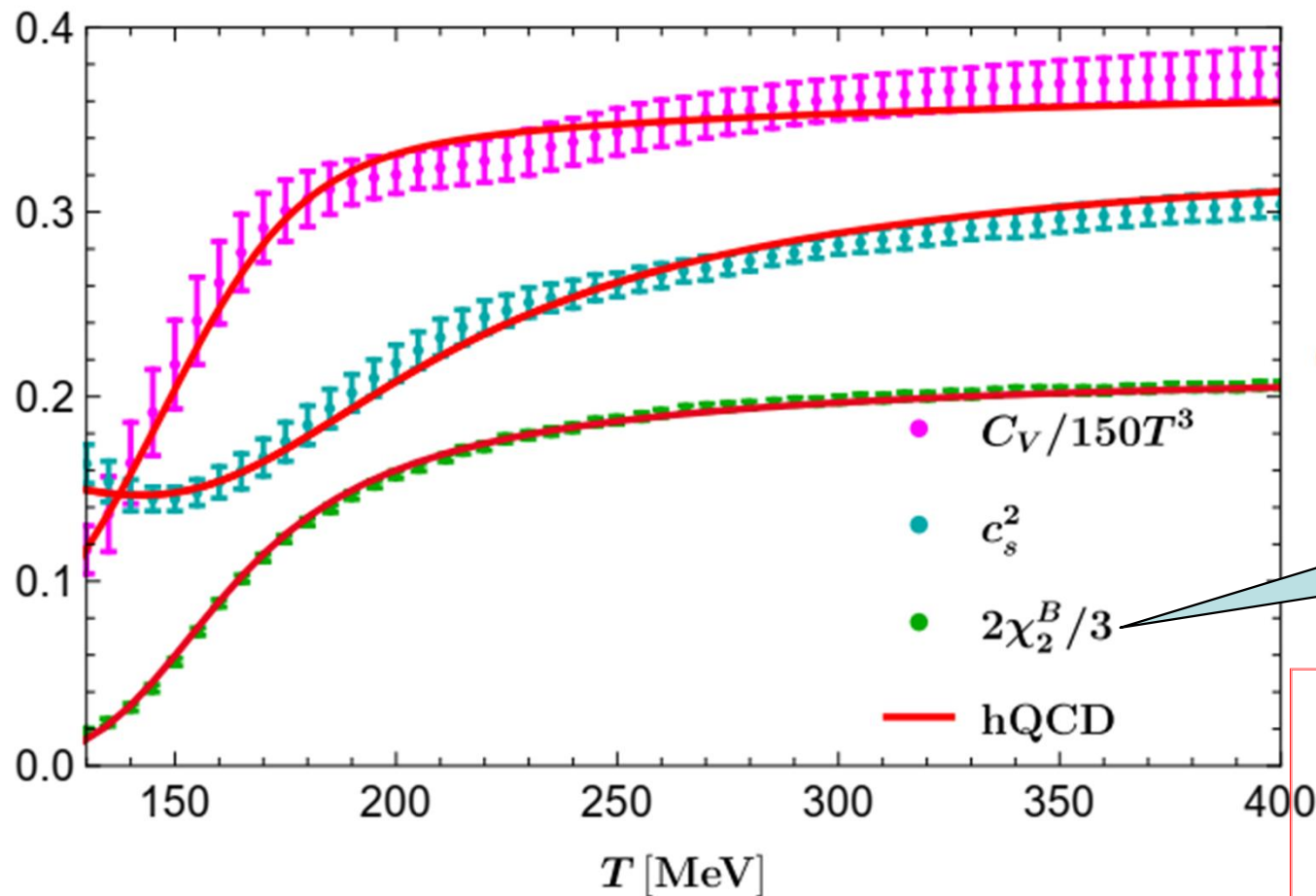
A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503[arXiv:1407.6387 [hep-lat]].

To fix model parameters by thermal dynamics

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

c_1, c_2, c_3, c_4, c_5 Effective Newton constant κ_N^2 Scalar source ϕ_s +Renormalization $\mathbf{b}$

Sound speed, specific heat, second-order baryon susceptibility **at vanishing chemical potential**



$$c_s = \sqrt{(dP/d\epsilon)_{\mu_B}}$$

$$C_V = (d\epsilon/dT)_{\mu_B}$$

$$\chi_2^B = (d\rho_B/d\mu_B)_T$$

Bazavov *et al.* [HotQCD], Phys.Rev. D 90 (2014), 094503[arXiv:1407.6387 [hep-lat]].

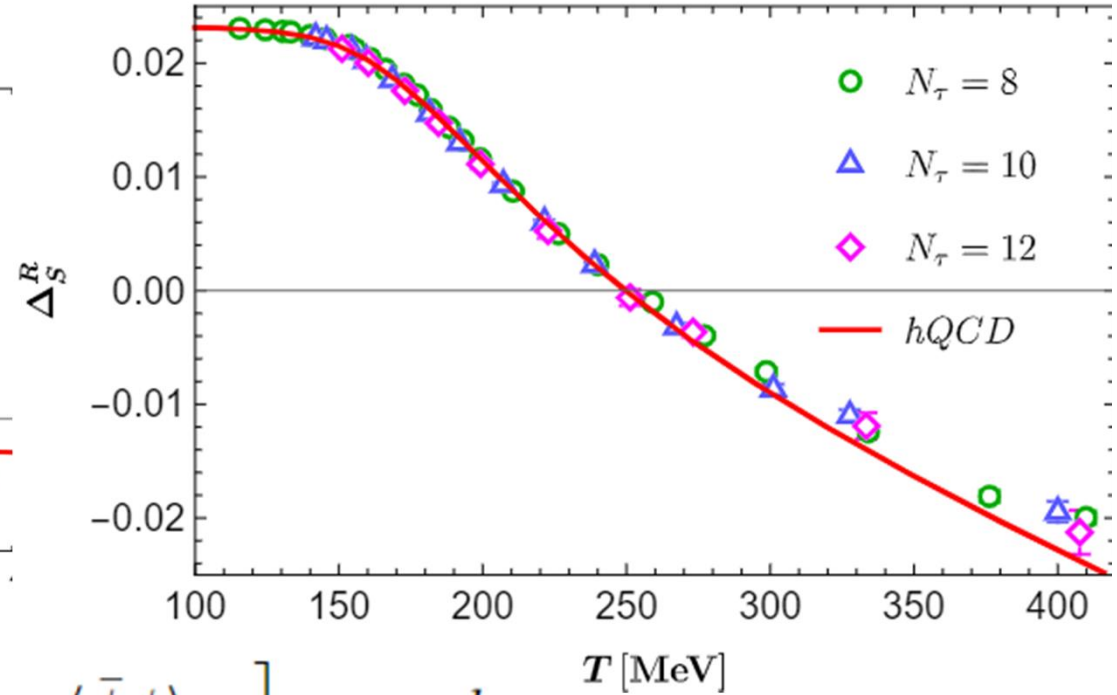
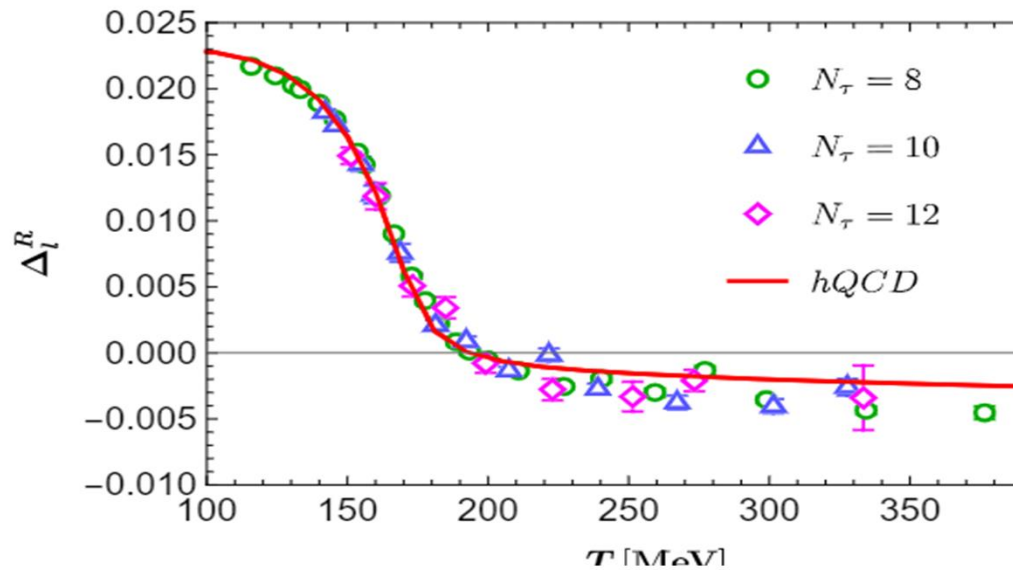
Predictions of hQCD model

Condensations, Finite Chemical Potential, CEP

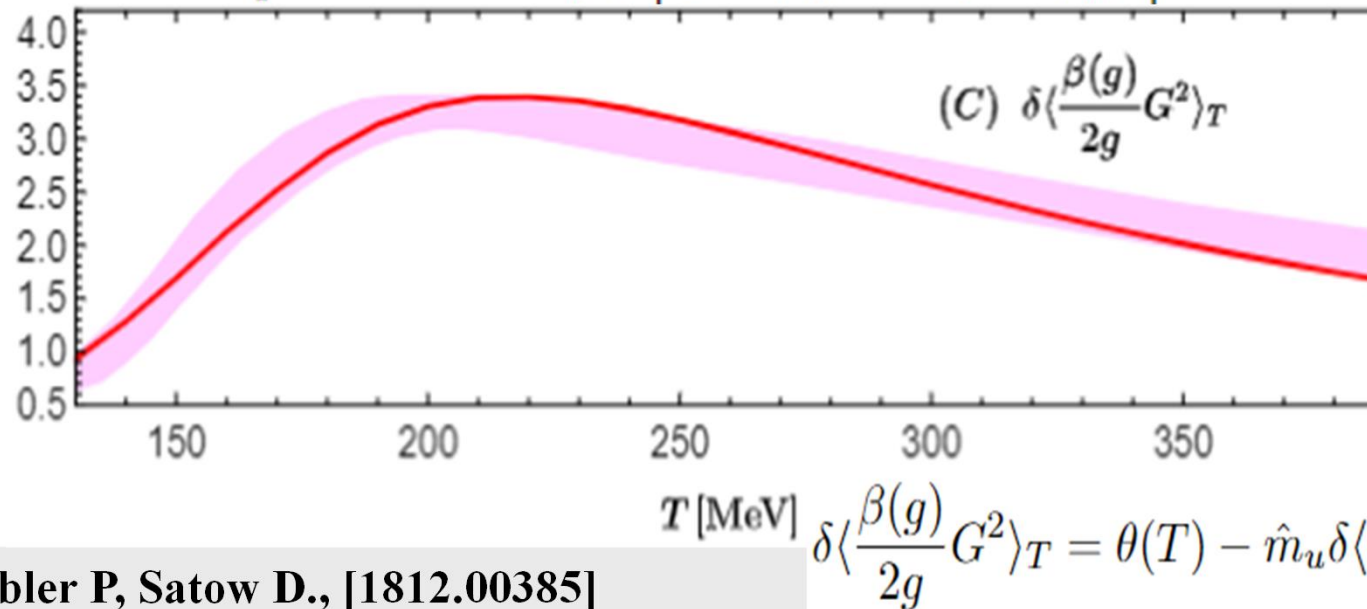
(2+1)-flavors

Condensation

$$m_u = m_d < m_s$$



$$\Delta_q^R = \hat{d} + 2m_q r_1^4 \left[\langle \bar{\psi}\psi \rangle_{q,T} - \langle \bar{\psi}\psi \rangle_{q,0} \right], \quad q = l, s,$$

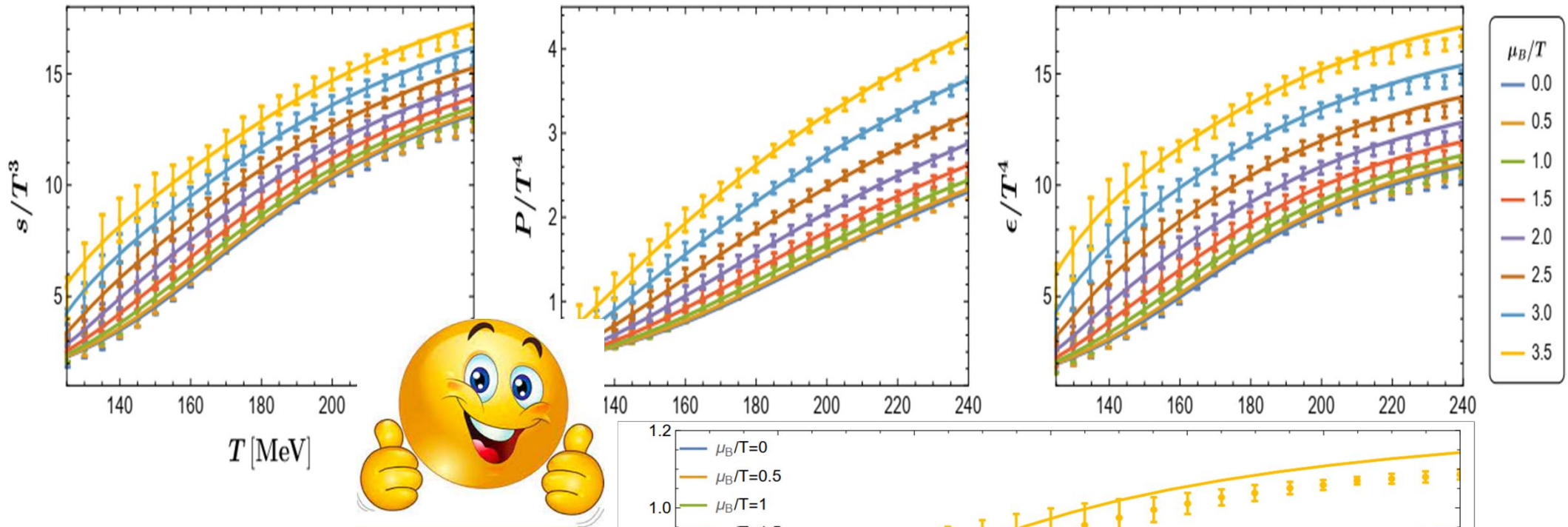


$$(C) \delta \left\langle \frac{\beta(g)}{2g} G^2 \right\rangle_T$$

$$\delta \left\langle \frac{\beta(g)}{2g} G^2 \right\rangle_T = \theta(T) - \hat{m}_u \delta \langle \bar{u}u \rangle_T - \hat{m}_d \delta \langle \bar{d}d \rangle_T - \hat{m}_s \delta \langle \bar{s}s \rangle_T$$

Predictions of thermal dynamical quantities at finite chemical potential

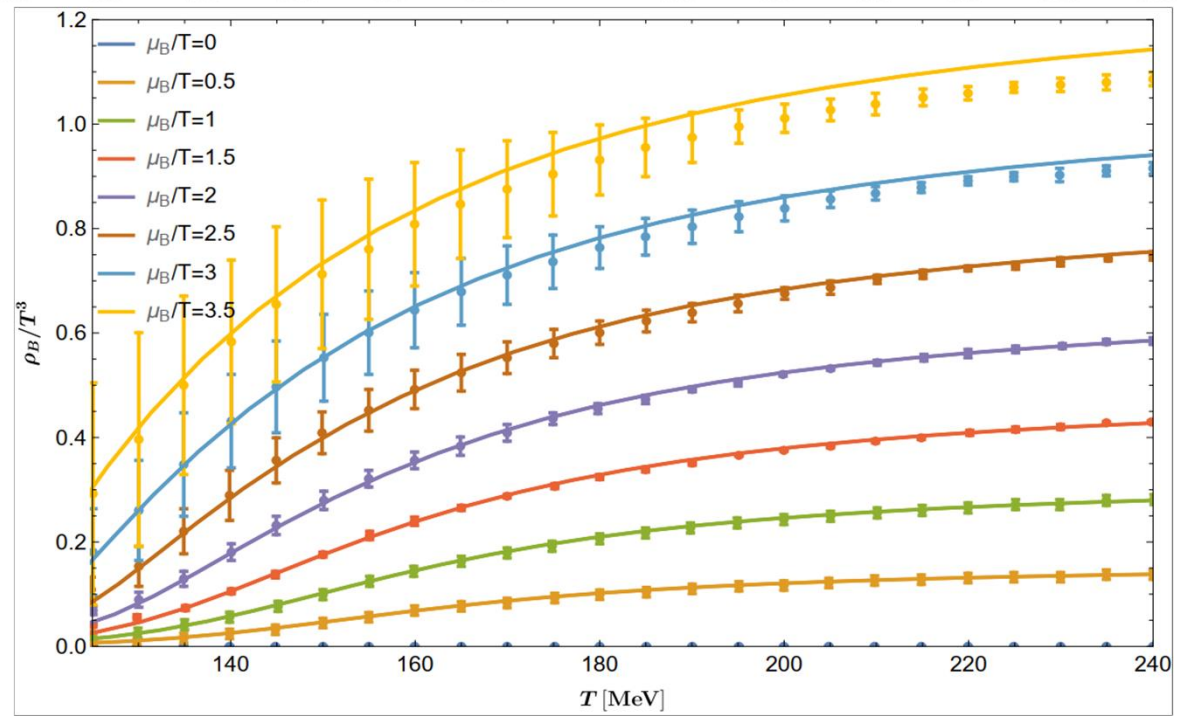
S. Bors'anyi, et al., Phys. Rev. Lett. 126
(2021) no.23, 232001
[arXiv:2102.06660 [hep-lat]]



More challenging fitting

Rho_B

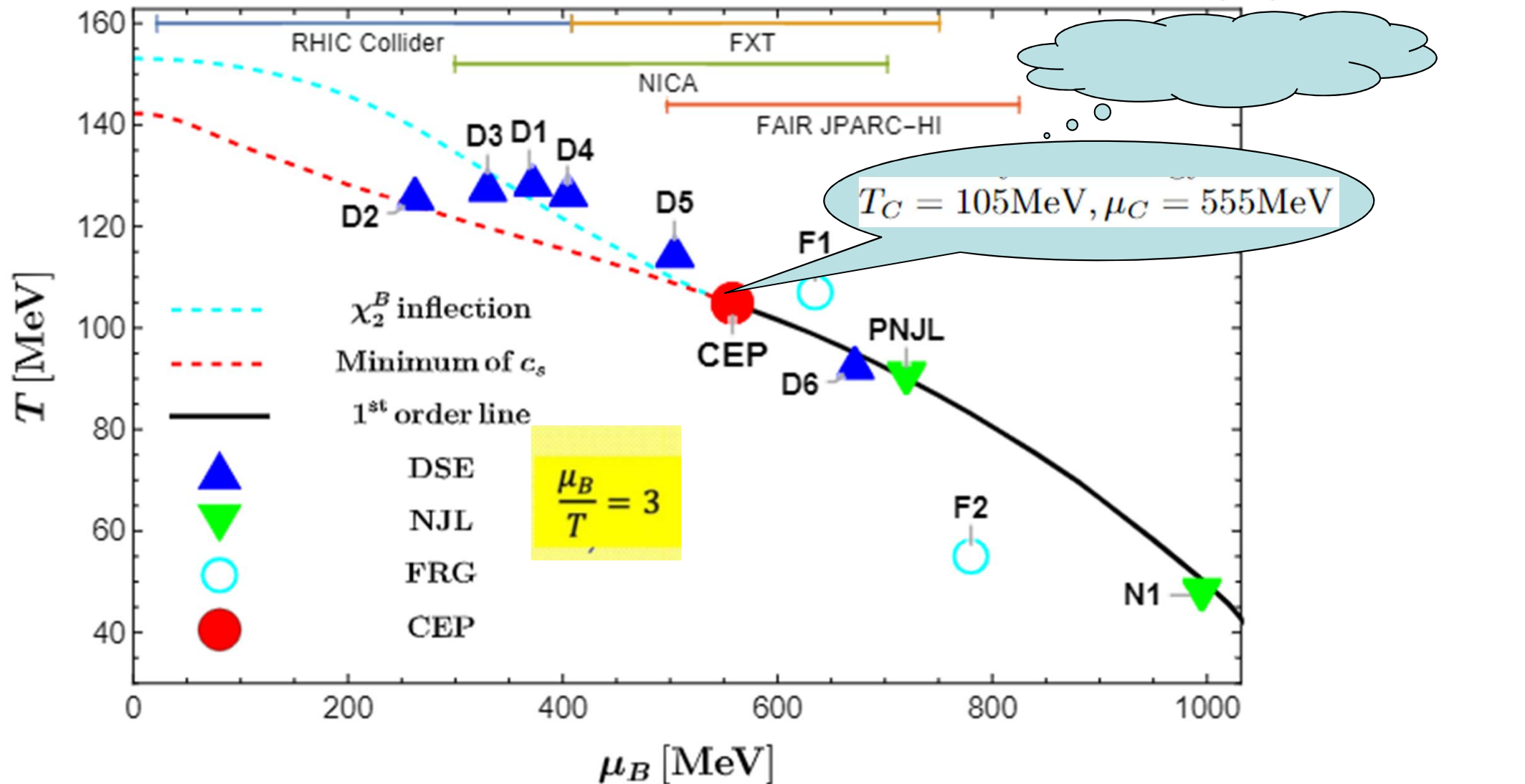
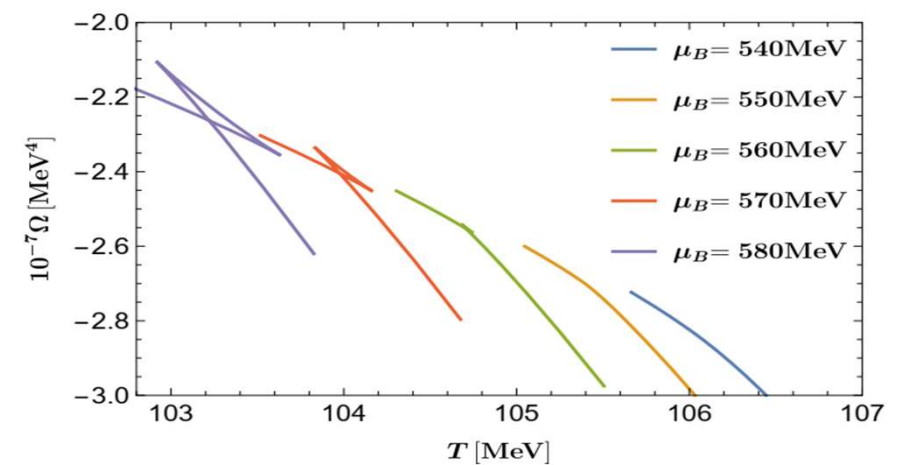
Rong-Gen Cai, Song He, Li Li,
Yuan-Xu Wang, 2201.02004



Predictions of QCD phase diagram

$\mu_B/T \leq 3$ & $\mu_B < 300$ MeV
excluded by lattice simulation

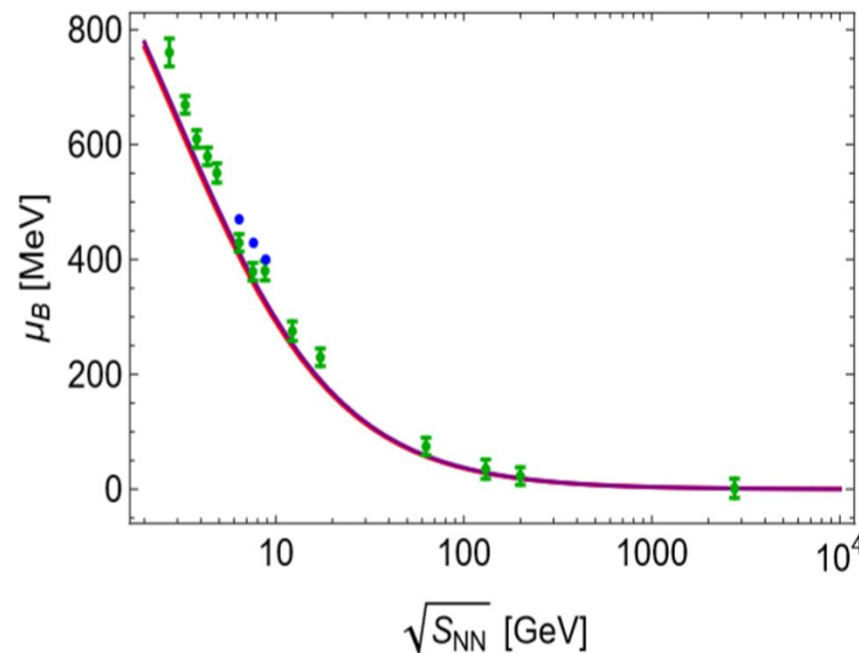
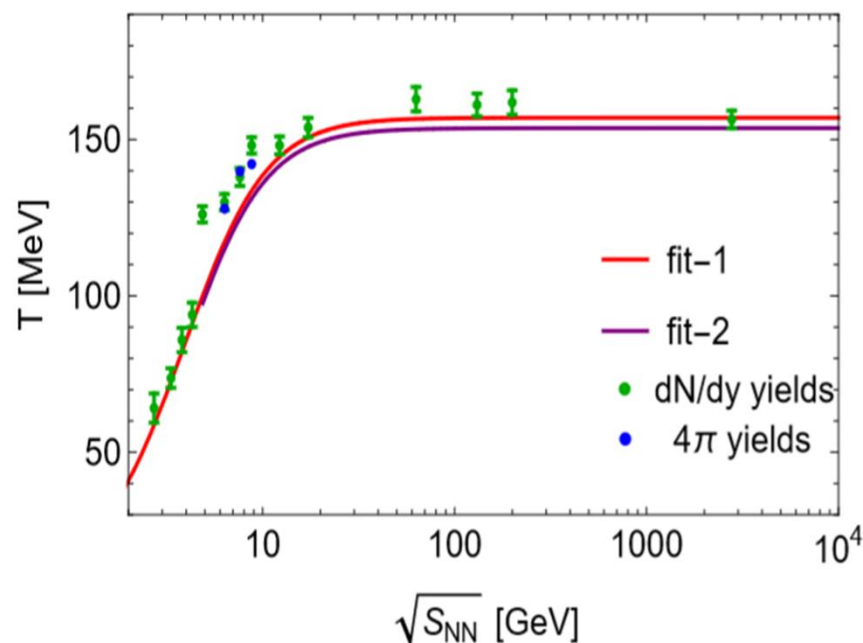
A. Bazavov, etc. Phys. Rev. D 95 (2017) no.5, 054504 [arXiv:1701.04325 [hep-lat]].



Emperical fitting formula from STAR Experimental group

w/. Li Li, Zhibin Li, Jing-Min Liang, 2305.13874

$$\mu_B = \frac{a}{1 + b\sqrt{S_{NN}}}, \quad T = \frac{T_{lim}}{1 + \exp [c - \ln (d\sqrt{S_{NN}} + e) / 0.45]},$$



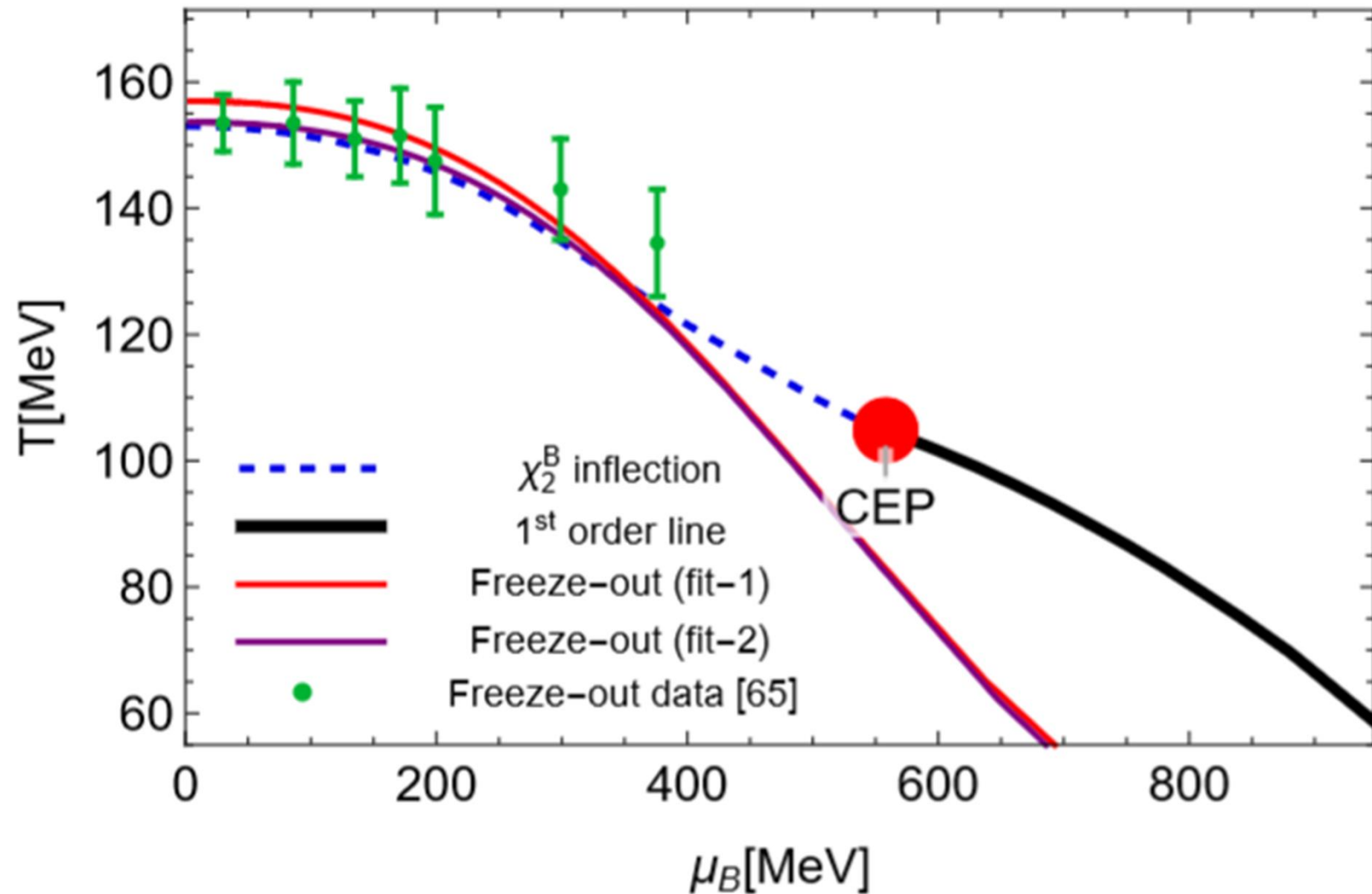
	a [MeV]	b	T_{lim} [MeV]	c	d	e
fit-1	1307.5	0.35	157.0	3.25	1	0.7
fit-2	1307.5	0.34	153.6	3.41	1.1	0.6

A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl. Phys. A 834 (2010) 237C–240C.

A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, Nature 561 no. 7723, (2018) 321–330.

Frozen line versus Phase boundary

w/. Li Li, Zhibin Li, Jing-Min Liang, 2305.13874

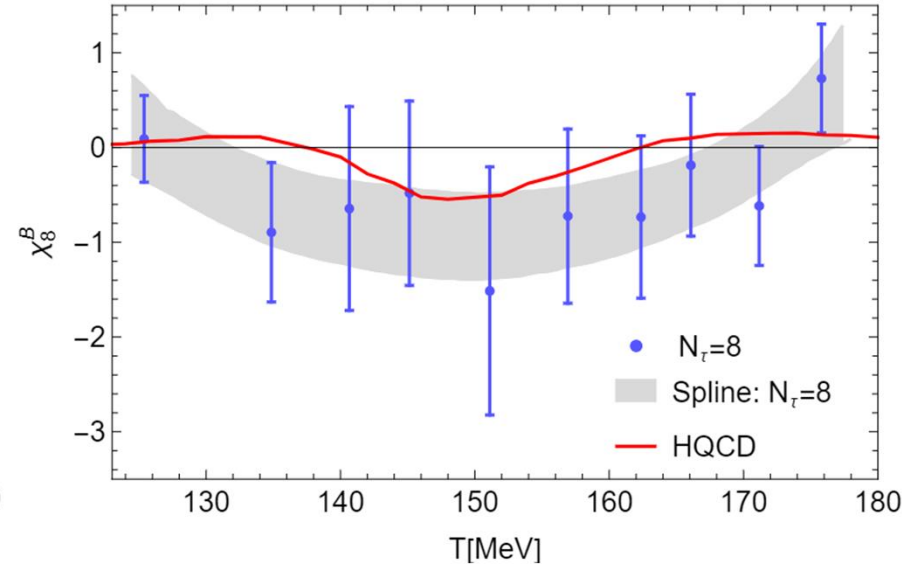
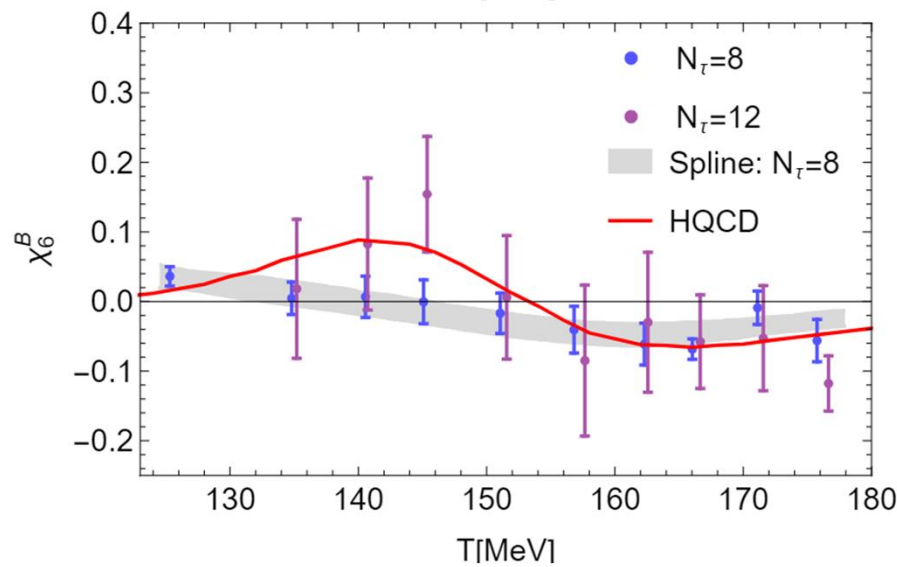
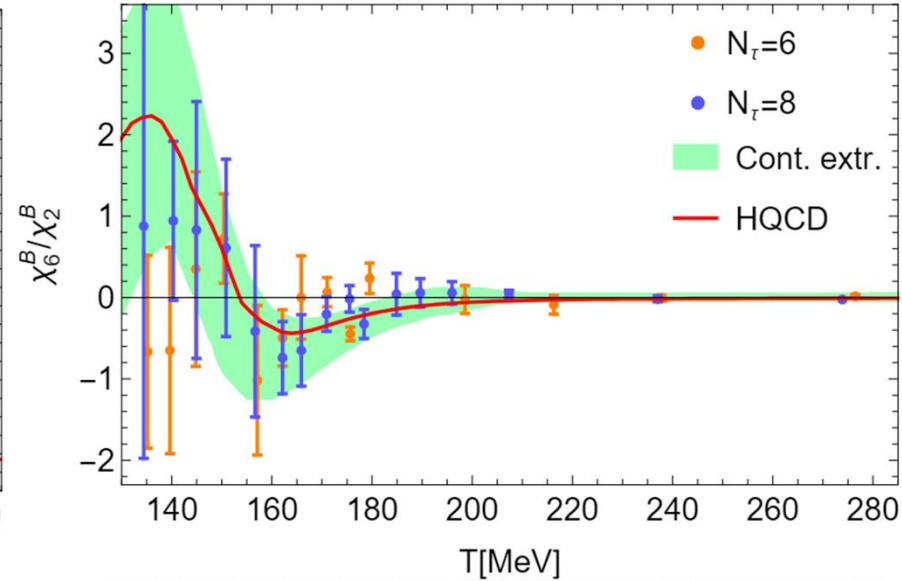
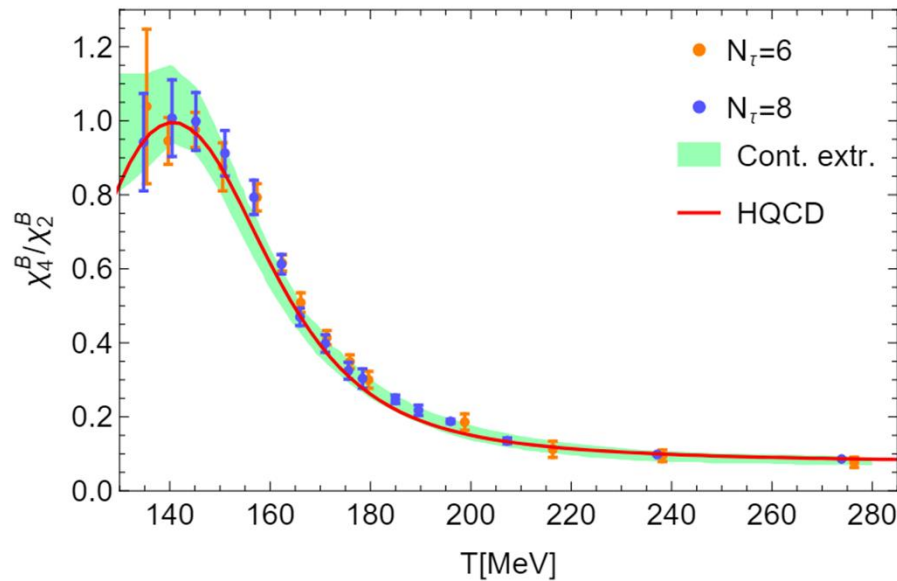


S. Gupta, D. Mallick, D. K. Mishra, B. Mohanty, and N. Xu, “Freeze-out and
A. thermalization in relativistic heavy ion collisions,” arXiv:2004.04681 [hep-ph]. 19

Higher precision checking EOS, Zero μ_B

Higher-order baryon susceptibility.

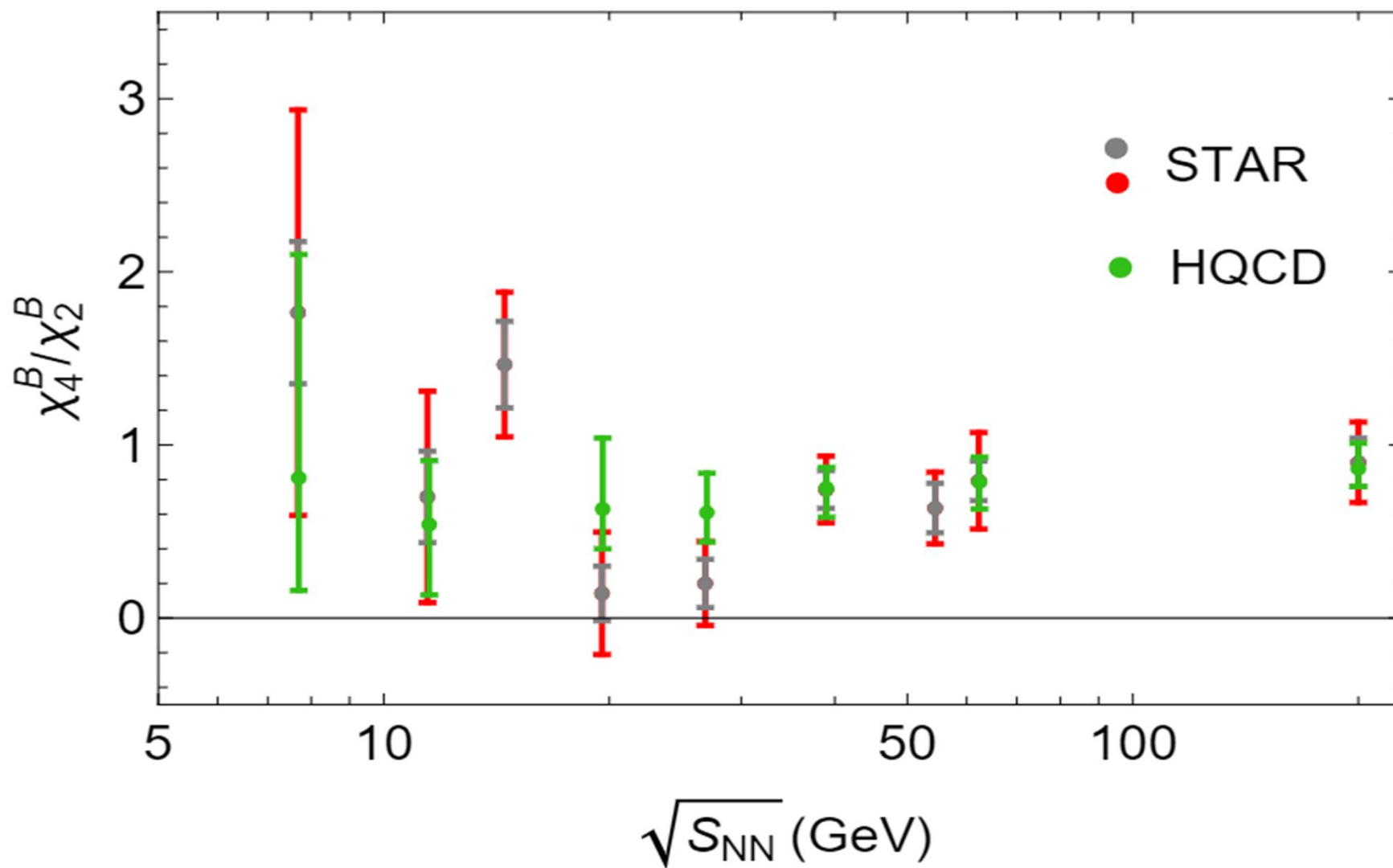
$$\chi_n^B = \frac{\partial^n P/T^4}{\partial \hat{\mu}_B^n}.$$



A. Bazavov *et al.* [HotQCD], Phys. Rev. D 95 (2017) 5, 05404 [arXiv:1701.04325[hep-lat]].

Higher precision checking EOS, finite μ_B
Higher-order baryon susceptibility.

$$\chi_n^B = \frac{\partial^n P/T^4}{\partial \hat{\mu}_B^n}.$$



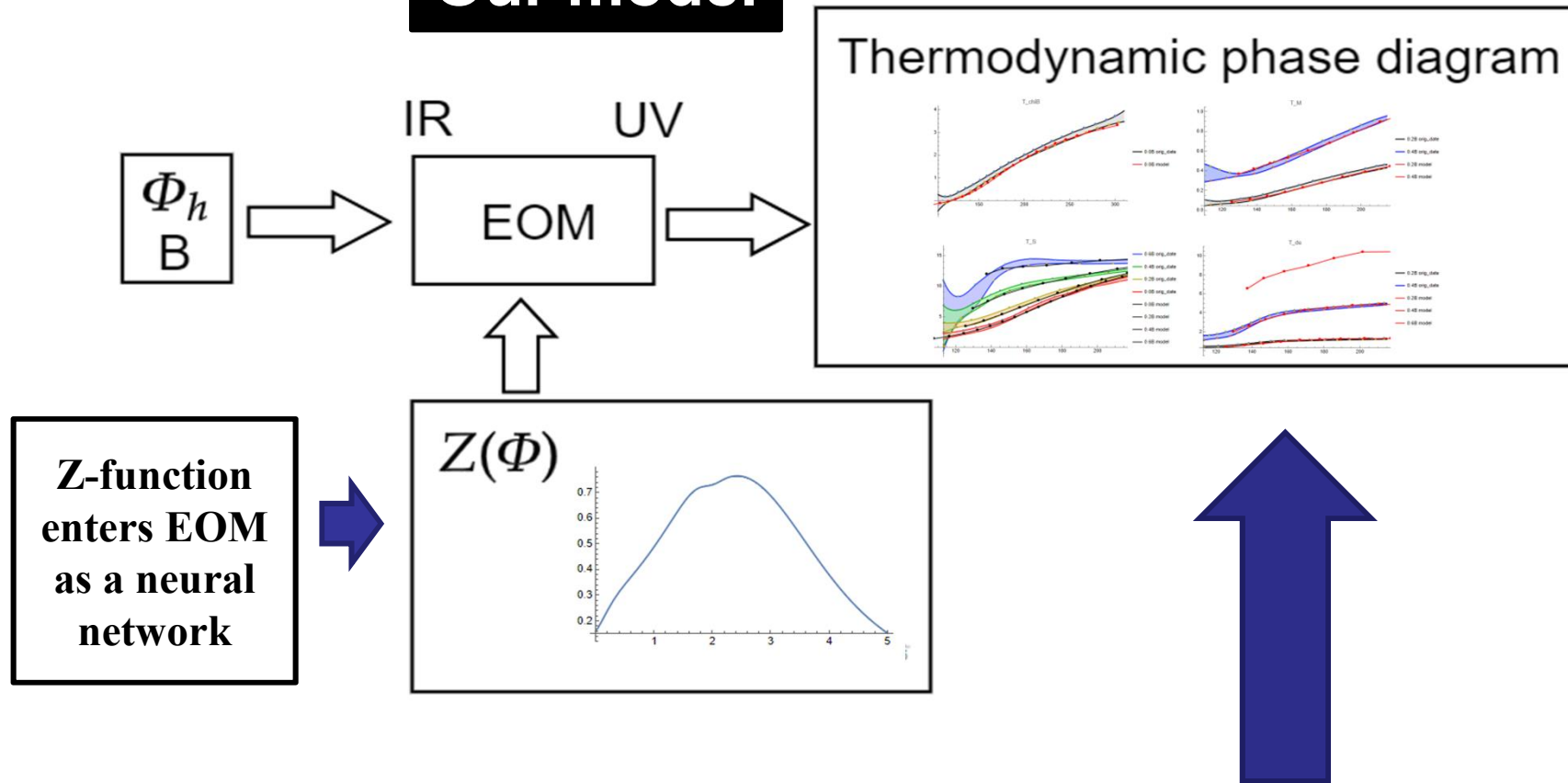
w/. Li Li, Zhibin Li, Jing-Min Liang, 2305.13874

hQCD model for 2+1 flavors with B field

Neural Network

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right],$$

Our model



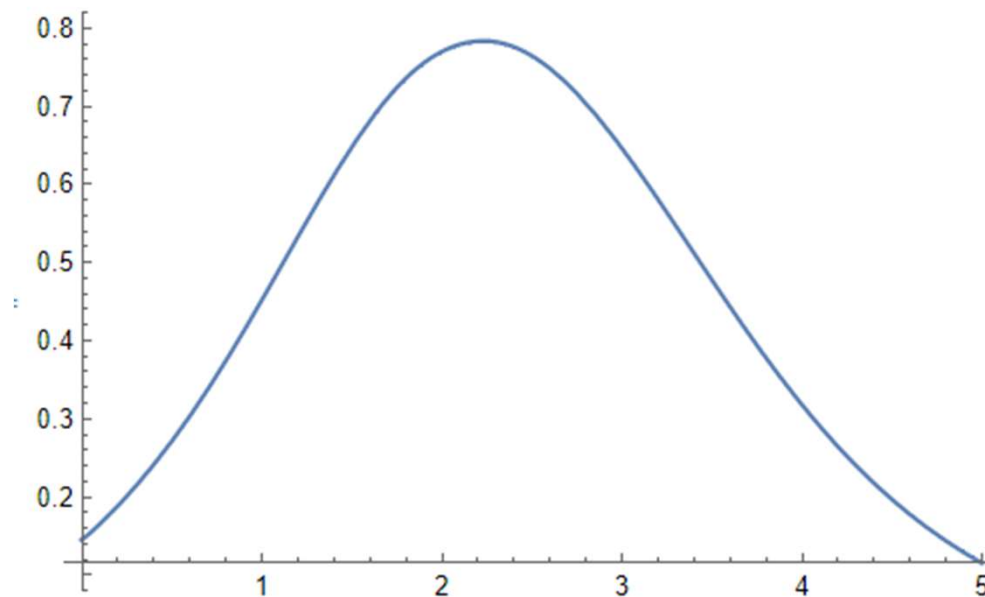
Target: To find a Z-function that matches the thermodynamic quantities of the boundary with the Lattice QCD data

Neural Network

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right],$$

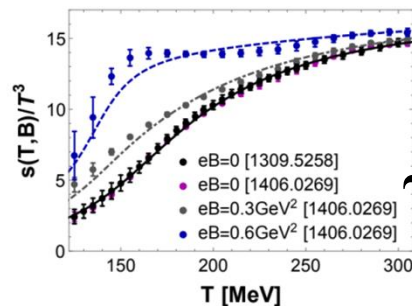
Z-function

result



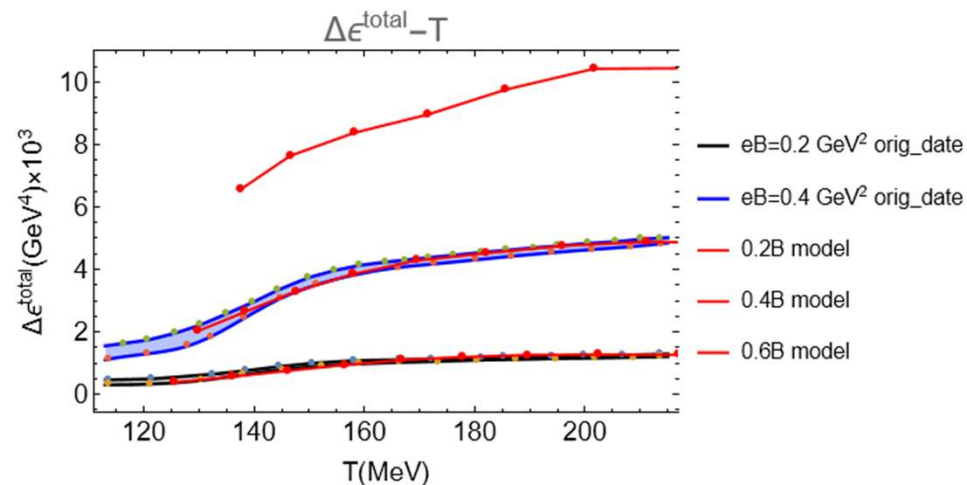
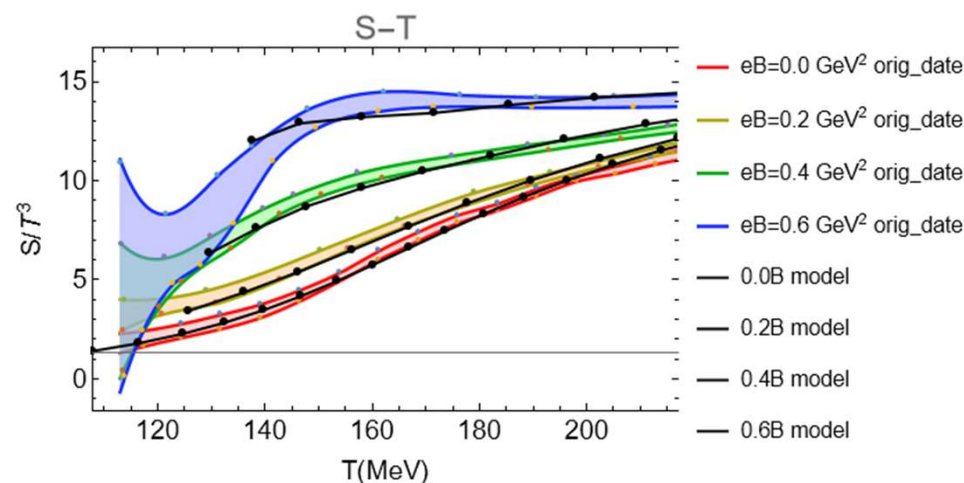
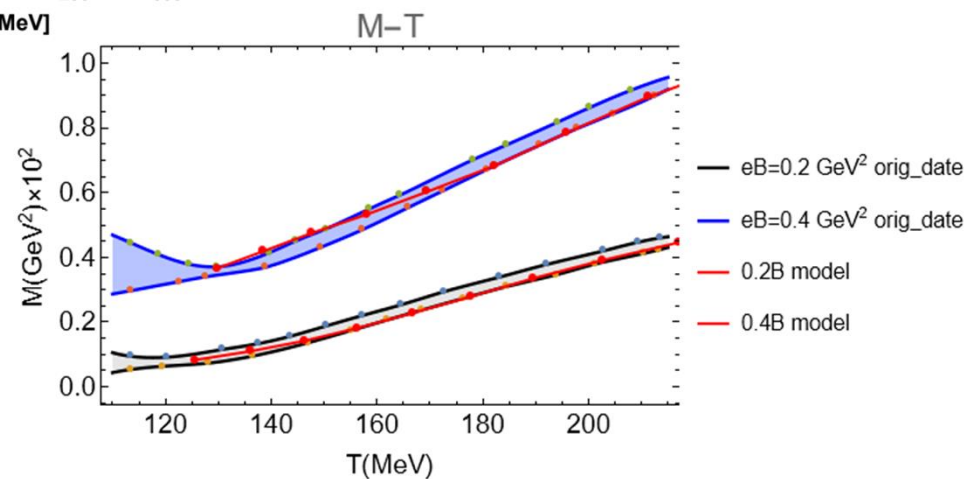
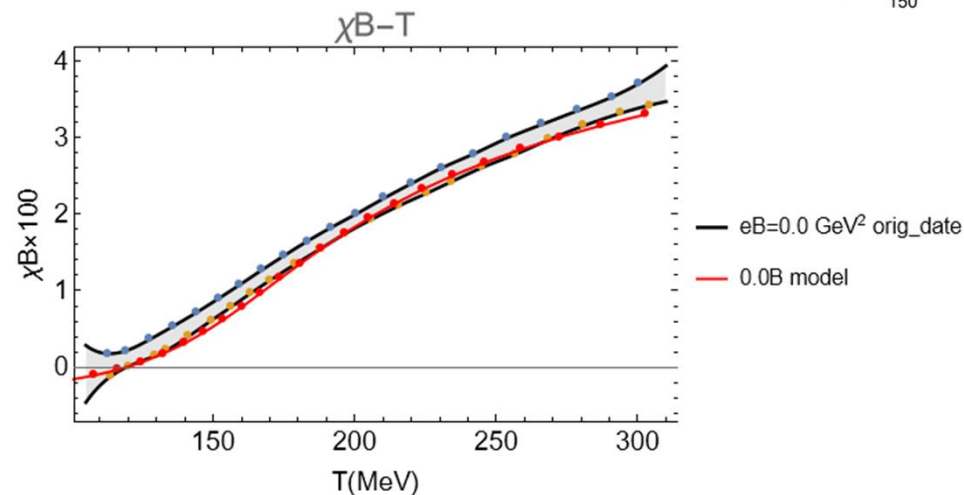
After some optimization of the neural network, the graph of the Z-function (neural network) is drawn as a graph above, and there is a small gap between the data obtained from this model and the lattice QCD data

Neural Network



2307.03885

result



Bali G S, Bruckmann F, Endrődi G, et al. The QCD equation of state in background magnetic fields[J]. Journal of High Energy Physics, 2014, 2014(8): 1-35.

w/. [Rong-Gen Cai, Li Li, Hongan Zeng](#)

hQCD model for $SU(3)$

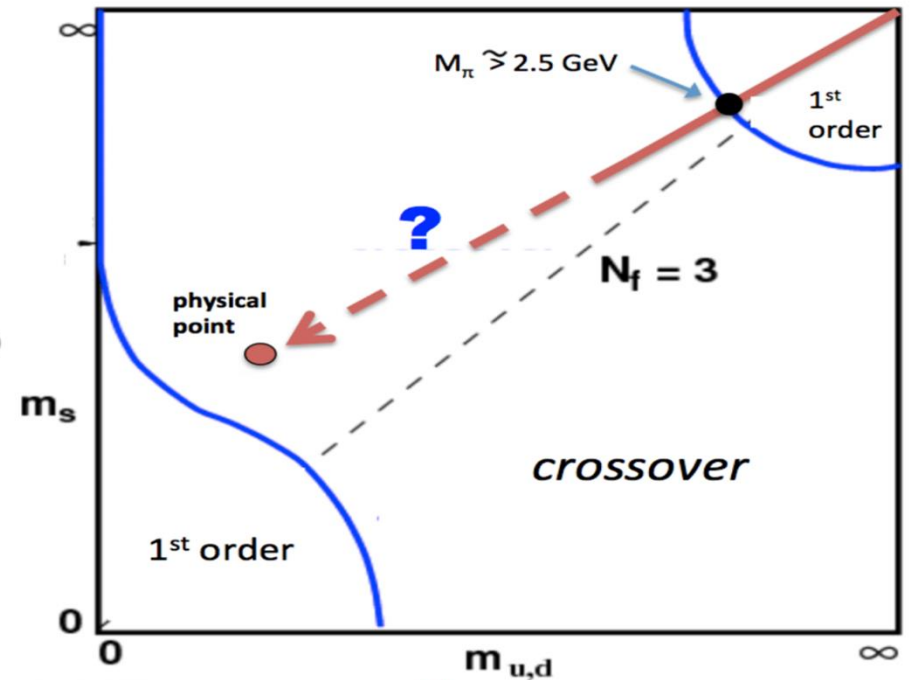
Pure gluon

HQCD model for Pure gluon syste in Einstein-Dilaton system

Motivation

[SH, Li Li, Zhibin Li, Shao-Jiang Wang, 2210.14094](#)

Gravity Action



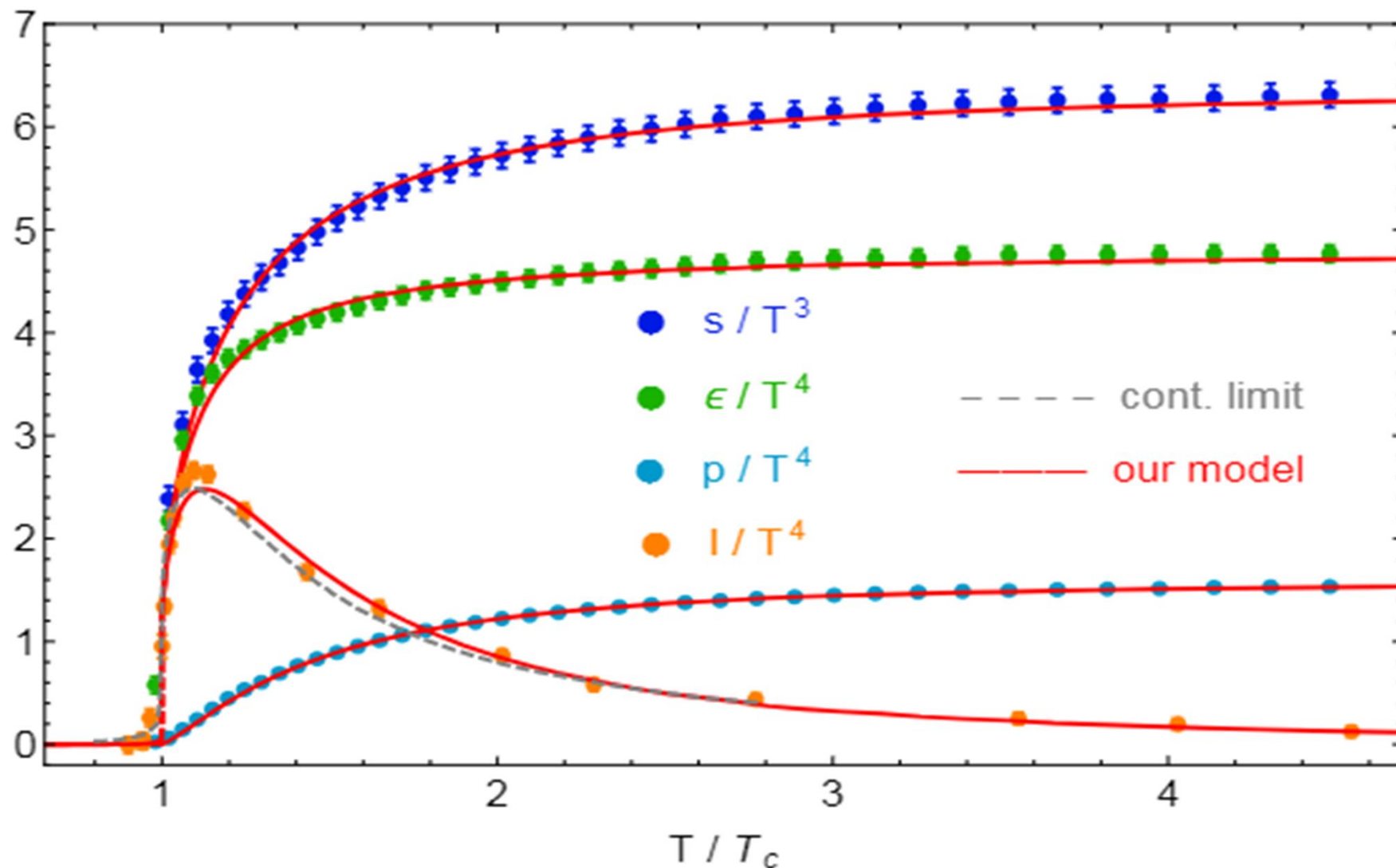
$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right]$$

$$ds^2 = -f(r)e^{-\eta(r)} dt^2 + \frac{dr^2}{f(r)} + r^2 d\mathbf{x}_3^2, \quad \phi = \phi(r),$$

$$V(\phi) = \left(6\gamma^2 - \frac{3}{2} \right) \phi^2 - 12 \cosh(\gamma\phi),$$

Break conformal
symmetry

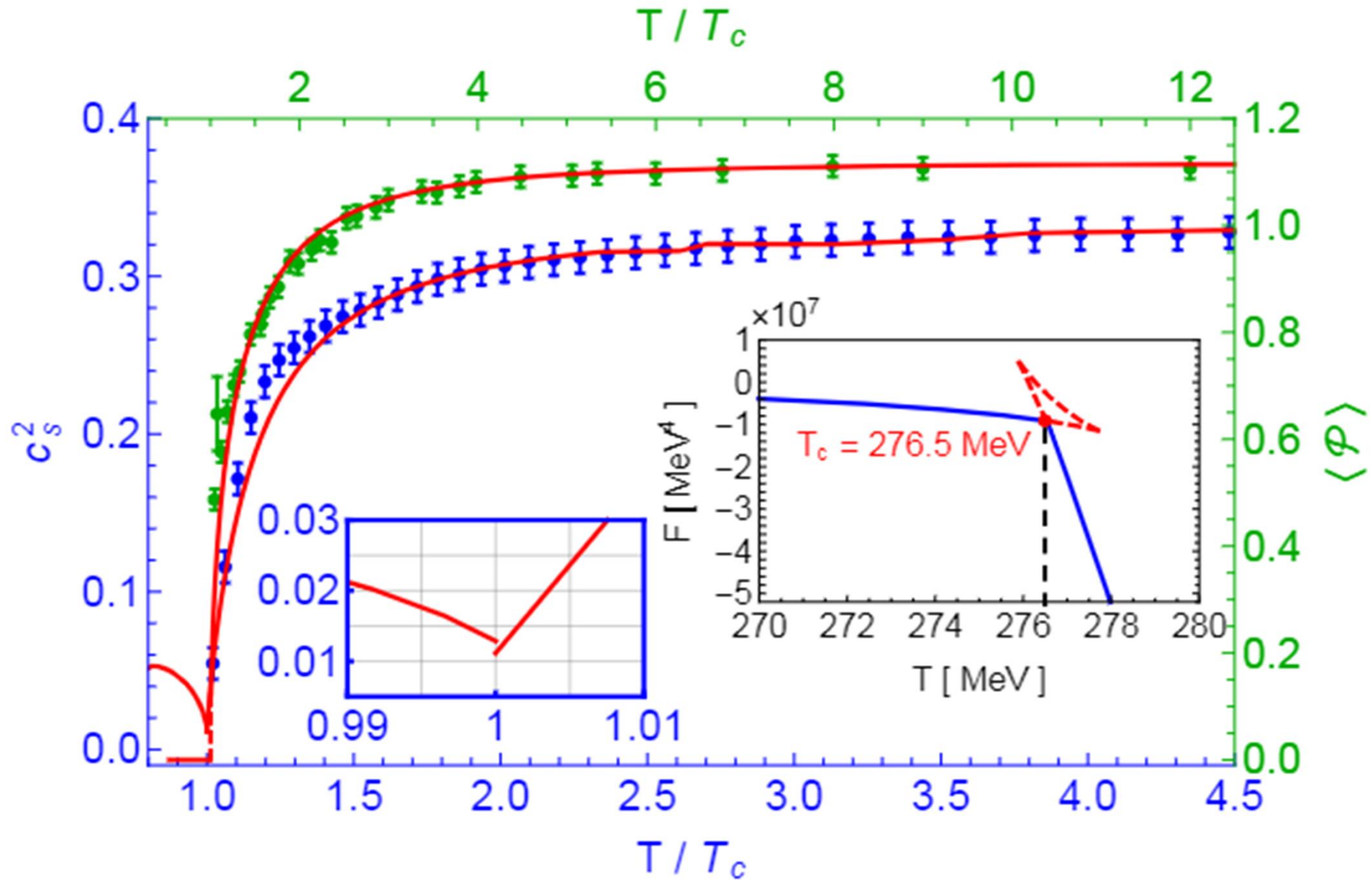
One parameter to fix all thermal dynamics lattice data



<http://arxiv.org/abs/hep-lat/9602007>

<http://arxiv.org/abs/0711.2251>

First order Phase transition vs Polyakov Loop



<http://arxiv.org/abs/hep-lat/9602007>

<http://arxiv.org/abs/0711.2251>

Gluon condensation- Probe

$$S = -\frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g^s} e^{-\sqrt{\frac{3}{8}}\phi} \left[\frac{1}{2} \nabla_\mu \chi \nabla^\mu \chi + \frac{1}{2} m_\chi^2 \chi^2 \right]. \quad (25)$$



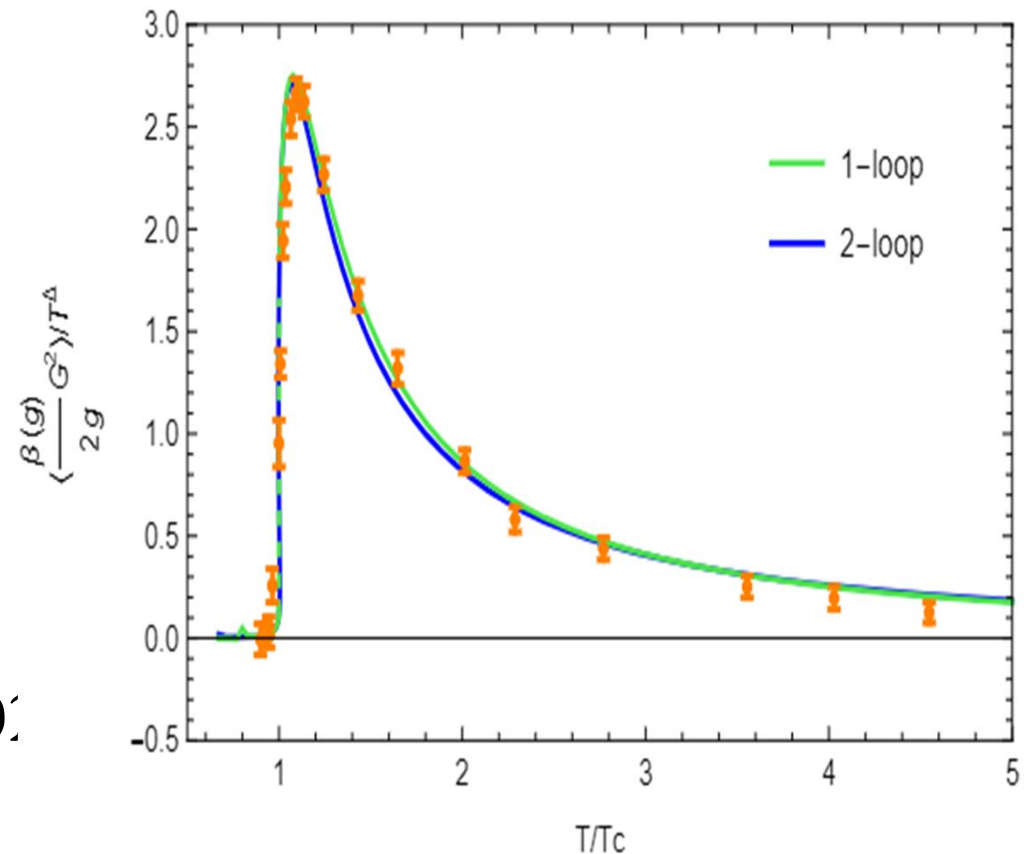
$$\chi(r) = \chi_0 r^{\Delta-4} + \dots + \chi_4 r^{-\Delta}$$

$$m_\chi^2 = \Delta(\Delta - 4)$$



$$\chi_4 = \langle G^2 \rangle$$

$$\left\langle \frac{\beta(g)}{2g} G^2 \right\rangle_T = \langle T_\mu^\mu \rangle = \epsilon - 3P$$

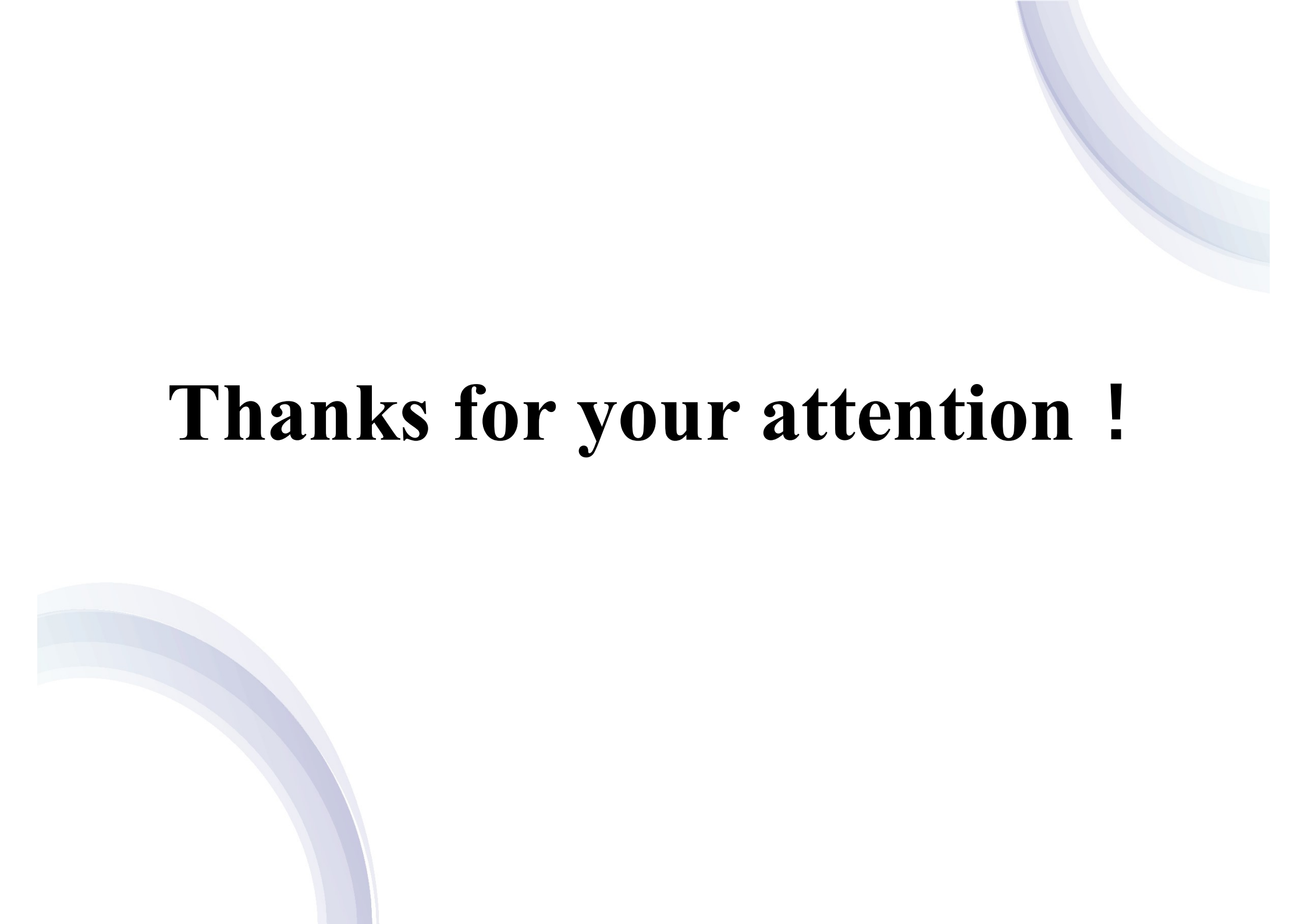


<http://arxiv.org/abs/hep-lat/9601>

<http://arxiv.org/abs/0711.2251>

Summary

- I. Propose a hQCD model on quantitative level to describe QCD phase diagram.**
- II. EOS confront with lattice simulations at zero/non-zero chemical potential.**
- III. Realize QCD CEP and quantitatively agrees with effective field results.**
- IV. Stochastic GW spectrum induced by QCD phase transition predicted.**
- V. Construct holographic pure gluon model.**
- VI.**

The image features a white background with decorative curved lines in the corners. In the top right corner, there is a light blue arc. In the bottom left corner, there is a multi-layered arc in shades of light blue and purple. Centered on the page is the text "Thanks for your attention !" in a bold, black, serif font.

Thanks for your attention !