Off diagonal elements of the spin density matrix of vector mesons in heavy ion collisions

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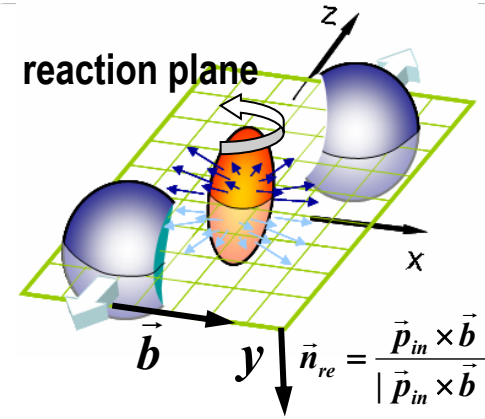
Outline

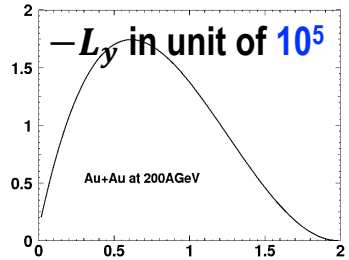
- Introduction
- The global vector meson spin alignment vs the global hyperon polarization
- The off diagonal elements of the spin density matrix of vector mesons
- Summary and outlook

Prediction of globally polarized QGP

ZTL & Xin-Nian Wang, PRL 94, 102301(2005); PLB 629, 20 (2005)

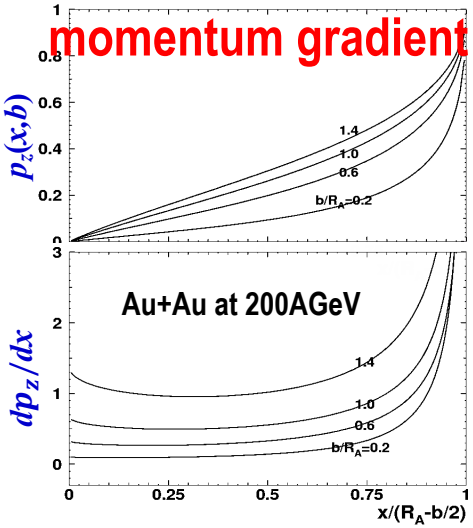


Huge orbital angular momentum

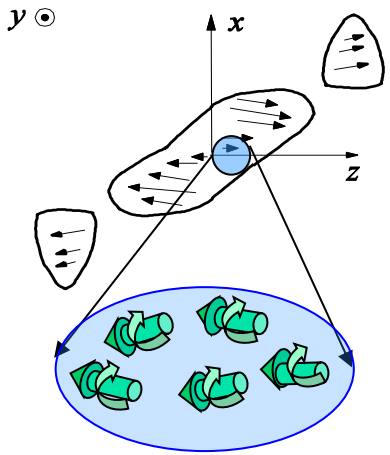


leads to

momentum gradient



QCD spin-orbital interaction



"global quark polarization"

hadronization

- **Global hyperon polarization**

$$P_H = P_{\bar{H}} = P_q = P_{\bar{q}}$$

- **Global vector spin alignment**

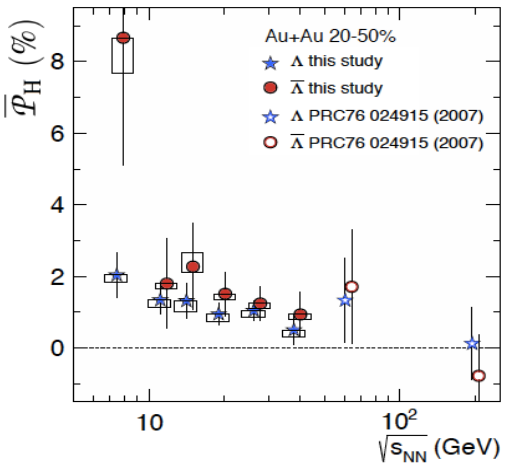
$$\rho_{00}^V = \frac{1 - P_q^2}{3 + P_q^2}$$

$$\rho_{00}^V - 1/3 \sim P_q^2$$

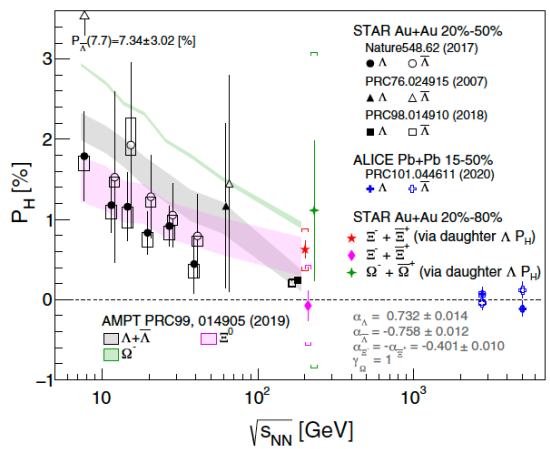
Global: the direction is fixed; the magnitude is approximately the same.

Great efforts of our experimental colleagues

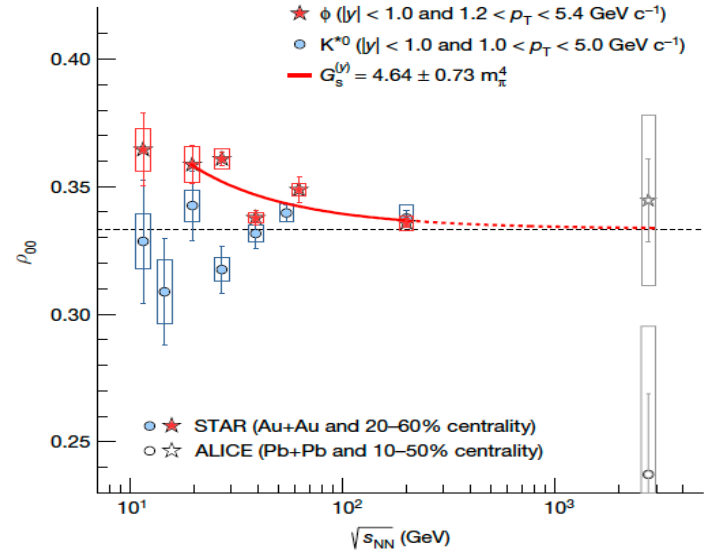
STAR, L. Adamczyk *et al.*, Nature 548, 62 (2017).

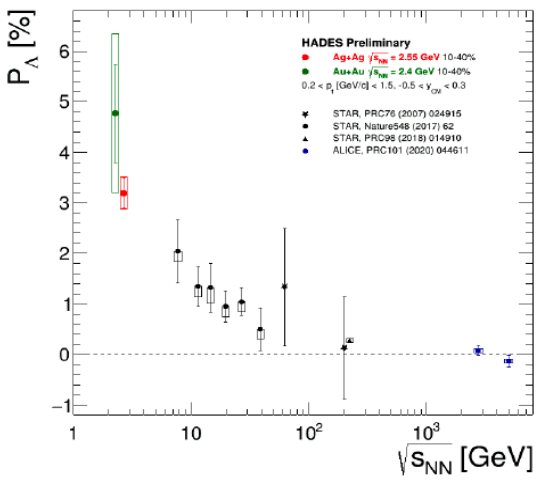


STAR, J. Adam *et al.*, PRL 126, 162301 (2021)



STAR, M.S. Abdallah *et al.*, Nature 614, 244 (2023).





HADES, R. Yassine *et al.*, PLB 835, 137506 (2022)

$$\left|\rho_{00}^{V} - \frac{1}{3}\right| \gg P_\Lambda^2 \sim P_q^2$$

What does it tell us?

How can we understand it?

Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

dominates at small and intermediate p_T

$$\hat{\rho}_{q_1 q_2 q_3} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{q_2} \otimes \hat{\rho}_3 \qquad \hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1 + P_q & 0 \\ 0 & 1 - P_q \end{pmatrix}$$

$$\rho_H(m, m') = \langle j_H, m' | \hat{\rho}_{q_1 q_2 q_3} | j_H, m \rangle$$

$$= \sum_{m_i, m_i'} \rho_{q_1 q_2 q_3}(m_i, m_i') \langle j_H, m' | m_1', m_2', m_3' \rangle \langle m_1, m_2, m_3 | j_H, m \rangle$$

Clebsch-Gordon coefficients

normalization

$$\rho_H(m, m') = \frac{\sum_{m_i, m_i'} \rho_{q_1 q_2 q_3}(m_i, m_i') \langle j_H, m' | m_1', m_2', m_3' \rangle \langle m_1, m_2, m_3 | j_H, m \rangle}{\sum_{m, m_i, m_i'} \rho_{q_1 q_2 q_3}(m_i, m_i') \langle j_H, m | m_1', m_2', m_3' \rangle \langle m_1, m_2, m_3 | j_H, m \rangle}$$

Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

dominates at small and intermediate p_T

$$P_H = \rho_H\left(\frac{1}{2},\frac{1}{2}\right) - \rho_H\left(-\frac{1}{2},-\frac{1}{2}\right)$$

$$P_H = c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3}$$

c_i's are constants determined by C.G. coefficients.

hyperon	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
combination	P_s	$\frac{4P_u-P_s}{3}$	$\frac{2(P_u+P_d)-P_s}{3}$	$\frac{4P_d-P_s}{3}$	$\frac{4P_s-P_u}{3}$	$\frac{4P_s-P_d}{3}$

In the case that $P_u = P_d = P_s = P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}}$,

$P_H = P_{\bar{H}} = P_q$ for all H's and $\bar{H}$'s (global polarization)

Global vector meson spin alignment

ZTL & Xin-Nian Wang, PLB 629, 20 (2005).

Quark combination scenario $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\hat{\rho}_{q_1\bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2} \qquad \hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix} \qquad \hat{\rho}_{\bar{q}} = \frac{1}{2}\begin{pmatrix} 1+P_{\bar{q}} & 0 \\ 0 & 1-P_{\bar{q}} \end{pmatrix}$$

$$\rho_V(m,m') = \frac{\sum_{m_i,m_i'} \rho_{q_1\bar{q}_2}(m_i,m_i')\langle j_V, m' | m_1', m_2'\rangle\langle m_1, m_2 | j_V, m\rangle}{\sum_{m,m_i,m_i'} \rho_{q_1\bar{q}_2}(m_i,m_i')\langle j_V, m | m_1', m_2'\rangle\langle m_1, m_2 | j_V, m\rangle}$$

$$\rho_{00}^V = \frac{1-P_{q_1}P_{\bar{q}_2}}{3+P_{q_1}P_{\bar{q}_2}} = \frac{1-P_q^2}{3+P_q^2} \qquad \hat{\rho}^{\mathrm{V}} = \begin{pmatrix} \rho_{11}^V & \rho_{10}^V & \rho_{1-1}^V \\ \rho_{01}^V & \rho_{00}^V & \rho_{0-1}^V \\ \rho_{-11}^V & \rho_{-10}^V & \rho_{-1-1}^V \end{pmatrix}$$

In both calculations, we considered **only the spin degree of freedom** and took P_q **as a constant,** no fluctuation, no correlation etc.

What does it change if we take other degrees of freedom into account?

Take other degrees of freedom into account

We make a minimal step forward and consider other degree of freedom denoted by α

For $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

The basis state for a quark: $|m, \alpha_q\rangle$

The element of the spin density matrix: $\rho_{qm_q,m_q'}(\alpha_q, \alpha_q') = \langle m_q', \alpha_q' | \hat{\rho}_q | m_q, \alpha_q \rangle$

We consider the simple case:

- $\rho_{qm_q,m_q'}(\alpha_q, \alpha_q') = \rho_{qm_q,m_q'}(\alpha_q)\delta_{\alpha_q,\alpha_q'}$ diagonal w.r.t. α_q
- $\hat{\rho}_{q_1\bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2}$
- factorized: $\langle \alpha_{q_1}, m_{q_1}; \alpha_{\bar{q}_2}, m_{\bar{q}_2} | j_V, m_V, \alpha_V \rangle = \langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle \langle m_{q_1}, m_{\bar{q}_2} | j_V, m_V \rangle$

wave function of V with α_V

$$\rho^V_{mm'}(\alpha_V) = \rho^V_{mm'}(\alpha_V, \alpha_V) = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} \left| \langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle \right|^2 \rho^{V(l)}_{mm'}(\alpha_{q_1}, \alpha_{\bar{q}_2})$$

average inside V

$$\rho^{V(l)}_{mm'}(\alpha_{q_1}, \alpha_{\bar{q}_2}) = \sum_{m_{q_1}, m_{\bar{q}_2}, m_{q_1}', m_{\bar{q}_2}'} \langle j_V m' | m_{q_1}' m_{\bar{q}_2}' \rangle \rho^q_{m_{q_1} m_{q_1}'}(\alpha_{q_1}) \rho^q_{m_{\bar{q}_2}, m_{\bar{q}_2}'}(\alpha_{\bar{q}_2}) \langle m_{q_1}, m_{\bar{q}_2} | j_V m \rangle$$

similar to what we had when α-dependence were not considered.

We can also further average over α_V and obtain the α_V-averaged spin alignment.

Take other degrees of freedom into account

In this way, we obtain

$$\rho_{00}^{V}(\alpha_V) = \frac{1 - \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V}{3 + \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V}$$

average inside V

$$\left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} \left| \langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle \right|^2 P_{q_1}(\alpha_{q_1}) P_{\bar{q}_2}(\alpha_{\bar{q}_2})$$

We further average over α_V and obtain the α_V-averaged spin alignment.

$$\langle \rho_{00}^{V} \rangle = \frac{1 - \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle}{3 + \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle}$$

$$\left\langle P_{q_1} P_{\bar{q}_2} \right\rangle = \sum_{\alpha_V} f_V(\alpha_V) \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V$$

In general, α_V denotes a set of variables $(\alpha_{1V}, \alpha_{2V}, \dots \alpha_{jV})$.

We can integrate over only part of them and study the dependence on the others, i.e.,

$$\langle \rho_{00}^{V} \rangle (\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{kV}) = \frac{1 - \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}}}{3 + \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}}}$$

$$\left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}} = \sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV}) \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V \Big/ \sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV})$$

Take other degrees of freedom into account

The average is two folded:

$$\langle P_{q_1} P_{\bar{q}_2} \rangle = \left\langle \langle P_{q_1} P_{\bar{q}_2} \rangle_V \right\rangle_S$$

average inside the vector meson V

average over the whole system or a sub-system S

$$\langle P_{q_1} P_{\bar{q}_2} \rangle_V = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} \left| \langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle \right|^2 P_{q_1}(\alpha_{q_1}) P_{\bar{q}_2}(\alpha_{\bar{q}_2})$$

$$\langle P_{q_1} P_{\bar{q}_2} \rangle_{\alpha_{k+1V}, \ldots, \alpha_{jV}} = \frac{\sum_{\alpha_{k+1V}, \ldots, \alpha_{jV}} f_S(\alpha_{1V}, \alpha_{2V}, \ldots, \alpha_{jV}) \langle P_{q_1} P_{\bar{q}_2} \rangle_V}{\sum_{\alpha_{k+1V}, \ldots, \alpha_{jV}} f_S(\alpha_{1V}, \alpha_{2V}, \ldots, \alpha_{jV})}$$

Hyperon polarization v.s. vector meson spin alignment

For $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\rho_{00}^{V} = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle}$$

For $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

$$P_H = \left\langle \left\langle c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3} \right\rangle_H \right\rangle_S = \left\langle c_1 \langle P_{q_1} \rangle_H + c_2 \langle P_{q_2} \rangle_H + c_3 \langle P_{q_3} \rangle_H \right\rangle_S$$

$$= c_1 \left\langle \langle P_{q_1} \rangle_H \right\rangle_S + c_2 \left\langle \langle P_{q_2} \rangle_H \right\rangle_S + c_3 \left\langle \langle P_{q_3} \rangle_H \right\rangle_S = c_1 \langle P_{q_1} \rangle + c_2 \langle P_{q_1} \rangle + c_3 \langle P_{q_1} \rangle$$

The STAR data show that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

One has to take fluctuations into account, so that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

By studying P_H, we study the average of quark polarization P_q;
by studying ρ_{00}^{V}, we study the correlation between P_q and $P_{\bar{q}}$.

Local correlation or long range correlation

One has to take fluctuations into account, i.e.,: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

$$\langle P_q P_{\bar{q}} \rangle = \left\langle \langle P_q P_{\bar{q}} \rangle_V \right\rangle_S$$

average inside the vector meson V

average over the whole system or a sub-system S

(1) local correlation: $\langle P_q P_{\bar{q}} \rangle_V \neq \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$

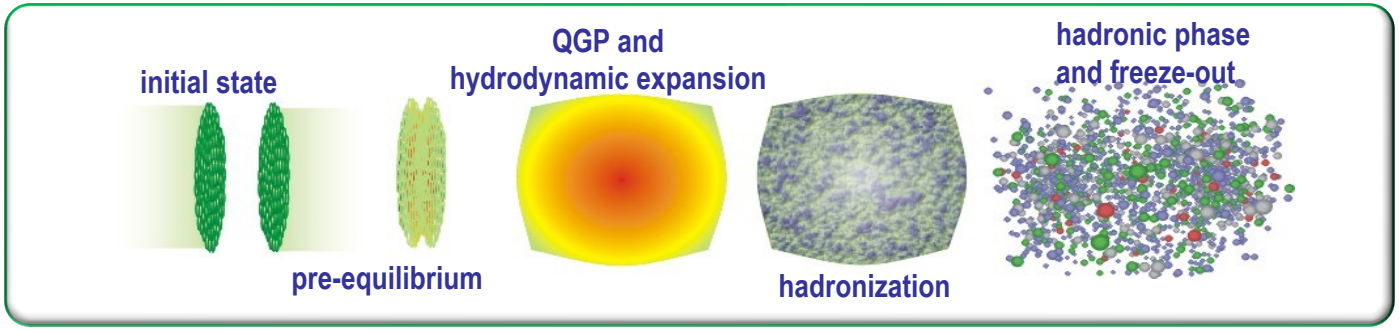
(2) long range correlation: $\langle P_q P_{\bar{q}} \rangle_V = \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$

$$\left\langle \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V \right\rangle_S \neq \left\langle \langle P_q \rangle_V \right\rangle_S \left\langle \langle P_{\bar{q}} \rangle_V \right\rangle_S$$

Vector meson spin alignment contains both contributions.

Vector meson spin alignment —— model

strong indication of phi-meson filed ⟹ strong local correlation



the only explanation yet

Strong interaction exhibits itself differently in different stages

$$P_s^\mu(x,p) \approx \frac{1}{4m_s}\varepsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_h}F^\phi_{\rho\sigma}\right)p_\nu$$

$$P_{\bar{s}}^\mu(x,p) \approx \frac{1}{4m_s}\varepsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_h}F^\phi_{\rho\sigma}\right)p_\nu$$

talks by Xin-Li Sheng and Qun Wang

[1] Yang-guang Yang, Ren-hong Fang, Qun Wang, and Xin-Nian Wang, PRC 97, 034917 (2018).

[2] Xin-Li Sheng, Lucia Oliva, and Qun Wang, PRD 101, 096005 (2020).

[3] Xin-Li Sheng, Qun Wang and Xin-Nian Wang, PRD 102, 056013 (2020).

[4] Xin-Li Sheng, Lucia Oliva, Zuo-tang Liang, Qun Wang and Xin-Nian Wang, 2205.15689 [hep-ph], to appear in PRL.

[5] Xin-Li Sheng, Lucia Oliva, Zuo-tang Liang, Qun Wang and Xin-Nian Wang, 2206.05868 [hep-ph].

Local correlation or long range correlation

Can we separate local or long rang correlation experimentally?

Study $\Lambda - \bar{\Lambda}$ or $\Lambda - \Lambda$ spin correlations

ZTL & X.N. Wang

$$C_{NN}^{\Lambda\Lambda} \equiv \frac{N_{\Lambda\bar{\Lambda}}^{\uparrow\uparrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\downarrow} - N_{\Lambda\bar{\Lambda}}^{\uparrow\downarrow} - N_{\Lambda\bar{\Lambda}}^{\downarrow\uparrow}}{N_{\Lambda\bar{\Lambda}}^{\uparrow\uparrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\downarrow} + N_{\Lambda\bar{\Lambda}}^{\uparrow\downarrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\uparrow}}$$

Even, in general, $H_i - \bar{H}_j$ or $H_i - H_j$ spin correlations

$$C_{NN}^{H_i\bar{H}_j} \equiv \frac{N_{H_i\bar{H}_j}^{\uparrow\uparrow} + N_{H_i\bar{H}_j}^{\downarrow\downarrow} - N_{H_i\bar{H}_j}^{\uparrow\downarrow} - N_{H_i\bar{H}_j}^{\downarrow\uparrow}}{N_{H_i\bar{H}_j}^{\uparrow\uparrow} + N_{H_i\bar{H}_j}^{\downarrow\downarrow} + N_{H_i\bar{H}_j}^{\uparrow\downarrow} + N_{H_i\bar{H}_j}^{\downarrow\uparrow}}$$

sensitive to the long range correlation

because H_i and $\bar{H}_j$ come from different phase space points

seems not so easy in experiments

Off-diagonal elements of $\hat{\rho}^V$?

- ZTL & Xin-Nian Wang, PRL 94, 102301 (2005)

considered the average $\langle\hat{\rho}_q\rangle = \frac{1}{2}\begin{pmatrix} 1+\langle P_q\rangle & 0 \\ 0 & 1-\langle P_q\rangle \end{pmatrix}$

i.e., $\langle P_{qy}\rangle = \langle P_q\rangle$, $\langle P_{qz}\rangle = \langle P_{qx}\rangle = 0$, also $\langle P_{q_1y}\, P_{\bar{q}_2y}\rangle = \langle P_{q_1}\rangle\langle P_{\bar{q}_2}\rangle$

- The STAR data show that: $\langle P_q P_{\bar{q}}\rangle \neq \langle P_q\rangle\langle P_{\bar{q}}\rangle$ $\quad \langle P_q P_{\bar{q}}\rangle \gg \langle P_q\rangle\langle P_{\bar{q}}\rangle$

indicates that the fluctuation $\Delta P_{qy}^2 \equiv \langle P_{qy}^2\rangle - \langle P_{qy}\rangle^2 \sim \langle P_{qy}^2\rangle \gg \langle P_{qy}\rangle^2$

i.e., compared to ΔP_{qy}^2, we can even take $\langle P_{qy}\rangle \sim \langle P_{qz}\rangle = \langle P_{qx}\rangle = 0$

Similar fluctuations $\langle P_{qz}^2\rangle$ and $\langle P_{qx}^2\rangle$ for $\langle P_{qz}\rangle$ and $\langle P_{qx}\rangle$?

- **take also the off-diagonal components into account**

$$\hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_{qy} & P_{qz}-iP_{qx} \\ P_{qz}+iP_{qx} & 1-P_{qy} \end{pmatrix} \qquad \hat{\rho}_{\bar{q}} = \frac{1}{2}\begin{pmatrix} 1+P_{\bar{q}y} & P_{\bar{q}z}-iP_{\bar{q}x} \\ P_{\bar{q}z}-iP_{\bar{q}x} & 1-P_{\overline{qy}} \end{pmatrix}$$

Off-diagonal elements of $\hat{\rho}^V$?

In this case, we obtain

$$\rho_{00}^V = \frac{1 + \vec{P}_q \cdot \vec{P}_{\bar{q}} - 2P_{qy}P_{\bar{q}y}}{3 + \vec{P}_q \cdot \vec{P}_{\bar{q}}}$$

also the off-diagonal elements of $\hat{\rho}^V$

$$\rho_{10}^V = \frac{P_{qz}(1 + P_{\bar{q}y}) + (1 + P_{qy})P_{\bar{q}z} - iP_{qx}(1 + P_{\bar{q}y}) - i(1 + P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{0-1}^V = \frac{P_{qz}(1 - P_{\bar{q}y}) + (1 - P_{qy})P_{\bar{q}z} - iP_{qx}(1 - P_{\bar{q}y}) - i(1 - P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{1-1}^V = \frac{P_{qz}P_{\bar{q}z} - P_{qx}P_{\bar{q}x} + i(P_{qx}P_{\bar{q}y} + P_{qy}P_{\bar{q}x})}{3 + \vec{P}_q \cdot \vec{P}_{\bar{q}}}$$

They should be sensitive to the local correlations.

Off-diagonal elements of $\hat{\rho}^V$?

Take the average

$$\langle \rho_{00}^V \rangle = \frac{1 + \langle P_{qz} P_{\bar{q}z} \rangle + \langle P_{qx} P_{\bar{q}x} \rangle - \langle P_{qy} P_{\bar{q}y} \rangle}{3 + \langle P_{qz} P_{\bar{q}z} \rangle + \langle P_{qx} P_{\bar{q}x} \rangle + \langle P_{qy} P_{\bar{q}y} \rangle}$$

$$\langle \rho_{10}^V \rangle = \frac{\langle P_{qz} P_{\bar{q}y} \rangle + \langle P_{qy} P_{\bar{q}z} \rangle - i\langle P_{qx} P_{\bar{q}y} \rangle - i\langle P_{qy} P_{\bar{q}x} \rangle}{\sqrt{2}(3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle)}$$

$$\langle \rho_{0-1}^V \rangle = \frac{-\langle P_{qz} P_{\bar{q}y} \rangle - \langle P_{qy} P_{\bar{q}z} \rangle + i\langle P_{qx} P_{\bar{q}y} \rangle + i\langle P_{qy} P_{\bar{q}x} \rangle}{\sqrt{2}(3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle)}$$

$$\langle \rho_{1-1}^V \rangle = \frac{\langle P_{qz} P_{\bar{q}z} \rangle - \langle P_{qx} P_{\bar{q}x} \rangle + i(\langle P_{qx} P_{\bar{q}y} \rangle + \langle P_{qy} P_{\bar{q}x} \rangle)}{3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle}$$

They should be sensitive to the local correlations.

Summary and outlook

- Global hyperon polarization and global vector meson spin alignment have been observed experimentally.
- The global hyperon polarization is a measure of the average value of the global quark polarization in the system, while the global vector meson spin alignment measures the correlation between quark and anti-quark polarization.
- Correlation between the polarization of hyperon-hyperon or hyperon-antihyperon can be sensitive to the long range correlation while off-diagonal elements of vector meson spin density matrix may provide important information on the local correlation.

Thank you for your attention!

Measurements

For $V \to 1 + 2$, where 1 and 2 are two pseudoscalar mesons, we have $S_A = 1, \lambda_1 = \lambda_2 = 0$

e.g., $\rho \to \pi\pi$

$$W(\theta,\varphi) = N \sum_{M_A, M'_A} |H_A|^2 D^{1*}_{M_A 0}(\varphi,\theta,-\varphi) D^{1}_{M'_A 0}(\varphi,\theta,-\varphi) \langle M_A | \hat{\rho}_A | M'_A \rangle$$

$$= \frac{3}{4\pi}\left\{\frac{1}{2}(\rho_{11} + \rho_{-1-1}) \sin^2\theta + \rho_{00}\cos^2\theta \right.$$

$$- \frac{1}{\sqrt{2}} \sin 2\theta \left[\cos\varphi\,(\mathrm{Re}\rho_{10} - Re\rho_{-10}) - \sin\varphi\,(\mathrm{Im}\rho_{10} + \mathrm{Im}\rho_{-10})\right]$$

$$\left. - \sin^2\theta\,(\cos 2\varphi\, \mathrm{Re}\rho_{1-1} - \sin 2\varphi\, \mathrm{Im}\rho_{1-1})\right\}$$

$$\int_0^{2\pi} d\varphi\, W(\theta,\varphi) = \frac{3}{4}\left[(1 - \rho_{00}) + (3\rho_{00} - 1)\cos^2\theta\right]$$

Measurements

For $V \to 1 + 2$, where 1 and 2 are two spin-1/2 Fermions, i.e., $S_A = 1, \lambda_1 = \pm\frac{1}{2},\ \lambda_2 = \pm\frac{1}{2}$

consider the case: (1) Helicity conservation: $\lambda_1 = -\lambda_2\ , \lambda = \pm 1$

(2) Space reflection invariance: $H_A(\lambda_1, \lambda_2) = H_A(-\lambda_1, -\lambda_2)$

only one independent helicity amplitude

e.g., $J/\psi \to e^+e^-$

$$W(\theta,\varphi) = \frac{3(1+\rho_{00})}{8\pi}[1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \cos 2\varphi + \lambda_{\theta\varphi} \sin 2\theta \cos\varphi$$
$$+\lambda_\varphi^\perp \sin^2\theta \sin 2\varphi + \lambda_{\theta\varphi}^\perp \sin 2\theta \sin\varphi]$$

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}} \qquad \lambda_\varphi = \frac{4\mathrm{Re}\rho_{1-1}}{1 + \rho_{00}} \qquad \lambda_{\theta\varphi} = \frac{\sqrt{2}\mathrm{Re}(\rho_{10} - \rho_{-10})}{1 + \rho_{00}}$$

$$\lambda_\varphi^\perp = \frac{4\mathrm{Im}\rho_{1-1}}{1 + \rho_{00}} \qquad \lambda_{\theta\varphi}^\perp = \frac{\sqrt{2}\mathrm{Im}(\rho_{10} - \rho_{-10})}{1 + \rho_{00}}$$