Relativistic spin dynamics for vector mesons

Xin-Li Sheng

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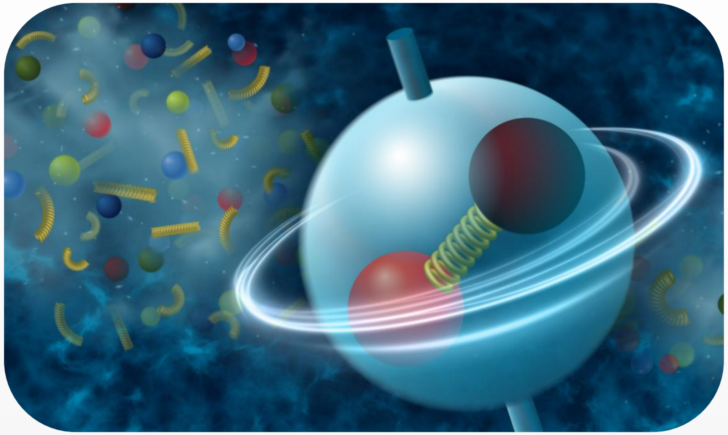
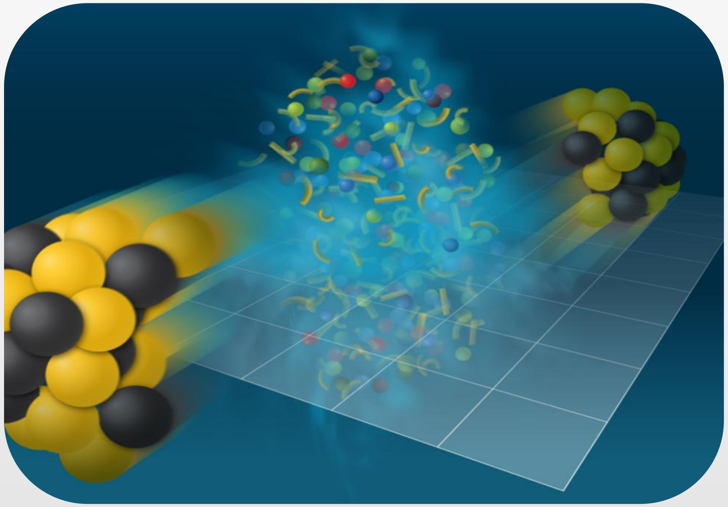
“The 7th International Conference on Chirality, Vorticity, and Magnetic Field in Heavy Ion Collisions”

Jul. 15-19, 2023





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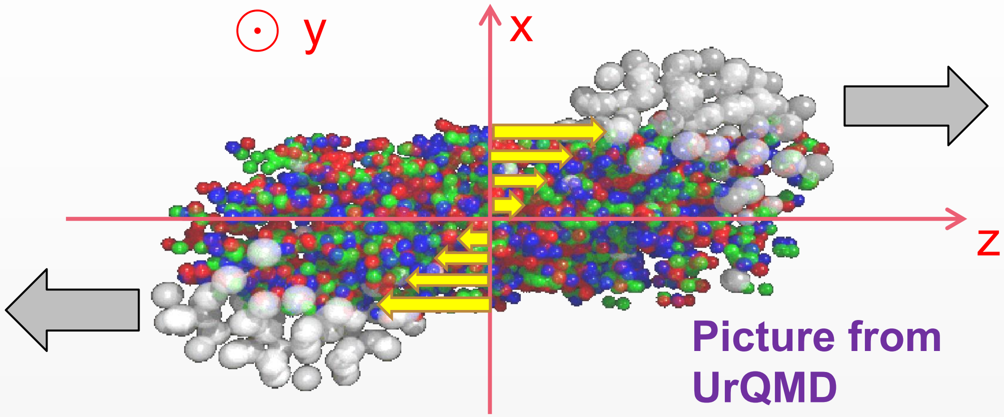
- Relativistic kinetic theory with spin for vector mesons
- ϕ meson's spin alignment
- Summary

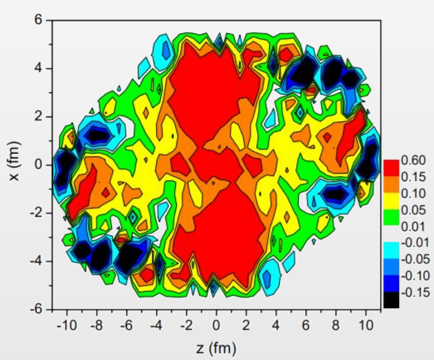
XLS, L. Oliva, Q. Wang, PRD 101, 096005 (2020); PRD 105, 099903 (2022) (erratum)
XLS, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, arXiv: 2205.15689, 2206.05868

Heavy-ion collisions



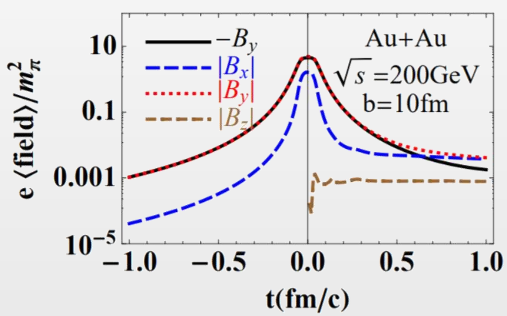
Picture from UrQMD

Vorticity fields



F. Becattini, L. Csernai, D.J. Wang, PRC 88, 034905 (2013); PRC 93, 069901 (2016)

Electromagnetic fields



W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012).

Vorticity, magnetic field

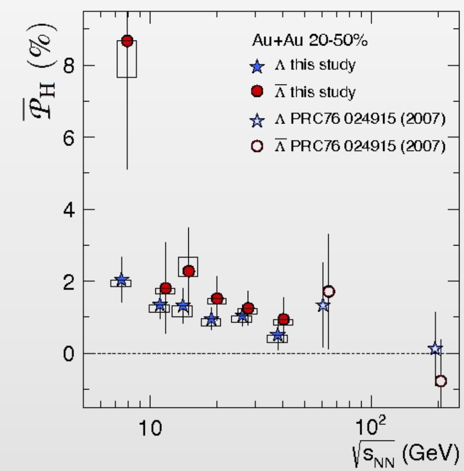
spin-orbit / magnetic coupling

Spin of hadrons

weak / strong p-wave decay

Measurements

Λ's global polarization



L. Adamczyk, et al. (STAR), Nature 548 (2017) 62.

Spin alignment

- Spin alignment for a vector meson ($J^P = 1^-$) is 00-element ρ_{00} of its normalized spin density matrix

  ρ_{00}=1/3 if spin does not have a preferred direction

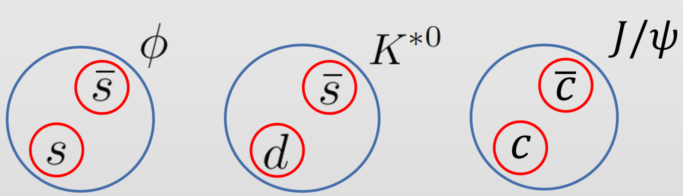
$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

- Spin alignment is measured through polar angle distribution of decay products

Processes	Examples	Polar angle distribution $W(\theta)$	Spin is converted to
Pairty-odd strong decay	$K^{*0} \to K^+ + \pi^-$ $\phi \to K^+ + K^-$	$\frac{3}{4}\left[1-\rho_{00}+(3\rho_{00}-1)\cos^2\theta\right]$	OAM
Dilepton decay	$J/\psi \to \mu^+ + \mu^-$	$\frac{3}{8}\left[1+\rho_{00}+(1-3\rho_{00})\cos^2\theta\right]$	Spin

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].

P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)



Global spin alignment

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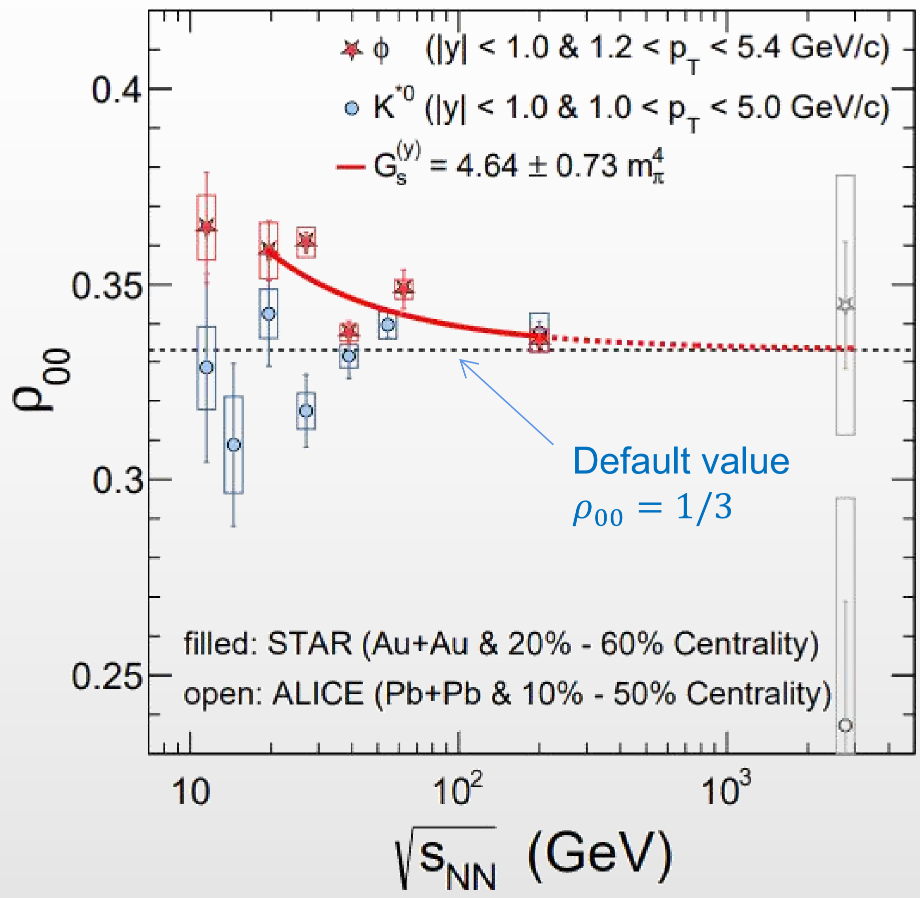
Article | Published: 18 January 2023

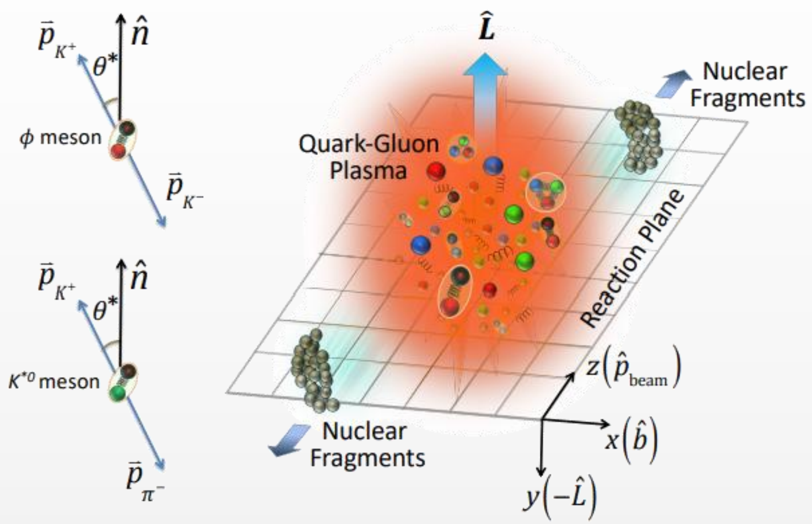
Pattern of global spin alignment of ϕ and K^{*0} mesons in heavy-ion collisions

STAR Collaboration

Nature **614**, 244–248 (2023) | Cite this article

Global spin alignment





Spin alignment along direction of global angular momentum

STAR, Nature 614, 244 (2023)

Talk by Subhash Singha

Theory prediction:
XLS, L. Oliva, Q. Wang, PRD 101, 096005 (2020); PRD 105, 099903 (2022) (erratum)

Experiment results

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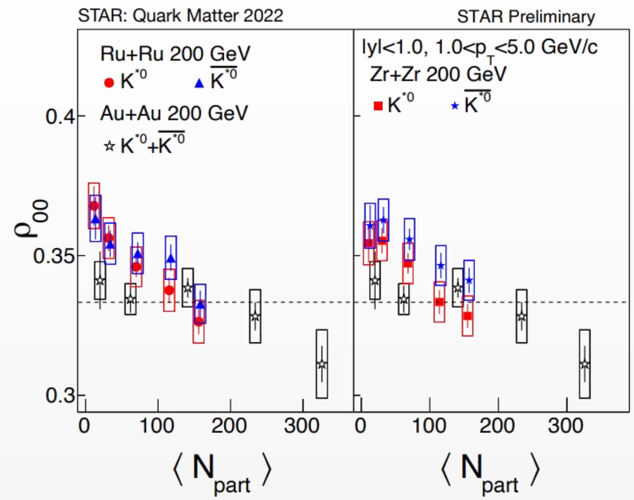
- Spin alignment in isobar collisions

  ➔ Talk by Subhash Singha

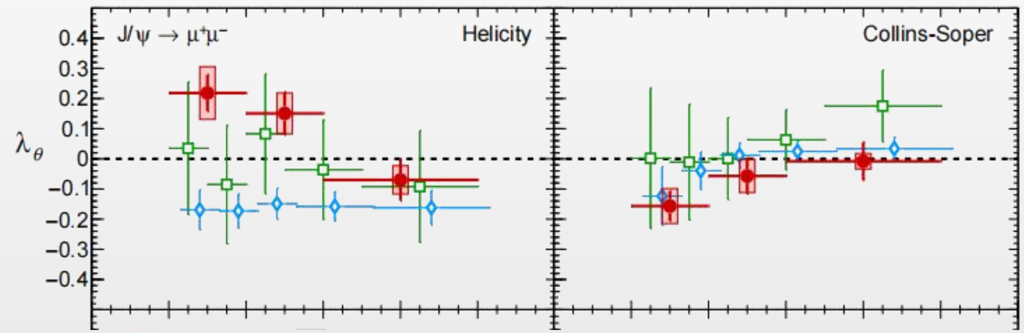


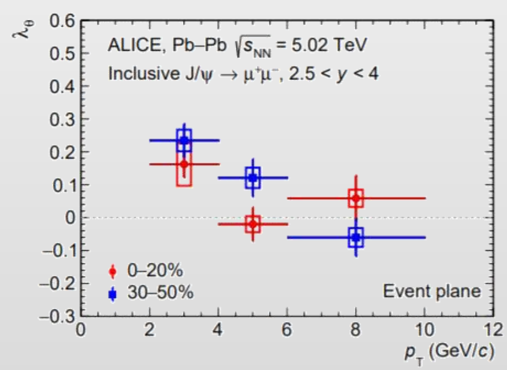
- Spin alignment of J/ψ in pp and Pb-Pb collisions

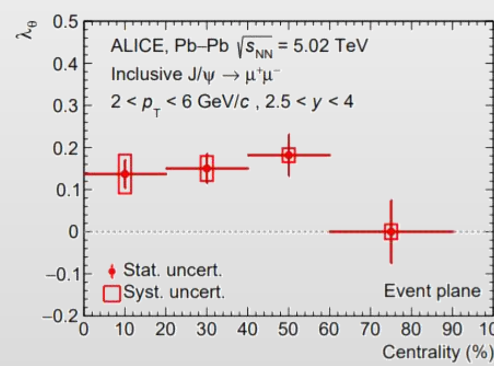
$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \approx -\frac{9}{4}\left(\rho_{00} - \frac{1}{3}\right)$$

ALICE, PLB 815 (2021) 136146;
e-Print: 2204.10171

➔ Talk by Xiao-Zhi Bai







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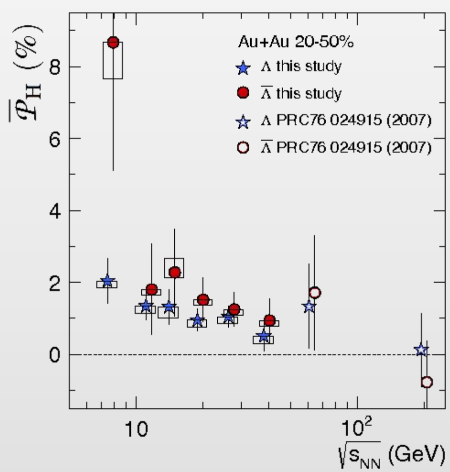
Spin Alignment of Vector Mesons in Non-central $A + A$ Collisions

PLB 629, 20 (2005).

Zuo-Tang Liang[1] and Xin-Nian Wang[2,1]
[1]*Department of Physics, Shandong University, Jinan, Shandong 250100, China*
'uclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 947
(Dated: November 5, 2018)

- Spin alignment of vector meson is determined by spin polarizations of constitute quark/antiquark

$$\boxed{\rho_{00}^{V\,(\mathrm{rec})} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9} P_q P_{\bar{q}}}$$



- Estimation using global quark spin polarization

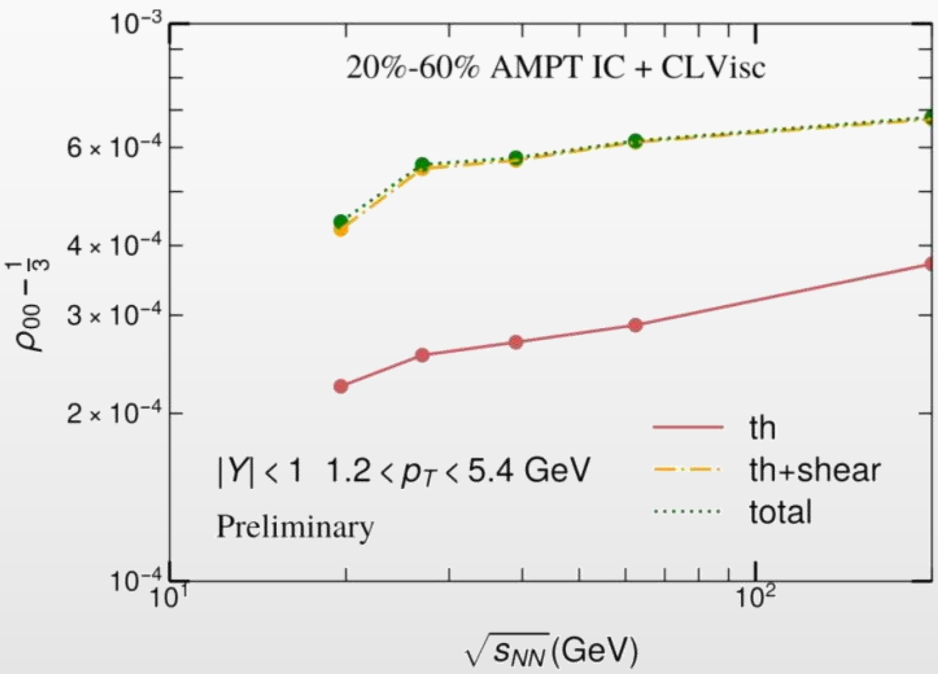
$$P_s \approx P_\Lambda \approx 2\%$$
$$P_{\bar{s}} \approx P_{\bar{\Lambda}} \approx 2\%$$

$$\rho_{00}^{\phi} - \frac{1}{3} \approx -1.7 \times 10^{-4}$$

Negative deviation from 1/3, order of 10^{-4}

- Numerical simulation for ϕ meson's spin alignment induced by local quark polarization

  (thermal vorticity, shear tensor)



Talk by Cong Yi, July 19th, 11:50 a.m. (Parallel Session A)

"Helicity polarization and vorticity contribution to the spin alignment in hydrodynamic approaches"

Positive deviation from 1/3, order of 10^{-4}

Spin alignment

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Cannot explain large positive deviation from 1/3

$$\rho_{00} \approx \frac{1}{3} + c_{\mathrm{hydro}} + c_{\mathrm{EM}} + c_F + c_A + c_h + c_{\mathrm{strong}}$$

Hydrodynamic gradient (vorticity, acceleraction, shear tensor)[1-9]

Talks by Cong Yi, Minghua Wei, Shuai Liu

Electromagnetic fields [3,4,9]

Talks by Shuyun Yang, Yan-Qing Zhao

Fragmen-tation [1]

Turbulent color field [10]

Helicity polarization [11]

Strong force (ϕ meson field, gluon fields) [4,12,13]

[1] Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005)

[2] F. Becattini, L. Csernai, D.-J. Wang, PRC 88, 034905 (2013)

[3] Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)

[4] XLS, L. Oliva, Q. Wang, PRD 101, 096005 (2020)

[5] X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)

[6] F. Li, S. Liu, arXiv: 2206.11890

[7] D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)

[8] M. Wei, M. Huang, arXiv:2303.01897

[9] XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872

[10] B. Muller, D.-L. Yang, PRD 105, 1 (2022).

[11] J.-H. Gao, PRD 104, 076016 (2021)

[12] XLS, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, arXiv: 2205.15689, 2206.05868

[13] A. Kumar, B. Muller, D.-L. Yang, arXiv: 2304.04181

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Green functions

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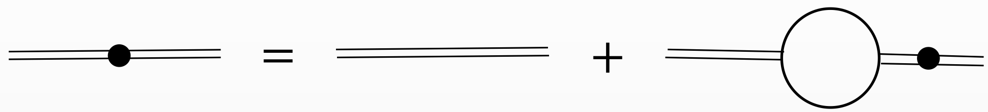
- Dyson-Schwinger equation

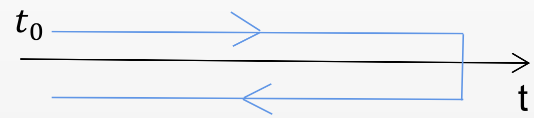


Full Green function   Free Green function   Self-energy

$$G = G_0 + G_0 \Sigma G$$

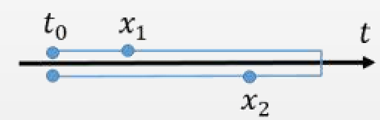
$$G = G_0 + G \Sigma G_0$$

- Two-point Green function on the closed-time path



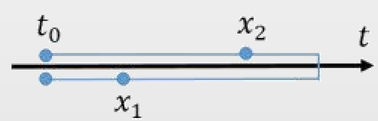
$$G^{\mu\nu}_{\rm CTP}(x_1, x_2) \equiv \left\langle T_C A^{\mu}_V(x_1) A^{\nu\dagger}_V(x_2) \right\rangle$$

$$G^F_{\mu\nu}(x_1, x_2)$$

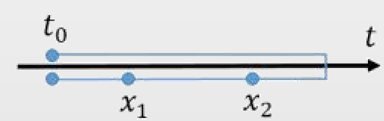


$$G^<_{\mu\nu}(x_1, x_2)$$

Wigner function



$$G^>_{\mu\nu}(x_1, x_2)$$



$$G^{\overline{F}}_{\mu\nu}(x_1, x_2)$$

P. Martin, J. S.Schwinger, PR 115 (1959) 1342.

L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).

L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515.

Kadanoff-Baym equation

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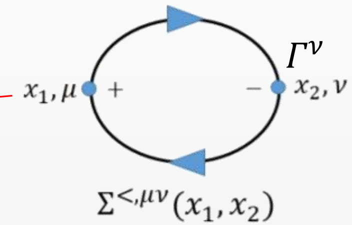
- With help of Schwinger-Keldysh (closed-time path) formalism, we derive Kadanoff-Baym equation at leading order in spatial gradient

$$
\begin{aligned}
&L^{\mu}_{\eta} G^{<,\mu\eta}(x,p) \\
&= -\frac{i\hbar}{2}\int \frac{d^4p'}{(2\pi\hbar)^4}\left\{ G^{<,\mu}{}_{\alpha}(x,p)\,\mathrm{Tr}\left[\Gamma^{\alpha} S^{>}(x,p+p')\,\Gamma^{\nu} S^{<}(x,p')\right]\right. \\
&\quad \left. -G^{>,\mu}{}_{\alpha}(x,p)\,\mathrm{Tr}\left[\Gamma^{\alpha} S^{<}(x,p+p')\,\Gamma^{\nu} S^{>}(x,p')\right]\right\} + \mathcal{O}(\hbar^2)
\end{aligned}
$$

Green functions on the closed-time path contour



One-loop self-energy

$$
L^{\mu}_{\eta} \equiv -g^{\mu}_{\eta}\left(p^2 - m_V^2\right) + p^{\mu}p_{\eta} + i\hbar\left[g^{\mu}_{\eta}\, p\cdot\partial_x - \frac{1}{2}\left(p_{\eta}\partial^{\mu}_{x} + p^{\mu}\partial^{x}_{\eta}\right)\right]
$$

- Comparing Kadanoff-Baym equation with its Hermitian conjugate, we are able to derive

Mass-shell condition $\quad -(p^2 - m_V^2)G^{<,\mu\nu} + (p^{\mu}p_{\eta}G^{<,\eta\nu} + p^{\nu}p_{\eta}G^{<,\mu\eta}) = \cdots$

Boltzmann equation $\quad p\cdot\partial_x G^{<,\mu\nu} - \frac{1}{4}(p^{\mu}\partial^{x}_{\eta}G^{<,\eta\nu} + p^{\nu}\partial^{x}_{\eta}G^{<,\mu\eta}) = \cdots$

Kadanoff-Baym equation

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- Two-point Green function can be expressed in terms of matrix valued spin-dependent distributions (MVSD)

$$G^{<}_{\mu\nu}(x,p) = \int d^4y\, e^{ip\cdot y/\hbar} \langle A^\dagger_\nu(x_2) A_\mu(x_1)\rangle$$

$$A^\mu_V(x) = \sum_{\lambda=0,\pm1} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} \frac{1}{2E^V_{\mathbf{p}}} \times \left[\underbrace{\epsilon^\mu(\lambda,\mathbf{p})}_{\text{polarization vector for a meson with spin } \lambda} \underbrace{a_V(\lambda,\mathbf{p})}_{\text{creation/anihilation operator}} e^{-ip\cdot x/\hbar} + \epsilon^{*\mu}(\lambda,\mathbf{p}) \underbrace{a^\dagger_V(\lambda,\mathbf{p})}_{\text{creation/anihilation operator}} e^{ip\cdot x/\hbar}\right]$$

polarization vector for a meson with spin λ

creation/anihilation operator
a_V, $b^\dagger_V$ if meson is not self-conjugate

$$\boxed{\begin{aligned} G^{<}_{\mu\nu}(x,p) = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_V^2\right) & \times \left\{\theta(p^0)\epsilon_\mu(\lambda_1,\mathbf{p})\,\epsilon^*_\nu(\lambda_2,\mathbf{p})\, f_{\lambda_1\lambda_2}(x,\mathbf{p}) \right. \\ & + \theta(-p^0)\epsilon^*_\mu(\lambda_1,-\mathbf{p})\,\epsilon_\nu(\lambda_2,-\mathbf{p}) \\ & \left. \times \left[\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1}(x,-\mathbf{p})\right]\right\}, \end{aligned}}$$

- MVSD for vector meson

$$f_{\lambda_1\lambda_2}(x,\mathbf{p}) \equiv \int \frac{d^4u}{2(2\pi\hbar)^3} \delta(p\cdot u) e^{-iu\cdot x/\hbar} \left\langle a^\dagger_V\left(\lambda_2, \mathbf{p} - \frac{\mathbf{u}}{2}\right) a_V\left(\lambda_1, \mathbf{p} + \frac{\mathbf{u}}{2}\right)\right\rangle$$

$$= 2E^V_{\mathbf{p}} \int \frac{dp^0}{2\pi\hbar} \theta(p^0) \epsilon^{*\mu}(\lambda_1,\mathbf{p}) \epsilon^\nu(\lambda_2,\mathbf{p}) G^{<}_{\mu\nu}(x,p)$$

Relation to Wigner function

$$= 3 f(x,\mathbf{p}) \rho_{\lambda_1\lambda_2}(x,\mathbf{p})$$

Relation to spin-averaged distribution and normalized density matrix

$$\boxed{f(x,\mathbf{p}) \equiv \frac{1}{3} \sum_{\lambda=0,\pm1} f_{\lambda\lambda}(x,\mathbf{p}), \qquad \sum_{\lambda=0,\pm1} \rho_{\lambda\lambda}(x,\mathbf{p}) = 1}$$

Boltzmann equation

- Dyson-Schwinger equation
  - ⇒ Kadanoff-Baym equation for Wigner function
  - ⇒ Matrix-form Boltzmann equation

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k}) - f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\mathcal{C}_{\rm diss}(x,\mathbf{k})\right]$$

Dilute gas limit

$$f_q \sim f_{\bar{q}} \sim f_V \ll 1$$

Meson polarization vectors

Coalescence

$$q+\bar{q}\to V$$

Dissociation (independent from quark distributions)

$$V\to q+\bar{q}$$

- Contribution from coalescence

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\rm coal}(x,\mathbf{k}) = & \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar{q}}_{\mathbf{p}'} - E^{q}_{\mathbf{k}-\mathbf{p}'}\right)\\ & \times \mathrm{Tr}\left\{\Gamma^\nu(p'\cdot\gamma - m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right]\right.\\ & \left.\times\Gamma^\mu[(k-p')\cdot\gamma + m_q]\left[1+\gamma_5\gamma\cdot P^{q}(x,\mathbf{k}-\mathbf{p}')\right]\right\}\\ & \times f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}'),\end{aligned}$$

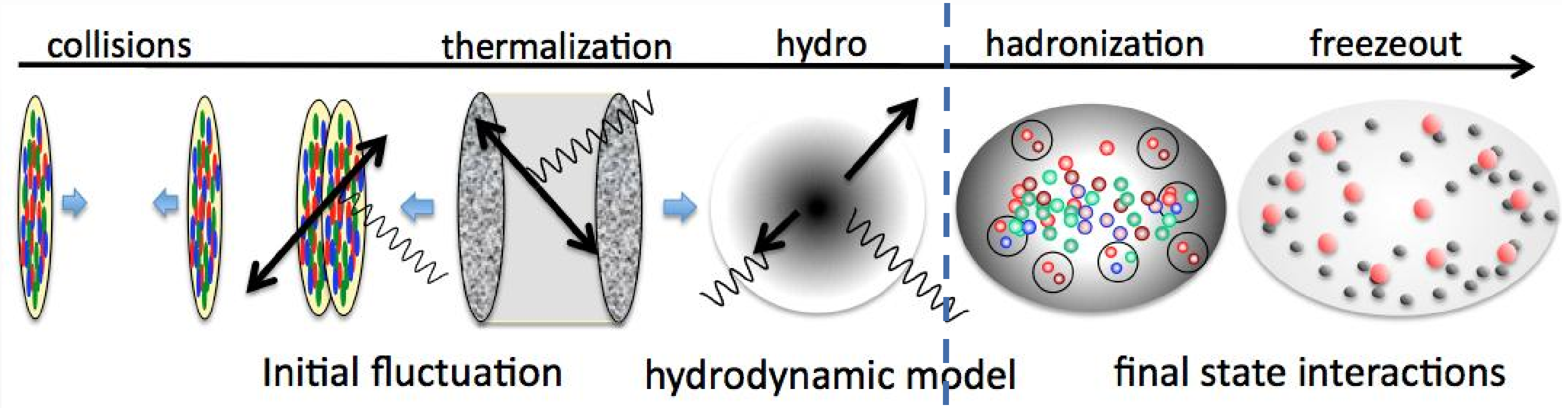
Energy conservation (all particles are on their normal mass shells)

Quark-antiquark-meson vertex

Polarizations of quark/antiquark

unpolarized quark/antiquark distributions

Spin alignment



No vector meson $f^V_{\lambda_1\lambda_2} = 0$

t_0

- Neglecting space-derivatives and assuming that $f^V_{\lambda_1\lambda_2} = 0$ before hadronization stage t_0, we obtain formal solution

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1-\exp[-\mathcal{C}_{\mathrm{diss}}(x,\mathbf{k})\Delta t]}{\mathcal{C}_{\mathrm{diss}}(x,\mathbf{k})}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})\right] \qquad \Delta t = t - t_0$$

- Spin alignment only depend on coalescence process

$$\boxed{\rho_{00} \equiv \frac{f^V_{00}}{f^V_{+1,+1}+f^V_{00}+f^V_{-1,-1}} = \frac{\epsilon^*_\mu(0,\mathbf{k})\epsilon_\nu(0,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})}}$$

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

Mass splitting

- Dyson-Schwinger equation

  ➡ Kadanoff-Baym equation for Wigner function

  ➡ Mass-shell condition

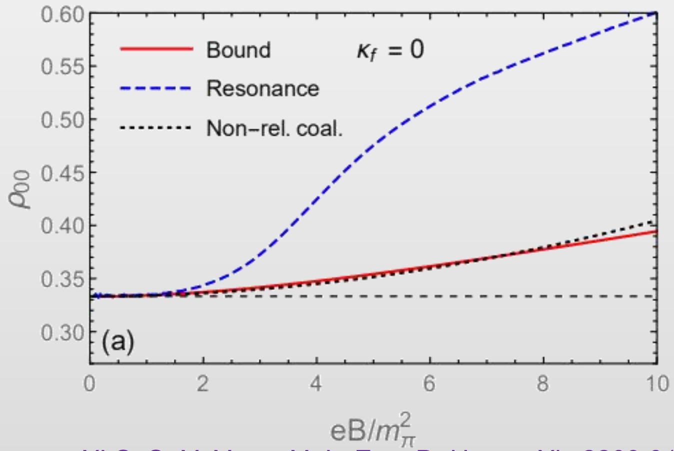
$$(p^2 - m_V^2) f^V_{\lambda_1\lambda_2}(x, \mathbf{k}) = \delta M_{\lambda_1\lambda_2}$$

mass corrections induced by interactions

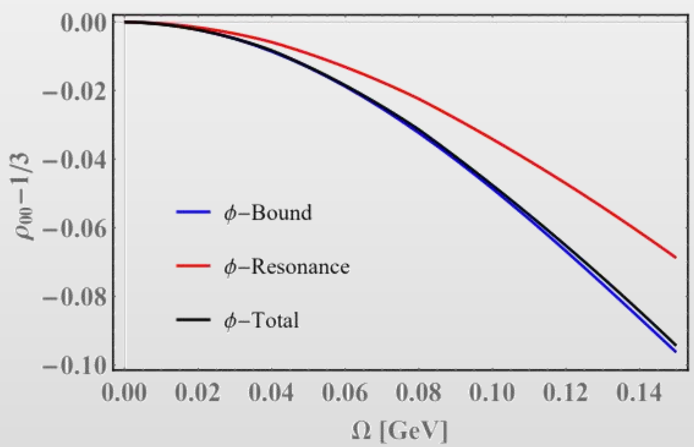
- Mass-splitting and spin alignment (for a thermodynamic equilibrium system)

$$\left.\begin{aligned} M_{V,0} &= \overline{M} + \Delta \\ M_{V,\pm} &= \overline{M} - \tfrac{\Delta}{2} \end{aligned}\right\} \quad f_\lambda \sim \frac{1}{\exp\left(M_{V,\lambda}/T\right) - 1} \quad \Rightarrow \quad \rho_{00} \approx \frac{1}{3} - \frac{\Delta}{3T} + \mathcal{O}\left[\left(\frac{\Delta}{T}\right)^2\right]$$

- Also see: talks by Shuyun Yang and Minghua Wei for spin alignment of vector mesons induced by magnetic field and rotation in framework of NJL model



XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872



M. Wei, M. Huang, arXiv:2303.01897

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Quark polarization

- Polarizations of strange quark/antiquark in a thermal equilibrium system

$$P_s^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} + \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$

$$P_{\bar{s}}^\mu(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\omega_{\rho\sigma} - \frac{Q_s}{(u\cdot p)T}F_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T}F^\phi_{\rho\sigma}\right]$$

F.Becattini, V.Chandra, L.Del Zanna, E.Grossi, Annals Phys. 338, 32 (2013)

Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys.Rev.C 97, 3 (2018).

XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020);

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

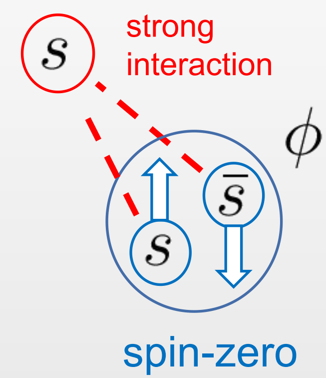
thermal vorticity field (rotation and acceleration)

classical electromagnetic field

$$\frac{e^2}{4\pi} \sim \frac{1}{137}$$

vector ϕ field (long wave-length components)

$$\frac{g_\phi^2}{4\pi} \sim \mathcal{O}(1) \gg \frac{e^2}{4\pi}$$



- Vector ϕ field has been used to explain the difference between polarizations of Λ and $\bar{\Lambda}$

L.P.Csernai, J.I.Kapusta, T.Welle, PRC 99, 021901 (2019)

Spin alignment

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- Spin alignment of the ϕ meson in its rest frame measuring along the direction of ϵ_0

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)}.$$

$$\begin{aligned}
\rho_{00} \approx\ & \frac{1}{3} + C_1 \left[\frac{1}{3}\omega' \cdot \omega' - (\epsilon_0 \cdot \omega')^2\right] \\
& + C_1 \left[\frac{1}{3}\varepsilon' \cdot \varepsilon' - (\epsilon_0 \cdot \varepsilon')^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\epsilon_0 \cdot \mathbf{B}'_\phi)^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_2 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\epsilon_0 \cdot \mathbf{E}'_\phi)^2\right]
\end{aligned}$$

Rotation and acceleration

$< 10^{-3}$ in heavy-ion collisions

Vector ϕ field

Mean value is zero, but can incorporate large fluctuations

Temperature at hadronization time

- Contribution from classical electromagnetic field to spin alignment is $< 10^{-3}$

XLS, L.Oliva, Q.Wang, PRD 101, 096005 (2020);

- Important features:

  - Cancellation for mixing terms (because of CP and reflection symmetries)

  - All fields appear in squares, spin alignment measures anisotropy of fluctuations in meson's rest frame

    e.g., contribution from $\mathbf{B}'_\phi$ to spin alignment along *y*-direction $\propto (B'_{\phi,y})^2 - \frac{(B'_{\phi,x})^2 + (B'_{\phi,z})^2}{2}$

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689, 2206.05868.

Heavy-ion collisions

INFN
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- Lorentz transformation between lab frame and meson's rest frame

$$\mathbf{B}'_\phi = \gamma \mathbf{B}_\phi - \gamma \mathbf{v} \times \mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$

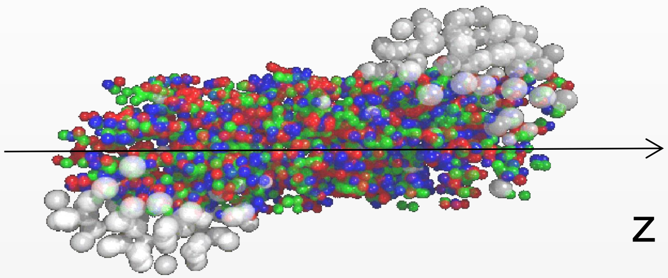
$$\mathbf{E}'_\phi = \gamma \mathbf{E}_\phi + \gamma \mathbf{v} \times \mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v},$$

$$\boldsymbol{\omega}' = \gamma \boldsymbol{\omega} - \gamma \mathbf{v} \times \boldsymbol{\varepsilon} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\omega}}{v^2}\mathbf{v},$$

$$\boldsymbol{\varepsilon}' = \gamma \boldsymbol{\varepsilon} + \gamma \mathbf{v} \times \boldsymbol{\omega} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\varepsilon}}{v^2}\mathbf{v},$$

$$\boxed{\gamma \equiv \frac{E^\phi_{\mathbf{k}}}{m_\phi}, \quad \mathbf{v} \equiv \frac{\mathbf{k}}{E^\phi_{\mathbf{k}}}}$$

Fluctuations along transverse direction are assumed to be different from fluctuations along longitudinal directions



- Assumptions for field fluctuations

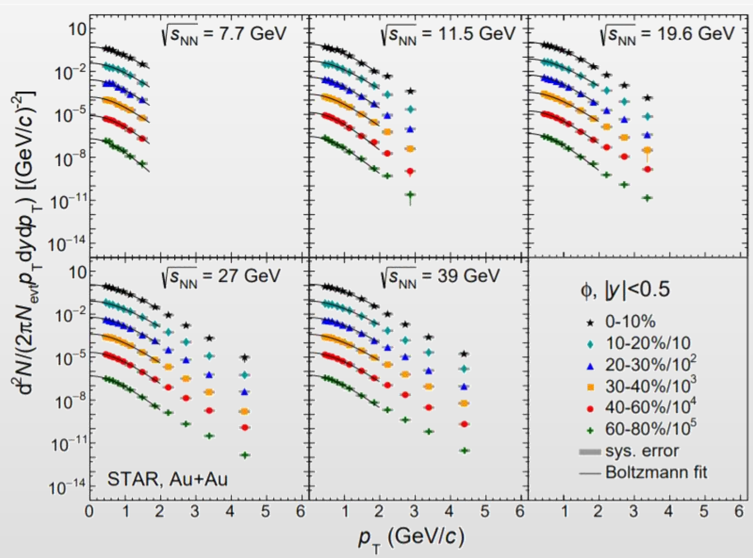
$$\left\langle (\omega_i)^2 \right\rangle = \left\langle (\varepsilon_i)^2 \right\rangle = 0$$

$$\left\langle (g_\phi \mathbf{B}^\phi_{x,y}/T_{\mathrm{h}})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x,y}/T_{\mathrm{h}})^2 \right\rangle \equiv F_T^2$$

$$\left\langle (g_\phi \mathbf{B}^\phi_{z}/T_{\mathrm{h}})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{z}/T_{\mathrm{h}})^2 \right\rangle \equiv F_z^2$$

- Spectra of ϕ meson

$$\frac{dN}{d^2\mathbf{k}_T dy} = \frac{1}{4\pi}\left[1 + 2v_2(k_T)\cos(2\phi)\right]\frac{dN}{k_T dk_T dy}$$



STAR collaboration, PRL 99,112301 (2007);
PRC 79, 064903 (2009); PRC 88,014902 (2013);
PRC 102, 034909(2020).

Fitting experiment datas

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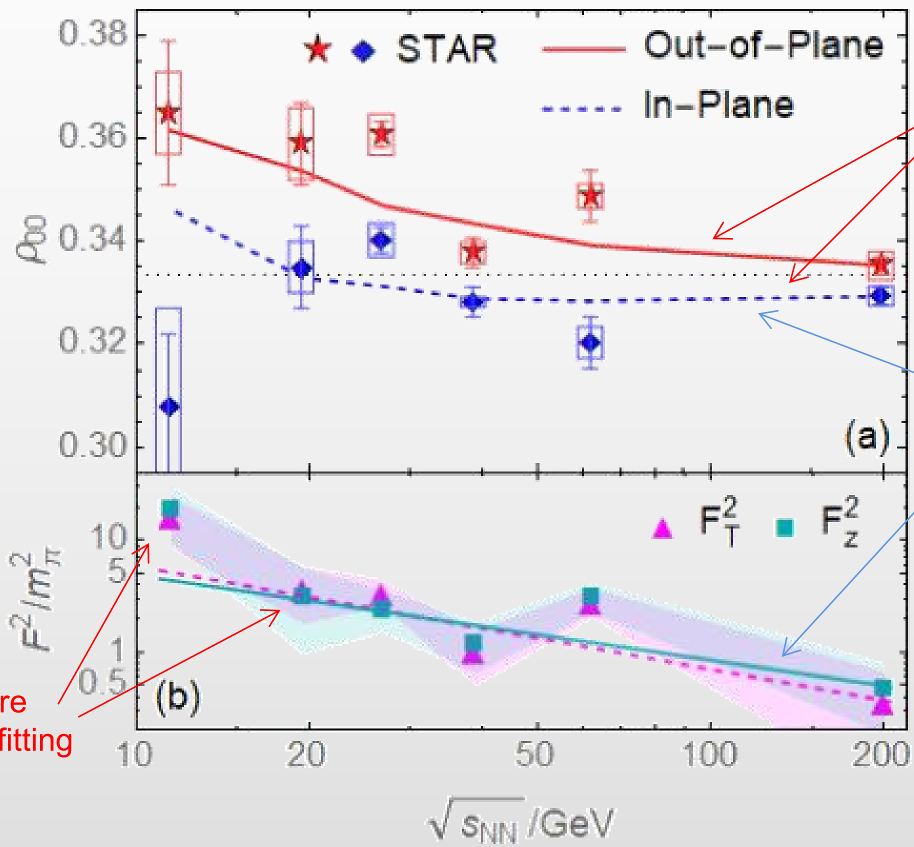
- Taking fluctuations of transverse and longitudinal fields as two independent parameters.

$$\left\langle (g_\phi \mathbf{B}^\phi_{x,y}/T_\mathrm{h})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x,y}/T_\mathrm{h})^2 \right\rangle \equiv F_T^2 \qquad \left\langle (g_\phi \mathbf{B}^\phi_{z}/T_\mathrm{h})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{z}/T_\mathrm{h})^2 \right\rangle \equiv F_z^2$$



Difference induced by v_2

Energy-dependent parameters fitted by

$$\ln(F_T^2/m_\pi^2) = 3.90 - 0.924 \ln \sqrt{s_{\mathrm{NN}}}$$
$$\ln(F_z^2/m_\pi^2) = 3.33 - 0.760 \ln \sqrt{s_{\mathrm{NN}}}$$

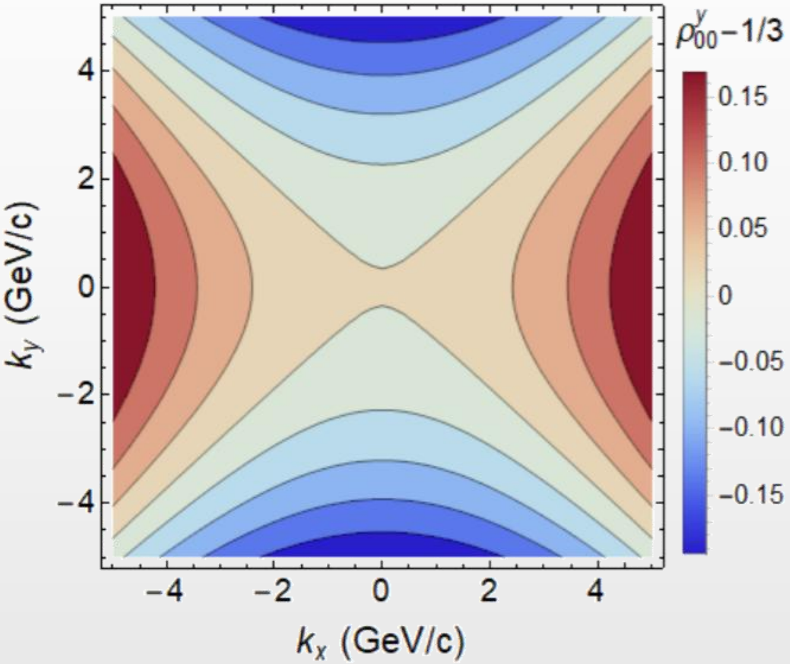
Parameters are evaluated by fitting STAR data

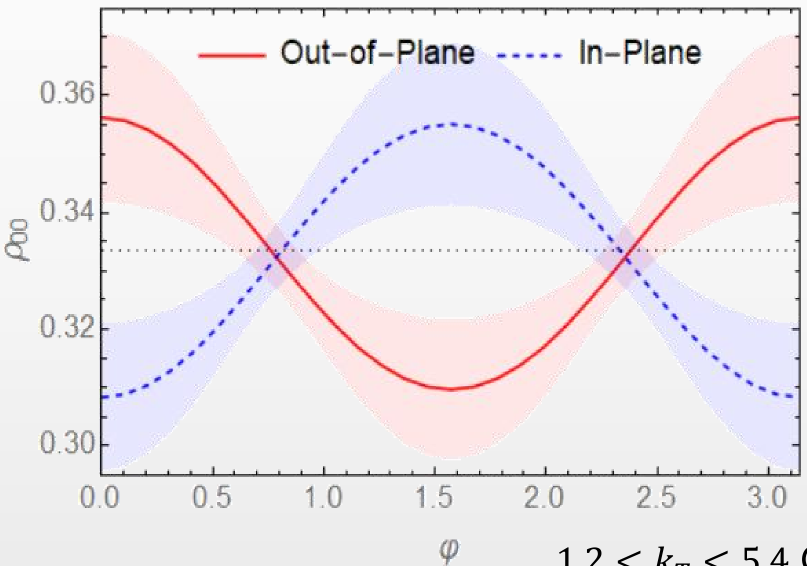
STAR, Nature 614, 244 (2023)

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

Model predictions

- With our theoretical model, we predict transverse momentum and azimuthal angle dependence of ϕ meson's spin alignment, which can be verified by future experiments.





$1.2 < k_T < 5.4$ GeV
Au-Au collisions
at 200 GeV/A

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

k_T dependence

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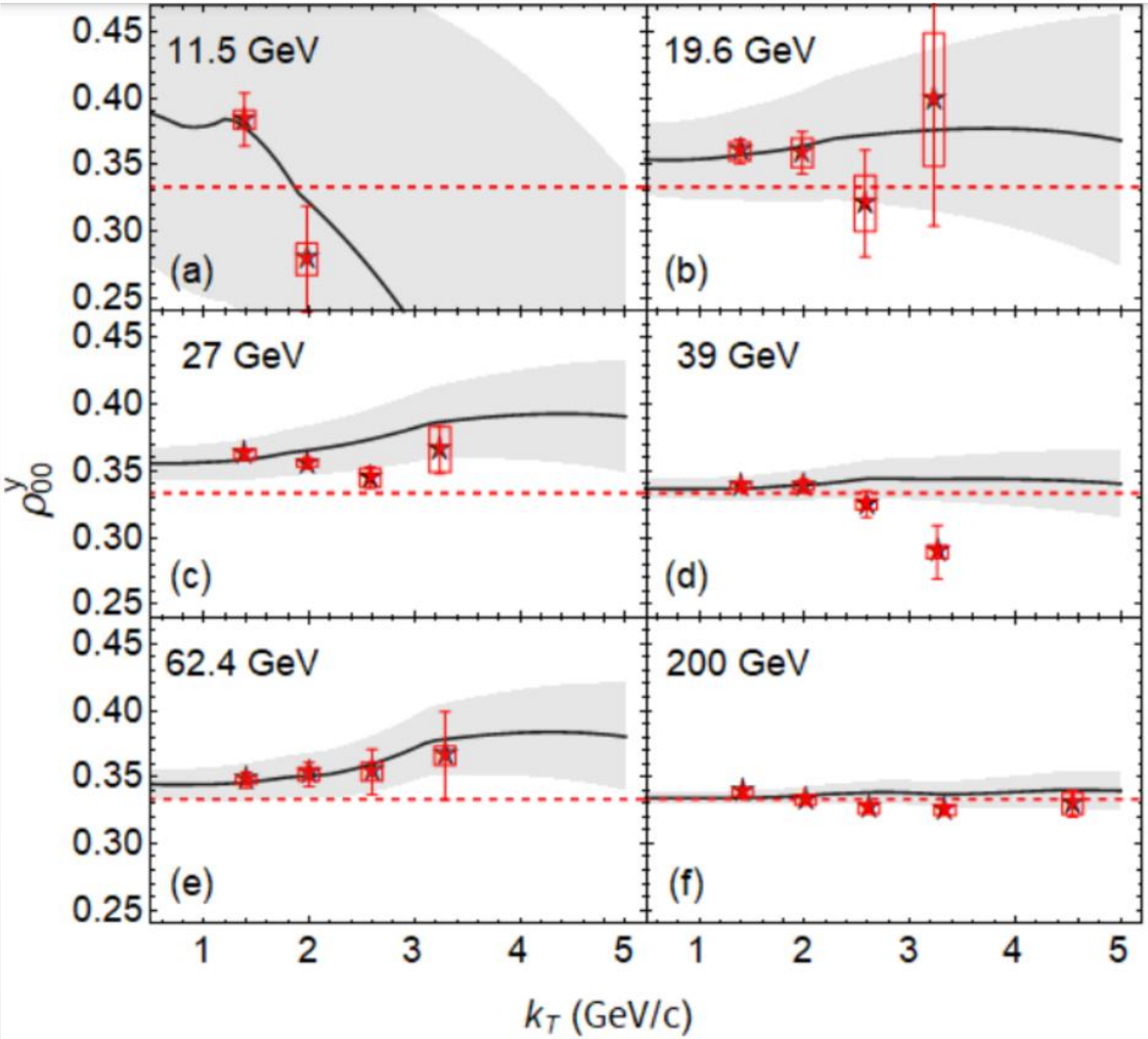

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, arXiv:2205.15689.

Calculated ρ_{00}^{y} as functions of ϕ meson's transverse momentum, in comparison with STAR data for Au+Au collisions in 0-80% centrality region.

STAR, Nature 614, 244 (2023)

Shaded error bands from uncetainties of extracted parameters F_T^2 and F_z^2

Summary

- We derive a relativistic Boltzmann equation for vector mesons with spin
- Spin alignment measures anisotropy of fluctuations in meson's rest frame
- Using two parameters (fluctuations for transverse and longitudinal components of strong ϕ field), we reproduce most of recent STAR data for ϕ meson spin alignment
- With our model, we predict azimuthal angle dependence of ϕ meson's spin alignment

Outlook

- First-principle calculation for fluctuations of ϕ fields
- Spin alignment of heavy quarkonium (J/ψ in Pb-Pb collisions)

  → anisotropy of gluon field fluctuations ?

Related talks

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Experi-
ment

Spin alignment STAR measurements
Subhash Singha 7/15 3:00 p.m.

Vector meson polarization measurements in pp and Pb--Pb collisions with ALICE at the LHC
Xiaozhi Bai 7/19 10:10 a.m. (Parallel Session A)

Tensor polarization and spectral properties of vector meson in QCD medium
Shuai Liu 7/17 4:00 p.m.

Mass splitting and spin alignment for ϕ mesons in a magnetic fielin NJL model
Shuyun Yang 7/18 2:50 p.m. (Parallel Session B)

Holographic spin alignment of J/ψ in anisotropic plasma
Yan-Qing Zhao 7/18 3:43 p.m. (Parallel Session A)

Spin alignment of vector mesons from quark dynamics in a rotating medium
Minghua Wei 7/18 5:55 p.m. (Parallel Session A)

Helicity polarization and vorticity contribution to the spin alignment in hydrodynamic approaches
Cong Yi 7/19 11:50 a.m. (Parallel Session A)

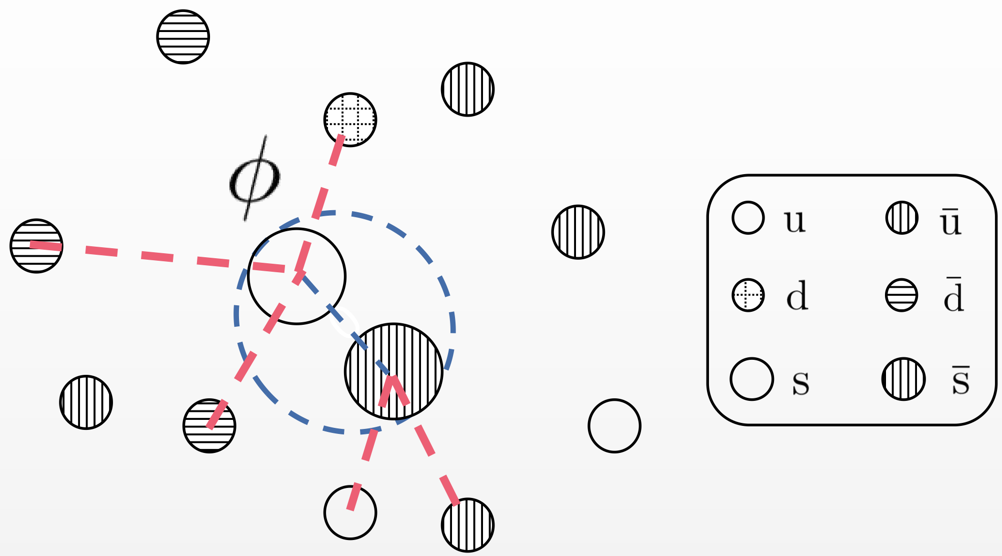
Spin alignment formula for vector bosons at local equilibrium
Zhong-Hua Zhang 7/19 12:05 p.m. (Parallel Session A)

Thank you!

Strong force

- Strong force is a fundamental interaction that acts between quarks.
- At high temperatures, strong interactions are mediated by gluons. (Quantum Chromodynamics)
- At low temperatures, strong interactions are mediated by mesons, proposed by Yukawa in 1935.

H. Yukawa, Proc. Phys. Math. Soc. Jap. 17, 48 (1935)



- Effective Lagrangian for a quark-meson model with scalar and vector mesons.

$$\mathcal{L}_{\text{eff}}(x) = \overline{\psi}(x)\left[i\partial\cdot\gamma - (m_0 + g_\sigma\sigma) - g_V\gamma\cdot V\right]\psi(x) + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_V^2 V_\mu V^\mu - \frac{1}{4}V^{\mu\nu}V_{\mu\nu}$$

strong interactions between $s/\bar{s}$ quarks are mediated by vector ϕ field

→ Short wave-length: quantum fields (particles)

→ Long wave-length: classical fields

Relation with quark polarization

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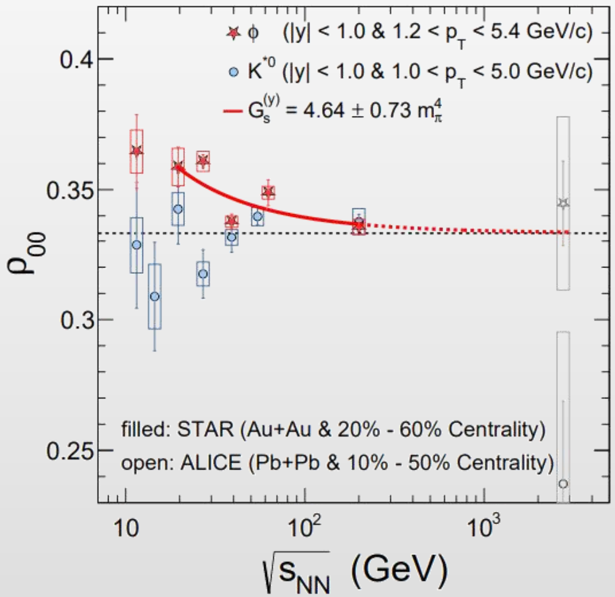
- Contributions from polarizations along all three space directions

$$\rho_{00}^{y} = \frac{1 - \langle P_q^y P_{\bar{q}}^y \rangle + \langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^z P_{\bar{q}}^z \rangle}{3 + \langle P_q^y P_{\bar{q}}^y \rangle + \langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^z P_{\bar{q}}^z \rangle}$$

$$\approx \frac{1}{3} - \frac{4}{9} \left[\langle P_q^y P_{\bar{q}}^y \rangle - \frac{\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^z P_{\bar{q}}^z \rangle}{2} \right]$$

Average over space-time
and meson's wave function

X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)



In order to explain significant deviation from 1/3, we need

1. Local polarizations for quark/antiquark are large enough
   $$\left|\mathbf{P}_{q/\bar{q}}\right| \gtrsim 0.1 \quad \text{if} \quad \rho_{00} - \frac{1}{3} \approx 0.01$$

2. Significant anisotropy
   $$\langle P_q^y P_{\bar{q}}^y \rangle \ \gg \text{or} \ll \ \frac{\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^z P_{\bar{q}}^z \rangle}{2}$$