

# Spin hydrodynamics

Dirk H. Rischke

thanks to

Nils Saß, David Wagner, Xin-li Sheng, Enrico Speranza, Nora Weickgenannt,  
Hannah Elfner, Qun Wang

based on

PRD 106 (2022) 096014, PRD 106 (2022) L091901, PRD 106 (2022) 116021

7<sup>th</sup> Int. Conf. on Chirality, Vorticity, and Magnetic Fields in Heavy-Ion Collisions  
UCAS, Beijing, China, July 15 – 19, 2023

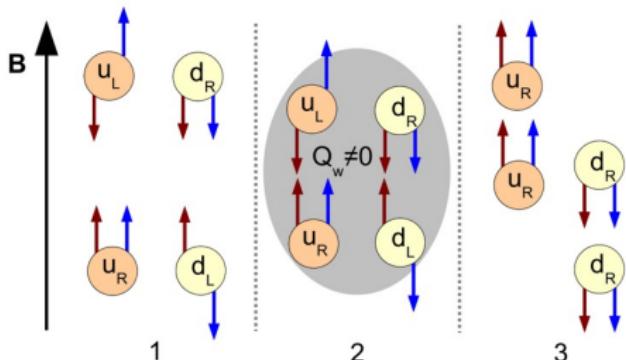


Investigation of chirality-related effects in heavy-ion collisions took off with prediction of  
“Chiral Magnetic Effect” (CME)

L.D. McLerran, D.E. Kharzeev, H.J. Warringa,  
PRD 78 (2008) 074033

K. Fukushima, D.E. Kharzeev, H.J. Warringa,  
NPA 803 (2008) 227

⇒ anomaly-induced generation of electric current  
in magnetic field



## “Chiral Magnetohydrodynamics” (CMHD)

D.E. Kharzeev, H.-U. Yee, PRD 84 (2011) 045025

PHYSICAL REVIEW D 84, 045025 (2011)

Anomalies and time reversal invariance in relativistic hydrodynamics: The second order and higher dimensional formulations

Dmitri E. Kharzeev<sup>1,2,\*</sup> and Ho-Ung Yee<sup>1,†</sup>

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(Received 22 June 2011; published 25 August 2011)

- theory of second-order dissipative conformal anomalous magnetohydrodynamics
- conformal symmetry imposes strong constraints on transport coefficients

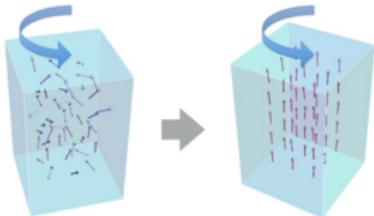


However: for massless particles, spin (helicity) is determined by momentum

→ spin as independent dynamical degree of freedom requires massive particles

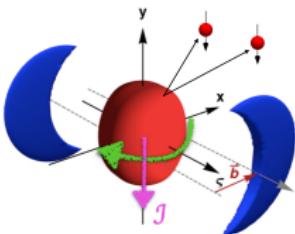
## Condensed matter: **Barnett effect**

- spins align under rotation  
(due to spin-orbit coupling)
- non-vanishing magnetization



## Non-central heavy-ion collisions: strong orbital angular momentum

Barnett effect in heavy-ion collisions?



- hadrons are polarized by vorticity of matter

Z.-T. Liang, X.-N. Wang, PRL 94 (2005) 102301

PRL 94, 102301 (2005)

PHYSICAL REVIEW LETTERS

week ending

18 MARCH 2005

### Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

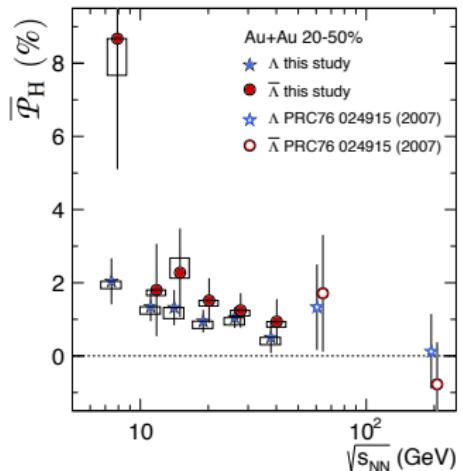
Zuo-Tang Liang<sup>1</sup> and Xin-Nian Wang<sup>2,1</sup>

<sup>1</sup>Department of Physics, Shandong University, Jinan, Shandong 250100, China

<sup>2</sup>Nuclear Science Division, MS 70R0319, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

(Received 25 October 2004; published 14 March 2005)



$\Lambda$  polarization along angular-momentum direction ("global polarization")


L. Adamczyk et al. (STAR), Nature 548 (2017) 62

→ QGP is "most vortical fluid ever observed"

$$\omega \simeq (9 + 1) \times 10^{21} \text{ s}^{-1}$$



For comparison:

- Great Red Spot of Jupiter  $\omega \simeq 10^{-4} \text{ s}^{-1}$
- turbulent flow in superfluid He-II  $\omega \sim 150 \text{ s}^{-1}$
- superfluid nanodroplets  $\omega \sim 10^7 \text{ s}^{-1}$

Assuming local equilibrium on freeze-out hypersurface,  
hydrodynamics describes global polarization quite well . . .

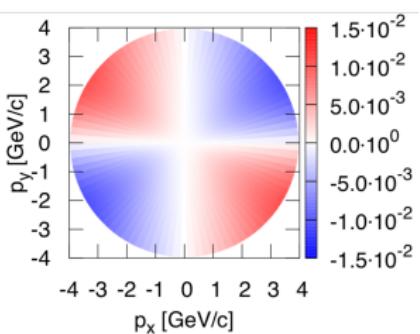
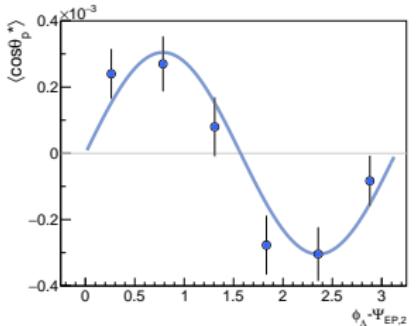
$$\Pi^\mu \sim \epsilon^{\mu\nu\rho\sigma} k_\nu \int_{\Sigma_{\text{f.o.}}} d\Sigma \cdot k \varpi_{\rho\sigma} f_{0k}$$

where  $\varpi_{\rho\sigma} \equiv -\frac{1}{2} (\partial_\rho \beta_\sigma - \partial_\sigma \beta_\rho)$  thermal vorticity,  
 $\beta^\mu \equiv u^\mu / T$ ,  $f_{0k}$  local-equilibrium distribution function

I. Karpenko, F. Becattini, NPA 967 (2017) 764

. . . but fails to describe azimuthal-angle dependence of polarization along the beam direction ("local longitudinal polarization")

F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70 (2020) 395



Recently, further (non-dissipative?) contributions to the polarization were found

F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

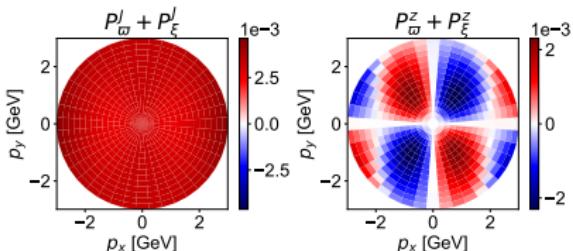
$$\Pi^\mu \sim \epsilon^{\mu\nu\rho\sigma} k_\nu \int_{\Sigma_{\text{f.o.}}} d\Sigma \cdot k \left( \varpi_{\rho\sigma} + 2\hat{t}_\rho \xi_{\sigma\lambda} \frac{k^\lambda}{k_0} \right) f_{0k}$$

where  $\xi_{\sigma\lambda} \equiv \frac{1}{2} (\partial_\sigma \beta_\lambda + \partial_\lambda \beta_\sigma)$  thermal shear tensor

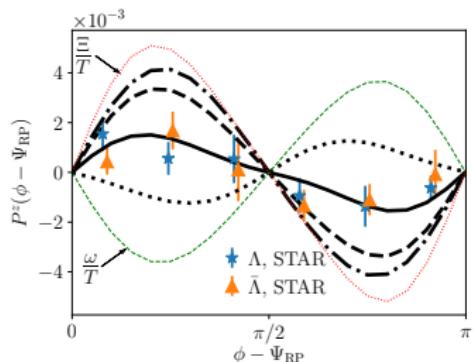
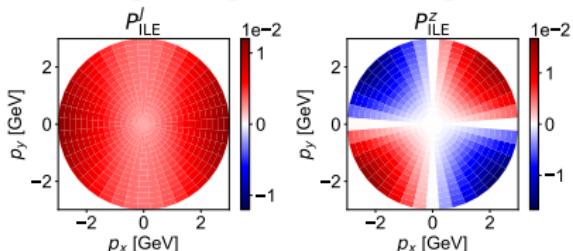
(see also S.Y.F. Liu, Y. Yin, JHEP 07 (2021) 188)

→ this does not quite do the job ...

see talks by F. Becattini, B. Fu, ...



... but neglecting temperature gradients on  $\Sigma_{\text{f.o.}}$  does!



## Observations:

- derivation of formula for polarization ( $\Pi^\mu \sim \varpi_{\rho\sigma}$ ) is strictly valid only for rotating global-equilibrium state  
F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Annals Phys. 338 (2013) 32  
⇒ application of formula requires (infinitely) fast equilibration of spin degrees of freedom (relative to timescale of collision)
- dissipative effects influence (almost) all other observables in a heavy-ion collision, even seem to appear explicitly in new term  $\sim \xi_{\sigma\lambda}$  in formula for polarization

## ⇒ Questions:

- (I) How fast do spin degrees of freedom equilibrate?
- (II) How is polarization influenced by dissipative effects?

⇒ requires a theory of second-order dissipative spin hydrodynamics!

N. Weickgenannt, D. Wagner, E. Speranza, DHR,  
PRD 106 (2022) 096014, PRD 106 (2022) L091901

Remark: for tensor polarization and spin-1 particles, see D. Wagner's talk

## Particle number conservation

$$\partial_\mu N^\mu = 0$$

where  $N^\mu$  particle four-current

## Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

where  $T^{\mu\nu}$  energy-momentum tensor

## Angular-momentum conservation

$$\partial_\mu J^{\mu,\nu\lambda} = 0$$

where  $J^{\mu,\nu\lambda}$  angular-momentum tensor

with  $J^{\mu,\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu} + \hbar S^{\mu,\nu\lambda}$  and energy-momentum conservation:

## Equation of motion for spin tensor

$$\hbar \partial_\mu S^{\mu,\nu\lambda} = T^{[\lambda\nu]}$$

where  $a^{[\lambda} b^{\nu]} \equiv a^\lambda b^\nu - a^\nu b^\lambda$

## Particle number conservation

$$\partial_\mu N^\mu = 0$$

1 equation, 4 unknowns

## Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

4 equations, (at least) 10 unknowns

## Angular-momentum conservation

$$\partial_\mu J^{\mu,\nu\lambda} = 0$$

with  $J^{\mu,\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu} + \hbar S^{\mu,\nu\lambda}$  and energy-momentum conservation:

## Equation of motion for spin tensor

$$\hbar \partial_\mu S^{\mu,\nu\lambda} = T^{[\lambda\nu]}$$

6 equations, 24 unknowns

where  $a^{[\lambda} b^{\nu]} \equiv a^\lambda b^\nu - a^\nu b^\lambda$

Assume local equilibrium

Single-particle distribution function (to order  $\mathcal{O}(\hbar)$ )



$$f_{\text{eq}, \mathbf{k}} = f_0 \mathbf{k} \left( 1 + \frac{\hbar}{4} \Omega_{\mu\nu} \Sigma_{\mathbf{k}}^{\mu\nu} \right), \quad f_0 \mathbf{k} = \exp(\alpha - \beta \cdot k)$$

where

- $\alpha \equiv \frac{\mu}{T}$  Lagrange multiplier for particle-number conservation
- $\beta^\mu \equiv \frac{u^\mu}{T}$  Lagrange multiplier for energy-momentum conservation,  $u^\mu$  fluid 4-velocity
- $\Omega_{\mu\nu}$  spin potential, Lagrange multiplier for angular-momentum conservation
- $\Sigma_{\mathbf{k}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathbf{s}_\beta$  dipole-moment tensor of particle with mass  $m$ , (on-shell) 4-momentum  $k_\alpha$ , and spin 4-vector  $\mathbf{s}_\beta$

Assume local equilibrium

Single-particle distribution function (to order  $\mathcal{O}(\hbar)$ )



$$f_{\text{eq}, \mathbf{k}} = f_0 \mathbf{k} \left( 1 + \frac{\hbar}{4} \Omega_{\mu\nu} \Sigma_{\mathbf{k}}^{\mu\nu} \right), \quad f_0 \mathbf{k} = \exp(\alpha - \beta \cdot k)$$

where

- $\alpha \equiv \frac{\mu}{T}$  Lagrange multiplier for particle-number conservation  
1 parameter
- $\beta^\mu \equiv \frac{u^\mu}{T}$  Lagrange multiplier for energy-momentum conservation,  $u^\mu$  fluid 4-velocity  
4 parameters
- $\Omega_{\mu\nu}$  spin potential, Lagrange multiplier for angular-momentum conservation  
6 parameters
- $\Sigma_{\mathbf{k}}^{\mu\nu} \equiv -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathbf{s}_\beta$  dipole-moment tensor of particle with mass  $m$ ,  
(on-shell) 4-momentum  $k_\alpha$ , and spin 4-vector  $\mathbf{s}_\beta$

Extend phase space by spin degrees of freedom (for more details, see D. Wagner's talk):

$$dK \equiv \frac{d^3 k}{(2\pi)^3 k^0} \longrightarrow d\Gamma \equiv dK \, dS(k)$$

with  $dS(k) \equiv \sqrt{\frac{k^2}{3\pi^2}} d^4 \mathbf{s} \delta(\mathbf{k} \cdot \mathbf{s}) \delta(\mathbf{s} \cdot \mathbf{s} + 3)$ ,

such that  $\int dS(k) = 2$ ,  $\int dS(k) s^\mu = 0$ ,  $\int dS(k) s^\mu s^\nu = -2 \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right)$

### Fluid-dynamical currents

$$\begin{aligned} N^\mu &\equiv \langle k^\mu \rangle \\ T^{\mu\nu} &\equiv \langle k^\mu k^\nu \rangle + \mathcal{O}(\hbar^2) \\ S^{\mu,\nu\lambda} &\equiv \left\langle k^\mu \left( \frac{1}{2} \Sigma_{\mathbf{k}\mathbf{s}}^{\nu\lambda} - \frac{\hbar}{4m^2} k^{[\nu} \partial^{\lambda]} \right) \right\rangle + \mathcal{O}(\hbar^2) \end{aligned}$$

where  $\langle \cdots \rangle \equiv \int d\Gamma \cdots f(x, k, \mathbf{s})$

$\Rightarrow$  for  $\langle \cdots \rangle_{\text{eq}} \equiv \int d\Gamma \cdots f_{\text{eq}, \mathbf{k}\mathbf{s}}$   $\Rightarrow$  equations of motion are closed!

see, e.g., W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709

## Dissipative spin hydrodynamics

- ⇒ provide additional equations of motion
- ⇒ start from underlying microscopic theory, apply method of moments  
see, e.g., G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047

From equation of motion for the Wigner function, derive to first order in  $\hbar$ :

N. Weickgenannt, E. Speranza, X.-I. Sheng, Q. Wang, DHR,

PRL 127 (2021) 052301, PRD 104 (2021) 016022      for more details, see D. Wagner's talk

## Boltzmann equation for spin-1/2 particles with nonlocal collision term

$$\mathbf{k} \cdot \partial f(\mathbf{x}, \mathbf{k}, \textcolor{brown}{s}) = \mathfrak{C}[f]$$

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' d\bar{S}(\mathbf{k}) \mathcal{W}_{\mathbf{k}\mathbf{k}' \rightarrow \mathbf{k}_1 \mathbf{k}_2}^{\bar{s}\bar{s}' \rightarrow s_1 s_2} (2\pi\hbar)^4 \delta^{(4)}(\mathbf{k} + \mathbf{k}' - \mathbf{k}_1 - \mathbf{k}_2)$$

$$\times [f(\mathbf{x} + \Delta_1 - \Delta, \mathbf{k}_1, \textcolor{brown}{s}_1)f(\mathbf{x} + \Delta_2 - \Delta, \mathbf{k}_2, \textcolor{brown}{s}_2) - f(\mathbf{x}, \mathbf{k}, \bar{s})f(\mathbf{x} + \Delta' - \Delta, \mathbf{k}', \textcolor{brown}{s}')]$$

where nonlocal position shift     $\Delta^\mu = \frac{\hbar}{2m(\mathbf{k} \cdot \hat{\mathbf{t}} + m)} \epsilon^{\mu\nu\alpha\beta} \hat{t}_\nu k_\alpha \textcolor{brown}{s}_\beta$

↔ Berry connection! see, e.g., M. Stone, V. Dwivedi, T. Zhou, PRD 91 (2015) 025004

Note:  $\Delta \sim \frac{\hbar}{m}$  Compton wavelength! ⇒ quantum length scale!

Nonlocal collisions: allow mutual conversion of orbital angular momentum and spin



Position shift  $\Delta^\mu$  depends on frame vector  $\hat{t}_\nu = (1, 0, 0, 0)$

- ⇒ does not transform covariantly!
- ⇒ breaking of Lorentz covariance due to (invalid) approximations made in calculation
- ⇒ corrected in D. Wagner, N. Weickgenannt, DHR, PRD 106 (2022) 116021

For NJL-type 4-fermion interaction  $\sim (\bar{\psi} \Gamma^{(a)} \psi)^2$

covariant position shift

$$\Delta^\mu = \frac{\hbar}{4m} \frac{\text{Im} \left\{ \text{Tr} \left[ h \gamma^\mu \Gamma^{(b)} h_2 \Gamma^{(a)} \right] \text{Tr} \left[ \Gamma^{(b)} h_1 \Gamma^{(a)} h' \right] - \text{Tr} \left[ h \gamma^\mu \Gamma^{(b)} h_1 \Gamma^{(a)} h' \Gamma^{(b)} h_2 \Gamma^{(a)} \right] \right\}}{\text{Re} \left\{ \text{Tr} \left[ h \Gamma^{(d)} h_2 \Gamma^{(c)} \bar{h} \right] \text{Tr} \left[ \Gamma^{(d)} h_1 \Gamma^{(c)} h' \right] - \text{Tr} \left[ h \Gamma^{(d)} h_1 \Gamma^{(c)} h' \Gamma^{(d)} h_2 \Gamma^{(c)} \bar{h} \right] \right\}}$$

where  $h \equiv h(k, \mathfrak{s}) \equiv \frac{1}{4m} (\mathbb{1} + \gamma_5 \not{s})(\not{k} + m)$

and similarly for  $(k_1, \mathfrak{s}_1), (k_2, \mathfrak{s}_2), (k', \mathfrak{s}'), (k, \bar{\mathfrak{s}})$

Usually, local-equilibrium distribution function  $f_{\text{eq}}$  is defined by condition

Local equilibrium

$$\mathcal{C}[f_{\text{eq}}] \equiv 0$$

However, as shown in N. Weickgenannt, E. Speranza, X.-I. Sheng, Q. Wang, DHR, PRL 127 (2021) 052301, PRD 104 (2021) 016022, nonlocal collision term vanishes only in

Global equilibrium

$$\alpha = \text{const.}$$

$$\partial^\mu \beta^\nu + \partial^\nu \beta^\mu = 0$$

$$\Omega_{\mu\nu} = \varpi_{\mu\nu} = \text{const.}$$

⇒ appears too restrictive: nonlocality  $\Delta \lesssim r_{\text{int}} \ll \lambda_{\text{mfp}} \ll L_{\text{hydro}}$

⇒ (quantum) nonlocality scale  $\Delta$  much smaller than hydrodynamic scale  $L_{\text{hydro}}$

Generalized local equilibrium



$$\mathcal{C}[f_{\text{eq}}] \sim \mathcal{O}(\Delta/L_{\text{hydro}})$$

Define hydrodynamic scale  $L_{\text{hydro}}$  by

$$\frac{1}{m} k \cdot \partial f_{\text{eq}, \mathbf{k}} \sim \frac{1}{L_{\text{hydro}}} f_{\text{eq}, \mathbf{k}}$$

$$\begin{aligned}\partial_\mu \alpha &\sim \mathcal{O}(L_{\text{hydro}}^{-1}) \\ \partial^\mu \beta^\nu + \partial^\nu \beta^\mu &\sim \mathcal{O}((k L_{\text{hydro}})^{-1}) \\ \partial_\lambda \Omega_{\mu\nu} &\sim \mathcal{O}(L_{\text{hydro}}^{-1})\end{aligned}$$

using conservation of total angular momentum  $J^{\mu\nu} \equiv \Delta^{[\mu} k^{\nu]} + \frac{\hbar}{2} \Sigma_{\mathbf{k}}^{\mu\nu}$  in binary collisions, order  $\mathcal{O}(\hbar)$  contribution to nonlocal collision term:

$$\sim \frac{\hbar}{4} \Omega_{\mu\nu} \Sigma_{\mathbf{k}}^{\mu\nu} + \Delta^{\mu} \partial_\mu (\alpha - \beta^\nu k^\nu) = \frac{1}{2} \Delta^{[\mu} k^{\nu]} (\varpi_{\mu\nu} - \Omega_{\mu\nu}) + \mathcal{O}(\Delta/L_{\text{hydro}})$$

for generalized local equilibrium:  $\Omega_{\mu\nu} \equiv \varpi_{\mu\nu} + \mathcal{O}((k L_{\text{hydro}})^{-1})$

consistent, as in global equilibrium  $L_{\text{hydro}} \rightarrow \infty$  and thus  $\Omega_{\mu\nu} \rightarrow \varpi_{\mu\nu}$

Usually, all gradients of fluid-dynamical quantities are  $\mathcal{O}(L_{\text{hydro}}^{-1})$

However, global-equilibrium conditions do not restrict value of thermal vorticity  $\varpi_{\mu\nu}$   
 see, e.g., F. Becattini, L. Tinti, Annals Phys. 325 (2010) 1566

⇒ vorticity does not follow usual power counting,  $\varpi_{\mu\nu} \not\sim \mathcal{O}((kL_{\text{hydro}})^{-1})$

⇒ define scale  $\ell_{\text{vort}}$  set by vorticity:  $\varpi_{\mu\nu} \sim \mathcal{O}((k\ell_{\text{vort}})^{-1})$

In principle,  $\ell_{\text{vort}}$  can take any value from  $\ell_{\text{vort}} \ll L_{\text{hydro}}$  to  $\ell_{\text{vort}} \sim L_{\text{hydro}}$

However, in order for  $\hbar$ -expansion to apply:  $\hbar\Omega_{\mu\nu}\Sigma_{\mathbf{k}s}^{\mu\nu} \sim \frac{\hbar}{m}\varpi_{\mu\nu}\epsilon^{\mu\nu\alpha\beta}k_{\alpha}s_{\beta} \sim \frac{\Delta}{\ell_{\text{vort}}} \ll 1$

⇒  $\ell_{\text{vort}}$  cannot be arbitrarily small (as in global equilibrium)

⇒ remember  $\Delta \lesssim r_{\text{int}} \ll \lambda_{\text{mfp}} \ll L_{\text{hydro}}$  ⇒  $\ell_{\text{vort}}$  could be as small as  $\lambda_{\text{mfp}}!$

⇒ for the sake of simplicity assume

$$\frac{\Delta}{\ell_{\text{vort}}} \sim \frac{\lambda_{\text{mfp}}}{L_{\text{hydro}}} \equiv Kn \ll 1$$

Extend method of moments developed in G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047 by spin degrees of freedom

## Single-particle distribution function

$$f(x, k, \mathbf{s}) = f_{\text{eq}, \mathbf{k}\mathbf{s}} + \delta f_{\mathbf{k}\mathbf{s}}$$

$$\delta f_{\mathbf{k}\mathbf{s}} = f_{0\mathbf{k}} \sum_{\ell=0}^{\infty} \sum_{n \in \mathbb{S}_{\ell}} \mathcal{H}_{\mathbf{k}n}^{(\ell)} \left[ \rho_n^{\mu_1 \dots \mu_{\ell}} - \tau_n^{\langle \mu \rangle, \mu_1 \dots \mu_{\ell}} \left( g_{\mu\nu} - \frac{k_{\langle \mu \rangle} u_{\nu}}{E_{\mathbf{k}}} \right) \mathbf{s}^{\nu} \right] k_{\langle \mu_1} \dots k_{\mu_{\ell} \rangle}$$

where

- $k_{\langle \mu_1 \dots k_{\mu_{\ell}} \rangle}$  irreducible tensors
- $\rho_n^{\mu_1 \dots \mu_{\ell}} \equiv \left\langle E_{\mathbf{k}}^n k^{\langle \mu_1} \dots k^{\mu_{\ell} \rangle} \right\rangle_{\delta}$  irreducible moments,  $\langle \dots \rangle_{\delta} \equiv \langle \dots \rangle - \langle \dots \rangle_{\text{eq}}$
- $\tau_n^{\mu, \mu_1 \dots \mu_{\ell}} \equiv \left\langle \mathbf{s}^{\mu} E_{\mathbf{k}}^n k^{\langle \mu_1} \dots k^{\mu_{\ell} \rangle} \right\rangle_{\delta}$  spin moments
- $E_{\mathbf{k}} \equiv k \cdot u$  energy of particle in fluid rest frame
- $A^{\langle \mu \rangle} \equiv \Delta^{\mu\nu} A_{\nu}$ , with  $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^{\mu} u^{\nu}$  projector onto 3-space orthogonal to  $u^{\mu}$
- $A^{\langle \mu_1 \dots \mu_{\ell} \rangle} \equiv \Delta_{\nu_1 \dots \nu_{\ell}}^{\mu_1 \dots \mu_{\ell}} A^{\nu_1 \dots \nu_{\ell}}$ ,  
 $\Delta_{\nu_1 \dots \nu_{\ell}}^{\mu_1 \dots \mu_{\ell}}$  symmetric, traceless rank- $2\ell$  projection operators built from  $\Delta^{\mu\nu}$
- $\mathcal{H}_{\mathbf{k}n}^{(\ell)}$  polynomials in  $E_{\mathbf{k}}$  of rank  $N_{\ell}$

Inserting  $f(x, k, \mathfrak{s}) = f_{\text{eq}, \mathbf{k}\mathfrak{s}} + \delta f_{\mathbf{k}\mathfrak{s}}$  into definition of spin tensor

### Spin tensor



$$S^{\mu, \nu, \lambda} = u^\mu \tilde{\mathfrak{N}}^{\nu, \lambda} + \tilde{\mathfrak{P}}^{\langle \mu \rangle, \nu, \lambda} + u_\alpha \tilde{\mathfrak{H}}^{\mu, \nu, \lambda, \alpha} + u^\mu \tilde{\mathfrak{H}}_\alpha^{\nu, \lambda, \alpha} + \tilde{\mathfrak{Q}}^{\mu, \nu, \lambda} - \frac{\hbar}{4m^2} \partial^{[\lambda} T^{\nu]\mu}$$

where

- $\tilde{\mathfrak{N}}^{\nu, \lambda} \equiv \epsilon^{\nu, \lambda, \alpha, \beta} \mathfrak{N}_{\alpha\beta}$ ,  
with  $\mathfrak{N}^{\alpha\beta} \equiv -\frac{1}{2m} u^\alpha \left[ \left\langle E_{\mathbf{k}}^2 \mathfrak{s}^\beta \right\rangle_{\text{eq}} + \tau_2^\beta \right]$  spin energy tensor
- $\tilde{\mathfrak{P}}^{\mu, \nu, \lambda} \equiv \epsilon^{\mu, \nu, \lambda, \alpha} \mathfrak{P}_\alpha$ ,  
with  $\mathfrak{P}^\alpha \equiv -\frac{1}{6m} \left[ \left\langle (m^2 - E_{\mathbf{k}}^2) \mathfrak{s}^\alpha \right\rangle_{\text{eq}} + m^2 \tau_0^\alpha - \tau_2^\alpha \right]$  spin pressure vector
- $\tilde{\mathfrak{H}}^{\mu, \nu, \lambda, \alpha} \equiv \epsilon^{\nu, \lambda, \alpha, \beta} \mathfrak{H}^\mu{}_\beta$ ,  
with  $\mathfrak{H}^{\mu\beta} \equiv -\frac{1}{2m} \left[ \left\langle E_{\mathbf{k}} k^{\langle \mu} \mathfrak{s}^{\beta \rangle} \right\rangle_{\text{eq}} + \tau_1^{\beta, \mu} \right]$  spin diffusion tensor
- $\tilde{\mathfrak{Q}}^{\mu, \nu, \lambda} \equiv \epsilon^{\nu, \lambda, \alpha, \beta} \mathfrak{Q}^\mu{}_{\alpha\beta}$ ,  
with  $\mathfrak{Q}^{\mu\alpha\beta} \equiv -\frac{1}{2m} \left[ \left\langle k^{\langle \mu} k^{\alpha \rangle} \mathfrak{s}^\beta \right\rangle_{\text{eq}} + \tau_0^{\beta, \mu\alpha} \right]$  spin stress tensor

Define local-equilibrium state via

### Landau matching conditions

$$\begin{aligned} N^\mu u_\mu &= N_{\text{eq}}^\mu u_\mu \\ T^{\mu\nu} u_\nu &= T_{\text{eq}}^{\mu\nu} u_\nu \\ J^{\mu,\nu\lambda} u_\mu &= J_{\text{eq}}^{\mu,\nu\lambda} u_\mu \end{aligned}$$

- ⇒ relates  $\alpha$ ,  $\beta^\mu$ , and  $\Omega_{\mu\nu}$  in  $f_{\text{eq},k_5}$  to fluid-dynamical variables
- ⇒ determine  $\alpha$ ,  $\beta^\mu$ , and  $\Omega_{\mu\nu}$  via conservation laws!
- ⇒ still need to derive equations of motion for dissipative currents  $\Pi$ ,  $n^\mu$ , and  $\pi^{\mu\nu}$ , as well as spin moments  $\tau_0^\alpha$ ,  $\tau_2^\alpha$ ,  $\tau_1^{\beta,\mu}$ , and  $\tau_0^{\beta,\mu\alpha}$  appearing in  $S^{\mu,\lambda\nu}$

Equations of motion for standard dissipative currents  $\Pi$ ,  $n^\mu$ , and  $\pi^{\mu\nu}$

- ⇒ see G.S. Denicol, H. Niemi, E. Molnár, DHR, PRD 85 (2012) 114047
- ⇒ relaxation-type equations, e.g.,  $\dot{\Pi} + \frac{1}{\tau_\Pi} \Pi = \dots$ , with  $\dot{\Pi} \equiv u^\mu \partial_\mu \Pi$

Take spin moments of Boltzmann equation:

## Equations of motion for spin moments

$$\Rightarrow \dot{\tau}_n^{\langle\mu\rangle,\langle\mu_1\cdots\mu_\ell\rangle} - \mathfrak{C}_{n-1}^{\langle\mu\rangle,\mu_1\cdots\mu_\ell} = \dots$$

$$\mathfrak{C}_{n-1}^{\mu,\mu_1\cdots\mu_\ell} = \int d\Gamma E_{\mathbf{k}}^{n-1} k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \mathfrak{s}^\mu \mathfrak{C}[f]$$

## Linearized collision integral

$$\mathfrak{C}_{n-1}^{\mu,\mu_1\cdots\mu_\ell} = \mathfrak{C}_{n-1,\text{local}}^{\mu,\mu_1\cdots\mu_\ell} + \mathfrak{C}_{n-1,\text{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell}$$

$$\mathfrak{C}_{n-1,\text{local}}^{\mu,\mu_1\cdots\mu_\ell} = - \sum_{r \in \mathbb{S}_\ell} B_{nr}^{(\ell)} \tau_r^{\mu,\mu_1\cdots\mu_\ell}$$

$$\mathfrak{C}_{n-1,\text{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell} = \int d\Gamma_1 d\Gamma_2 d\Gamma d\Gamma' \mathcal{W}_{\mathbf{k}\mathbf{k}' \rightarrow \mathbf{k}_1 \mathbf{k}_2}^{\mathfrak{s}\mathfrak{s}' \rightarrow \mathfrak{s}_1 \mathfrak{s}_2} E_{\mathbf{k}}^{n-1} f_{0\mathbf{k}} f_{0\mathbf{k}'}$$

$$\times k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \mathfrak{s}^\mu \left[ \frac{\hbar}{4} (\varpi_{\alpha\beta} - \Omega_{\alpha\beta}) \Sigma_{\mathbf{k}\mathfrak{s}}^{\alpha\beta} + \xi_{\alpha\beta} \Delta^\alpha k^\beta \right]$$

- $\mathfrak{C}_{n-1,\text{local}}^{\mu,\mu_1\cdots\mu_\ell} \Rightarrow$  inverting  $B_{nr}^{(\ell)}$  yields relaxation times
- $\mathfrak{C}_{n-1,\text{nonlocal}}^{\mu,\mu_1\cdots\mu_\ell} \Rightarrow$  gives rise to Navier-Stokes terms

Infinite set of moment equations needs to be truncated

⇒ lowest-order truncation:

14 standard fluid-dynamical moments + 24 moments for components of spin tensor

⇒ (14+24)-moment approximation

Independent spin moments:  $p^\mu \equiv \tau_0^{\langle\mu\rangle}$ ,  $\mathfrak{z}^{\mu\nu} \equiv \tau_1^{\langle\mu\rangle,\nu} + \tau_1^{\langle\nu\rangle,\mu}$ ,  $q^{\mu\nu\lambda} \equiv \tau_0^{\langle\mu\rangle,\nu\lambda}$

3      +      6                  +                  15 components

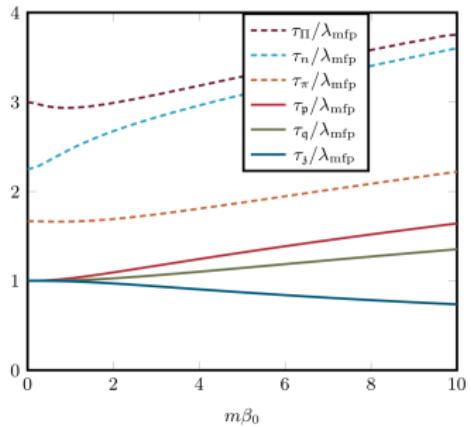
### Equations of motion for independent spin moments

$$\Rightarrow \begin{aligned} \tau_p \dot{p}^{\langle\mu\rangle} + p^\mu &\sim \epsilon^{\mu\nu\alpha\beta} (\varpi_{\alpha\beta} - \Omega_{\alpha\beta}) u_\nu + \dots \\ \tau_{\mathfrak{z}} \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\mu\nu} &\sim \dots \\ \tau_q \dot{q}^{\langle\mu\rangle\langle\nu\lambda\rangle} + q^{\mu\nu\lambda} &\sim \frac{1}{T} \sigma_\alpha^{\langle\nu} \epsilon^{\lambda\rangle\mu\alpha\beta} u_\beta + \dots \end{aligned}$$

where  $\sigma^{\mu\nu} \equiv \partial^{\langle\mu} u^{\nu\rangle}$  fluid shear tensor

for more details, see D. Wagner's talk

## Spin relaxation times



- ⇒ spin relaxation times of the same order (but somewhat smaller) than relaxation times for  $\Pi, n^\mu, \pi^{\mu\nu}$
- ⇒ spin degrees of freedom equilibrate (i.e., approach their Navier-Stokes values) as fast (or even faster) than  $\Pi, n^\mu, \pi^{\mu\nu}$
- ⇒ answers Question (I)

## Pauli-Lubanski vector (spin polarization vector!) in Navier-Stokes limit

$$\begin{aligned} \Pi_{\text{NS}}^\mu &\sim \int_{\Sigma_{\text{f.o.}}} d\Sigma \cdot k f_{0k} \left\{ \epsilon^{\mu\nu\rho\sigma} k_\nu \Omega_{\rho\sigma} + \left( \delta_\nu^\mu - \frac{u^\mu k_{\langle\nu\rangle}}{E_k} \right) \right. \\ &\times \left. \left[ \kappa_p \epsilon^{\nu\rho\alpha\beta} (\Omega_{\alpha\beta} - \varpi_{\alpha\beta}) u_\rho + \frac{\kappa_q}{T} \sigma_\alpha^{\langle\rho} \epsilon^{\sigma\rangle\nu\alpha\beta} u_\beta k_{\langle\rho} k_{\sigma\rangle} \right] \right\} \end{aligned}$$

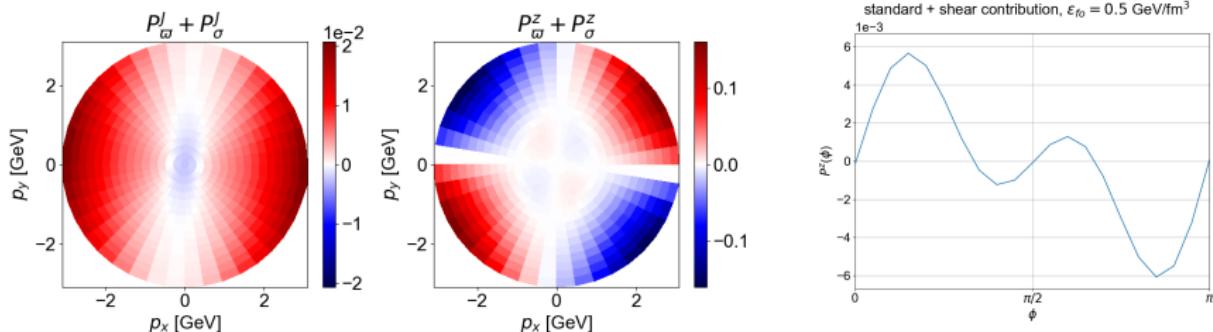
- ⇒ novel dissipative(?) corrections  $\sim \Omega_{\alpha\beta} - \varpi_{\alpha\beta}$  and  $\sigma_{\alpha\beta}$  ⇒ answers Question (II)

Note: no temperature gradients in term  $\sim \frac{\kappa_q}{T} \sigma_\alpha^{\langle\rho} \epsilon^{\sigma\rho\alpha\beta} u_\beta k_{\langle\rho} k_\sigma \rangle!!$

→ just what we need according to

F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

also S.Y.F. Liu, Y. Yin, JHEP 07 (2021) 188 features no temperature gradients



N. Saß, D. Wagner, H. Elfner, DHR, in preparation

qualitative agreement with experimental data!

still need to dynamically solve spin hydrodynamics, to account for  $\Omega_{\alpha\beta} \neq \varpi_{\alpha\beta}$

- Starting from Boltzmann equation with nonlocal collision term, and using method of moments, derived equations of motion of relativistic second-order dissipative spin hydrodynamics in (14+24)-moment approximation
- Spin degrees of freedom relax as fast as usual dissipative quantities
- Polarization vector is influenced by dissipative(?) corrections
- Corrections could potentially solve sign problem of longitudinal polarization!