The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Instabilities in chiral plasmas

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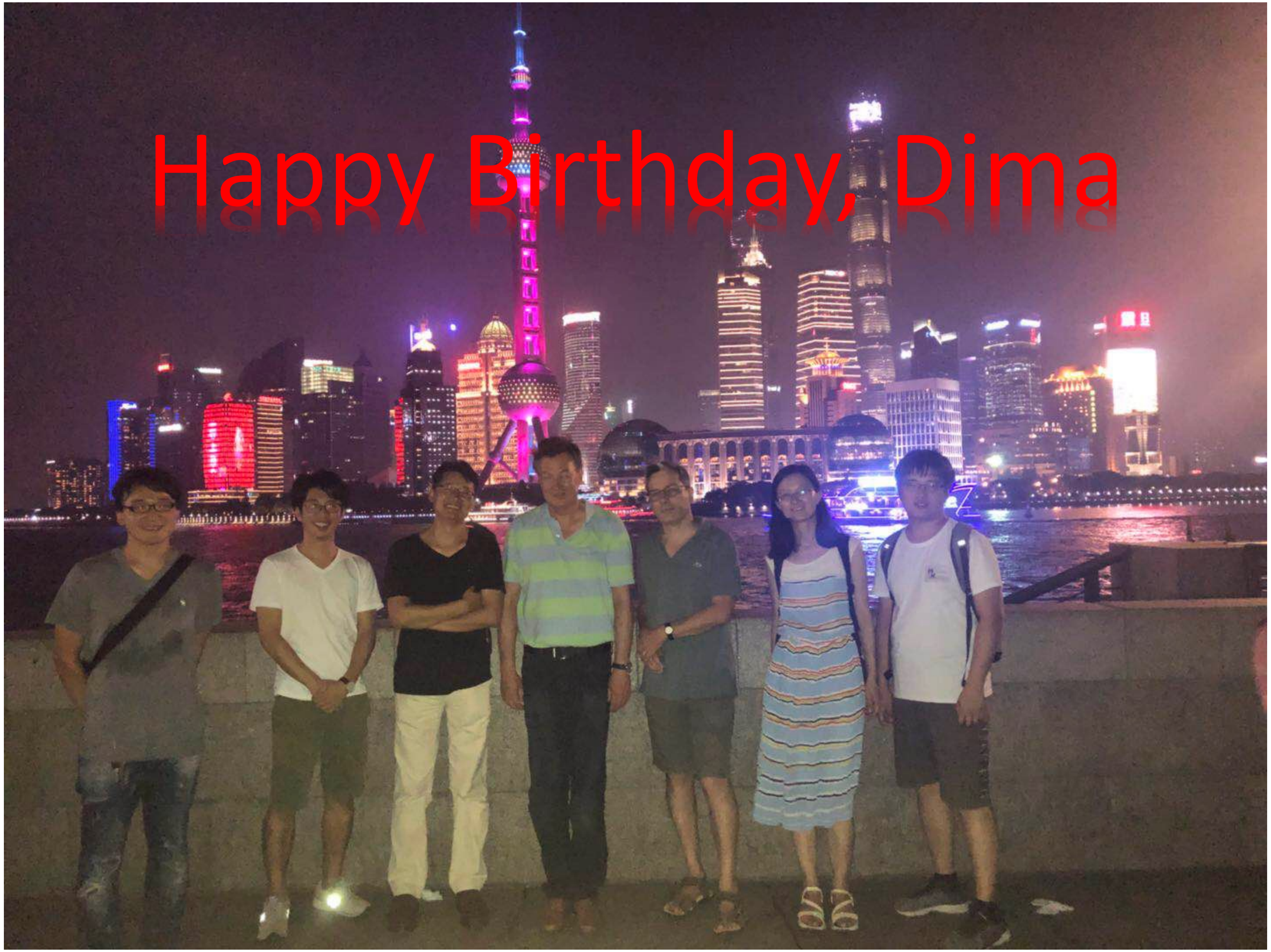

Happy Birthday, Dima

Content

- Introduction

- Instability induced by chiral magnetic effect

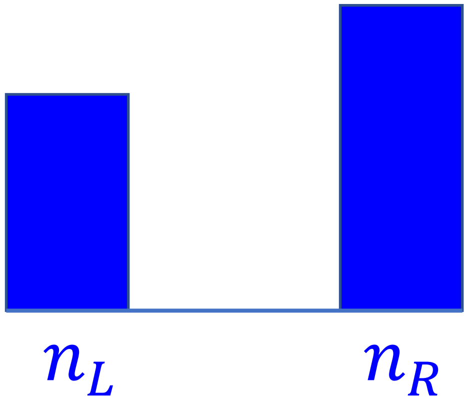
- Instabilities induced by chiral vortical effect
  - Chiral vortical instability in background B field
  - Chiral vortical instability at finite viscosity

- Summary

Introduction

Chiral plasmas

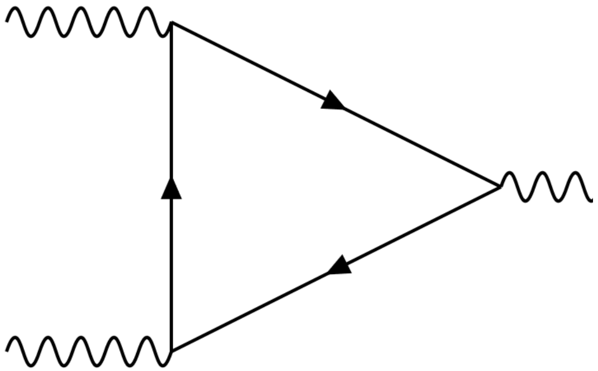
- Chiral plasmas: constituent particles exhibit net chirality



$$n_5 = n_R - n_L > 0$$

$$\text{or} \;\; \mu_5 > 0$$

- Chiral plasmas allow anomaly-induced transports



$$\mathbf{J} = \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

(charge q=1)

Chiral magnetic effect (CME)

(Kharzeev etal 2007; … …)

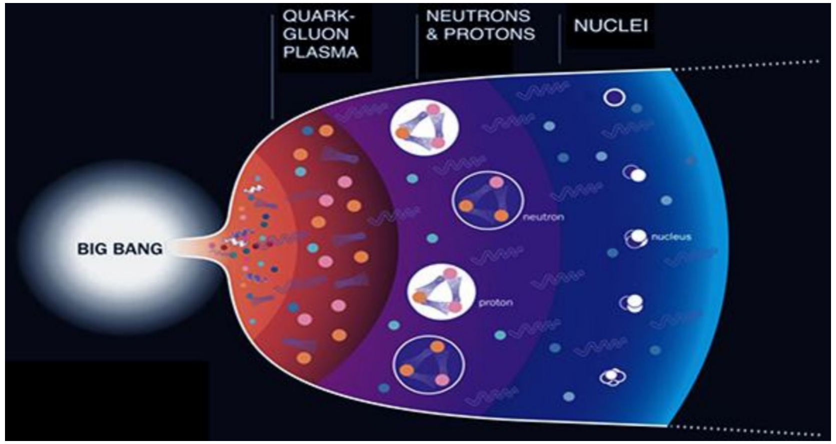
$$\mathbf{J} = \frac{\mu_5}{\pi^2}\mu\boldsymbol{\omega}$$

Chiral vortical effect (CVE)

(Vilenkin 1979; Erdmenger etal 2009; Son, Surowka 2009; … …)

Chiral plasmas

- Example: Early Universe



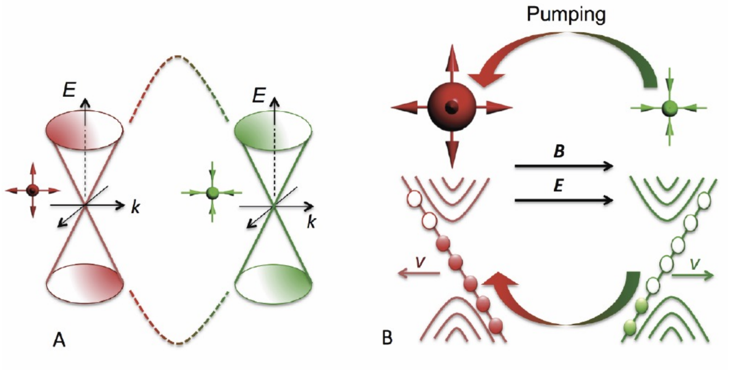
μ_5 uncertain, e.g. 100 GeV

(Brandenburg etal 2017)

May be seen in polarization in microwave background or gravitational wave background or helicity in galaxy spirals

(Yu etal 2020)

- Example : Electron “plasma” in Weyl/Dirac semimetals



$$\partial_\mu J_5^\mu = C_A \vec{E}\cdot\vec{B}$$

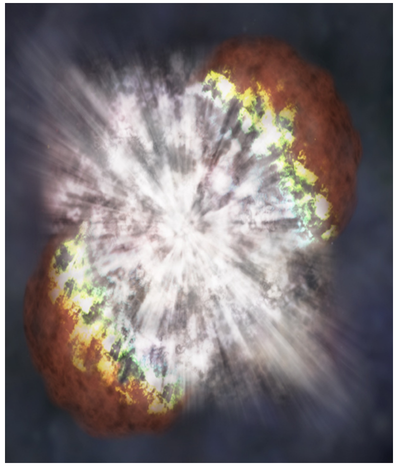
$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B} - \frac{\rho_5}{\tau_V}$$

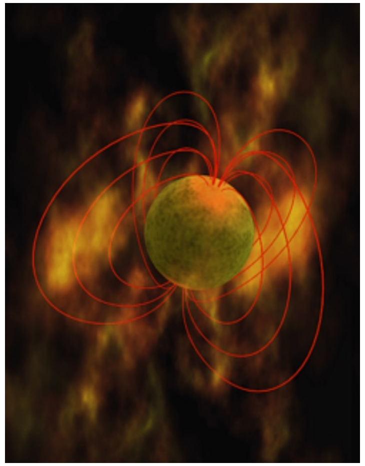
$$\Downarrow$$

$$\mu_5 = \frac{3}{4}\frac{v^3}{\pi^2}\frac{e^2}{\hbar^2 c}\frac{\vec{E}\cdot\vec{B}}{T^2+\frac{\mu^2}{\pi^2}}\tau_V$$ (Li-Kharzeev etal 2014)

Chiral plasmas

- Example: Core-collapse supernovas and neutron stars





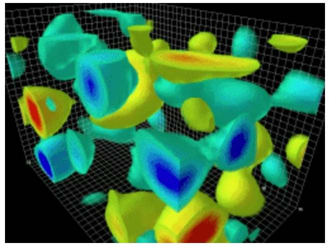
Electron capture $p + e_L^- = n + \nu_L$

➡ $\mu_5 \sim 100$ MeV (Ohnishi-Yamamoto 2014)

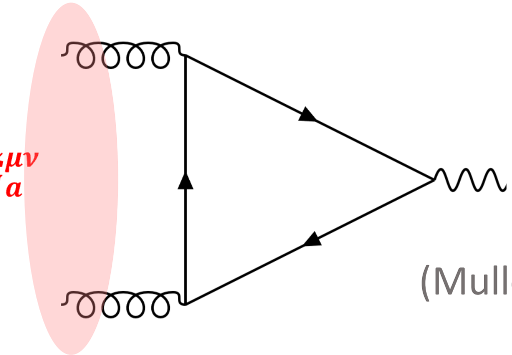
May be suppressed by mass effect

(Grabowska-Kaplan-Reddy 2014)

- Example: Quark gluon plasma (QGP) in heavy ion collisions



$$Q = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



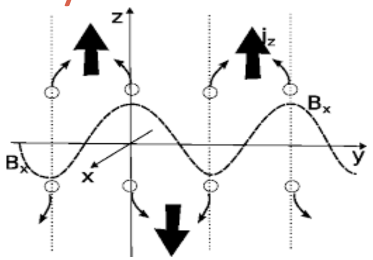
$\mu_5 \sim$10-100 MeV

(Muller-Schafer 2010; Hirono-Hirano-Kharzeev 2014)

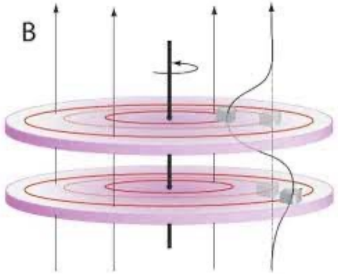
Topological transition between different QCD theta vacua

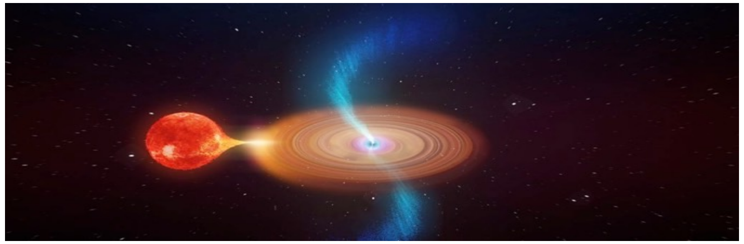
Instabilities in plasmas

- In the usual plasmas, various instabilities are known
  - Weibel instability

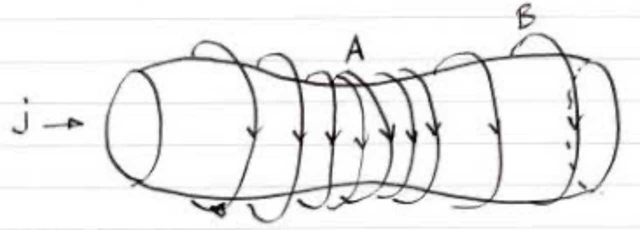


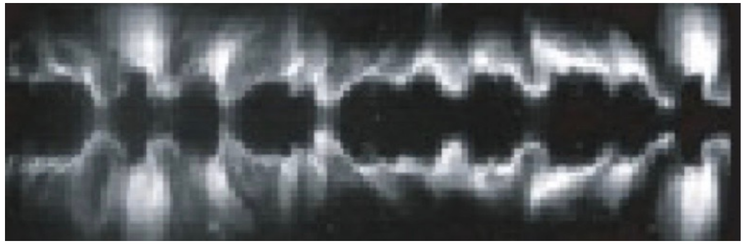
  - Magentorotational instability





  - Sausage instability





- We want to discuss instabilities unique for chiral plasmas

CME induced instability

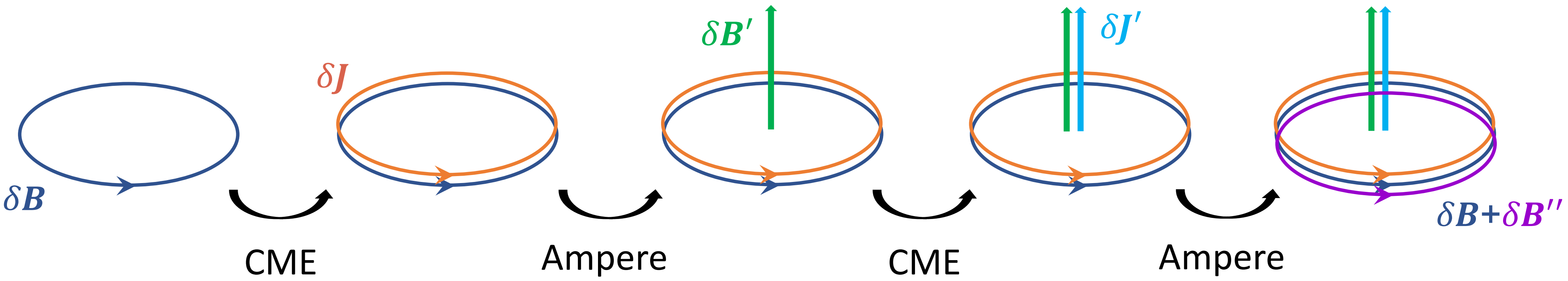
CME induced instability

- Constitutive relation for electric current:

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \xi_B \boldsymbol{B}$$

Ohm current   CME $\xi_B \propto \mu_5$

- Interplay of CME and Ampere's law (constant ξ_B):



➡ $\delta\boldsymbol{B}''$ added to $\delta\boldsymbol{B}$ in the same direction

➡ Chiral plasma instability (CPI) or chiral dynamo instability

(Joyce-Shaposhnikov 1997; Boyarsky-Frohlich-Ruchayskiy 2012; Akamatsu-Yamamoto 2013)

CME induced instability

- CPI from modified Maxwell equations

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$
$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \sigma \boldsymbol{E} + \xi_B \boldsymbol{B}$$
$$\Rightarrow \quad \partial_t \boldsymbol{B} + \eta \partial_t^2 \boldsymbol{B} = \eta \nabla^2 \boldsymbol{B} + \eta \xi_B \boldsymbol{\nabla} \times \boldsymbol{B}$$

  - High frequency region ($\omega/\sigma \gg 1$): CME-modified electromagnetic waves

$$\partial_t^2 \boldsymbol{B} = \nabla^2 \boldsymbol{B} + \xi_B \boldsymbol{\nabla} \times \boldsymbol{B}$$

  - Low frequency region ($\omega/\sigma \ll 1$): Instability
    Right-handed and left-handed modes $\boldsymbol{B} = B_k^+ \boldsymbol{e}_+(\boldsymbol{k}) + B_k^- \boldsymbol{e}_-(\boldsymbol{k})$:

$$\partial_t B_k^\pm = \underbrace{\boxed{-\eta k^2 B_k^\pm}}_{\text{Magnetic diffusion}} \pm \underbrace{\boxed{\eta \xi_B B_k^\pm}}_{\text{Instability}} \quad \Rightarrow \quad B_k^\pm(t) = B_k^\pm(0) e^{\boxed{-\eta k^2 t \pm \eta \xi_B k t}}$$

Exponential growth for $k < \xi_B$

- A dynamo action for low-momentum magnetic fields but: $|\xi_B| \propto |\mu_5|$ must be reduced

CME induced instability

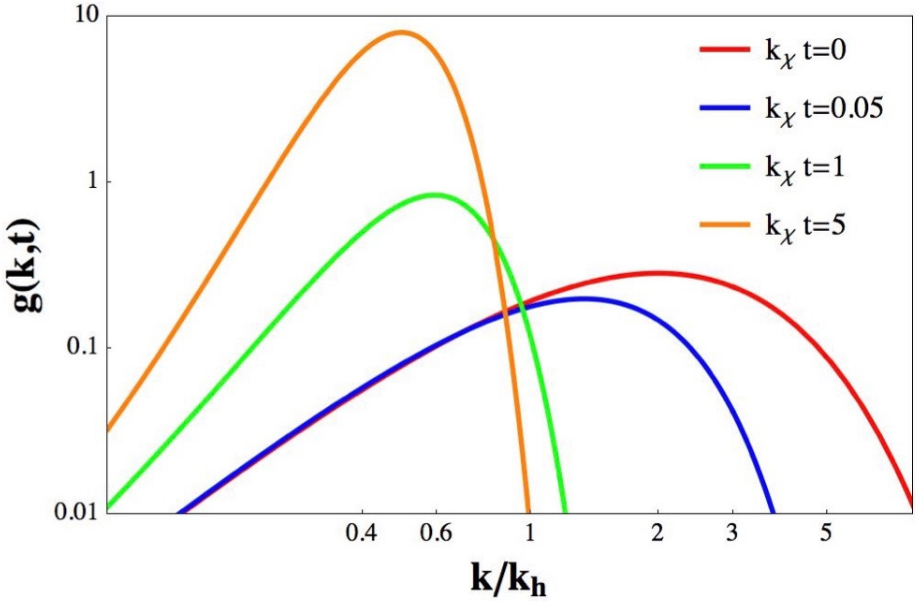
- Dynamics of ξ_B: chiral anomaly

$$\frac{d}{dt}\int d^3\boldsymbol{x} n_5 = C\int d^3\boldsymbol{x}\boldsymbol{E}\cdot\boldsymbol{B} = -\frac{C}{2}\frac{d}{dt}\boxed{\int d^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B}}$$

Magnetic helicity $\mathcal{H}_b$

$$\frac{d\xi_B}{dt} = -C_{\xi_B}\frac{d}{dt}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{k}\left(|B_k^+|^2 - |B_k^-|^2\right)$$

Interpreted as a helicity conservation law



- CPI drives ξ_B to decay while the magnetic helicity to grow
- For mode $k < \xi_B$: Instability stops when $\xi_B = k$ reached
- Chirality transferred into magnetic helicity
- At mean time, magnetic helicity transferred from high-k modes to low-k modes
- The magnetic energy also transferred from high-k modes to low-k modes (Inverse cascade)
- Profound consequences in astrophysics and cosmology

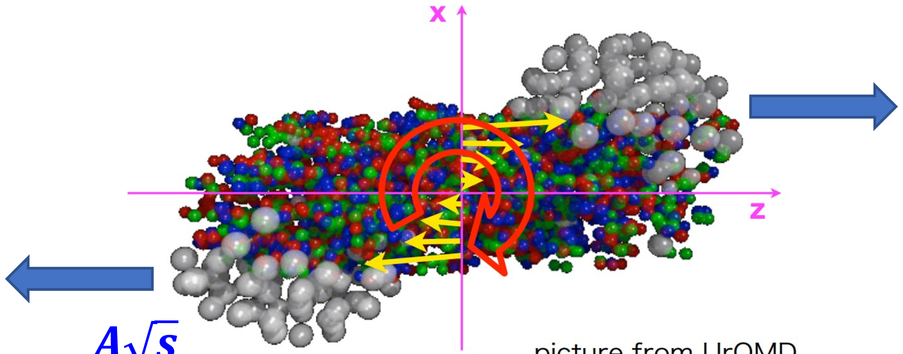
(Kharzeev-Hirono-Yin 2016; and others)

CVE induced instability I: Chiral magnetovortical instability

(Wang-Huang 2023)

Vorticity in quark gluon plasma

- CVE's role in chiral plasma evolution is less considered. But it may be important when:
    - CVE leads to new instability
    - The plasma has very strong vorticity
- Example: "Most vortical fluid" in heavy ion collisions



$$P_z \sim \frac{A\sqrt{s}}{2}$$

picture from UrQMD

Angular momentum

$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

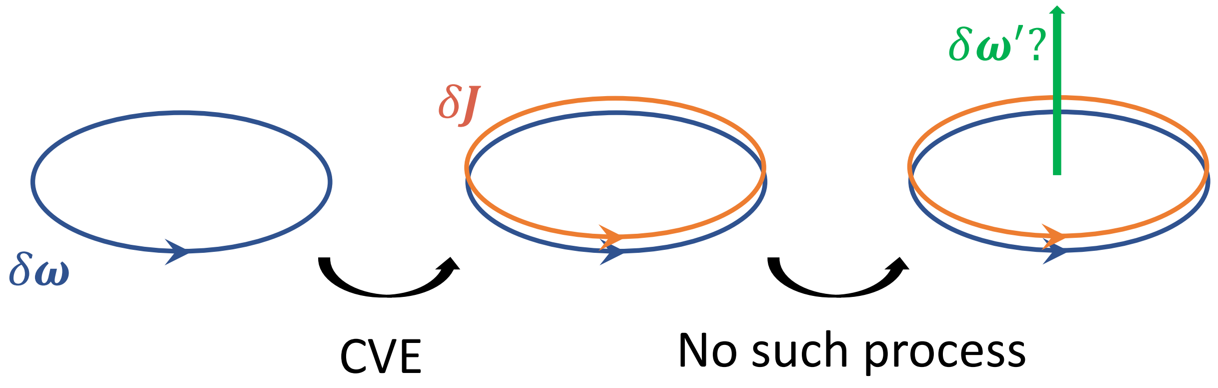
CVE induced instability

- Question: Any CVE induced plasma instability?
- Constitutive relation for electric current:

$$\boldsymbol{J} = \sigma \boldsymbol{E} + \xi_\omega \boldsymbol{\omega}$$

Ohm current CVE $\xi_\omega \propto \mu\mu_5$

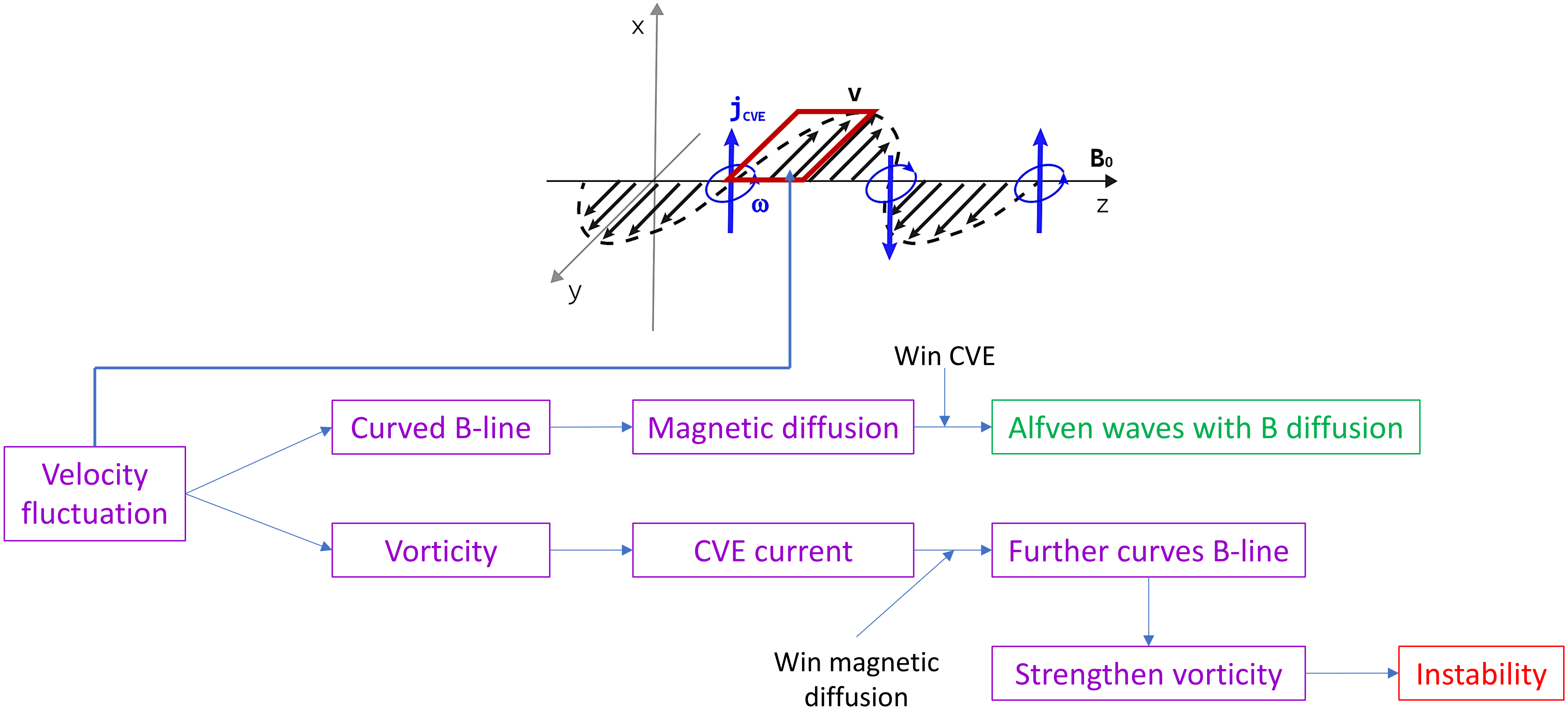
- No direct coupled evolution between fluctuations of vorticity and current



➡ No CVE analog of chiral plasma instability

CVE induced instability

- However, in the presence of external B field: Chiral magnetovortical instability (CMVI)



Chiral magnetovortical instability

- Chiral magnetohydrodynamics (MHD) equations

$$\rho\,(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} = -\boldsymbol{\nabla}P + \boxed{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}$$

Lorentz force

$$\partial_t\boldsymbol{B} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}) + \boxed{\eta\nabla^2\boldsymbol{B}} + \boxed{\eta\xi_\omega\boldsymbol{\nabla}\times\boldsymbol{\omega}}$$

Magnetic diffusion CVE

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = \boldsymbol{0}$$
$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = \boldsymbol{0}$$

- Consider a background $\boldsymbol{B}_0$. Small perturbed $\boldsymbol{v}$ and magnetic field $\boldsymbol{b}$ transverse to $\boldsymbol{B}_0$.

Linearized chiral MHD:

$$\rho\partial_t\boldsymbol{v} = \boldsymbol{B}_0\cdot\boldsymbol{\nabla}\boldsymbol{b}$$
$$\partial_t\boldsymbol{b} = \boldsymbol{B}_0\cdot\boldsymbol{\nabla}\boldsymbol{v} + \eta\nabla^2\boldsymbol{b} - \eta\xi_\omega\nabla^2\boldsymbol{v}$$

➡ Dispersion relation of eigenmodes: $\omega_\pm \equiv -i\frac{\eta}{2}\boldsymbol{k}^2 \pm \sqrt{\underline{\frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho} - \frac{\eta^2\boldsymbol{k}^4}{4} - i\eta\xi_\omega\boldsymbol{k}^2\frac{\boldsymbol{B}_0\cdot\boldsymbol{k}}{\rho}}}$

Instability ⬅ Negative when $|\xi_\omega| > \sqrt{\rho}$

Chiral magnetovortical instability

- Long-wavelength (low-k) modes $k_z, k_t \ll v_A/\eta$:

$$\omega_\pm \approx \pm\frac{\boldsymbol{B}_0\cdot\boldsymbol{k}}{\sqrt{\rho}} - i\frac{\eta}{2}\left(1 \pm \frac{\xi_\omega}{\sqrt{\rho}}\right)\boldsymbol{k}^2$$

Alfven waves
$v_A = |\boldsymbol{B}_0|/\sqrt{\rho}$

Magnetic diffusion

CVE

- Short-wavelength (high-k) modes $v_A/\eta \ll k_z, k_t \ll 1/\eta$:

$$\omega_\pm = \boxed{\pm\frac{\xi_\omega}{\rho}\boldsymbol{B}_0\cdot\boldsymbol{k}} - \frac{i}{2}[1 \mp (-1)]\eta\boldsymbol{k}^2 \mp i\frac{\boldsymbol{B}_0^2(\xi_\omega^2-\rho)}{\eta\rho^2}$$

Chiral Alfven waves

(Yamamoto 2015): No dynamical magnetic field

- Similarly with CPI, CMVI would drive $|\mu_5|$ to drop and finally be tamed

The fate of CMVI

- Dynamics of ξ_ω:

$$\partial_t J_5^0 + \nabla \cdot \boldsymbol{J}_5 = C\boldsymbol{E}\cdot\boldsymbol{B}$$

$$J_5^0 = n_5 + \boxed{\kappa_B \boldsymbol{v}\cdot\boldsymbol{B}} + \boxed{\kappa_\omega \boldsymbol{v}\cdot\boldsymbol{\omega}}$$

Chiral separation effect $\kappa_B \propto \mu$

Axial CVE $\kappa_\omega \propto T^2 + \#\mu^2$

Magnetic helicity $\mathcal{H}_b = \langle \boldsymbol{A}\cdot\boldsymbol{b}\rangle$

Kinetic helicity $\mathcal{H}_v = \langle \boldsymbol{v}\cdot\boldsymbol{\omega}\rangle$

Cross helicity $\mathcal{H}_c = \langle \boldsymbol{v}\cdot\boldsymbol{b}\rangle$

Chirality-flipping rate

$$\chi_5 \partial_t \mu_5 = -\frac{C}{2}\partial_t \mathcal{H}_b - \kappa_B \partial_t \mathcal{H}_c - \kappa_\omega \partial_t \mathcal{H}_v - \Gamma\chi_5\mu_5$$

- Eigenmodes of Chiral MHD: CVE-modified Elsasser fields $\boldsymbol{z}_{1,2} = \sum_{s=\pm} z_{1,2s}\boldsymbol{e}_s(\boldsymbol{k})$

$$z_{1\pm} \approx \left(1 - \frac{i\eta\xi'_\omega \boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm - \left(1 - \frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm$$

$$z_{2\pm} \approx \left(1 - \frac{i\eta\xi'_\omega \boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm + \left(1 + \frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm$$

$$\partial_t z_{1\pm}(t,\boldsymbol{k}) = -i\omega_+ z_{1\pm}(t,\boldsymbol{k})$$

$$\partial_t z_{2\pm}(t,\boldsymbol{k}) = -i\omega_- z_{2\pm}(t,\boldsymbol{k})$$

Primed: scale by mass density ρ

$\boldsymbol{B}'_0 = \boldsymbol{B}_0/\sqrt{\rho}, \boldsymbol{b}' = \boldsymbol{b}/\sqrt{\rho}$, and $\xi'_\omega = \xi_\omega/\sqrt{\rho}$

The fate of CMVI

- Consider $\xi_\omega > 0$:

$$\begin{cases} \boldsymbol{z}_{1,2}(t,\boldsymbol{k}) = exp\left[-i\int_0^t \omega_\pm(t',\boldsymbol{k})dt'\right]\boldsymbol{z}_{1,2}(0,\boldsymbol{k}) \\ \omega_\pm \approx \pm\frac{\boldsymbol{B}_0\cdot\boldsymbol{k}}{\sqrt{\rho}} - i\frac{\eta}{2}\left(1\pm\frac{\xi_\omega}{\sqrt{\rho}}\right)\boldsymbol{k}^2 \end{cases}$$

$$\begin{cases} z_{1\pm} \approx \left(1-\frac{i\eta\xi'_\omega\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm - \left(1-\frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm \\ z_{2\pm} \approx \left(1-\frac{i\eta\xi'_\omega\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)v_\pm + \left(1+\frac{i\eta\boldsymbol{k}^2}{2\boldsymbol{B}'_0\cdot\boldsymbol{k}}\right)b'_\pm \end{cases}$$

- CVE catalyzes magnetic diffusion of $\boldsymbol{z}_1$ modes

➡ A new mechanism for rapid alignment of velocity and magnetic field (Alfvenic state)

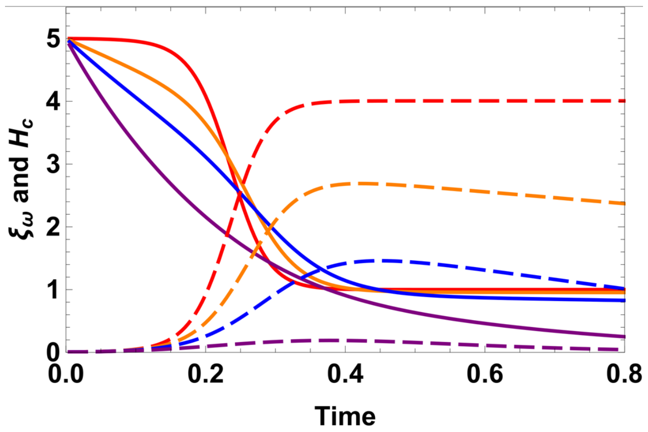
(Sudan 1979; Matthaeus etal 2008)

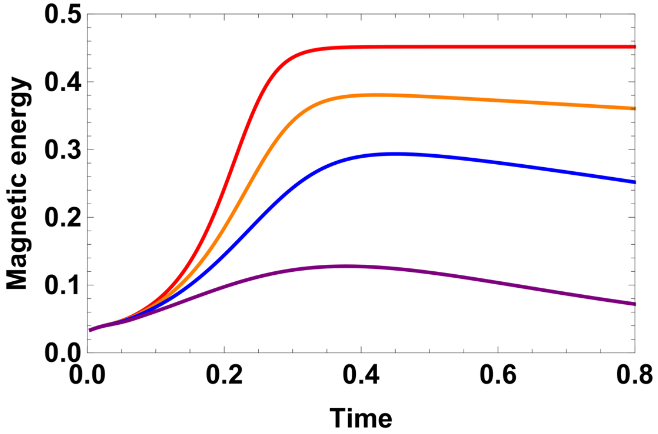
➡ Rapid arising of cross helicity

- Drives ξ_ω to decay to $\xi'_\omega = 1$ stationary state (if $\Gamma = 0$)
- Magnetic energy rapidly arises. A new dynamo action similar to turbulent cross-helicity dynamo (Yoshizawa 1990)

- Numerical simulation with zero initial magnetic and kinetic helicities

Red to purple: Chirality flipping rate from zero to $0.002/\eta$





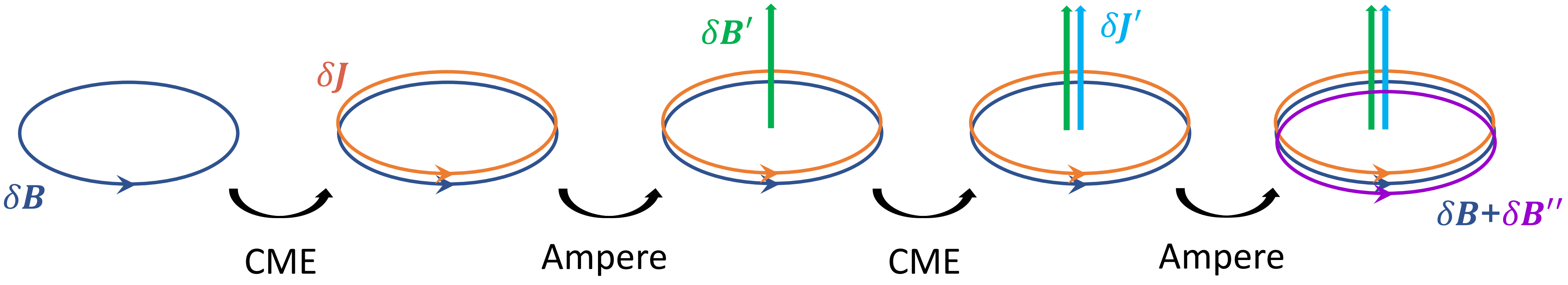
See talk by Shuai Wang

CVE induced instability II: Viscosity induced CVE instability

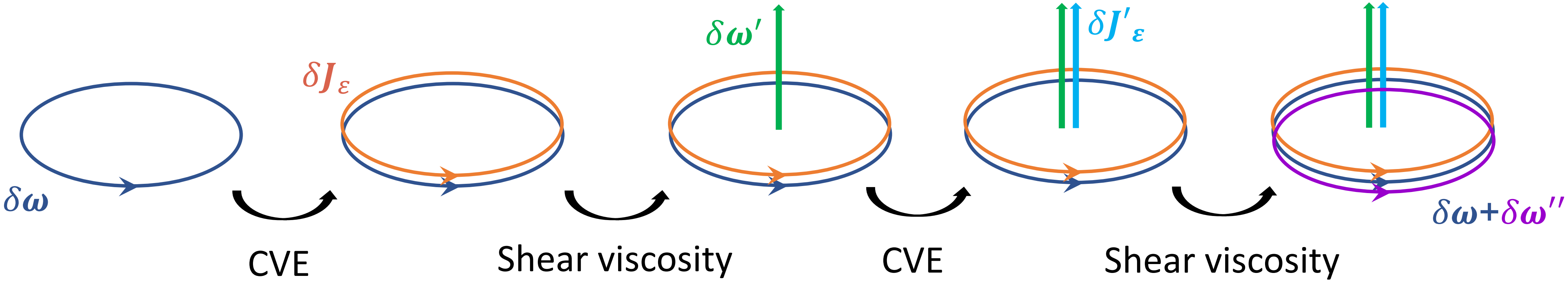
(Hattori-Huang, In preparation)

CVE instability at finite shear viscosity

- Look at again the CME induced instability



- Consider CVE induced energy current at finite shear viscosity



➡ Thus it is possible for CVE analog of chiral plasma instability but at finite shear viscosity

CVE instability at finite shear viscosity

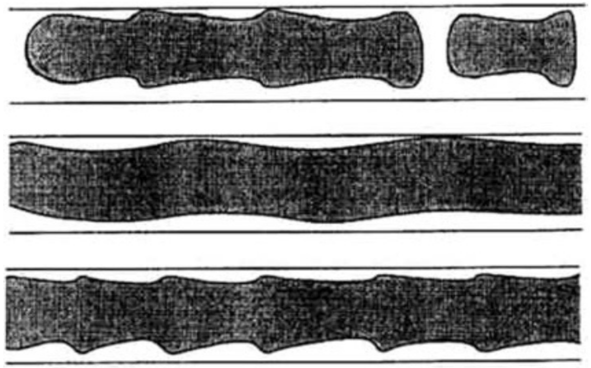
- Indeed, we can find that one of the shear modes is unstable:

$$\omega = -i\nu\left(1 \pm \frac{\lambda_\omega}{\rho}|\boldsymbol{k}|\right)\boldsymbol{k}^2 \longleftarrow \nu = \frac{\mu}{\rho}$$: Kinematic viscosity, $\lambda_\omega \propto \mu_5$: CVE for energy current

Compare with CPI by CME: $\omega = -i\eta\left(|\boldsymbol{k}| \pm \xi_B\right)|\boldsymbol{k}|$

- Unlike CPI, this instability is active for large k mode. It can induce normal cascade of fluid energy and kinetic helicity.
- Note: Though viscosity usually stabilizes the flow, there are known viscous instabilities:

Saffman-Taylor instability



Oil in pipes

<u>Summary</u>

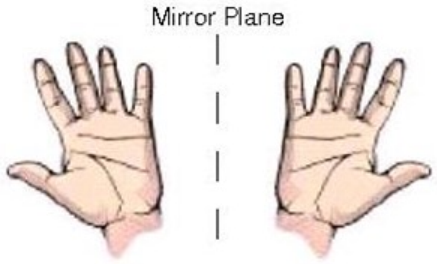
Summary

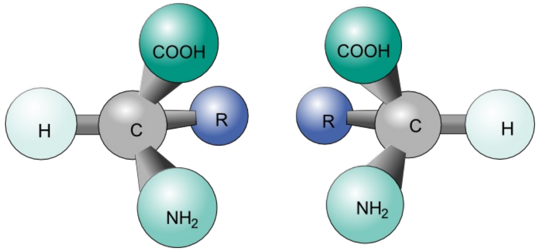
- There are new instabilities due to chiral anomaly in chiral plasma
- The CVE can induce instabilities under magnetic field and due to shear viscosity
- It will be interesting to study their consequences
    - Effects on the energy and cross-helicity spectra cascade
    - CVE induces instabilities in kinetic theory
    - The dynamo mechanism simulation for astrophysical systems
    - Axion dark matter
    - Quark-gluon plasma
    - … …

Thank you

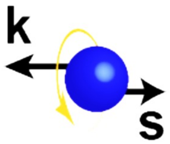
Chiral plasmas: what and where

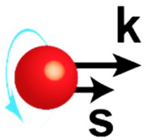
- Chirality: handedness





- For massless fermions: helicity





Chirality currents: $J_L^\mu = \bar{\psi}_L \gamma^\mu \psi_L$ $\quad J_R^\mu = \bar{\psi}_R \gamma^\mu \psi_R$

- Classically

$$\partial_\mu J^\mu = 0 = \partial_\mu J_5^\mu \quad \text{with} \quad J^\mu, J_5^\mu = J_R^\mu \pm J_L^\mu$$