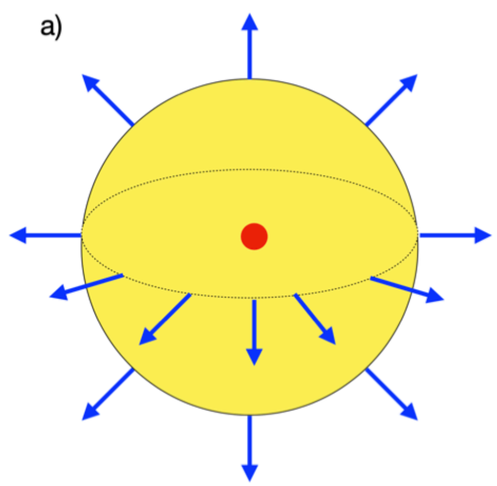
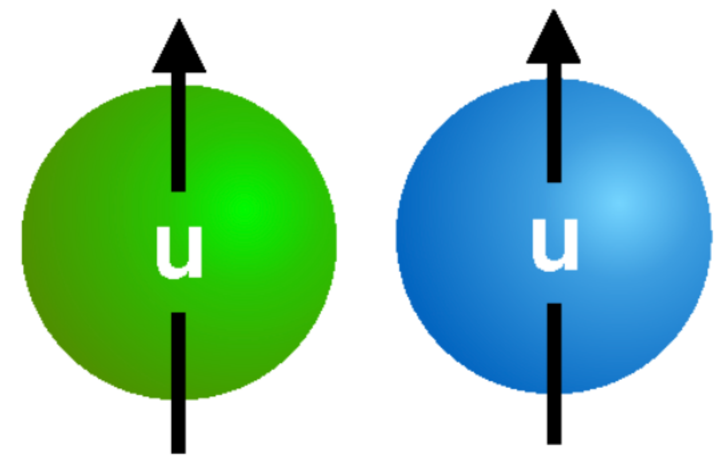
Berry monopole and color superconductor





Yi Yin

Institute of Modern Physics (IMP), Lanzhou, Chinese academy of sciences

Noriyuki Sogabe, YY, to appear

Chirality workshop, July.16th, 2023

Personal recollection

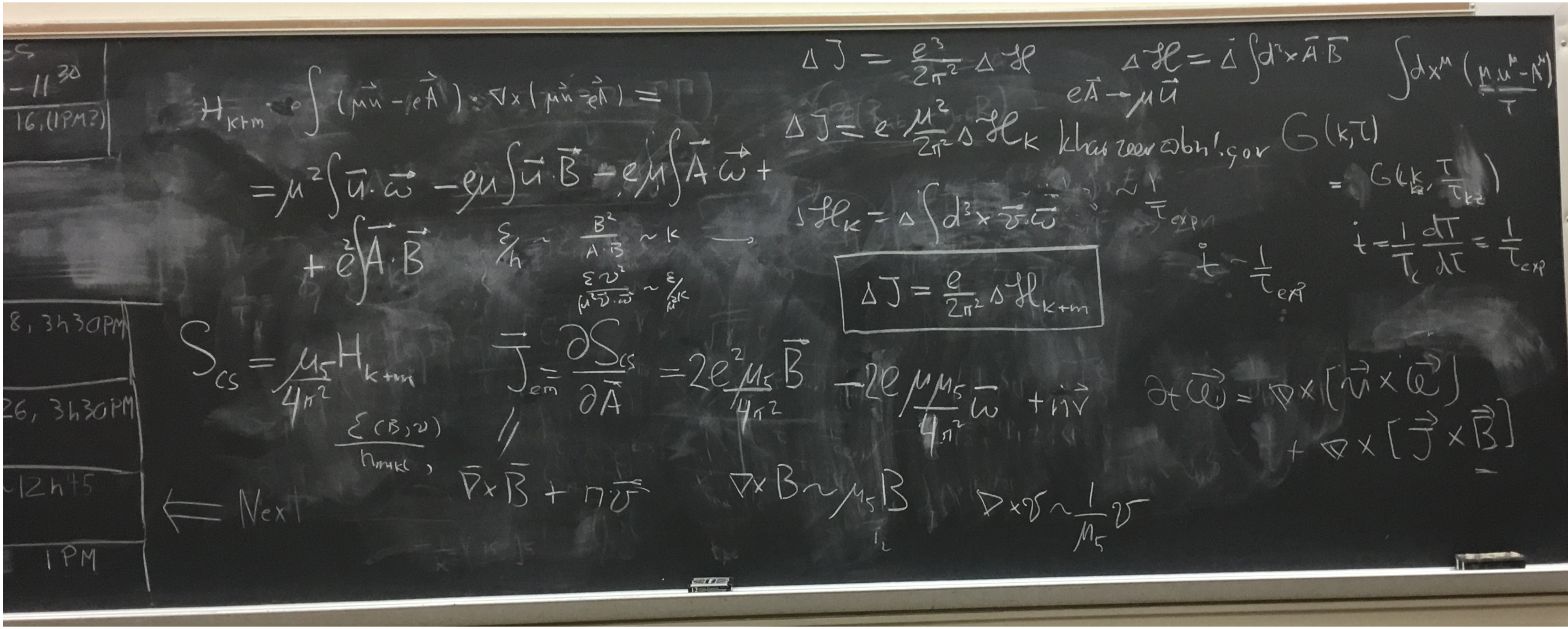
Dima hosted my (first ever) seminar at Stonybrook on Aug. 8th, 2013 when I was a Ph. D student,.

We start working together when I was a visiting graduate at Stonybrook and then a postdoc at BNL (2014-16).

Dima is a inspiring, generous, trustworthy supervisor and had a crucial impact on my career.

We also had joyful times when attending social in conferences.

QM 19 in Wuhan



The blackboard in Gerry Brown room, after a discussion with Dima, Derek Teaney, Ho-Ung Yee and Yuji in 2016

We are working on the dynamical conversion among the chirality of fermions and topologically configurations of gauge fields.

Kharzeev-Hirono-YY, PRD 2015'; PRL 2016

But perhaps more importantly, I am inspired by his

- passion for the physics,
- unique thinking style,

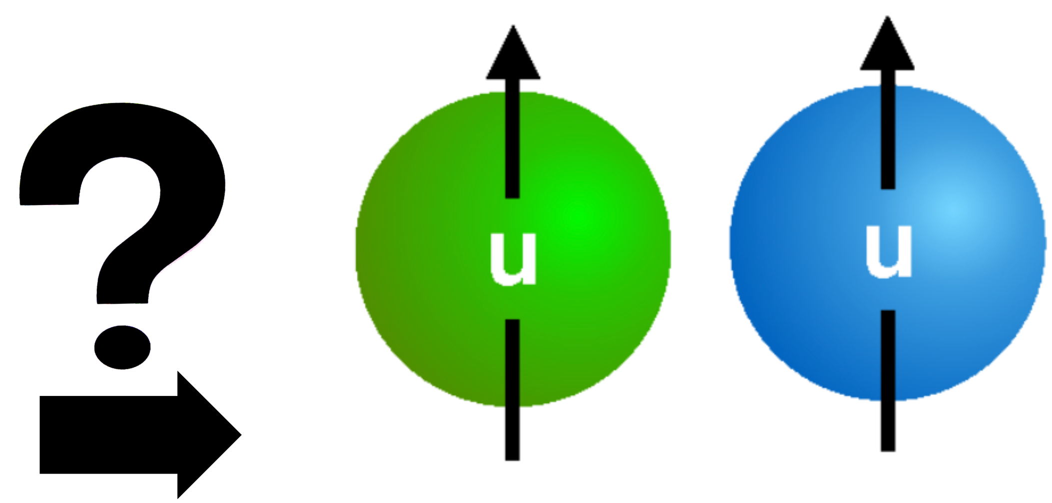
"He (Dima) is unusual in that he has a smooth intelligence that allows him to immediately understand ideas at an intuitive level."—Larry McLerran

- vision that "different subfields in physics are deeply connected".

Today, I would talk about topological aspect of color super-conductor (SC) in the light of recent development in condensed matter physics.

Topological Nodal Cooper Pairing in Doped Weyl Metals

Yi Li and F. D. M. Haldane
Phys. Rev. Lett. **120**, 067003 – Published 9 February 2018

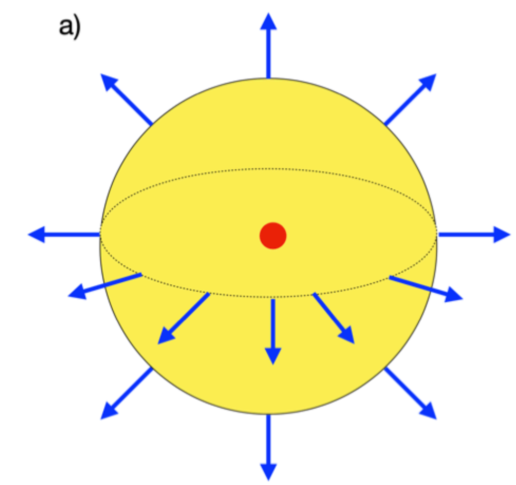


Berry structure

Berry monopole

$$\vec{a}(\hat{k}) \equiv i\phi^{\dagger}(\hat{k})\,\vec{\nabla}_k \phi(\hat{k}) \qquad \vec{b} \equiv \vec{\nabla}_k \times \vec{a}$$

Berry gauge field *wave-fun.* *Berry magnetic field*



Flux of $\vec{b}$ on sphere = $4\pi q$ (monopole charge)

$q \neq 0 \rightarrow \phi(\hat{k})$ is only well-defined on the patches of F. S. . (q characterizes topologically different ways to glue patches.)

(e.g. for R/L chiral fermions carry $q = \pm 1/2$)

- plays important role in the describing anomaly-induced effects in chiral kinetic theory.
- But Fermi surface (F.S.) is not stable against attractive interaction. What happens to Berry structure in superconducting (SC) phase?

$$\hat{H}_{\mathrm{BCS}} = a_1^\dagger(\hat{k})\,\phi_1^*(\hat{k})\,M(\hat{k})\,a_2^\dagger(\hat{k})\,\phi_2(\hat{k}) + c\,.\,c$$

grad. matrix

- Operator $\hat{H}$ does not depend the "choice" of phase of $\phi_{1,2}$, so

$$\phi_{1,2} \to e^{i\alpha_{1,2}}\phi_{1,2} \qquad M \to e^{i(\alpha_1-\alpha_2)}M$$

- The phase of gap matrix can not be well-defined globally when "pairing" monopole charge $q_{\mathrm{pair}} = q_2 - q_1$ is non-zero.

Murakami-Nagaosa, PRL 03

  - e.g. oppo-chirality pairing has $q_{\mathrm{pair}} = 1$

Implications?

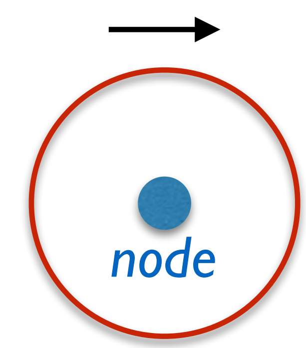
Topological nodes

- Nodes: points k_0 on the F.S. where gap vanishes $M(\hat{k}_0) = 0$, i.e. the excitations become gapless around k_0

- Define the gradient of the phase of $M(\hat{k})$ as the "velocity field". Its winding no. around the node is topologically.

The real-space gradient of the phase of the gap gives the super-fluid/spin velocity.

$$v(\hat{k}) = i\underbrace{\nabla_k \log \det(M(\hat{k}))}_{\text{grad. of the phase}} \qquad n_w = \int_C dl \cdot v$$



- The winding no. assigns the chirality of the gapless excitations near the node.

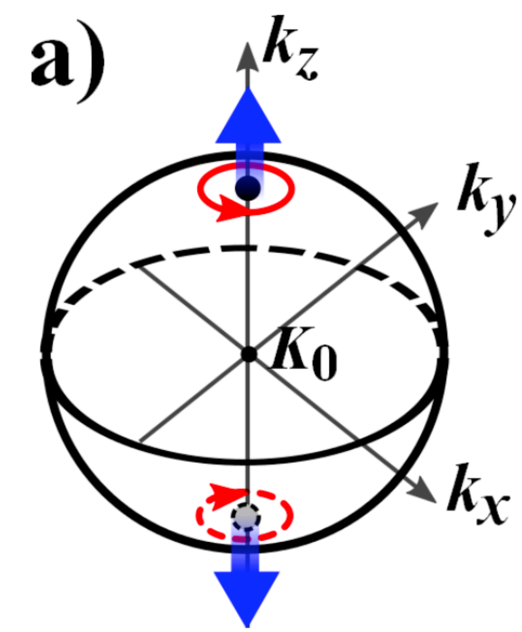
- Topological nodal SC features various interesting transport phenomena e.g. anomalous spin and thermal Hall effects.

Review by Schnyder-Brydon, 2015

Li-Haldane relation

Li-Haldane, PRL 18'

$$q_{\text{pari}} = \int_{F.S.} \nabla \times \Big(\underbrace{a_1(\hat{k}) - a_2(\hat{k}) + v(\hat{k})}_{\text{gauge-invariant comb.}} \Big) = \frac{1}{2} \sum_{\text{node}} n_w$$



$$\boxed{\text{Paring monopole charge } = \text{sum of wind no. of the nodes}}$$

- Non-zero q_{pair} forces the emergence of the topological nodes.

Color super-conductivity (SC)

Color super-conductivity

Alford, Schmitt, Rajagopal, Schaffer, Rev. Mod. Phys. 2008

- Baryonic matter at high density: color SC in color-flavor locking phase.
- Single flavor pairing (this talk)
    - might be favored at environments in stars.
    - spin-one pairing.
    - opposite chirality pairing is energetically favored.
    - extensively studied but topological aspects has been overlooked for years.

e.g. works by Mei Huang, Defu Hou, T. Schaffer, A. Schmitt, Qun Wang, Pengfei Zhuang and many others

Oppo-chirality pairing

$$\hat{H}_{\mathrm{BCS}} \propto \left[\Psi_L^\dagger(\hat{k})\, M(\hat{k})\, \Psi_R^\dagger(\hat{k}) \ + (c\,.\,c)\right] + (L \leftrightarrow R)$$

color-spin matrix $M(\hat{k}) \propto J_i\, O^i_j\, \sigma^j_\perp$

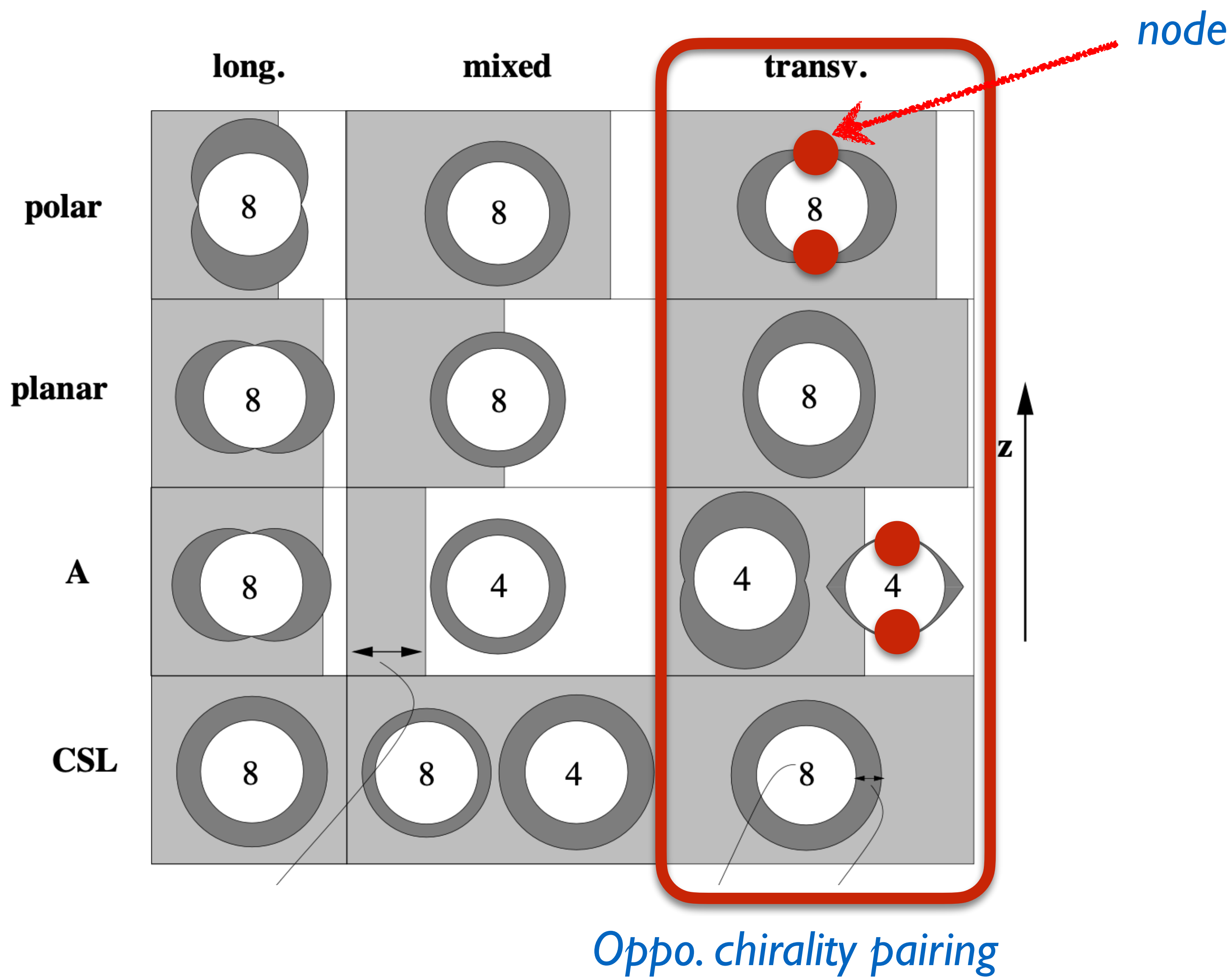
collection of anti-symmetric Gell-Mann matrices

Specify different phases

- In many ways, the one-flavor SC is analogous to ^{3}He superfluid ($J_i \to \hat{k}_i$).
- Since ^{3}He is essentially massive fermions, they have qualitative difference from the topological perspective.

Phases of one-flavor CSC

Schmitt, PRD 2005



- Some phases (e.g. polar phase) have nodes. Are they topologically?
- CSL/planar phase is fully gapped. Why no nodes?

Next: examining the pairing Berry monopole.

Computing pairing monopole

- L/R eigen-modes: $N_{color} = 3$ modes at each given helicity.

$$MM^\dagger \phi_L^\lambda = \lambda^2 \phi_L^\lambda, \qquad M^\dagger M \phi_R^\lambda = \lambda^2 \phi_R^\lambda, \qquad E_{\vec{k}} = \sqrt{k^2 + \Delta_0^2 \lambda^2(\hat{k})}$$

- Two non-zero modes $(\lambda \neq 0)$
  - participating the pairing and determine the pair monopole charge.

$$\phi_R^\lambda = \left(c_1^\lambda, c_2^\lambda, c_3^\lambda\right)^T \otimes \xi_R \qquad a[\phi_L^\lambda] - a[\phi_R^\lambda] \to q_{\text{pair}}$$

(Both color and spin structure may contribute.)

  - determine gapped excitations: $\psi^\lambda = \left(b\,\phi_L, b'\,\phi_R\right)^T$

- One zero mode $\lambda = 0 \to$ gapless excitation: $\psi^0 \propto \left(\phi_L^0, 0\right)^T$

Polar phase

$$M \propto \begin{pmatrix} 0 & \sigma_\perp^3 & 0 \\ -\sigma_\perp^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \propto J_3$$

color rotation

- Modes with different J_3 eigenvalues j_3 do not mix.

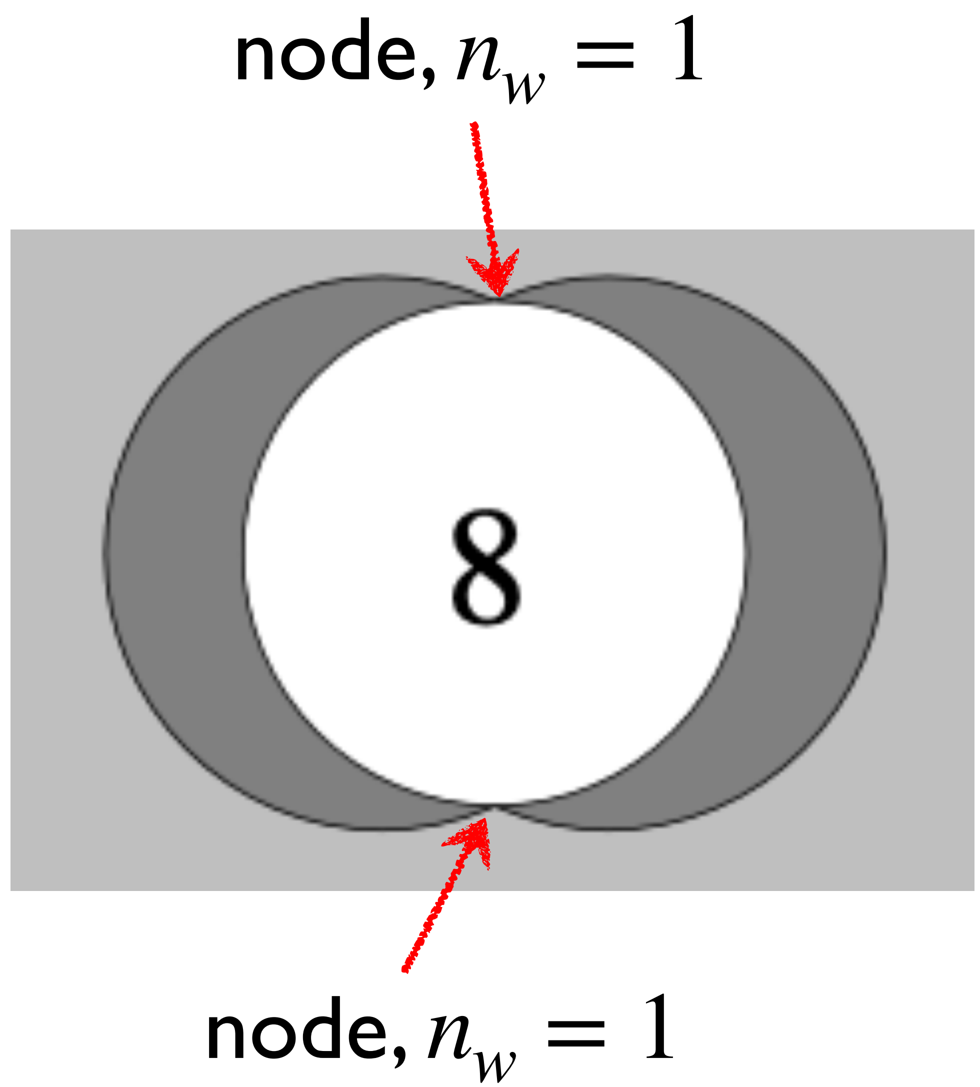
$j_3 = 0 \qquad \phi_R^0 = (0,0,1)^T \otimes \xi_R$ *(Green quark does not participant the pairing)*

$j_3 = \pm 1 \qquad \phi_R^\lambda = (1,\pm i,0)^T \otimes \xi_R\,,$

$q_{\text{pair}} = 1$ *No. contribution from color*

$\lambda(\hat{k}) = \sin\theta \longrightarrow$ Nodes at $\theta = 0,\pi$

Polar phase



- Polar phase is a nodal color SC. (NB: material realization of topological SC is very rare to date)

- Qualitatively different from A-phase of ^{3}He.

$$n_w(\theta = 0) = 1, n_w(\theta = \pi) = -1 \rightarrow \sum n_w = 0$$

Color-spin locking (CSL) phase

$$M \propto \begin{pmatrix} 0 & -\sigma_\perp^3 & \sigma_\perp^2 \\ \sigma_\perp^3 & 0 & -\sigma_\perp^1 \\ -\sigma_\perp^2 & \sigma_\perp^1 & 0 \end{pmatrix}$$

- For each helicity, two degenerate and mixed non-zero modes.

$$\phi_R^{\lambda I} = \left(c_1^{\lambda I}(\hat{k}), c_2^{\lambda I}(\hat{k}), c_3^{\lambda I}(\hat{k}) \right)^T \otimes \xi_R \,, \qquad I = 1{,}2$$

$\lambda^2 = 2$ (no node at all!)

- Computing non-Abelian generalization of Berry gauge field.

Wilzeck-Zee, PRL 1984

$$a \to \mathrm{tr}(a^{IJ})$$

CSL:

- Monopole charge cancels between the spin and color.

$$q^{\lambda}_{R,color} + q_{R,spin} = -1 + (1/2) \times 2 = 0 \;\rightarrow\; q_{\text{pair}} = 0$$

consistent with the fact that CSL has no nodes. "*But signature is the absence of a signature!*"

- Gapped excitations carry non-trivial non-Abelian Berry flux in color!

- The gapless excitations carry monopole charge $\pm 3/2$. (c.f. for the L-L/R-R pairing, $q^0 = \pm 1/2$)

$$q^{\lambda}_{color} + q^{0}_{color} = 0 \rightarrow q^{0}_{R,color} + q^{0}_{R,spin} = 3/2$$

Energetic consideration

- Two possibilities with non-zero (spin) pairing monopole charge:

  a) topologically nodes (polar and A phase). *Li-Haldane, PRL 18'*

  b) excitations with non-trivial Berry flux in color (CSL and planar phase) *Sogabe-YY, to appear*

- Possibility b) might be energetically favorable:

$$\text{Condensation energy diff} \propto \int_{\hat{k}} \lambda^2(\hat{k})$$

Discussion and summary

Summary and outlook

- For one-flavor color SC, we examine non-trivial topological structure for the quark pairs with opposite chirality.
  - Topological nodes;
  - No nodes, but excitations feature non-trivial color Berry flux.
- Based on symmetry breaking pattern, Schaffer argued that in one flavor QCD, the low and high density phases are continuously connected. But those two phases may be topologically different.
- Berry monopole plays an important role in describing effects of quantum anomaly in Fermi liquid. How about SC? (Anomaly matching for dense QCD).
- Cross-fertilization among different sub-fields of physics.

Back-up

Sogabe-YY, to appear

Phase	$\mathcal{S}_0$	TR	PH	SL	Class	Defects	δ	Topo no.
Polar M	$\{J_3\}$	-	$+$	0	DIII	no	3	Z_2
Polar L	$\{R_5, J_3\}$	-	$+$	0	DIII	nodal line	1	0
Polar T	$\{R_5', J_3\}$	-	0	0	AII	node	2	Z_2
A-phase M	$\{J_3\}$	0	$+$	0	D	no	3	0
A-phase L	$\{R_5, J_3\}$	0	$+$	0	D	node	2	Z_2
A-phase T	$\{R_5', J_3\}$	0	0	0	A	node	2	Z_2
CSL M	$\emptyset$	-	$+$	$+$	DIII	no	3	Z
CSL L	$\{R_5\}$	-	$+$	$+$	DIII	no	3	Z_2
CSL T	$\{R_5'\}$	-	0	0	AII	no	3	Z_2

TABLE V. The summary of topological classification for various different phases of CSC in ultra-relativistic limit. Here $\delta = d - D$ where d denotes the spatial dimension and D denotes the co-dimension of the defects. In all cases, $\mathcal{S}_1$ is empty.

- The SC can be classified into different topologically classes based on its discrete symmetry properties.
  - CFL phase is found to be class DIII. *Nishida, PRD 2010*
- By analyzing R-L, R-R/L-L and mixed paring, we found one-flavor SC has very rich topological structure.

Dima's style as I see it

I told Dima that I can derive B from A.

Dima typically would not check the derivation. Instead, he may derive C assuming B is correct.

If C does not make sense, Dima will tell me there must be somethink wrong in my derivation.

If C is correct, Dima would suggest to derive C from A (which is always easier and elegant).

If C is physically interesting, Dima would ask me to forget about A and try to understand C

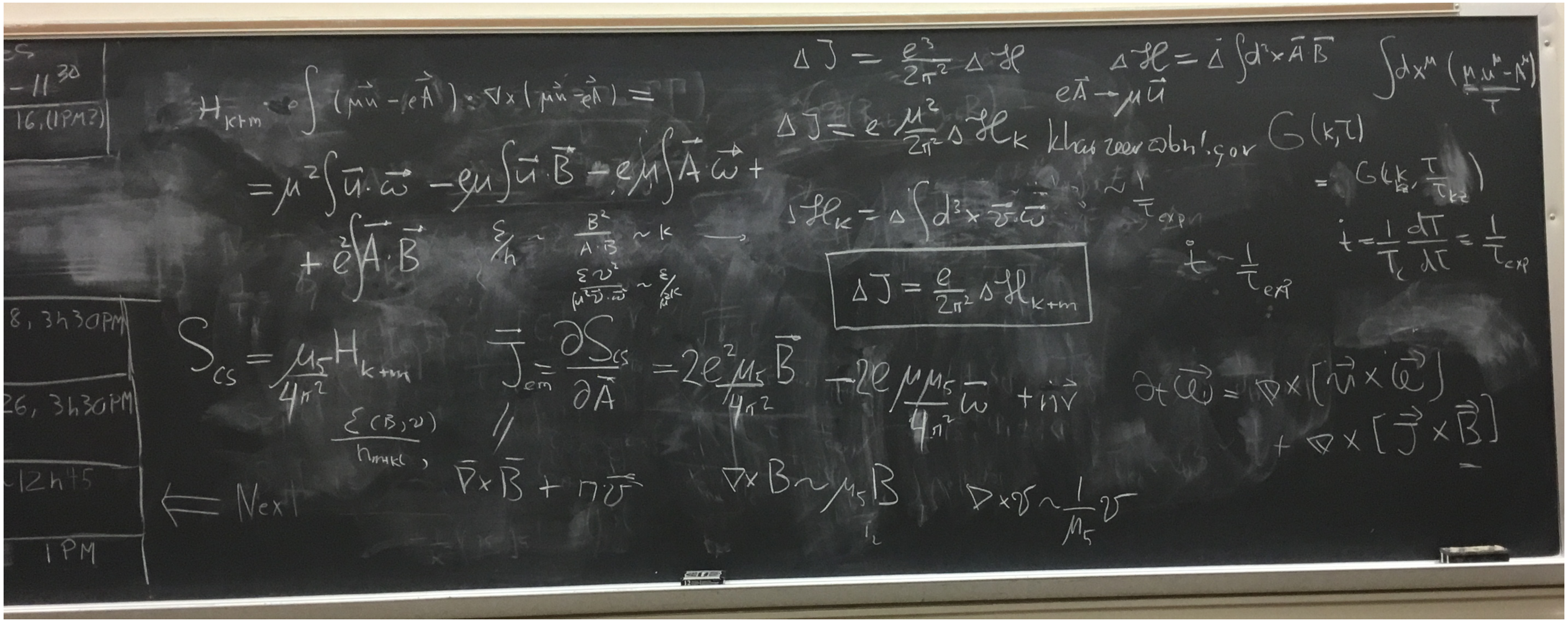
When I was a Ph. D student, Dima hosted my (first ever official) seminar at Stonybrook on Aug. 8, 2013.

Since the discussion with him and his group members (Gocke Basar, Frasher Loshaj) went long, Dima drove me BNL where I was staying.

I asked the problems he thought were interesting, I vividly remembered the his voice suddenly became very energetic and enthusiastic:

"I need a blackboard to explain !"

We start working together when I was a visiting graduate at Stonybrook and then a postdoc at BNL (2014-16).



The blackboard in Gerry Brown room, after a discussion with Dima, Derek Teaney, Ho-Ung Yee and Yuji in 2016

I typically went to his office around 1 or 2 pm and discussion extends towards the dinner time. (I still remember that once, after three-hours of discussion, Dima suddenly said to me and Yuji, look at the beautiful sun-set!).

QCD phase diagram and topology

- Key question: identify possible phases of QCD.
- Traditional paradigm (Landau-Ginzburg): based on symmetry breaking pattern.
- Topological aspects: important in understanding and characterizing quantum phases.