Spin polarization in Wigner function approach

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The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions , July 15-19, 2023, UCAS, Beijing

Special symposium

Outline

- Introduction to polarization phenomena in HIC
- Spin distributions from Wigner functions
- Spin dynamics for vector mesons in quantum kinetic theory
- Ideal spin hydrodynamics from Wigner functions
- Summary

Barnet effect and Einstein-de Haas effect

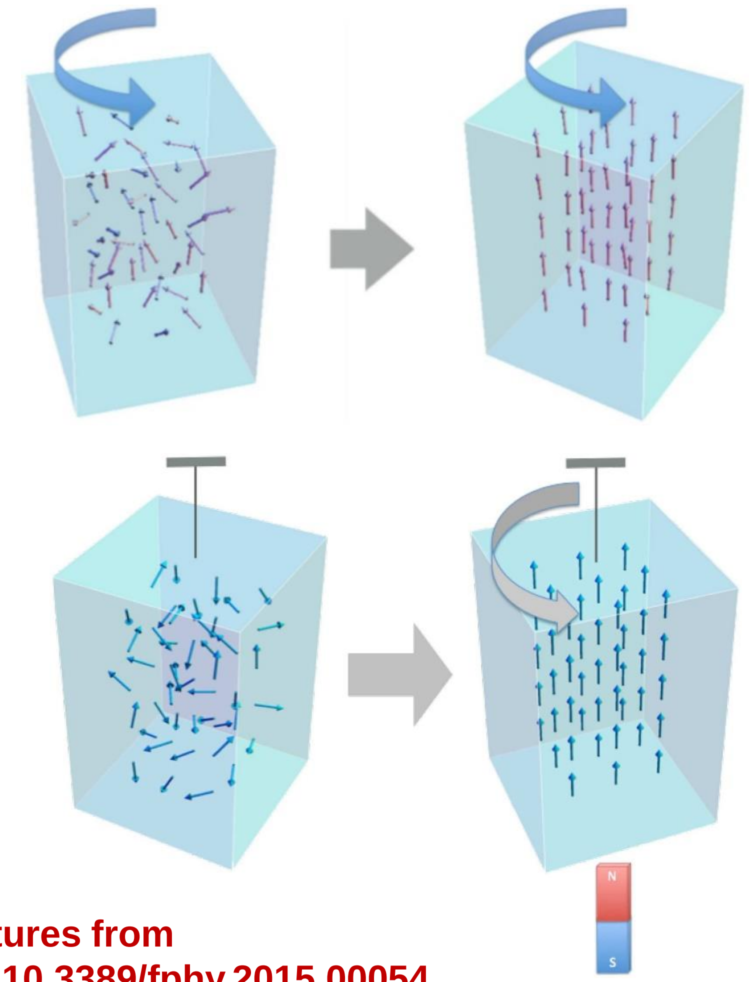
<u>Barnett effect:</u>

Barnett, Magnetization by rotation, Phys Rev. 6, 239-270 (1915).

Spin-orbit (LS) coupling!

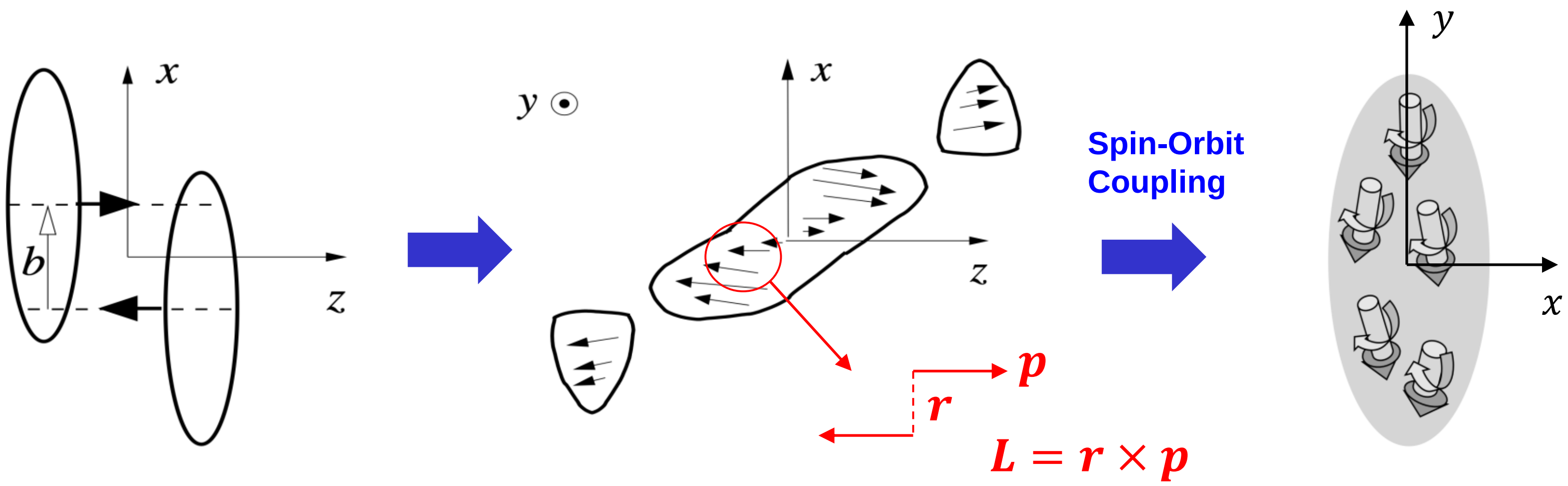
<u>Einstein-de Haas effect:</u>

Einstein, de Haas, Experimental proof of the existence of Ampere’s molecular currents, Verhandl. Deut. Phys. Ges. 17, 152–170 (1915).



Pictures from doi:10.3389/fphy.2015.00054

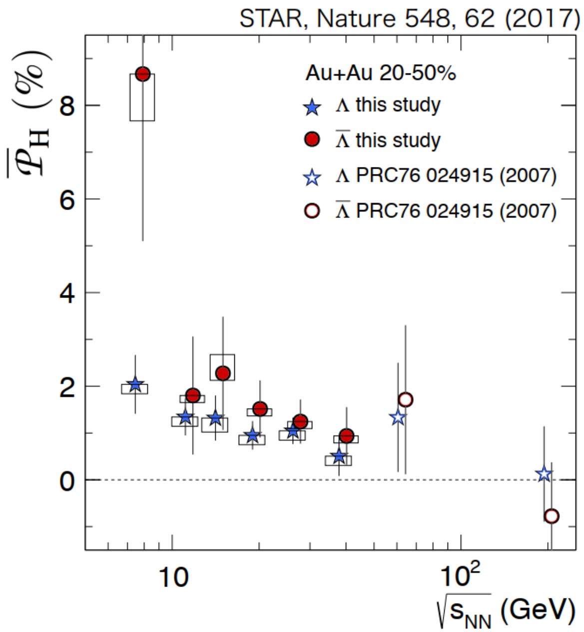
Global OAM and polarization in HIC



Global OAM leads to global polarization of Λ hyperons through spin-orbit coupling

Liang and Wang, PRL 94,102301(2005); Betz, Gyulassy, Torrieri, PRC (2007); Becattini, Piccinini, Rizzo, PRC (2008); Gao, Chen, Deng, Liang, QW, Wang, PRC (2008)

STAR: global polarization of Λ hyperon



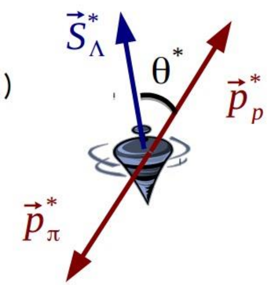
parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

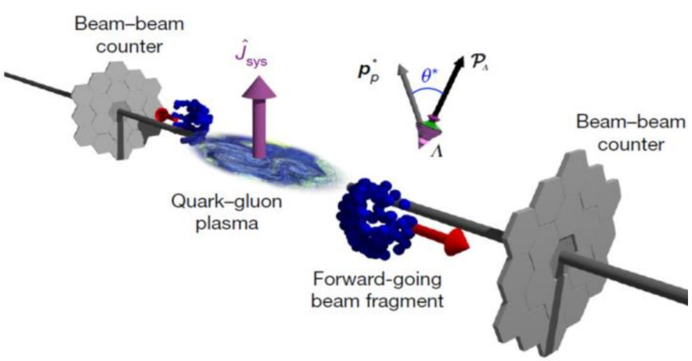
α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

Updated by BES III, PRL129, 131801 (2022)



$$\Lambda \to p + \pi^+$$

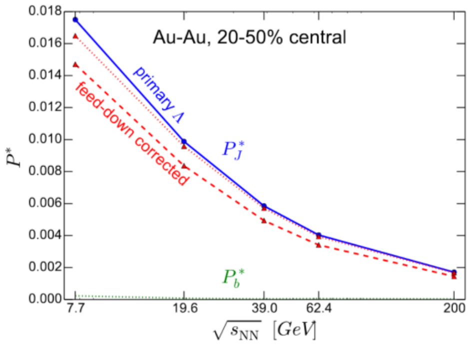
(BR: 63.9%, cτ~7.9 cm)



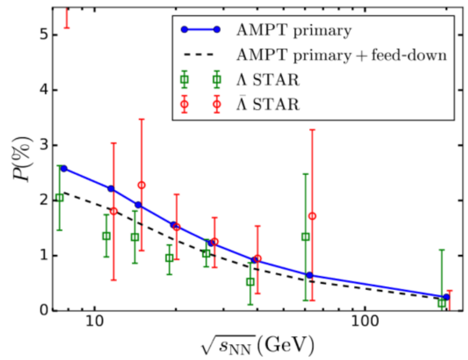
ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

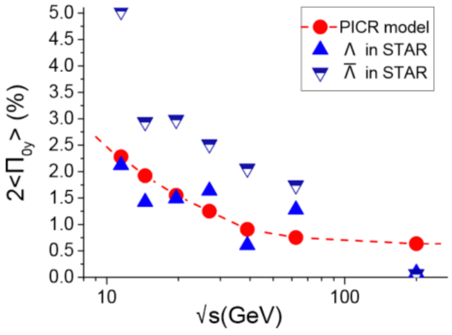
Theoretical model calculations: global polarization of Λ hyperon



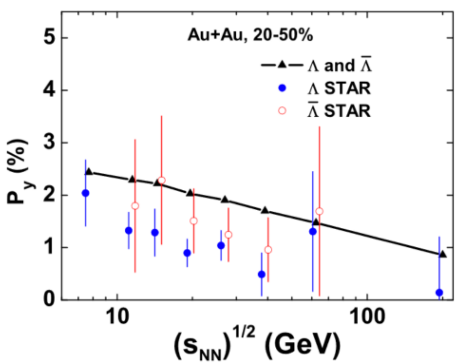
Karpenko, Becattini, EPJC(2017)



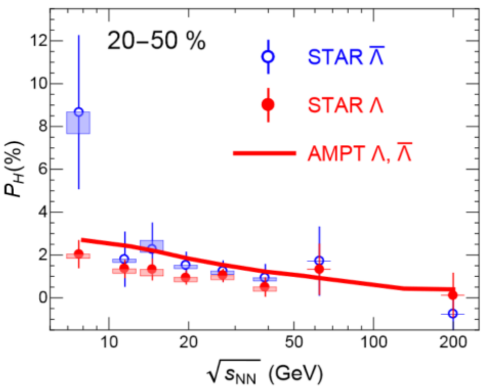
Li, Pang, Wang, Xia PRC(2017)



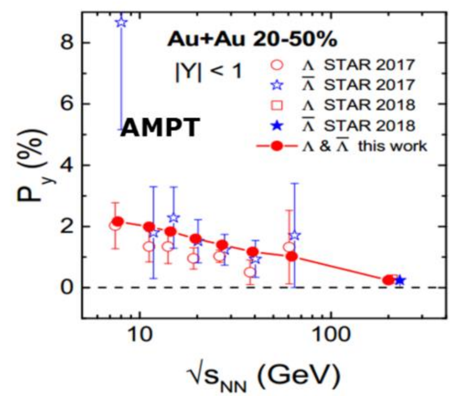
Xie, Wang, Csernai, PRC(2017)



Sun, Ko, PRC(2017)



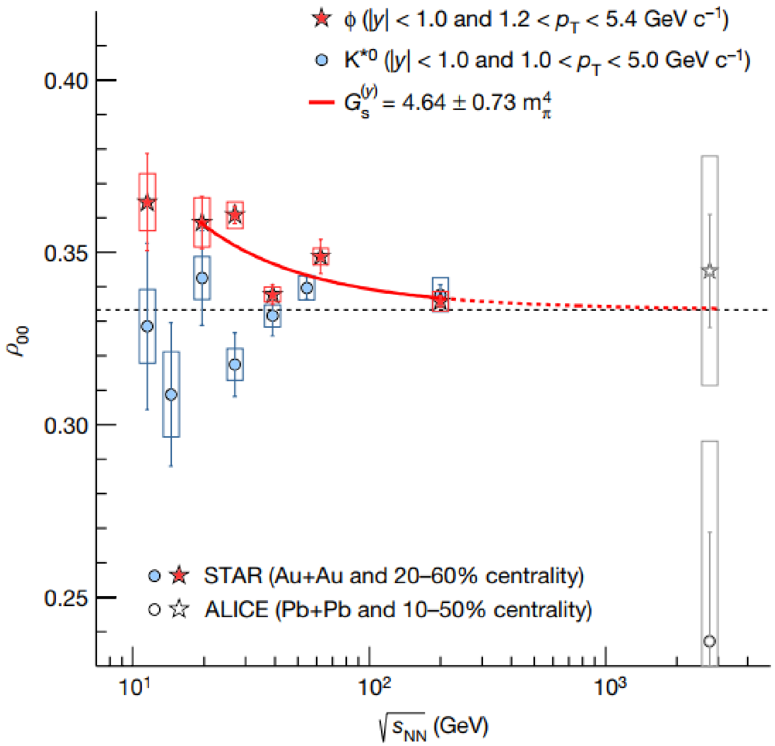
Shi, Li, Liao, PLB(2018)

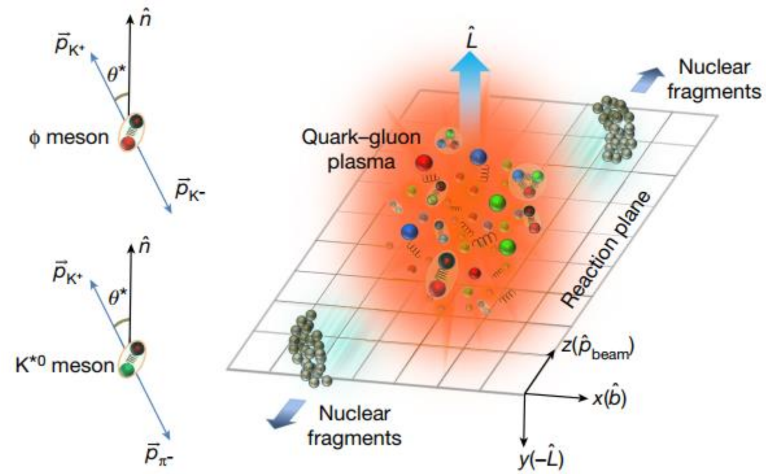


Wei, Deng, Huang, PRC(2019)

STAR: global spin alignments of vector mesons

STAR Collab., Nature 614, 244 (2023)





Theory prediction:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2022).

Implication of local correlation or fluctuation of strong force fields
[S. Singha's and X.L. Sheng's talks on July 15]

Single-particle distribution function in classical theory (no spin)

- **Single particle distribution function in phase space $f(t,x,p)$**

$$f(t,\mathbf{x},\mathbf{p})d^3xd^3p$$

particle number in phase space volume d^3xd^3p

- **The evolution of $f(t,x,p)$ is governed by the semi-classical Boltzmann equation**

$$\begin{aligned}\frac{d}{dt}f(t,\mathbf{x},\mathbf{p}) &= \left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{E_p}\cdot\nabla+\mathbf{F}\cdot\nabla_{\mathbf{p}}\right)f(t,\mathbf{x},\mathbf{p})=\mathcal{C}[f] \\ \mathcal{C}[f] &= \int_{124} d\widetilde{\Gamma}_{1,2\to p,4}(f_1f_2-f_pf_4)\end{aligned}$$

Classical feature: x and p of the particle can be determined at the same time !

QKT for massive fermions in Wigner functions

- **Wigner function (4x4 matrix) for spin 1/2 massive fermions**

$$W_{\alpha\beta}(x,p)=\int d^4y\exp\left(\frac{i}{\hbar}p\cdot y\right)\left\langle\overline{\psi}_\beta\left(x-\frac{y}{2}\right)\psi_\alpha\left(x+\frac{y}{2}\right)\right\rangle$$

Heinz, PRL 51, 351 (1983);
Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987)

- **Wigner function decomposition in 16 generators of Clifford algebra**

spin 4-vector

$$W=\frac{1}{4}\left[\mathscr{F}+i\gamma^5\mathscr{P}+\gamma^\mu\mathscr{V}_\mu+\gamma^5\gamma^\mu\mathscr{A}_\mu+\frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu=\int d^4p\mathscr{V}^\mu,\qquad j_5^\mu=\int d^4p\mathscr{A}^\mu,\qquad T^{\mu\nu}=\int d^4p\,p^\mu\mathscr{V}^\nu$$

Recent reviews:
Hidaka-Pu-QW-Yang, PPNP (2022)
Gao-Liang-QW, IJMPA (2021)

Vasak-Gyulassy-Elze, Ann. Phys. 173, 462 (1987);
Elze-Gyulassy-Vasak, Nucl. Phys. B 276, 706 (1986);

Continuous spin variable in quantum kinetic theory (with collisions)

Extended phase space: $(\boldsymbol{x}, \boldsymbol{p}) \Longrightarrow (\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{s})$

$$f(x,p,\mathfrak{s})\delta(p^2-m^2)d^4p\,dS(p)$$

$$dS(p)=\frac{1}{\pi}\sqrt{\frac{p^2}{3}}d^4\mathfrak{s}\,\underline{\delta(\mathfrak{s}^2+3)\delta(p\cdot\mathfrak{s})}$$

continuous spin variable s is space-like, which can be normalized as $\boldsymbol{s}^2=-3$ and normal to momentum $\boldsymbol{p}\cdot\boldsymbol{s}=0$ ($\boldsymbol{p}$ is time-like)

Weickgenannt, Speranza, Sheng, QW, Rischke (2021); Florkowski et al. (2019);
Spin DOF as Grassmann variables:
Mueller, Venugopalan (2019)
[D. Wagner's talk on July 17]

Boltzmann equation with non-local collisions

$$p\cdot\partial f=\underline{\mathfrak{C}[f]}$$

Non-local collision term

$$\underline{\mathfrak{C}[f]}=\int \underline{d\Gamma_1 d\Gamma_2 d\Gamma'}\,\mathcal{W}\left[f(x+\underline{\Delta_1},p_1,\mathfrak{s}_1)f(x+\underline{\Delta_2},p_2,\mathfrak{s}_2)\right.$$
$$\left.-f(x+\underline{\Delta},p,\mathfrak{s})f(x+\underline{\Delta'},p',\mathfrak{s}')\right]$$

$$d\Gamma\equiv d^4p\,\delta(p^2-m^2)dS(p)$$

Phase space measure

Space-time shift: "side-jump" [Chen, Son, Stephanov (2015)]

Spin DOF: Matrix Valued Spin Dependent Distributions (MVSD)

Relativistic MVSD for fermion in QFT

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q\cdot x\right) \delta(\underline{p\cdot q}) \left\langle a^\dagger(\underline{s,\mathbf{p}_2}) a(\underline{r,\mathbf{p}_1})\right\rangle$$

Relativistic MVSD can be parameterized in un-polarized distributions and Spin Density Matrix (polarization part)

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2}\underline{f_q(x,\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^q(x,\mathbf{p})}\,\underline{n_j^{(+)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2}\underline{f_{\bar{q}}(x,-\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^{\bar{q}}(x,-\mathbf{p})}\,\underline{n_j^{(-)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

un-polarized dist.

spin polarization dist.

Four-vectors of three basis directions in rest frame of q and $\bar{q}$ (one is the spin quantization direction)

MVSD:
Becattini et al. (2013)
Sheng, Weickgenannt, et al. (2021)
Sheng, QW, Rischke (2022)

Conversion between distribution with continuous spin variable and MVSD

- **From MVSD to distribution with continuous spin variable**

$$P^{\mu}(x,\mathbf{p})f(x,\mathbf{p}) = \sum_{i,r,s} n_i^{\mu}\tau_{sr}^{i} f_{rs}(x,\mathbf{p})$$

$$f(x,\mathbf{p},\mathfrak{s}) \sim \left[1-\mathfrak{s}\cdot P(x,\mathbf{p})\right] f(x,\mathbf{p})$$

- **From distribution with continuous spin variable to MVSD**

$$P^{\mu}(x,\mathbf{p}) \sim \int d^4p\, dS(p)\delta(p^2-m^2)\, \mathfrak{s}^{\mu} f(x,\mathbf{p},\mathfrak{s})$$

$$f_{rs}(x,\mathbf{p}) = \frac{1}{2} f(x,\mathbf{p})\left[\delta_{rs} - P(x,\mathbf{p})\cdot n_i \tau_{rs}^{i}\right]$$

Wigner functions in CTP for massive fermions

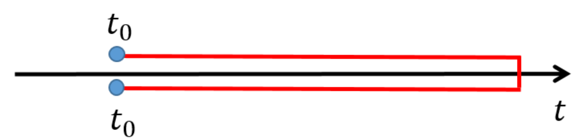
- **We use Closed-Time-Path (CTP) or Schwinger-Keldysh formalism.**

$$\begin{pmatrix} G^{++}(x_1,x_2) & G^{+-}(x_1,x_2) \\ G^{-+}(x_1,x_2) & G^{--}(x_1,x_2) \end{pmatrix} = \begin{pmatrix} G^{F}(x_1,x_2) & \underline{G^{<}(x_1,x_2)} \\ \underline{G^{>}(x_1,x_2)} & G^{\overline{F}}(x_1,x_2) \end{pmatrix}$$

Chou, Su, Hao, Yu, Phys. Rep. (1985); Blaizot, Iancu, Phys. Rep. (2002)

$$G^{<}_{\alpha\beta}(x,p) \equiv -\int d^4y e^{ip\cdot y/\hbar}\left\langle \bar{\psi}_{\beta}\left(x-\frac{y}{2}\right)\psi_{\alpha}\left(x+\frac{y}{2}\right)\right\rangle$$

Wigner transformation for spin-1/2 fermions



- **Wigner function in terms of MVSD at leading and next-to-leading order**

$$\begin{aligned} G^{<,(0)}_{\alpha\beta}(x,p) \quad = \quad & -2\pi\hbar\,\theta(p_0)\delta\left(p^2-m^2\right)\sum_{r,s} u_{\alpha}(r,p)\,\overline{u}_{\beta}(s,p)\,\underline{f^{(+,0)}_{rs}(x,p)} \\ & -2\pi\hbar\,\theta(-p_0)\delta\left(p^2-m^2\right)\sum_{r,s} v_{\alpha}(s,\overline{p})\,\overline{v}_{\beta}(r,\overline{p})\left[\delta_{rs}-\underline{f^{(-,0)}_{rs}(x,\overline{p})}\right] \end{aligned}$$

$\overline{\boldsymbol{p}} = (\boldsymbol{E_p}, -\boldsymbol{p})$

$$G^{<,(0)}_{\alpha\beta}[\underline{f^{(+,0)}_{rs}\to f^{(+,1)}_{rs}},\ \underline{\delta_{rs}-f^{(-,0)}_{rs}\to -f^{(-,1)}_{rs}}] \Longrightarrow G^{<,(1)}_{\alpha\beta}(x,p)$$

Sheng, Weickgenannt, et al. (2021)

Spin Boltzmann equation for massive fermions (with collisions)

- **Kadanoff-Baym's equation in terms of on-shell two-point function (Wigner function)**

Mrowczynski-Heinz (1994); Schonhofen-Cubero-Friman-Norenberg-Wolf (1994); ……

$$
\begin{aligned}
&\left(i\frac{1}{2}\hbar\gamma_\mu\partial_x^\mu+\gamma_\mu p^\mu-m\right)G^<(x,p)\\
=\;& -i\frac{1}{2}\hbar\left[\Sigma^<(x,p)G^>(x,p)-\Sigma^>(x,p)G^<(x,p)\right]\\
&-\frac{1}{4}\hbar^2\left[\left\{\Sigma^<(x,p),G^>(x,p)\right\}_{\mathrm{PB}}-\left\{\Sigma^>(x,p),G^<(x,p)\right\}_{\mathrm{PB}}\right]
\end{aligned}
$$

- **With two-point functions being expressed in terms of MVSDs, the Boltzmann equation with spin DOF can be derived from Kadanoff-Baym equation**

$$
\Sigma^{\lessgtr}\Rightarrow G^{\lessgtr}\Rightarrow f_{rs}^{(\pm)}(x,p)
$$

Spin Boltzmann equation for massive fermions

- **At leading order spin Boltzmann equation (SBE) with local collision terms**

$$\begin{aligned}\frac{1}{E_p}p\cdot\partial_x\mathrm{tr}\left[f^{(0)}(x,p)\right] &= \mathscr{C}_{\mathrm{scalar}}\left[f^{(0)}\right]\\ \frac{1}{E_p}p\cdot\partial_x\mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(0)}(x,p)\right] &= \mathscr{C}_{\mathrm{pol}}\left[f^{(0)}\right]\end{aligned} \Longrightarrow \boldsymbol{f_{rs}^{(0)}(x,p)}$$

- **At next-to-leading order, SBE describes how $\boldsymbol{f^{(1)}(x,p)}$ evolves for given $\boldsymbol{f^{(0)}(x,p)}$ with space-time derivatives of $\boldsymbol{f^{(0)}(x,p)}$**

$$\begin{aligned}\frac{1}{E_p}p\cdot\partial_x\mathrm{tr}\left[f^{(1)}(x,p)\right] &= \mathscr{C}_{\mathrm{scalar}}\left[\underline{f^{(0)},\partial_x f^{(0)}},f^{(1)}\right]\\ \frac{1}{E_p}p\cdot\partial_x\mathrm{tr}\left[n_j^{(+)\mu}\tau_j f^{(1)}(x,p)\right] &= \mathscr{C}_{\mathrm{pol}}\left[\underline{f^{(0)},\partial_x f^{(0)}},f^{(1)}\right] \longrightarrow \boxed{\partial_\mu u_\nu,\ \partial_\mu T,\ \partial_\mu \mu_B}\end{aligned}$$

determined by leading order SBE

Convenient for simulation !

Sheng, Speranza, Rischke, QW, Weickgenannt (2021)

spin transport for massive fermions from KB equation was also studied in:

Yang, Hattori, Hidaka (2020); Wang, Zhuang (2021)

Polarization from different sources in QKT with Wigner functions (without collisions)

Axial vector component of WF (spin vector) has many contributions

$$\mathcal{J}_5^\mu = \mathcal{J}^\mu_{\text{thermal}} + \mathcal{J}^\mu_{\text{shear}} + \mathcal{J}^\mu_{\text{accT}} + \mathcal{J}^\mu_{\text{chemical}} + \mathcal{J}^\mu_{\text{EB}},$$

Becattini, et al, (2021); Fu, Liu, et al., (2021);

[Becattini's and Fu's talks on July 15]

Thermal vorticity

$$\mathcal{J}^\mu_{\text{thermal}} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$

Shear viscous tensor

$$\mathcal{J}^\mu_{\text{shear}} = -a\frac{1}{(u\cdot p)T}\underline{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\partial_{<\sigma}u_{\nu>}}$$

Fluid acceleration

$$\mathcal{J}^\mu_{\text{accT}} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha(Du_\beta - \frac{1}{T}\partial_\beta T).$$

Gradient of chemical potential

$$\mathcal{J}^\mu_{\text{chemical}} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta\partial_\nu\frac{\mu}{T},$$

Electromagnetic fields

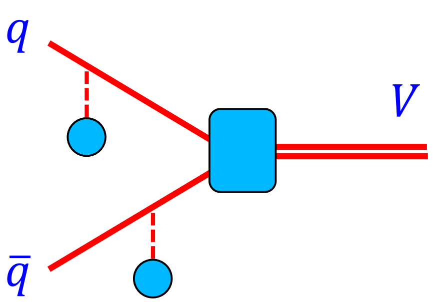
$$\mathcal{J}^\mu_{\text{EB}} = a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta E_\nu + a\frac{B^\mu}{T},$$

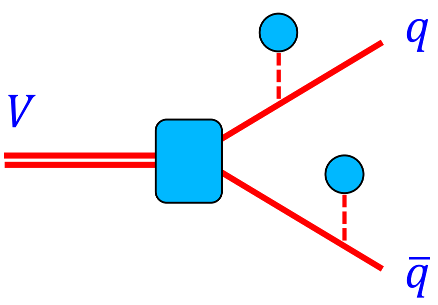
Hidaka, Pu, Yang (2018); Yi, Pu, Yang (2021); Shi Pu's talk on July 17;

Relativistic Spin Dynamics based on Spin Kinetic Equation (SKE) with MVSDs for vector mesons

Sheng, Oliva, et al., 2206.05868 (PRL in press), 2205.15689

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989





RSBE in MVSD for vector meson: fusion and dissociation process in

- **Relativistic MVSD for vector meson in QFT**

$$f^V_{\lambda_1\lambda_2} = \int \frac{d^4q}{2(2\pi\hbar)^3} \exp\left(-i\frac{q\cdot x}{\hbar}\right)\delta(p\cdot q)\left\langle a^\dagger_V(\lambda_2,\mathbf{p}_2)\, a_V(\lambda_1,\mathbf{p}_1)\right\rangle$$

- **RSBE for fusion (coalescence) and dissociation process $q\bar{q} \leftrightarrow V$ can be simplified as**

Coalescence collision kernel

In rest frame of vector meson: ϵ_λ is polarization 3-vector and n_x, n_y, n_z are three basis directions

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\left[\underline{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})}\underline{\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(x,\mathbf{k})}\right.$$
$$\left. - \underline{\mathcal{C}_{\mathrm{diss}}(\mathbf{k})} f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\right],$$

Dissociation collision kernel

$$\begin{aligned}
\boldsymbol{\epsilon}_0 &= \mathbf{n}_y \\
\boldsymbol{\epsilon}_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\
\boldsymbol{\epsilon}_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x)
\end{aligned}$$

$$\underline{\epsilon^\mu(\lambda,\mathbf{k})} = \left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V}, \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda}{m_V(E^V_\mathbf{k}+m_V)}\mathbf{k}\right) \Longrightarrow k_\mu\epsilon^\mu(\lambda,\mathbf{k}) = 0$$

polarization vector of vector meson

MVSD or spin density matrix element for vector mesons

Forml solution to MVSD (spin density matrix) for vector mesons

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\text{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\text{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})$$

where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k}) = & \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^{q}_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\bar{q}}_{\mathbf{p}'}-E^{q}_{\mathbf{k}-\mathbf{p}'}\right)\\ & \times \mathrm{Tr}\left\{\underline{\Gamma^\nu}\left(p'\cdot\gamma-m_{\bar{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\bar{q}}(x,\mathbf{p}')}\right]\right.\\ & \left.\times\underline{\Gamma^\mu}\left[(k-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot\underline{P^q(x,\mathbf{k}-\mathbf{p}')}\right]\right\}\\ & \times \underline{f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}')},\end{aligned}$$

Covariant polarization phase space distributions for q and $\bar{q}$

BS Wave function for VM [Roberts et al (2019, 2021)]

un-polarized distributions for q and $\bar{q}$

$$\boxed{\Gamma^\alpha \approx g_V B(\mathbf{k}-\mathbf{p}',\mathbf{p}')\gamma^\alpha}$$

Spin density matrix element for vector mesons

Spin density matrix (normalized MVSD) for vector mesons

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}}$$

For ϕ meson, covariant polarization phase space distributions for s and $\bar{s}$ appearing in $C^{\mu\nu}_{coal}$ have the form

$$P^\mu_s(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\text{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

field strength tensor of ϕ field

Spin density matrix element for vector mesons

The fusion (coalescence) collision kernel $C_{coal}^{\mu\nu}$ can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}

$$
\begin{aligned}
\rho_{00}(x,\underline{\mathbf{0}}) \approx & \frac{1}{3} + C_1 \left[\frac{1}{3}\boldsymbol{\omega}' \cdot \boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0 \cdot \boldsymbol{\omega}')^2\right] \\
& + C_2 \left[\frac{1}{3}\boldsymbol{\varepsilon}' \cdot \boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0 \cdot \boldsymbol{\varepsilon}')^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\underline{\boldsymbol{\epsilon}_0} \cdot \mathbf{B}'_\phi)^2\right] \\
& - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} C_2 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\underline{\boldsymbol{\epsilon}_0} \cdot \mathbf{E}'_\phi)^2\right],
\end{aligned}
$$

rest frame of ϕ meson

$$
C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)},
$$

$$
C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2 (m_\phi^2 + 2m_s^2)}.
$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

Features: (1) Perfect factorization of x and p dependence; (2) Perfect cancellation for mixing terms (protected by symmetry): all fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields. Surprising results!

Lorentz transformation for ϕ fields

We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons through Lorentz transformation

$$\mathbf{B}'_\phi = \gamma\mathbf{B}_\phi - \gamma\mathbf{v}\times\mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$

$$\mathbf{E}'_\phi = \gamma\mathbf{E}_\phi + \gamma\mathbf{v}\times\mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v},$$

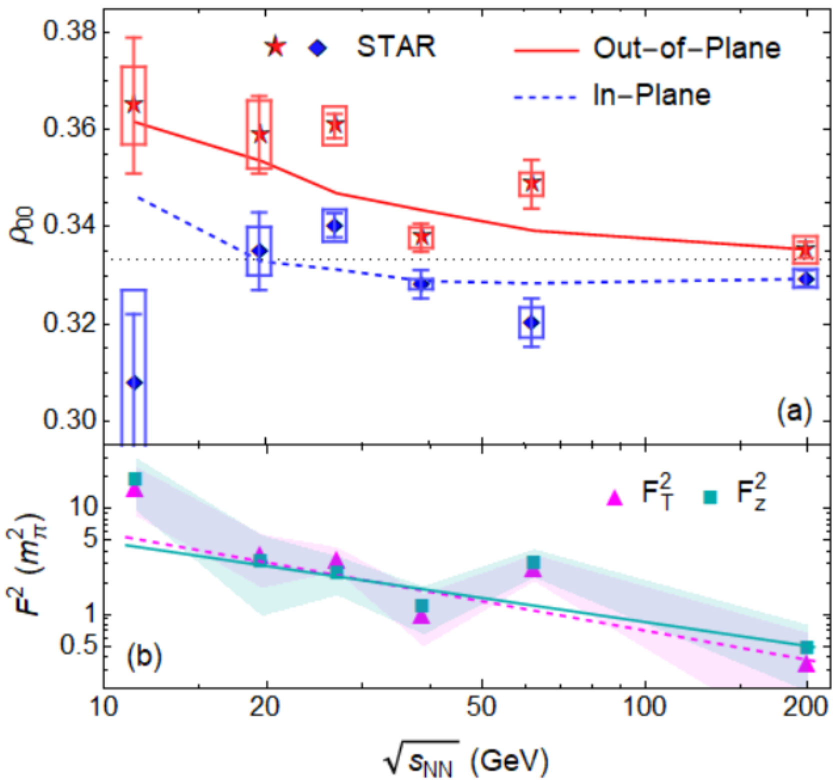
where $\gamma = E_{\mathbf{k}}^{\phi}/m_\phi$ **and** $\mathbf{v} = \mathbf{k}/E_{\mathbf{k}}^{\phi}$

Then we obtain factorization form $\left\langle \rho_{00}^{\phi} \right\rangle$ in terms of lab-frame fields

$$\left\langle \overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \right\rangle_{x,\mathbf{p}} \approx \frac{1}{3} + \frac{1}{3}\underbrace{\sum_{i=1,2,3}}_{\text{three basis directions in lab frame}} \underbrace{\langle I_{B,i}(\mathbf{p})\rangle}_{\text{momentum average}} \frac{1}{m_\phi^2}\left[\left\langle \boldsymbol{\omega}_i^2 \right\rangle - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\left\langle (\mathbf{B}_i^\phi)^2 \right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3} \langle I_{E,i}(\mathbf{p})\rangle \frac{1}{m_\phi^2}\left[\left\langle \boldsymbol{\varepsilon}_i^2 \right\rangle - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\left\langle (\mathbf{E}_i^\phi)^2 \right\rangle\right]$$

space-time average

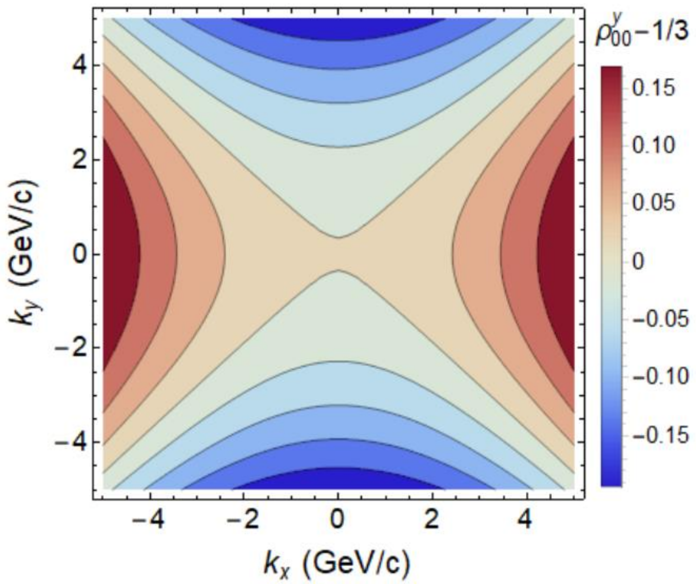
Spin density matrix element for vector mesons



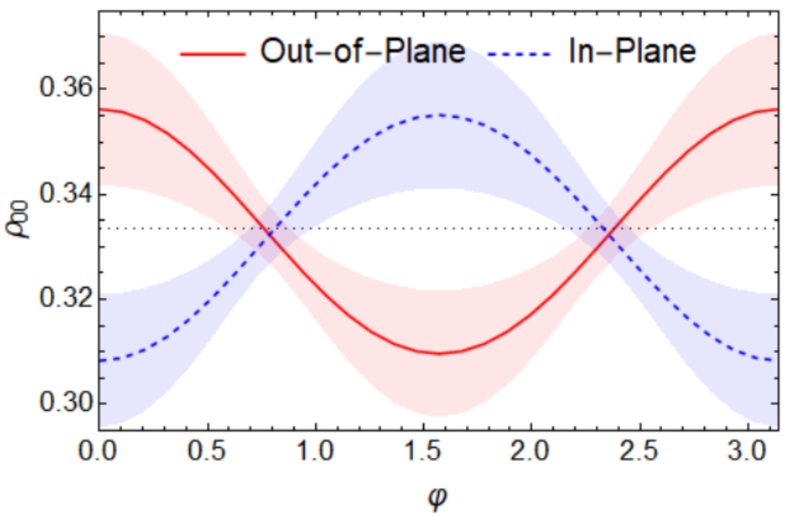
(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (c). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$ (see the text).

Spin density matrix element for vector mesons

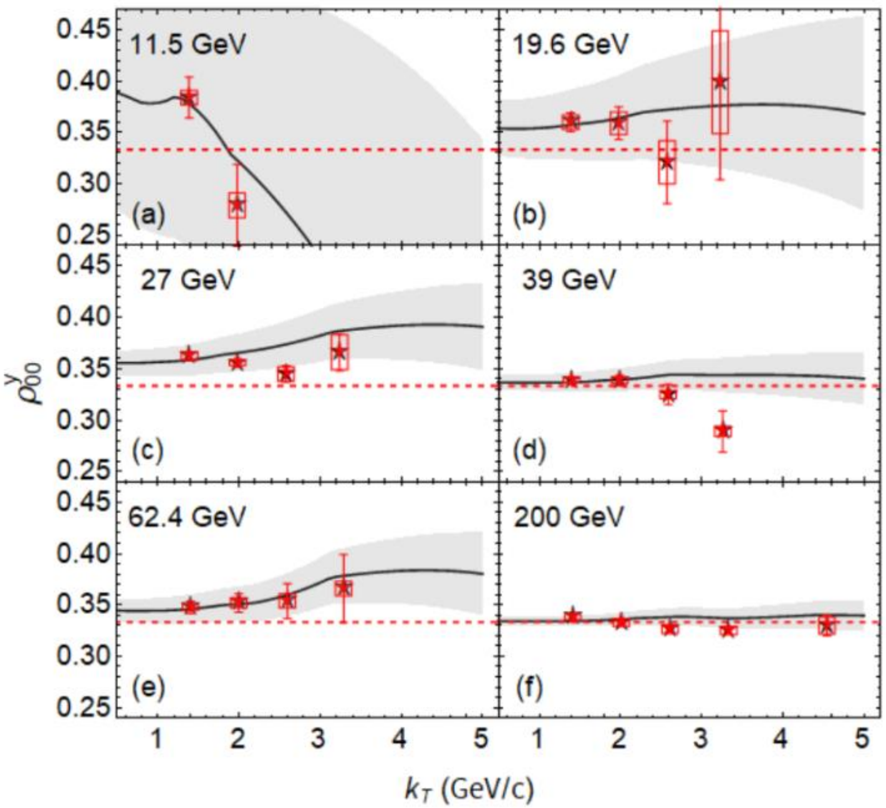


Contour plot of $\rho^y_{00} - 1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.



Calculated ρ^y_{00} (out-of-plane) and ρ^x_{00} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Shaded error bands are from the extracted parameters F_T^2 and F_z^2.

Spin density matrix element for vector mesons



Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Vector fields in Chiral quark model

Nuclear Physics B234 (1984) 189–212

Citations: 2232 (till April 2023)

Fernandez, Valcarce, Straub, Faessler (1993)
Zhang, et al, (1997); Li, Ye, Lu (1997); Zhao, Li, Bennhold (1998)

CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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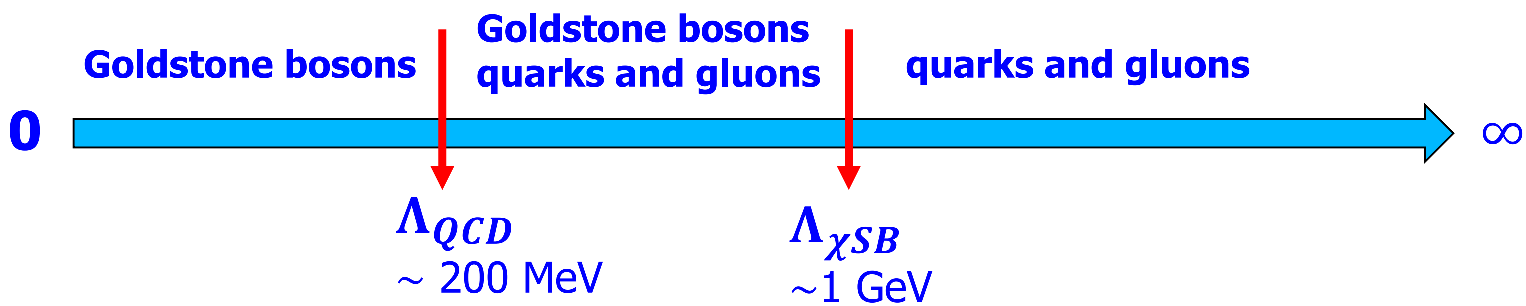
Received 18 July 1983

We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

Vector fields in Chiral quark model

- **Scale for strong interaction in dynamical process**



- **SU(3) Goldstone bosons by 3×3 matrix Σ and ξ,**

$$\begin{aligned}\Sigma &= \exp\left(i\frac{2\chi}{f}\right)\\ &= \exp\left(i\frac{\chi}{f}\right)\exp\left(i\frac{\chi}{f}\right)\\ &= \xi\xi\end{aligned}$$

f **= 93 MeV**

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Vector field in Chiral quark model

- **Σ and ξ transform under $SU_L(3) \times SU_R(3)$ as**

$$\Sigma \to L\Sigma R^\dagger, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger$$

- **A set of color and flavor triplet quarks** $\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \qquad \psi = U\psi$
- **Lagrangian is invariant under $SU_L(3) \times SU_R(3)$ transformation**

$$\begin{aligned} \mathcal{L} &= \overline{\psi}\left[i\gamma_\mu\left(\partial^\mu + igG^\mu\right) + \underline{g_V\gamma_\mu V^\mu}\right]\psi + g_A\overline{\psi}\gamma_\mu V^\mu\psi \\ &\quad + \frac{1}{4}f^2\mathrm{Tr}\left(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right) - \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} \end{aligned}$$

3x3 matrix

$$\boxed{V^\mu = \frac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)}$$

$$A^\mu = \frac{1}{2}i\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)$$

Effective vector fields induced by gradients of Goldstone boson fields

Take-home message and Questions for discussions

Take-home message

- P_Λ measures the fields (net mean field), ρ_{00}^{ϕ} measures field squared (field correlation or fluctuation).
- The ϕ field is induced by current of pseudo-Goldstone boson during the hadronization

Questions to be answered in the future:

- Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?
- Implication for J/Psi polarization (gluon fields)?
- Any connection with effects from glasma fields? (Kuma, Mueller, Yang, 2023)
- Other contributions from hydro quantities [Li, Liu (2022); Wagner, Weickgenannt, Speranza (2022)]

Ideal spin hydrodynamics with Wigner functions

H.H. Peng, J.J. Zhang, X.L. Sheng, QW, Chin. Phys. Lett. 38, 116701 (2021)
[Feature: (1) Rigorous power counting scheme; (2) Analytical solution of Wigner function to the 2nd order; (3) Exact evolution equations for spin hydro variables to the 2nd order]

Earlier works:
Florkowski, Friman, Jaiswal, Speranza, Phys.Rev. C97, 041901 (2018)
Florkowski, Friman, Ryblewski, Speranza, Phys.Rev. D97, 116017 (2018)

Review:
Florkowski, Kumar, Ryblewski, Prog.Part.Nucl.Phys. 108, 103709 (2019)

Quantum kinetic equation and Wigner functions

- **The kinetic equation of Wigner function can be derived from the Dirac equation**

$$\left[\gamma_\mu\left(p^\mu+\frac{i}{2}\partial^\mu\right)-m\right]W(x,p)=0$$

- **Power counting**

$$\mathrm{Kn}\sim\left|\frac{\partial_\mu O}{O}\right|\ll 1$$

$$\chi_s\sim\frac{\left|S^{\lambda,\mu\nu}\right|}{n}\sim\frac{\left|M^\mu\right|}{n}\ll 1$$

Wigner function at O(1)

$$\begin{aligned}W_0(x,p)=&\frac{1}{(2\pi)^3}\delta(p^2-m^2)\\&\times\sum_{rs}\left[\theta(p_0)u(r,\mathbf{p})\overline{u}(s,\mathbf{p})f^+_{rs}(x,\mathbf{p})\right.\\&\left.-\theta(-p_0)v(r,-\mathbf{p})\overline{v}(s,-\mathbf{p})f^+_{rs}(x,-\mathbf{p})\right]\end{aligned}$$

Weickgenannt, Sheng, Speranza, QW, Rischke (2019)
Sheng, Weickgenannt, Speranza, Rischke, QW (2021)

The 1st and 2nd solutions to Wigner functions

- **The 1st and 2nd order corrections in space-time gradient for the Wigner function can be obtained by solving the kinetic equation**

$$\delta W = \frac{i}{4m}\left[\gamma^{\mu}, \partial_{\mu} W_0\right] + \frac{1}{16m^2}(\gamma\cdot\partial)W_0(\gamma\cdot\overleftarrow{\partial}) + \frac{\gamma\cdot p + m}{8m(p^2 - m^2)}\partial^2 W_0$$

$$W = W_0 + \delta W$$

Peng, J.-J. Zhang, X.-L. Sheng, QW (2021)

- **The appearance of δW is a result of the uncertainty principle for quantum particles with non-local correlation. These corrections include the electric dipole moment induced by an inhomogeneous charge distribution, the magnetization current, and the off-mass-shell correction.**

MVSDs and conservation law

- **The MVSDs in thermal equilibrium are assumed to be in the form [Becattini, Chandra, Del Zanna, Grossi, Ann. Phys. (2013)]**

$$f^{+}_{\mathrm{eq},rs}(x,\mathbf{p}) = \frac{1}{2m}\overline{u}(r,\mathbf{p})\left(e^{\beta\cdot p-\xi-\underline{\omega_{\mu\nu}\sigma^{\mu\nu}/4}}+1\right)^{-1}u(s,\mathbf{p})$$

$$f^{-}_{\mathrm{eq},rs}(x,-\mathbf{p}) = -\frac{1}{2m}\overline{v}(r,-\mathbf{p})\left(e^{\beta\cdot \overline{p}-\xi-\underline{\omega_{\mu\nu}\sigma^{\mu\nu}/4}}+1\right)^{-1}v(s,-\mathbf{p})$$

$$p = (E_p,\mathbf{p})$$
$$\overline{p} = (E_p,-\mathbf{p})$$

- **The current density, the energy-momentum tensor (density), and the spin tensor (density) can be obtained from vector and axial vector components of WF**

$$J^{\mu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = \int d^4p\mathcal{V}^{\mu}(x,p)$$

$$T^{\mu\nu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = \int d^4p p^{\nu}\mathcal{V}^{\mu}(x,p)$$

$$S^{\lambda,\mu\nu}\left[\beta^{\rho},\xi,\omega^{\rho\sigma}\right] = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}\int d^4pA_{\rho}(x,p)$$

$\Longrightarrow$

$$\partial_\mu J^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\lambda S^{\lambda,\mu\nu} = T^{\mu\nu}-T^{\nu\mu}$$

Evolution equations for hydro variables

- **Constitutive relations for current, energy momentum and spin tensor to second order in Kn and χ_s**

$$J^{\mu}_{\text{eq}} = n_{\text{eq}} u^{\mu} + \underline{\delta j^{\mu}},$$

$$T^{\mu\nu}_{\text{eq}} = \epsilon_{\text{eq}} u^{\mu} u^{\nu} - P_{\text{eq}} \Delta^{\mu\nu} + \underline{\delta T^{\mu\nu}_{S} + \delta T^{\mu\nu}_{A}}$$

$$\boxed{S^{\lambda,\mu\nu}_{\text{eq}}(x) = \frac{1}{4}\left(u^{\lambda}\omega^{\mu\nu} + 2u^{[\mu}\omega^{\nu]\lambda}\right) K_1 \cosh\xi}$$

depend on spin potential $\omega_{\mu\nu}$

spin tensor

- **The equations of motions for β^{μ}, ξ, $\omega_{\mu\nu}$**

$$\dot{\beta} = \frac{K_2 + \beta^{-1} K_1 \cosh^2\xi}{K_1 K_3 \cosh^2\xi - K_2 K_2 \sinh^2\xi} K_1 \theta\,,$$

$$\dot{\xi} = \frac{\left(K_2 + \beta^{-1} K_1\right) K_2 - K_1 K_3}{K_1 K_3 \cosh^2\xi - K_2 K_2 \sinh^2\xi} \theta \sinh\xi \cosh\xi$$

$$\dot{u}^{\mu} = \frac{K_1}{K_1 + \beta K_2} \tanh\xi \nabla^{\mu}\xi - \frac{1}{\beta}\nabla^{\mu}\beta\,,$$

$$\frac{d}{d\tau} \equiv u_{\mu}\partial^{\mu}, \quad \nabla^{\mu} \equiv \Delta^{\mu\nu}\partial_{\nu}$$

$$K_n(\beta) \equiv \frac{8}{(2\pi)^3} \int \frac{d^3\mathbf{p}}{2E_{\mathbf{p}}} E^n_{\mathbf{p}} e^{-\beta E_{\mathbf{p}}}$$

$$\boxed{\dot{\omega}^{\mu\nu} = \Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\dot{\omega}^{\alpha\beta} - u^{\mu}\dot{\omega}^{\nu\alpha}u_{\alpha} + u^{\nu}\dot{\omega}^{\mu\alpha}u_{\alpha}}$$

evolution equation for spin potential

MVSDs and conservation law

- **Here the terms in l.f.s. of the evolution equition for $\omega^{\mu\nu}$ are**

$$
\begin{aligned}
\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\dot{\omega}^{\alpha\beta} &= C_3\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}\omega^{\alpha\beta} + C_4\Delta^{[\mu}_{\beta}\sigma_h^{\nu]\rho}\omega^{\beta}{}_{\rho} \\
&\quad -\frac{1}{2}C_4(\nabla^{[\mu}\omega^{\nu]}{}_{\rho})u^{\rho} + C_2C_4u^{\rho}\omega^{[\mu}{}_{\rho}\nabla^{\nu]}\xi \\
\dot{\omega}^{\mu\nu}u_{\nu} &= C_1\omega^{\mu\nu}u_{\nu} + C_2\Delta^{\mu}_{\rho}\omega^{\rho\nu}\nabla_{\nu}\xi \\
&\quad +\sigma_h^{\mu\nu}\omega_{\nu\rho}u^{\rho} + \frac{1}{2}\Delta^{\mu}_{\rho}(\nabla^{\nu}\omega^{\rho}{}_{\nu}),
\end{aligned}
$$

- **where C_i $(i = 1, 2, 3, 4)$ are analytical function of hydro variables $(\beta,\ \xi,\ \theta,\ \dot{\beta},\ \dot{\xi})$**

Peng, J.-J. Zhang, X.-L. Sheng, QW (2021)

Viscous spin hydrodynamics

Hattori, Hongo, et al., PLB(2019); Li, Stephanov, Yee, PRL(2021); Fukushima, Pu, PLB (2021); Bhadury, Florkowski, et al., PRD(2021); Weickgenannt, Wagner, et al., PRD(2022); She, Huang, et al., Sci.Bull. (2022); many others ……..

Talks by
D. Rischke
K. Hattori

First order viscous spin hydrodynamics

1. **Introduce spin potential term $\omega_{\mu\nu}S^{\mu\nu}$ into Gibbs-Duhem relation, assume constitutive relation for spin tensor $S^{\mu\nu}[u^\alpha, \omega^{\alpha\beta}]$**
2. **Introduce anti-symmetric term into EM tensor $T^{\mu\nu}_{asym}[q^\alpha, \phi^{\alpha\beta}]$**
3. **From entropy principle (divergence of entropy current should be non-negative), one obtains expressions for $q^\mu[u^\alpha, \omega^{\alpha\beta}]$ and $\phi^{\mu\nu}[u^\alpha, \omega^{\alpha\beta}]$.**

Summary

Covariant Wigner function building-block of quantum kinetic theory

Spin Boltzmann equation with local and non-local collisions

Spin hydrodynamics with local and global equilibrium of spin