



# Generalized Chiral Kinetic Equations & Revisit Spin Effects by Thermal Vorticity

→ S.X. Ma, JHG , arXiv: 2209.10737

JHG, arXiv: XXXX.XXXX ←

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The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

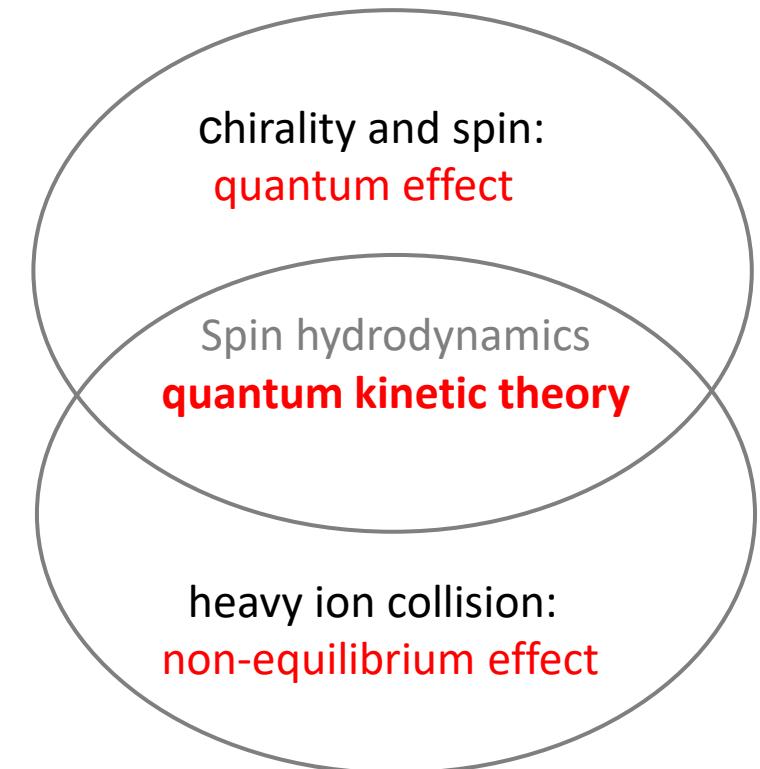
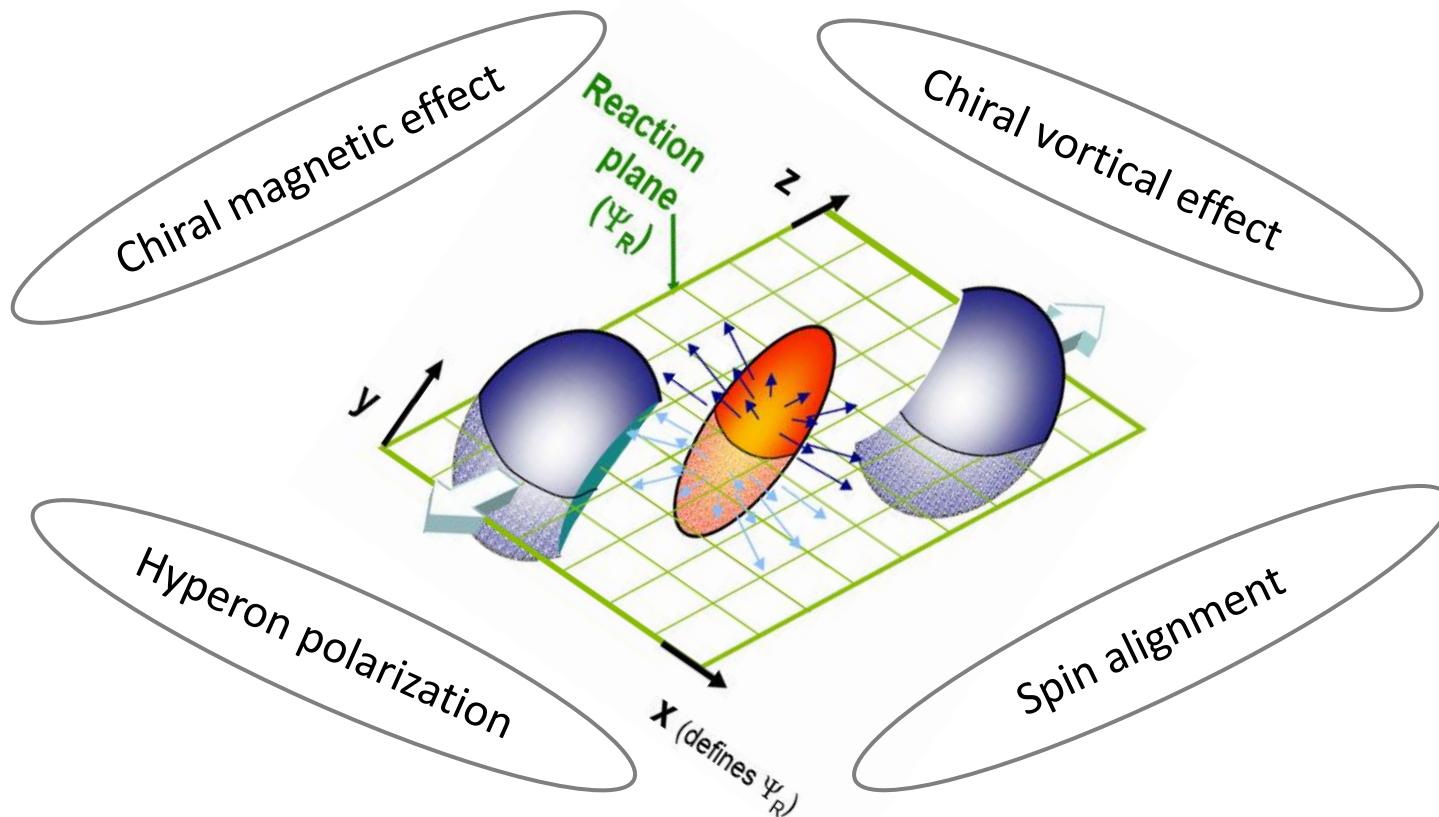
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# Generalized Chiral Kinetic Equations (GCKE)

- Introduction and motivation
- Derive the GCKE in Wigner function formalism
- The result of the GCKE and simple applications

# Introduction

- Chiral and spin effects in heavy-ion collisions



# Introduction

- Quantum Kinetic Theory for **massless** particles:
- Stephanov, Yin PRL2012;  
Chen, Pu, Q. Wang , X. Wang PRL2013,  
Hidaka,Pu, Yang PRD 2017;  
Huang, Shi, Jiang, Liao, Zhuang PRD2018;  
Gao, Liang, Q.Wang, X.Wang PRD2018;  
Liu, Gao, Mamed, Huang PRD2019;  
Lin, Yang PRD2020  
Luo, Gao JHEP2021  
.... .... ....
- Quantum Kinetic Theory for **massive** particles:
- Weickgenannt, Sheng, Speranza , Wang PRD2019;  
Gao, Liang PRD2019;  
Hattori, Hidaka, Yang PRD2019;  
Wang, Guo, Shi , Zhuang PRD2019  
Liu, Mamed, Huang CPC2021  
Sheng, Wang, Rischke PRD2022  
Weickgenannt, Speranza, Sheng, Wang PRD2019  
Lin, PRD2022  
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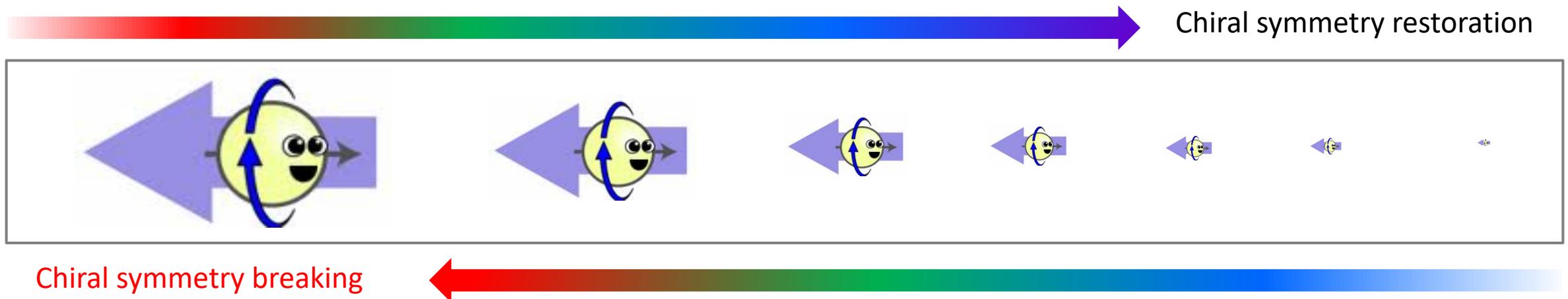
- Talks on quantum kinetic theory in this workshop:

Next talk: X.L. Sheng Morning session-I 17/07: Q. Wang, S. Pu, K. Mameda Morning session-II 17/07: S. Lin, Z. Wang, X. Guo  
Afternoon session-I 17/07: A. Mizher, D. Wagner Parallel A Afternoon session-I 18/07: J.Y. Tian, S. Fang, Z.L. Mo, P.W. Yu  
Parallel B Afternoon session-II 18/07: L.X. Yang

# Introduction

- Smooth transition from **massive** to **massless**: **Not Trivial !**

Hattori, Hidaka, Yang PRD2019; Wang, Guo, Shi , Zhuang PRD2019; Sheng, Wang, Huang PRD2020; Guo CPC2020



- Quantum kinetic equation with a trivial smooth transition and closest form to CKE ?

Chiral Kinetic equation



Generalized Chiral Kinetic equations

# Wigner function formalism

- Wigner function for spin-1/2 fermion in gauge field: Vasak, Gyulassy, Elze, Annals Phys. 1987

$$W(x, p) = \left\langle \int \frac{d^4y}{(2\pi)^4} e^{-ip \cdot y} \bar{\psi} \left( x + \frac{y}{2} \right) U \left( x + \frac{y}{2}, x - \frac{y}{2} \right) \psi \left( x - \frac{y}{2} \right) \right\rangle$$

- 16 independent Wigner functions:  $W = \frac{1}{4} \left[ \mathcal{F} + i\gamma^5 \mathcal{P} + \gamma^\mu \mathcal{V}_\mu + \gamma^5 \gamma^\mu \mathcal{A}_\mu + \frac{1}{2} \sigma^{\mu\nu} \mathcal{S}_{\mu\nu} \right]$

- 32 Wigner equations in background EM field at  $O(\hbar)$  : high constrained!

$\begin{aligned} 0 &= p^\mu \mathcal{A}_\mu \\ 0 &= \nabla^\mu \mathcal{V}_\mu \\ 0 &= \frac{1}{2} \nabla_\mu \mathcal{F} - p^\nu \mathcal{S}_{\mu\nu} \\ 0 &= p_\mu \mathcal{P} + \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} \nabla^\nu \mathcal{S}^{\rho\sigma} \\ 0 &= (p_\mu \mathcal{V}_\nu - p_\nu \mathcal{V}_\mu) + \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \nabla^\rho \mathcal{A}^\sigma \end{aligned}$	$\begin{aligned} m\mathcal{F} &= p^\mu \mathcal{V}_\mu \\ m\mathcal{P} &= -\frac{1}{2} \nabla^\mu \mathcal{A}_\mu \\ m\mathcal{V}_\mu &= p_\mu \mathcal{F} + \frac{1}{2} \nabla^\nu \mathcal{S}_{\mu\nu} \\ m\mathcal{A}_\mu &= \frac{1}{2} \nabla_\mu \mathcal{P} - \epsilon_{\mu\nu\rho\sigma} p^\nu \mathcal{S}^{\rho\sigma} \\ m\mathcal{S}_{\mu\nu} &= -\epsilon_{\mu\nu\rho\sigma} p^\rho \mathcal{A}^\sigma + \frac{1}{2} (\nabla_\mu \mathcal{V}_\nu - \nabla_\nu \mathcal{V}_\mu) \end{aligned}$	$\begin{aligned} \mathcal{F} &= \frac{1}{m} p^\mu \mathcal{V}_\mu \\ \mathcal{P} &= \frac{1}{m} \left( -\frac{1}{2} \nabla^\mu \mathcal{A}_\mu \right) \quad \text{singularity } \frac{1}{m} \\ \mathcal{V}_\mu &= \frac{1}{m} \left( p_\mu \mathcal{F} + \frac{1}{2} \nabla^\nu \mathcal{S}_{\mu\nu} \right) \\ \mathcal{A}_\mu &= \frac{1}{m} \left( \frac{1}{2} \nabla_\mu \mathcal{P} - \epsilon_{\mu\nu\rho\sigma} p^\nu \mathcal{S}^{\rho\sigma} \right) \\ \mathcal{S}_{\mu\nu} &= \frac{1}{m} \left( -\epsilon_{\mu\nu\rho\sigma} p^\rho \mathcal{A}^\sigma + \frac{1}{2} (\nabla_\mu \mathcal{V}_\nu - \nabla_\nu \mathcal{V}_\mu) \right) \end{aligned}$
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# How to derive GCKE

- Introduce an auxiliary time-like 4-vector  $n^\mu$  with normalization  $n^2 = 1$ :

Vector decomposition:  $X_\mu = X_n n^\mu + \bar{X}^\mu$        $\bar{X}_\mu = \Delta_{\mu\nu} \bar{X}^\nu$        $\Delta_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$

Tensor decomposition:  $F^{\mu\nu} = E^\mu n^\nu - E^\nu n^\mu - \bar{\epsilon}^{\mu\nu\sigma} B_\sigma$        $\mathcal{I}^{\mu\nu} = \mathcal{K}^\mu n^\nu - \mathcal{K}^\nu n^\mu - \bar{\epsilon}^{\mu\nu\sigma} \mathcal{M}_\sigma$        $\bar{\epsilon}^{\mu\nu\sigma} \equiv \epsilon^{\mu\rho\nu\sigma} n_\rho$

**Express one Wigner function as another one without  $1/m$  singularity :**

Example 1:  $m\mathcal{F} = p^\mu \mathcal{V}_\mu \quad \Rightarrow \quad \mathcal{F} = \frac{1}{m} p^\mu \mathcal{V}_\mu$       VS       $m\mathcal{V}_\mu = p_\mu \mathcal{F} + \frac{1}{2} \nabla^\nu \mathcal{I}_{\mu\nu} \quad \Rightarrow \quad \mathcal{F} = \frac{m}{p_n} \mathcal{V}_n + \frac{1}{2p_n} \nabla^\nu \mathcal{K}_\nu$

Example 2:  $m\mathcal{P} = -\frac{1}{2} \nabla^\mu \mathcal{A}_\mu \quad \Rightarrow \quad \mathcal{P} = \frac{1}{m} \left( -\frac{1}{2} \nabla^\mu \mathcal{A}_\mu \right)$       VS       $0 = p_\mu \mathcal{P} + \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} \nabla^\nu \mathcal{I}^{\rho\sigma} \quad \Rightarrow \quad \mathcal{P} = \frac{1}{2p_n} \nabla^\nu \mathcal{M}_\nu$

- Introduce a chirality basis to be consistent with chiral kinetic theory:  $\mathcal{J}_\mu^s \equiv \frac{1}{2} (\mathcal{V}_\mu + s \mathcal{A}_\mu) \quad s = \pm 1$

$$\begin{aligned} \nabla^\mu \mathcal{J}_\mu^s &= -ms\mathcal{P} & \frac{1}{2} \nabla_\mu \mathcal{F} - p^\nu \mathcal{I}_{\mu\nu} &= 0 & p_\mu \mathcal{P} + \frac{1}{2} \nabla^\nu \tilde{\mathcal{I}}_{\mu\nu} &= 0 & \tilde{\mathcal{I}}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \mathcal{I}_{\rho\sigma} \\ p^\mu \mathcal{J}_\mu^s &= \frac{1}{2} m \mathcal{F} & \frac{1}{2} \nabla_\mu \mathcal{P} - p^\nu \tilde{\mathcal{I}}_{\mu\nu} &= m \sum_s s \mathcal{J}_\mu^s & p_\mu \mathcal{F} + \frac{1}{2} \nabla^\nu \mathcal{I}_{\mu\nu} &= m \sum_s \mathcal{J}_\mu^s \end{aligned}$$

$$(p_\mu \mathcal{J}_\nu^s - p_\nu \mathcal{J}_\mu^s) + \frac{1}{2} s \epsilon_{\mu\nu\rho\sigma} \nabla^\rho \mathcal{J}^{s\sigma} = ms \tilde{\mathcal{I}}_{\mu\nu}$$

# GCKE in 8-dimensional phase space

- Choose 4 independent distribution functions:  $\mathcal{J}_n^s(x, p)$      $\mathcal{M}_\perp^\mu(x, p)$      $n \cdot \mathcal{M} = 0$      $\bar{p} \cdot \mathcal{M}_\perp^\mu = 0$      $\mathcal{M}^\mu = \mathcal{M}_\parallel^\mu + \mathcal{M}_\perp^\mu$
- Some Wigner equations express the other Wigner functions in terms of  $\mathcal{J}_n^s$  and  $\mathcal{M}_\perp^\mu$ :

$$\mathcal{M}_\parallel^\mu = \frac{m\bar{p}^\mu}{\bar{p}^2} \sum_s s \mathcal{J}_{sn}, \quad \bar{\mathcal{J}}_s^\mu = (\Delta^{\mu\sigma} + \frac{s}{2p_n} \bar{\epsilon}^{\mu\rho\sigma} \nabla_\rho) (\frac{\bar{p}_\sigma}{p_n} \mathcal{J}_{sn} - \frac{sm}{2p_n} \mathcal{M}_\sigma), \quad \mathcal{F} = \frac{m}{p_n} \sum_s \mathcal{J}_{sn} + \frac{1}{2p_n} \bar{\epsilon}^{\mu\alpha\beta} \nabla_\mu (\frac{p_\alpha}{p_n} \mathcal{M}_\beta), \quad \mathcal{K}_\mu = \frac{1}{p_n} \bar{\epsilon}_{\mu\alpha\beta} p^\alpha \mathcal{M}^\beta + \frac{1}{2p_n} \bar{\nabla}_\mu (\frac{m}{p_n} \sum_s \mathcal{J}_{sn}), \quad \mathcal{P} = \frac{1}{2p_n} \nabla_\mu \mathcal{M}^\mu,$$

- Some Wigner equations give the modified on-shell conditions:

$$(p^2 - m^2) \frac{\mathcal{J}_{sn}}{p_n} = \frac{s}{p_n} B^\mu (\bar{p}_\mu \frac{\mathcal{J}_{sn}}{p_n} - \frac{sm}{2} \cdot \frac{\mathcal{M}_\mu}{p_n})$$



$$(p^2 - m^2) \frac{\mathcal{M}_\mu}{p_n} = \frac{m}{p_n} B_\mu \sum_s \frac{\mathcal{J}_{sn}}{p_n}$$

New distribution functions  $\mathcal{J}_{sn}$   $\mathcal{M}^\mu$



$$\mathcal{J}_{sn} = p_n \mathcal{J}_{sn} \delta(p^2 - m^2) - s B^\mu (\bar{p}_\mu \mathcal{J}_{sn} - \frac{sm}{2} \mathcal{M}_\mu) \delta'(p^2 - m^2)$$

$$\mathcal{M}_\mu = p_n \mathcal{M}_\mu \delta(p^2 - m^2) - m B_\mu \sum_s \mathcal{J}_{sn} \delta'(p^2 - m^2)$$

- Some Wigner equations are the GCKE in 8-dimensional phase space :

$$p^\mu \nabla_\mu (\frac{\mathcal{J}_{sn}}{p_n}) = -\frac{s}{2} \bar{\epsilon}^{\mu\rho\sigma} \nabla_\mu [\frac{1}{p_n} \nabla_\rho (\bar{p}_\sigma \frac{\mathcal{J}_{sn}}{p_n} - \frac{sm}{2} \cdot \frac{\mathcal{M}_\sigma}{p_n})] + \frac{ms}{2p_n} E_\mu \frac{\mathcal{M}^\mu}{p_n},$$

$$p^\nu \nabla_\nu (\frac{\mathcal{M}^\mu}{p_n}) = \frac{1}{p_n} (\bar{p}^\mu E^\nu - p_n \bar{\epsilon}^{\mu\nu\alpha} B_\alpha) \frac{\mathcal{M}_\nu}{p_n} - \frac{m}{2} \bar{\epsilon}^{\mu\nu\rho} \nabla_\nu [\frac{1}{p_n} \nabla_\rho (\sum_s \frac{\mathcal{J}_{sn}}{p_n})].$$

- All the other Wigner equations are satisfied automatically!

# GCKE in 7-dimensional phase space

- The GCKE in 7-dimensional phase space after integrating over  $p_n$ :

$$\begin{aligned}
 & (1 - sB \cdot \Omega) \partial_n^x \tilde{\mathcal{J}}_{sn} + [(1 - 2sB \cdot \Omega) \bar{v}^\mu + s\bar{\epsilon}^{\mu\rho\sigma} \Omega_\rho E_\sigma] \bar{\partial}_\mu^x \tilde{\mathcal{J}}_{sn} + [E^\mu + \mathcal{E}_{\bar{p}} \bar{\partial}_x^\mu (B \cdot \Omega) - (1 - 2sB \cdot \Omega) \bar{\epsilon}^{\rho\sigma\mu} \bar{v}_\rho B_\sigma - s(E \cdot B) \Omega^\mu] \partial_\mu^p \tilde{\mathcal{J}}_{sn} \\
 = & \frac{m}{2\mathcal{E}_{\bar{p}}^2} (1 - sB \cdot \Omega) E_\mu \tilde{\mathcal{M}}^\mu - \frac{m}{4\mathcal{E}_{\bar{p}}^3} (\bar{v}^\mu B^\nu + \bar{\epsilon}^{\mu\rho\nu} E_\rho) \bar{\partial}_\mu^x \tilde{\mathcal{M}}_\nu - \frac{m}{4\mathcal{E}_{\bar{p}}^3} [(E \cdot B) \Delta^{\mu\nu} - B^\mu E^\nu - B^\nu \bar{\epsilon}^{\mu\rho\sigma} \bar{v}_\rho B_\sigma - \mathcal{E}_{\bar{p}} \bar{\partial}_x^\mu B^\nu] \bar{\partial}_\mu^p \tilde{\mathcal{M}}_\nu \\
 & \partial_n^x \tilde{\mathcal{M}}_\perp^\mu + \bar{v}^\nu \bar{\partial}_\nu^x \tilde{\mathcal{M}}_\perp^\mu + (E^\nu - \bar{\epsilon}^{\rho\sigma\nu} \bar{v}_\rho B_\sigma) \partial_\nu^p \tilde{\mathcal{M}}_\perp^\mu + \frac{\mathcal{E}_{\bar{p}}}{\bar{p}^2} (\bar{v}^\mu E^\nu + \bar{v}^2 \bar{\epsilon}^{\mu\nu\alpha} B_\alpha) \tilde{\mathcal{M}}_\perp\nu \quad \Delta_\perp^{\mu\nu} = \Delta^{\mu\nu} - \bar{p}^\mu \bar{p}^\nu / \bar{p}^2 \quad E_\perp^\mu / B_\perp^\mu = \Delta_\perp^{\mu\nu} E^\nu / B^\nu \\
 = & -\frac{m}{\bar{p}^2} E_\perp^\mu \sum_s s \tilde{\mathcal{J}}_{sn} - \frac{m}{2\mathcal{E}_{\bar{p}} \bar{p}^2} [(\bar{v}^\mu \bar{\epsilon}^{\sigma\nu\rho} - \bar{v}^\sigma \bar{\epsilon}^{\mu\nu\rho}) \bar{v}_\sigma E_\rho - \bar{v}^2 B_\perp^\mu \bar{v}^\nu] \partial_\nu^x \sum_s \tilde{\mathcal{J}}_{sn} - \frac{m}{2\mathcal{E}_{\bar{p}}^3} [B_\perp^\mu E^\nu - (E \cdot B) \Delta_\perp^{\mu\nu} + B_\perp^\mu \bar{\epsilon}^{\rho\sigma\nu} \bar{v}_\rho B_\sigma + \mathcal{E}_{\bar{p}} (\partial_x^\nu B_\perp^\mu)] \bar{\partial}_\nu^p \sum_s \tilde{\mathcal{J}}_{sn}.
 \end{aligned}$$

- The distribution functions in 7-dimensional GCKE :

$$\tilde{\mathcal{J}}_{sn} \equiv \left( \mathcal{J}_{sn} + \frac{sB \cdot \bar{p}}{2\mathcal{E}_{\bar{p}}^2} \mathcal{J}'_{sn} - \frac{mB^\nu}{4\mathcal{E}_{\bar{p}}^2} \mathcal{M}'_\nu \right)_{p_n=\mathcal{E}_{\bar{p}}} \quad \tilde{\mathcal{M}}_\perp^\mu \equiv \left( \mathcal{M}_\perp^\mu + \frac{mB_\perp^\mu}{2\mathcal{E}_{\bar{p}}^2} \sum_s \mathcal{J}'_{sn} \right)_{p_n=\mathcal{E}_{\bar{p}}}$$

Free particle's energy:  $\mathcal{E}_{\bar{p}} = \sqrt{m^2 + |\bar{p}^2|}$

Free particle's space velocity:

$$\bar{v}^\mu = \frac{\bar{p}^\mu}{\mathcal{E}_{\bar{p}}}$$

$$\text{Berry curvature:} \quad \Omega^\mu = \frac{\bar{p}^\mu}{2\mathcal{E}_{\bar{p}}^3}$$

Modified invariant phase space:  $(1 - sB \cdot \Omega) d^3 \bar{p}$

**The GCKE has a trivial smooth transition and closest form to CKE !**

# Some simple applications

- Spin polarization vector:  $\int dp_n \bar{\mathcal{A}}^\mu = \frac{1}{2} \bar{v}^\mu \Delta \tilde{\mathcal{J}}_n + \frac{1}{\mathcal{E}_{\bar{p}}} [\mathcal{E}_{\bar{p}} \bar{\epsilon}^{\mu\rho\sigma} \partial_\rho^x - 2\bar{v}^\mu B^\sigma - \bar{\epsilon}^{\mu\rho\sigma} E_\rho + \mathcal{E}_{\bar{p}} (B^\sigma \bar{\partial}_p^\mu - B^\mu \bar{\partial}_p^\sigma)] \frac{\bar{p}_\sigma \tilde{\mathcal{J}}_n}{4\mathcal{E}_{\bar{p}}^2} - \frac{m(\mathcal{E}_p^3 \tilde{\mathcal{M}}^\mu - m B^\mu \tilde{\mathcal{J}}_n)}{2\mathcal{E}_p^4}$

- Electric charge separation:  $\int dp_n \bar{\mathcal{V}}^\mu = \frac{1}{2} \bar{v}^\mu \tilde{\mathcal{J}}_n + \frac{1}{\mathcal{E}_{\bar{p}}} [\mathcal{E}_{\bar{p}} \bar{\epsilon}^{\mu\rho\sigma} \partial_\rho^x - 2\bar{v}^\mu B^\sigma - \bar{\epsilon}^{\mu\rho\sigma} E_\rho + \mathcal{E}_{\bar{p}} (B^\sigma \bar{\partial}_p^\mu - B^\mu \bar{\partial}_p^\sigma)] \frac{\bar{p}_\sigma \Delta \tilde{\mathcal{J}}_n - m \tilde{\mathcal{M}}_\sigma}{4\mathcal{E}_{\bar{p}}^2}$

$$\begin{aligned}\tilde{\mathcal{J}}_n &= \tilde{\mathcal{J}}_{+1n} + \tilde{\mathcal{J}}_{-1n} \\ \Delta \tilde{\mathcal{J}}_n &= \tilde{\mathcal{J}}_{+1n} - \tilde{\mathcal{J}}_{-1n}\end{aligned}$$

- The distribution functions in global equilibrium with vorticity and electromagnetic field:

$$\mathcal{J}_{sn} = (p_n f + \frac{s}{2} \omega \cdot \bar{p} f') \delta(p^2 - m^2) - s B \cdot \bar{p} f \delta'(p^2 - m^2) \quad \mathcal{M}_\perp^\mu = m \omega_\perp^\mu f' \delta(p^2 - m^2) - 2m B_\perp^\mu f \delta'(p^2 - m^2) \quad f = \frac{1}{4\pi^3} \left[ \frac{\theta(p_0)}{e^{\beta \cdot p - \bar{\mu}}} + \frac{\theta(-p_0)}{-\beta \cdot p + \bar{\mu}} - \theta(-p_0) \right]$$

- The other Wigner functions:  $\left\{ \begin{array}{l} \mathcal{F} = 2m f \delta(p^2 - m^2) \\ \mathcal{J}_s^\mu = \left( p^\mu f - \frac{s}{2} \tilde{\Omega}^{\mu\nu} p_\nu f' \right) \delta(p^2 - m^2) + s \tilde{F}^{\mu\nu} p_\nu f \delta'(p^2 - m^2) \\ \mathcal{S}^{\mu\nu} = m \Omega^{\mu\nu} f' \delta(p^2 - m^2) - 2m F^{\mu\nu} f \delta'(p^2 - m^2) \\ \mathcal{P} = \frac{m}{8} \tilde{\Omega}^{\mu\nu} \Omega_{\mu\nu} f'' \delta(p^2 - m^2) - \frac{m}{2} \tilde{\Omega}^{\mu\nu} F_{\mu\nu} f' \delta'(p^2 - m^2) + \frac{m}{2} \tilde{F}^{\mu\nu} F_{\mu\nu} f'' \delta(p^2 - m^2) \end{array} \right.$

$$\tilde{\Omega}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \Omega_{\rho\sigma}$$

$$\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

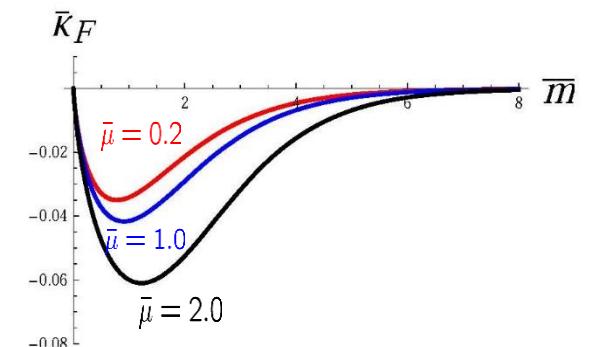
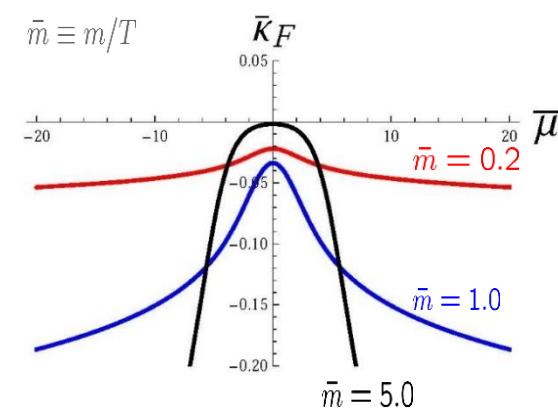
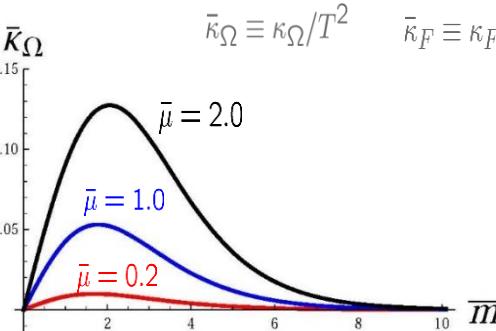
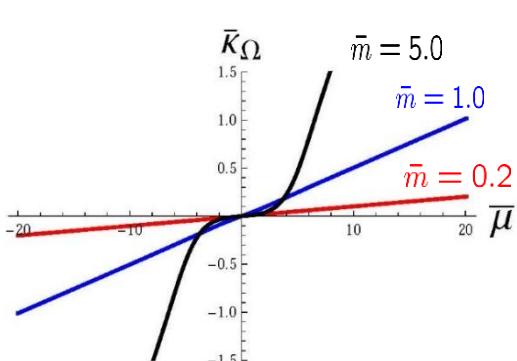
# Some simple applications

- Moment tensor density:  $S^{\mu\nu}(x) \equiv \int d^4 p \mathcal{S}^{\mu\nu} = \kappa_\Omega T \Omega^{\mu\nu} + \kappa_F F^{\mu\nu}$

$$\kappa_F^{\text{vac}} = \frac{m}{4\pi^2} \left( \frac{2}{\epsilon} - \ln m - \gamma + \mathcal{O}(\epsilon) \right) \quad \epsilon_p = \sqrt{m^2 + p^2}$$

$$\kappa_\Omega = -\frac{m}{2\pi^2} \int dp \left( \frac{1}{e^{\beta\mathcal{E}_p + \bar{\mu}} + 1} - \frac{1}{e^{\beta\mathcal{E}_p - \bar{\mu}} + 1} \right),$$

$$\kappa_F = -\frac{m}{2\pi^2} \int \frac{dp}{\mathcal{E}_p} \left( \frac{1}{e^{\beta\mathcal{E}_p + \bar{\mu}} + 1} + \frac{1}{e^{\beta\mathcal{E}_p - \bar{\mu}} + 1} \right) + \kappa_F^{\text{vac}}$$



- Pseudo-scalar density:  $P \equiv \int d^4 p \mathcal{P} = C_\Omega T^2 \tilde{\Omega}^{\mu\nu} \Omega_{\mu\nu} + C_M T \tilde{\Omega}^{\mu\nu} F_{\mu\nu} + C_F \tilde{F}^{\mu\nu} F_{\mu\nu}$

$$C_\Omega = -\frac{1}{8} \frac{\partial \kappa_\Omega}{\partial \mu}$$

$$C_M = -\frac{1}{4} \frac{\partial \kappa_F}{\partial \mu},$$

$$C_F = -\frac{m}{16\pi^2} \int \frac{dp}{p^2 \mathcal{E}_p} \left[ \frac{1}{e^{\beta\mathcal{E}_p + \bar{\mu}} + 1} + \frac{1}{e^{\beta\mathcal{E}_p - \bar{\mu}} + 1} - 1 \right]$$

## Revisit Spin Effects by Thermal Vorticity

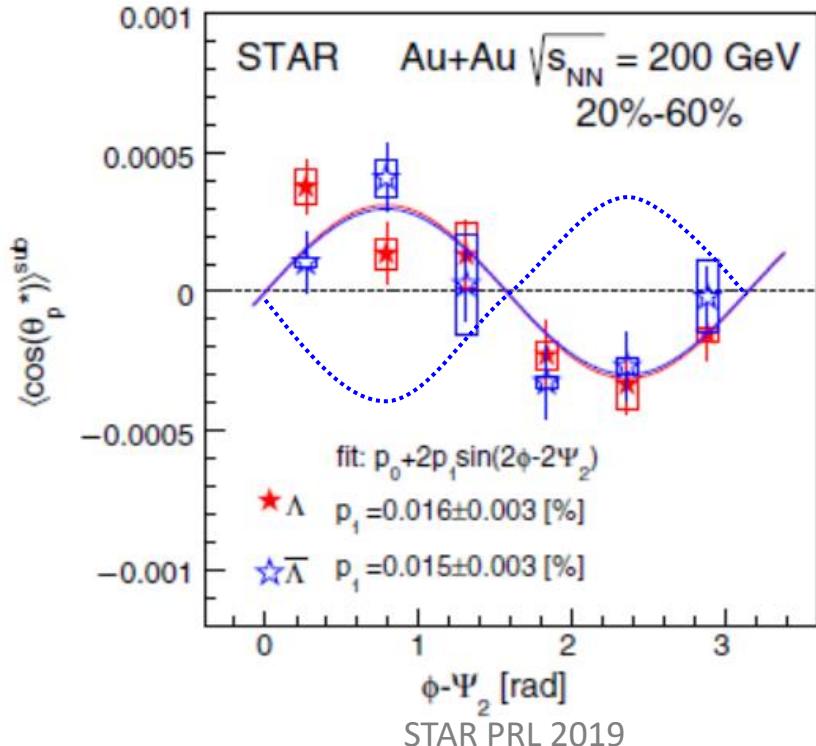
- Induction an motivation
- The spin density matrix and spin distribution functions
- Spin polarization for the spin-1/2 particles
- Spin alignment and polarization for the spin-1 particles

# Introduction

- Local hyperon polarization:

$$S^\mu(k) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} k_\nu \frac{\int d\Sigma_\lambda k^\lambda \boldsymbol{\varpi}_{\rho\sigma} n_F (1 - n_F)}{\int d\Sigma_\lambda k^\lambda n_F}$$

Becattini-Karpenko PRL 2018

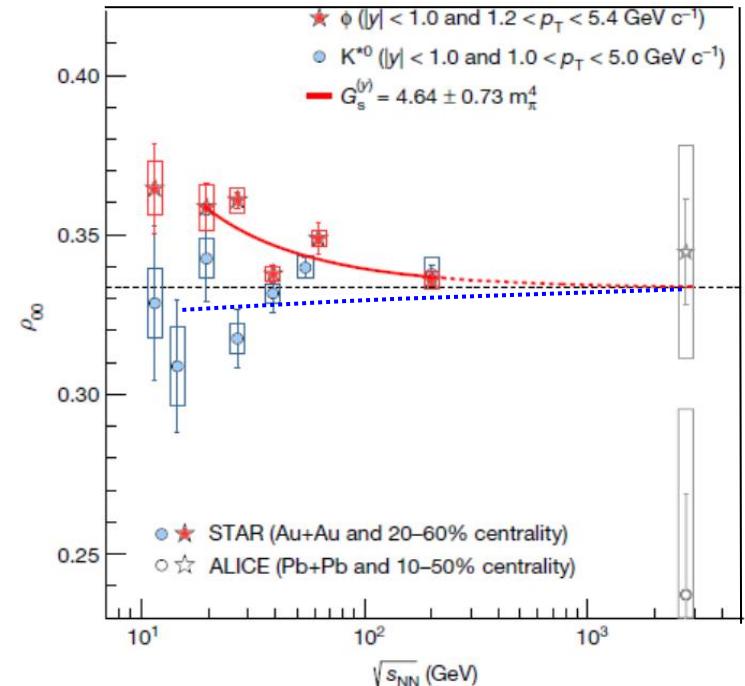


Spin puzzles

- Vector meson's spin alignment:

$$\rho_{00}^{V(\text{rec})} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9} P_q P_{\bar{q}} < \frac{1}{3}$$

Liang-Wang PLB 2005



# Introduction

Becattini-Karpenko PRL 2018

$$S^\mu(k) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} k_\nu \frac{\int d\Sigma_\lambda k^\lambda \varpi_{\rho\sigma} f_F(1-f_F)}{\int d\Sigma_\lambda k^\lambda \varpi_{\rho\sigma} f_F}$$

Spin polarization vector for hyperons



Spin alignment for vector mesons

$$\rho_{00}^{V(\text{rec})} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9} P_q P_{\bar{q}} < \frac{1}{3}$$

Liang-Wang PLB 2005

Liu-Yin JHEP2021,

Fu-Liu-Pang-Song-Yin PRL2021

Becattini-Buzzegoli-Palermo PLB2021,

Becattini-Buzzegoli-Palermo-Inghirami-Karpenko PRL2021

**Shear-induced term for hyperons**

Talks by F. Becattini and B.C. Fu 15/07

Talks by X.L. Sheng 15/07

**Strong force field for vector mesons**

Sheng-Oliva-Wang PRD2020,  
Sheng-Wang-Wang PRD2020

Sheng-Oliva-Liang-Wang-Wang arXiv2205.15689,

Sheng-Oliva-Liang-Wang-Wang arXiv2206.05868

- The spin polarization for hyperons within the formalism of relativistic hydrodynamics: Becattini-Chandra-Del Zanna-Grossi Annals Phys. 2013

$$S^\mu(x, k) = \frac{f'_F}{8mf_F} \epsilon^{\mu\nu\rho\sigma} k_\nu \varpi_{\rho\sigma} \quad \longrightarrow$$

Local spin vector at  $x$

$$S^\mu(k) = \frac{\int d\Sigma_\alpha k^\alpha S^\mu(x, k) f_F(x, k)}{\int d\Sigma_\alpha k^\alpha f_F(x, k)} \quad \longrightarrow$$

Integrate over freeze-out surface

$$S^{*\mu} = (0, \mathbf{S}^*) \quad \longrightarrow$$

$$P = \mathbf{n} \cdot \mathbf{S}^*$$

In the rest frame

Spin polarization along  $\mathbf{n}$

- The spin alignment for vector meson within the formalism of relativistic hydrodynamics?

# Particle distribution function with spin

- Spin density matrix:

$$\rho_{sr}(k) = \frac{\int d\Sigma_\alpha k^\alpha f_{sr}(x, k) f(x, k)}{\int d\Sigma_\alpha k^\alpha f(x, k)} \quad f(x, k) \equiv \sum_s f_{ss}(x, k)$$

De Groot-Van Leeuwen-Van Weert 1980

- Particle distribution functions with spin:

$$f_{sr}(x, k) = \int \frac{d^3 q}{(2\pi)^3} e^{-i(E_{k+q/2} - E_{k-q/2})t} e^{iq \cdot x} \langle a_{k-q/2}^{s\dagger} a_{k+q/2}^r \rangle$$

- Ensemble average in global equilibrium:

Constant scalar, vector, antisymmetric tensor  $\alpha, b_\mu, \omega_{\mu\nu}$

$$\langle a_p^{s\dagger} a_{\bar{p}}^r \rangle = \frac{1}{Z} \text{Tr} \left[ \exp \left( -b \cdot P + \alpha Q + \frac{1}{2} \omega_{\mu\nu} J^{\mu\nu} \right) a_p^{s\dagger} a_{\bar{p}}^r \right]$$

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- Commutation relations

$$[Q, a_p^{s\dagger}] = a_p^{s\dagger}, \quad [b \cdot P, a_p^{s\dagger}] = b \cdot p a_p^{s\dagger}, \quad \left[ \frac{1}{2} \omega_{\mu\nu} J^{\mu\nu}, a_p^{s\dagger} \right] = \sum_r \Lambda_p^{sr} a_p^{r\dagger}$$

- The formal result:

$$\langle a_p^{s\dagger} a_{\bar{p}}^r \rangle = (2\pi)^3 \left[ \frac{1}{e^{b \cdot p - \alpha - \Lambda_p} \pm 1} \right]^{sr} \delta(p - \bar{p})$$

A complicated operator

# Particle distribution function with spin $\frac{1}{2}$

- Dirac field with fixed spin quantization direction  $\mathbf{n}$ :

$$\langle a_{\mathbf{p}}^{s\dagger} a_{\bar{\mathbf{p}}}^r \rangle = (2\pi)^3 \left[ \frac{1}{e^{b \cdot p - \alpha - \Lambda_p} + 1} \right]^{sr} \delta(\mathbf{p} - \bar{\mathbf{p}}) \quad \Lambda_p^{sr} = \frac{1}{2} \left( \boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathbf{p}}{E_p + m} \right) \cdot \boldsymbol{\lambda}^{sr} + \frac{i\delta^{sr}}{2E_p} \boldsymbol{\varepsilon} \cdot \mathbf{p} + \delta^{sr} (E_p \boldsymbol{\varepsilon} + \boldsymbol{\omega} \times \mathbf{p}) \cdot i\nabla_p$$

$$\varepsilon^i = \omega^{i0}, \quad \omega^i = -\frac{1}{2} \epsilon^{ijk} \omega^{jk}, \quad \boldsymbol{\lambda}^{rs} = s \delta^{s,r} \mathbf{n} + \delta^{-s,r} \left( \mathbf{n}_{\perp}^{(1)} - i s \mathbf{n}_{\perp}^{(2)} \right), \quad \mathbf{n}_{\perp}^{(1)} = \frac{\hat{\mathbf{z}} \times \mathbf{n}}{|\hat{\mathbf{z}} \times \mathbf{n}|}, \quad \mathbf{n}_{\perp}^{(2)} = \mathbf{n}_{\perp}^{(1)} \times \mathbf{n}$$

- Expand the distribution function as the Taylor series of  $\boldsymbol{\omega}$  and  $\boldsymbol{\varepsilon}$

The 0<sup>th</sup> order:  $f_{sr}^{(0)}(x, k) = f_F(b \cdot p - \alpha) \delta^{sr}$   $f_F(b \cdot p - \alpha) = \frac{1}{e^{b \cdot p - \alpha} + 1}$  : Fermi-Dirac distribution

The 1<sup>st</sup> order:  $f_{sr}^{(1)}(x, k) = f'_F(b \cdot p - \alpha) [E_k \boldsymbol{\varepsilon} \cdot \mathbf{x} - \mathbf{k} \cdot (\boldsymbol{\varepsilon} t + \boldsymbol{\omega} \times \mathbf{x})] \delta^{sr} - \frac{1}{2} f'_F(b \cdot p - \alpha) \left( \boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathbf{k}}{E_k + m} \right) \cdot \boldsymbol{\lambda}^{sr}$

- “Resummation”:  $f_{sr}(x, k) = f_{sr}^{(0)}(x, k) + f_{sr}^{(1)}(x, k) \approx f_F(\beta \cdot p - \alpha) \delta^{sr} - \frac{1}{2} f'_F(\beta \cdot p - \alpha) \left( \boldsymbol{\omega} - \frac{\boldsymbol{\varepsilon} \times \mathbf{k}}{E_k + m} \right) \cdot \boldsymbol{\lambda}^{sr}$

- Thermal vorticity :  $b^\mu = (b_0, \mathbf{b}) \Rightarrow \beta^\mu = \frac{u^\mu}{T} = (b_0 + \boldsymbol{\varepsilon} \cdot \mathbf{x}, \mathbf{b} + \boldsymbol{\varepsilon} t + \boldsymbol{\omega} \times \mathbf{x}), \quad \omega^{\mu\nu} = \varpi^{\mu\nu} = -\frac{1}{2} (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu)$

# Polarization with fixed spin direction

- Spin polarization is defined by:

$$P(x, k) = \frac{f_{+1/2,+1/2}(x, k) - f_{-1/2,-1/2}(x, k)}{f_{+1/2,+1/2}(x, k) + f_{-1/2,-1/2}(x, k)}$$

- Polarization from present method:

$$P(x, k) = -\frac{f'_F}{2f_F} \left( \omega - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}} + m} \right) \cdot \mathbf{n}$$

- Polarization from previous method:

$$P(x, k) = -\frac{f'_F}{2f_F} \cdot \frac{E_{\mathbf{k}}}{2m} \left[ \omega - \frac{(\omega \cdot \mathbf{k})\mathbf{k}}{E_{\mathbf{k}}(E_{\mathbf{k}} + m)} - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}}} \right] \cdot \mathbf{n}$$

- Differences:

No  $1/m$  singularity

No  $(\omega \cdot \mathbf{k})\mathbf{k}$  term

The  $\boldsymbol{\epsilon} \times \mathbf{k}$  term suppressed

# Polarization with momentum-dependence direction

- The polarization receives **additional** contribution when the spin direction depends on the momentum

1. Helicity polarization

$$\mathbf{n} = \hat{\mathbf{p}}$$

$$P = -\frac{f'_F}{2f_F} \left( \omega - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E + m} \right) \cdot \mathbf{n} \\ + \frac{f'_F}{2f_F} \cdot \frac{1}{k} (E\boldsymbol{\epsilon} + \boldsymbol{\omega} \times \mathbf{k}) \cdot \mathbf{n}_\perp^{(1)} \cot\theta$$

2. Transverse polarization

$$\mathbf{n} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{p}}}{|\hat{\mathbf{z}} \times \hat{\mathbf{p}}|} = \hat{\mathbf{n}}_\perp^{(1)}$$

$$P = -\frac{f'_F}{2f_F} \left( \omega - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E + m} \right) \cdot \mathbf{n} \\ + 0$$

3. Transverse polarization

$$\mathbf{n} = \hat{\mathbf{n}}_\perp^{(1)} \times \hat{\mathbf{p}} = \hat{\mathbf{n}}_\perp^{(2)}$$

$$P = -\frac{f'_F}{2f_F} \left( \omega - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E + m} \right) \cdot \mathbf{n} \\ + \frac{f'_F}{2f_F} \cdot \frac{1}{k} (E\boldsymbol{\epsilon} + \boldsymbol{\omega} \times \mathbf{k}) \cdot \mathbf{n}_\perp^{(1)}$$

# Particle distribution function with spin 1

- Vector field with fixed spin quantization direction  $\mathbf{n}$ :

$$\langle a_p^{s\dagger} a_{\bar{p}}^r \rangle = (2\pi)^3 \left[ \frac{1}{e^{b \cdot p - \alpha - \Lambda_p} - 1} \right]^{sr} \delta(p - \bar{p}) \quad \Lambda_p^{sr} = i \left[ \omega - \frac{(\omega \times p) \times p}{mE} - \frac{\epsilon \times p}{E + m} \right] \cdot (\epsilon^{(s)} \times \epsilon^{(r)}) + \frac{i\delta^{sr}}{2E_p} \epsilon \cdot p + \delta^{sr} (E_p \epsilon + \omega \times p) \cdot i\nabla_p$$

- Orthonormal linear polarization vectors  $\epsilon^{(1)}, \epsilon^{(2)}, \epsilon^{(3)}$  with  $\epsilon^{(1)} \times \epsilon^{(2)} = \epsilon^{(3)} = \mathbf{n}$  :
- The spin distribution function with linear polarization index  $s, r = 1, 2, 3$  up to the first order :

$$f_{sr}(x, k) = f_B(\beta \cdot k - \alpha) \delta^{sr} - i f'_B(\beta \cdot k - \alpha) \left[ \omega - \frac{(\omega \times k) \times k}{mE_k} - \frac{\epsilon \times k}{E_k + m} \right] \cdot (\epsilon^{(s)} \times \epsilon^{(r)}) \quad f_B(\beta \cdot p - \alpha) = \frac{1}{e^{\beta \cdot p - \alpha} - 1}$$

Bose-Einstein distribution

- Orthonormal circular polarization vector  $\epsilon^{(+)}, \epsilon^{(-)}, \epsilon^{(0)}$  with  $\epsilon^{(\pm)} = (\epsilon^{(1)} \pm i\epsilon^{(2)})/\sqrt{2}$ ,  $\epsilon^{(0)} = \epsilon^{(3)} = \mathbf{n}$  :
- The spin distribution function with circular polarization index  $s, r = \pm 1, 0$  up to the first order :

$$f_{sr}(x, k) = f_B(\beta \cdot k - \alpha) \delta^{sr} - i f'_B(\beta \cdot k - \alpha) \left[ \omega - \frac{(\omega \times k) \times k}{mE_k} - \frac{\epsilon \times k}{E_k + m} \right] \cdot (\epsilon^{(s)} \times \epsilon^{(r)*})$$

# Spin polarization and alignment for vector mesons

- The spin density matrix can be measured by angular distribution of vector mesons' decay:

$$\frac{dN}{d\cos\theta^* d\varphi^*} = \frac{3}{8\pi} \left[ 1 - \rho_{00} + (3\rho_{00} - 1) \cos^2\theta^* - 2\text{Re}\rho_{+-} \sin^2\theta^* \cos(2\varphi^*) - 2\text{Im}\rho_{+-} \sin^2\theta^* \sin(2\varphi^*) \right. \\ \left. - \sqrt{2}\text{Re}(\rho_{+0} - \rho_{0-}) \sin\theta^* \cos\varphi^* + \sqrt{2}\text{Im}(\rho_{+0} - \rho_{0-}) \sin\theta^* \sin\varphi^* \right]$$

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- The 1<sup>st</sup> order:  $\rho_{00} = \frac{1}{3}$      $\rho_{+-} = 0$      $\rho_{+0} - \rho_{0-} = \frac{\sqrt{2}f'_B}{3f_B} \left[ \omega - \frac{(\omega \times \mathbf{k}) \times \mathbf{k}}{mE_{\mathbf{k}}} - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}} + m} \right] \cdot (\boldsymbol{\epsilon}^{(1)} + i\boldsymbol{\epsilon}^{(2)})$
- The 2<sup>nd</sup> order:  $\rho_{00} - \frac{1}{3} = -\frac{1}{6} \cdot \frac{f''_B}{f_B} \left\{ \left[ \left( \omega - \frac{(\omega \times \mathbf{k}) \times \mathbf{k}}{mE_{\mathbf{k}}} - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}} + m} \right) \cdot \mathbf{n} \right]^2 - \frac{1}{3} \left[ \omega - \frac{(\omega \times \mathbf{k}) \times \mathbf{k}}{mE_{\mathbf{k}}} - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}} + m} \right]^2 \right\}$
- Comparison with Dirac fermion:  $\omega - \frac{(\omega \times \mathbf{k}) \times \mathbf{k}}{mE_{\mathbf{k}}} - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E_{\mathbf{k}} + m}$     VS     $\omega - \frac{\boldsymbol{\epsilon} \times \mathbf{k}}{E + m}$
- Similarly, additional contribution will arise when the spin direction depends on momentum!

# Summary

- GCKE are applicable to the fermions with arbitrary mass which can be smoothly reduced to the CKE at chiral limit in a trivial way.
- Spin effects induced by thermal vorticity are calculated directly from spin density matrix instead of spin polarization vector.
- The off-diagonal element of the spin density matrix of vector mesons receives first-order contribution from thermal vorticity.
- The spin effects receive additional contribution when the spin direction depends on the momentum.

**Thanks for your attention !**