

Nonlinear Chiral Kinetic Theory

Phys. Rev. D 108, 016001 (2023)

Kazuya Mameda

Tokyo University of Science

Chiral transport phenomena

chiral magnetic effect

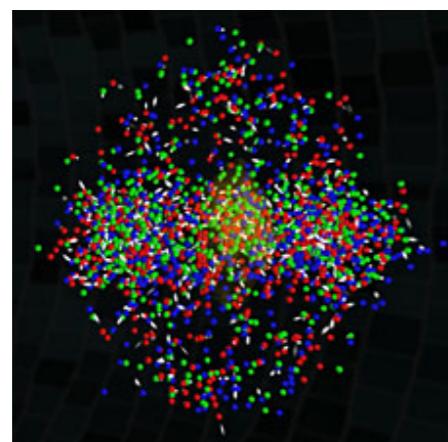
$$\vec{J} = \sigma_{\text{CME}} \vec{B}$$

chiral vortical effect

$$\vec{J} = \sigma_{\text{CVE}} \vec{\omega}$$

from **quantum anomaly** in Dirac theory

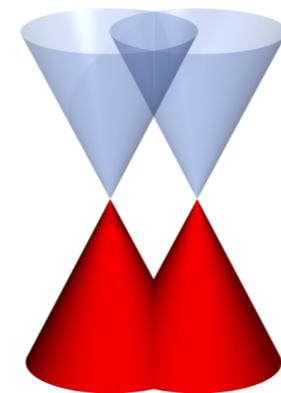
hep



quark-gluon plasma

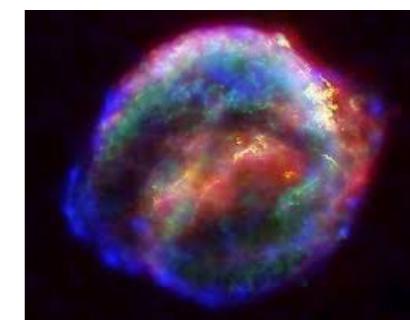
cond-mat

Vazifeh, Franz (2013)



Weyl semimetal

astro



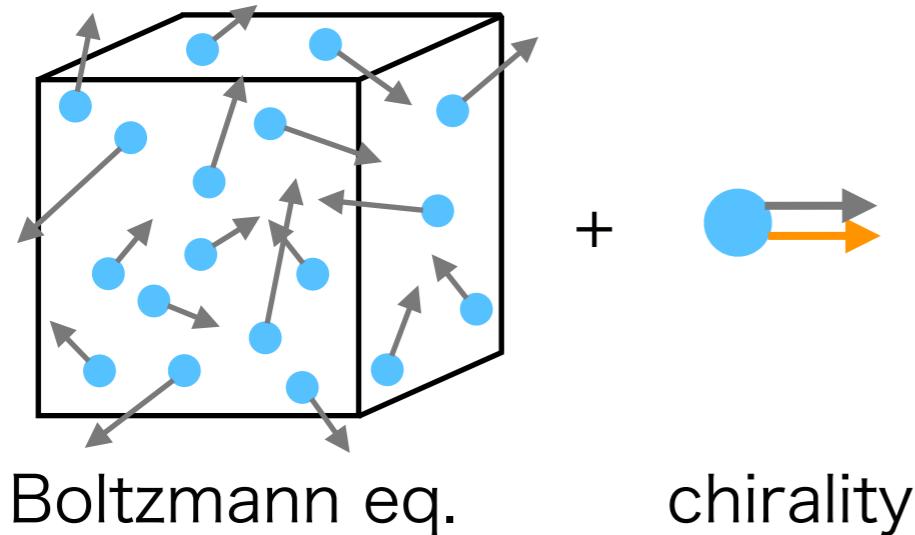
neutrino

for matter in early universe
(2010)



for any chiral matter
(present)

Chiral kinetic theory



Chiral Kinetic Theory(CKT)

Stephanov, Yin (2012)

Son, Yamamoto (2012)

Chen et al. (2013)

- Lorentz covariance
- collision
- mass correction
- spin polarization

- strong magnetic field
- gravitational field
- other derivations

Hidaka, Pu, Wang, Yang (2022)

Limitation 1 : magneto-vortical transport

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\partial_\mu - eA_\mu)\psi$$

$$\rightarrow O(\partial) \sim O(\hbar)$$

Limitation 1 : magneto-vortical transport

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\partial_\mu - eA_\mu)\psi$$

$$\rightarrow O(\partial) \sim O(\hbar)$$

$O(1)$

$O(\hbar)$

$$J^0 \sim \mu^3$$

Fermi sphere

Boltzmann

$$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$$

CME

CVE

linear CKT

$$B \sim \partial A \quad \omega \sim \partial u$$

Limitation 1 : magneto-vortical transport

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\partial_\mu - eA_\mu)\psi$$

$$\rightarrow O(\partial) \sim O(\hbar)$$

$O(1)$

$O(\hbar)$

$O(\hbar^2)$

Hattori, Yin (2016)

$$J^0 \sim \mu^3$$

Fermi sphere

Boltzmann

$$\vec{J} \sim \mu \vec{B}$$

CME

$$\vec{J} \sim \mu^2 \vec{\omega}$$

CVE

linear CKT

$$J^0 \sim \vec{B} \cdot \vec{\omega}$$

$$B \sim \partial A \quad \omega \sim \partial u$$

nonlinear CKT

Limitation 1 : magneto-vortical transport

$$O(\hbar)$$

$$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$$

anomaly, nondissipative

Limitation 1 : magneto-vortical transport

$$O(\hbar)$$

$$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$$

anomaly, nondissipative

$$O(\hbar^2)$$

$$J^0 \sim \vec{B} \cdot \vec{\omega}$$

anomaly? nondissipative?

- linear response Hattori, Yin (2016)
- rotating fermions Ebihara, Fukushima, KM (2017)

Limitation 1 : magneto-vortical transport

$$O(\hbar)$$

$$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$$

anomaly, nondissipative

$$O(\hbar^2)$$

$$J^0 \sim \vec{B} \cdot \vec{\omega}$$

anomaly? nondissipative?

- linear response Hattori, Yin (2016)
- Thermal Field Theory, but…

Limitation 1 : magneto-vortical transport

$$O(\hbar)$$

$$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$$

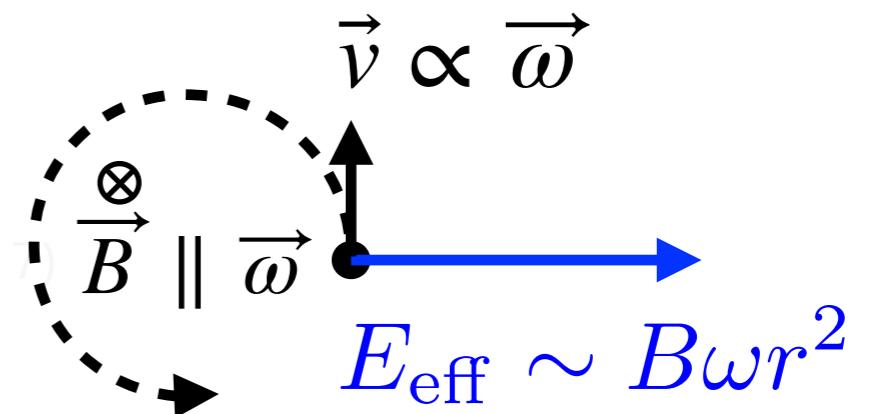
anomaly, nondissipative

$$O(\hbar^2)$$

$$J^0 \sim \vec{B} \cdot \vec{\omega}$$

anomaly? nondissipative?

- linear response Hattori, Yin (2016)
- Thermal Field Theory, but ...



Equilibrated?

Check it from off-equilibrium nonlinear CKT

Yang, Gao, Liang, Wang (2020), Lin, Yang (2021) : near-equilibrium

Limitation 2 : trace anomaly

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\partial_\mu - eA_\mu)\psi$$

$$\longrightarrow O(\partial) \sim O(\hbar)$$

$O(1)$

$$\partial_\mu J^\mu = 0$$

$$T^\mu{}_\mu = 0$$

Boltzmann

$O(\hbar)$

$$\partial_\mu J^\mu \sim \vec{E} \cdot \vec{B}$$

chiral anomaly

linear CKT

$$E, B \sim \partial A$$

Limitation 2 : trace anomaly

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\partial_\mu - eA_\mu)\psi$$



$$O(\partial) \sim O(\hbar)$$

$O(1)$

$$\begin{aligned}\partial_\mu J^\mu &= 0 \\ T^\mu{}_\mu &= 0\end{aligned}$$

Boltzmann

$O(\hbar)$

$$\begin{aligned}\partial_\mu J^\mu &\sim \vec{E} \cdot \vec{B} \\ \text{chiral anomaly}\end{aligned}$$

linear CKT

$$E, B \sim \partial A$$

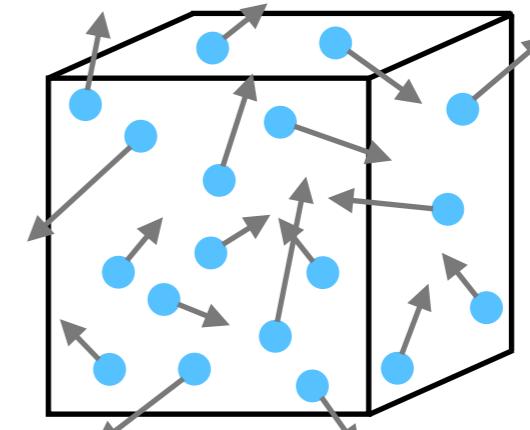
$O(\hbar^2)$

$$\begin{aligned}T^\mu{}_\mu &\sim \beta(e) F_{\mu\nu}^2 \\ \text{trace anomaly}\end{aligned}$$

nonlinear CKT

Limitation 2 : trace anomaly

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} i \not{D} \psi$$



nonlinear CKT

$O(\hbar)$

$$\partial_\mu J^\mu \sim \vec{E} \cdot \vec{B}$$



Berry monopole

$O(\hbar^2)$

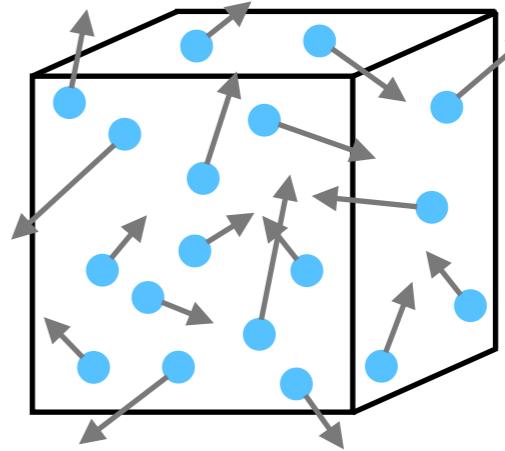
$$T^\mu{}_\mu \sim \beta(e) F_{\mu\nu}^2$$



???

Limitation 2 : trace anomaly

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4} F_{\mu\nu}^2 + \mathcal{L}_{\text{fer}} \quad \leftrightarrow \quad \begin{array}{c} \text{Euler-Heisenberg theory} \\ \boxed{\text{Diagram showing a series of loop corrections to the action}}} \end{array}$$



nonlinear CKT

$$O(\hbar) \quad \leftrightarrow \quad \partial_\mu J^\mu \sim \vec{E} \cdot \vec{B} \quad \text{Berry monopole}$$

$$O(\hbar^2) \quad \leftrightarrow \quad T^\mu{}_\mu \sim \beta(e) F_{\mu\nu}^2 \quad ???$$

Quantum kinetic theory

- classical

$$n_{\mathbf{p}}(t, \underline{\mathbf{x}}, \underline{\mathbf{p}})$$

- quantum field theory (relativistic)

$$W(x, p) = \int_y \underline{e^{-ip \cdot y / \hbar}} \langle \bar{\psi}(x + \frac{y}{2}) \psi(x - \frac{y}{2}) \rangle$$

uncertainty Green's function

Quantum kinetic theory

- classical

$$n_{\mathbf{p}}(t, \mathbf{x}, \mathbf{p})$$

- right-handed current density

$$\mathcal{R}^\mu(x, p) = \text{tr} \left[\gamma^\mu \frac{1 + \gamma^5}{2} W(x, p) \right] \sim (n_{\mathbf{p}}, \mathbf{v} n_{\mathbf{p}}) + O(\hbar)$$

Quantum kinetic theory

- classical

$$n_{\mathbf{p}}(t, \mathbf{x}, \mathbf{p})$$

- right-handed current density

$$\mathcal{R}^\mu(x, p) = \text{tr} \left[\gamma^\mu \frac{1 + \gamma^5}{2} W(x, p) \right] \sim (n_{\mathbf{p}}, \mathbf{v} n_{\mathbf{p}}) + O(\hbar)$$

- **chiral kinetic theory**

EoMs for $\mathcal{R}^\mu = \mathcal{R}_{(0)}^\mu + \hbar \mathcal{R}_{(1)}^\mu + \hbar^2 \mathcal{R}_{(2)}^\mu$

Zeroth-order Wigner function

- general solution

$$\mathcal{R}_{(0)}^\mu = 2\pi \underline{\delta(p^2)} p^\mu \underline{f_{(0)}(x, p)}$$

on-shell condition distribution function

- current

$$J_{(0)}^0 = \int_{\mathbf{p}} n_{\mathbf{p}} \quad J_{(0)}^i = \int_{\mathbf{p}} \frac{p^i}{|\mathbf{p}|} n_{\mathbf{p}}$$

$$n_{\mathbf{p}} = f_{(0)}(p_0 = |\mathbf{p}|) - f_{(0)}(p_0 = -|\mathbf{p}|)$$

Linear-order Wigner function

- general solution Hidaka, Pu, Yang (2016)

$$\mathcal{R}_{(1)}^\mu = 2\pi\delta(p^2) \left[p^\mu f_{(1)} + \left(\Sigma_n^{\mu\nu} \Delta_\nu - \frac{1}{p^2} \tilde{F}^{\mu\nu} p_\nu \right) f_{(0)} \right]$$

- spin tensor

$$\Sigma_n^{\mu\nu} = \frac{\varepsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{2p \cdot n}$$

Chen, Son, Stephanov (2015)

e.g. $n^\mu = (1, \mathbf{0})$

$$(\Sigma^{23}, \Sigma^{31}, \Sigma^{12}) = \frac{\mathbf{p}}{2p_0} \sim \frac{\hat{\mathbf{p}}}{2}$$

- $\mathcal{R}_{(1)}^\mu$ frame-independent

Linear-order equilibrium Wigner function

- solution of kinetic eq Hidaka, Pu, Yang (2016)

$$\mathcal{R}_\mu^{(1)} = \mathcal{R}_\mu^{(F)} + \mathcal{R}_\mu^{(\omega)}$$

- momentum integral

e.g. $J_{(1)}^\mu = \frac{\mu}{4\pi^2} B^\mu + \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right) \omega^\mu$

CME

CVE

- Nondissipative due to quantum anomaly

Nonlinear-order Wigner function

- general solution KM (2023)

$$\begin{aligned}\mathcal{R}_\mu^{(2)} &= 2\pi\delta(p^2) \left[p_\mu f_{(2)} + \left(\Sigma_{\mu\nu}^u \Delta^\nu - \frac{1}{p^2} \tilde{F}_{\mu\nu} p^\nu \right) f_{(1)} - \Sigma_{\mu\nu}^u \varepsilon^{\nu\rho\sigma\lambda} \Delta_\rho \frac{n_\sigma}{2p \cdot n} \Delta_\lambda f_{(0)} \right] \\ &\quad + 2\pi \frac{\delta(p^2)}{p^2} \left(\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} p^\nu \Delta^\rho + \frac{p_\mu p^\nu}{p^2} \tilde{F}_{\nu\sigma} - \tilde{F}_{\mu\sigma} \right) \left(\Sigma_n^{\sigma\lambda} \Delta_\lambda - \frac{1}{p^2} \tilde{F}^{\sigma\lambda} p_\lambda \right) f_{(0)} \\ &\quad + 2\pi \frac{\delta(p^2)}{p^2} \Sigma_{\mu\nu}^u \left[\Delta_\alpha \Sigma_n^{\alpha\nu} + \frac{n_\alpha}{p \cdot n} \tilde{F}^{\alpha\nu} + \frac{1}{p^2} \tilde{F}^{\nu\lambda} p_\lambda \right] p \cdot \Delta f_{(0)} \\ &\quad + \frac{2\pi}{p^2} \left[-p_\mu Q \cdot p + 2p^\nu Q_{[\mu} p_{\nu]} \right] f_{(0)} \delta(p^2) \sim \partial_\mu F_{\nu\rho}\end{aligned}$$

- another frame vector? u^μ Hayata, Hidaka, KM (2021)

- $\mathcal{R}_{(2)}^\mu$ frame-independent

Nonlinear-order equilibrium Wigner function

- solution of kinetic eq

$$\mathcal{R}_\mu^{(2)} = \mathcal{R}_\mu^{(\partial F)} + \mathcal{R}_\mu^{(FF)} + \mathcal{R}_\mu^{(F\omega)} + \mathcal{R}_\mu^{(\omega\omega)}$$

KM (2023)

Yang, Gao, Liang, Wang (2020)

- momentum integral

e.g. $J_{(F\omega)}^0 = -\frac{1}{8\pi^2} B \cdot \omega$

- Nondissipative

conformed from off-equilibrium Wigner function

UV divergence

- $f_{(0)}(x, p)$ contains **vacuum** contribution

$$\mathcal{R}_{(0)}^\mu = 2\pi\delta(p^2)p^\mu f_{(0)}(x, p)$$

- **vacuum** contribution to current

$$J_{(\partial F)} \sim \partial F \times \int_0^\infty \frac{dp}{p} [n_F(|\mathbf{p}| - \mu) + n_F(|\mathbf{p}| + \mu) - 1]$$

$+3 = +3$ $+0$

UV divergence

- $f_{(0)}(x, p)$ contains **vacuum** contribution

$$\mathcal{R}_{(0)}^\mu = 2\pi\delta(p^2)p^\mu f_{(0)}(x,p)$$

- vacuum contribution to current

- UV logarithmic divergence

$$n_F(|\mathbf{p}| - \mu) + n_F(|\mathbf{p}| + \mu) \rightarrow 0$$

Regularization problem

- Pauli-Villars $\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2}$

inapplicable: no mass parameter in CKT

Regularization problem

- Pauli-Villars $\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2}$

inapplicable: no mass parameter in CKT

Yang, Gao, Liang, Wang (2020)

- 't Hooft-Veltman (dimensional regularization) $\int d^4 k \rightarrow \int d^{4-\epsilon} k$

Regularization problem

- Pauli-Villars $\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2}$

inapplicable: no mass parameter in CKT

Yang, Gao, Liang, Wang (2020)

- 't Hooft-Veltman (dimensional regularization) $\int d^4 k \rightarrow \int d^{4-\epsilon} k$

inapplicable: $\mathcal{R}^\mu = \text{tr} \left[\gamma^\mu \frac{1 + \gamma^5}{2} W \right]$ incorrect in $d \neq 4$

$\gamma^\mu = (\gamma^0, \dots, \gamma^3, \gamma^4, \dots, \gamma^{d-1})$ commute with γ^5

Point-Splitting regularization in CKT

- inverse Wigner transformation

$$J^\mu(x, \textcolor{red}{y}) = \text{tr} \left\langle \bar{\psi}(x + \frac{\textcolor{red}{y}}{2}) \gamma^\mu \frac{1 + \gamma^5}{2} \psi(x - \frac{\textcolor{red}{y}}{2}) \right\rangle = \int_p e^{ip \cdot \textcolor{red}{y}/\hbar} \mathcal{R}^\mu(x, p)$$

Point-Splitting regularization in CKT

- inverse Wigner transformation

$$J^\mu(x, \textcolor{red}{y}) = \text{tr} \left\langle \bar{\psi}(x + \frac{\textcolor{red}{y}}{2}) \gamma^\mu \frac{1 + \gamma^5}{2} \psi(x - \frac{\textcolor{red}{y}}{2}) \right\rangle = \int_p e^{ip \cdot \textcolor{red}{y}/\hbar} \mathcal{R}^\mu(x, p)$$

→ Euclidian 4d integral

Point-Splitting regularization in CKT

- inverse Wigner transformation

$$J^\mu(x, \textcolor{red}{y}) = \text{tr} \left\langle \bar{\psi}(x + \frac{\textcolor{red}{y}}{2}) \gamma^\mu \frac{1 + \gamma^5}{2} \psi(x - \frac{\textcolor{red}{y}}{2}) \right\rangle = \int_p e^{ip \cdot \textcolor{red}{y}/\hbar} \mathcal{R}^\mu(x, p)$$

→ Euclidian 4d integral

- regularized vacuum part

$$\begin{array}{ccc} J^\mu_{(\partial F) \text{ vac}} & = & \frac{\mathcal{J}}{12\pi^2} \partial_\lambda F^{\mu\lambda} \\ +3 & = & +0 \quad \quad +3 \end{array}$$

$$\mathcal{J} = \int_0^{\textcolor{red}{y}^{-1}} \frac{dp}{p}$$

Point-Splitting regularization in CKT

- inverse Wigner transformation

$$J^\mu(x, \textcolor{red}{y}) = \text{tr} \left\langle \bar{\psi}(x + \frac{\textcolor{red}{y}}{2}) \gamma^\mu \frac{1 + \gamma^5}{2} \psi(x - \frac{\textcolor{red}{y}}{2}) \right\rangle = \int_p e^{ip \cdot \textcolor{red}{y}/\hbar} \mathcal{R}^\mu(x, p)$$

→ Euclidian 4d integral

- regularized vacuum part

$$\begin{array}{ccc} J_{(\partial F) \text{ vac}}^\mu & = & \frac{\mathcal{J}}{12\pi^2} \partial_\lambda F^{\mu\lambda} \\ +3 & = & +0 \quad +3 \end{array} \qquad \mathcal{J} = \int_0^{\textcolor{red}{y}^{-1}} \frac{dp}{p}$$

$$\begin{array}{ccc} T_{(FF) \text{ vac}}^{\mu\nu} & = & -\frac{\mathcal{J}}{12\pi^2} \left[F^\mu{}_\sigma F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}^2 \right] \\ +4 & = & +0 \quad +4 \end{array}$$

Compensation of PS regularization

- gauge invariance unbroken

$$\partial_\mu J_{(2)}^\mu = 0$$



- translational invariance **broken**

$$\partial_\mu T_{(2)}^{\mu\nu} + F^{\mu\nu} J_\mu^{(2)} \neq 0$$



Compensation of PS regularization

- gauge invariance unbroken

$$\partial_\mu J_{(2)}^\mu = 0$$



- translational invariance **broken**

$$\partial_\mu T_{(2)}^{\mu\nu} + F^{\mu\nu} J_\mu^{(2)} \neq 0$$



- conformal invariance **unbroken**

$$T^\mu{}_{\mu(2)} = 0$$



because of background EM fields?

Yang, Gao, Liang, Wang (2020)

Compensation of PS regularization

- gauge invariance unbroken

$$\partial_\mu J_{(2)}^\mu = 0$$



- translational invariance **broken**

$$\partial_\mu T_{(2)}^{\mu\nu} + F^{\mu\nu} J_\mu^{(2)} \neq 0$$



- conformal invariance **unbroken**

$$T^\mu{}_{\mu(2)} = 0$$



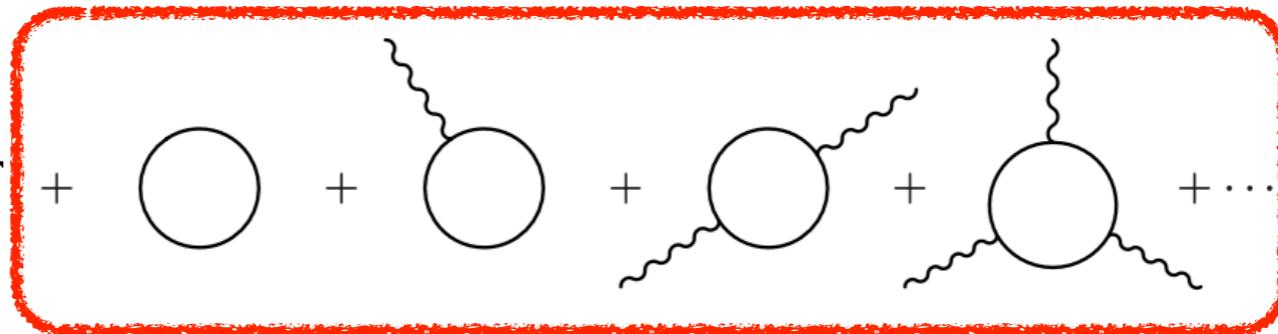
because of background EM fields?

Yang, Gao, Liang, Wang (2020)

regularization

Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} + \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots$$


Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} e^{-sm^2} \frac{\operatorname{Re} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]}{\operatorname{Im} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]} \mathcal{G}$$

$$\mathcal{F} := F_{\alpha\beta}^2/4, \quad \mathcal{G} := F^{\alpha\beta}\tilde{F}_{\alpha\beta}/4$$

Heisenberg, Euler (1936)
Schwinger (1950)

Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} e^{-sm^2} \frac{\operatorname{Re} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]}{\operatorname{Im} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]} \mathcal{G}$$

$$\mathcal{F} := F_{\alpha\beta}^2/4, \quad \mathcal{G} := F^{\alpha\beta}\tilde{F}_{\alpha\beta}/4$$

UV regulator $\sim y^2$

Heisenberg, Euler (1936)
Schwinger (1950)

Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} e^{-sm^2} \frac{\operatorname{Re} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]}{\operatorname{Im} \cosh [\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})}]} \mathcal{G}$$

$\mathcal{F} := F_{\alpha\beta}^2/4, \mathcal{G} := F^{\alpha\beta}\tilde{F}_{\alpha\beta}/4$

Heisenberg, Euler (1936)
Schwinger (1950)

UV regulator $\sim y^2$

- current & EM tensor

$$J_{\text{EH}(2)}^\mu \Big|_{m \rightarrow 0} = \frac{\mathcal{J}}{6\pi^2} \partial_\lambda F^{\mu\lambda} = \mathbf{2} J_{(\partial F) \text{ vac}}^\mu \quad \text{chirality}$$

$$T_{\text{EH}(2)}^{\mu\nu} \Big|_{m \rightarrow 0} = -\frac{\mathcal{J}}{6\pi^2} \left[F^\mu{}_\sigma F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}^2 \right] = \mathbf{2} T_{(FF) \text{ vac}}^{\mu\nu}$$

nonlinear CKT is consistent with EH theory

Physics in logarithm

- effective charge

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} e^{-sm^2} \frac{\operatorname{Re} \cosh \left[\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})} \right]}{\operatorname{Im} \cosh \left[\hbar es \sqrt{2(\mathcal{F} + i\mathcal{G})} \right]} \mathcal{G}$$

Physics in logarithm

- effective charge

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4} F_{\mu\nu}^2 \left(1 + \frac{e^2}{12\pi^2} \log \frac{s_0^{-1}}{m^2} \right) + \text{const.} + O(F^4)$$

$$= e_{\text{eff}}^2(s_0)/e^2$$

Schwartz (2014)

Physics in logarithm

- effective charge

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4}F_{\mu\nu}^2 \left(1 + \frac{e^2}{12\pi^2} \log \frac{s_0^{-1}}{m^2} \right) + \text{const.} + O(F^4)$$

$$= e_{\text{eff}}^2(s_0)/e^2$$

Schwartz (2014)

EH theory

$$\log s_0^{-1/2}$$



charge renormalization

$$\beta(e_{\text{eff}}) = M \frac{de_{\text{eff}}(M)}{dM} = \frac{e^3}{12\pi^2}$$



trace anomaly

$$T^\mu{}_\mu = \frac{\beta}{2e^2} F_{\mu\nu}^2 = \frac{e^2}{24\pi^2} F_{\mu\nu}^2$$

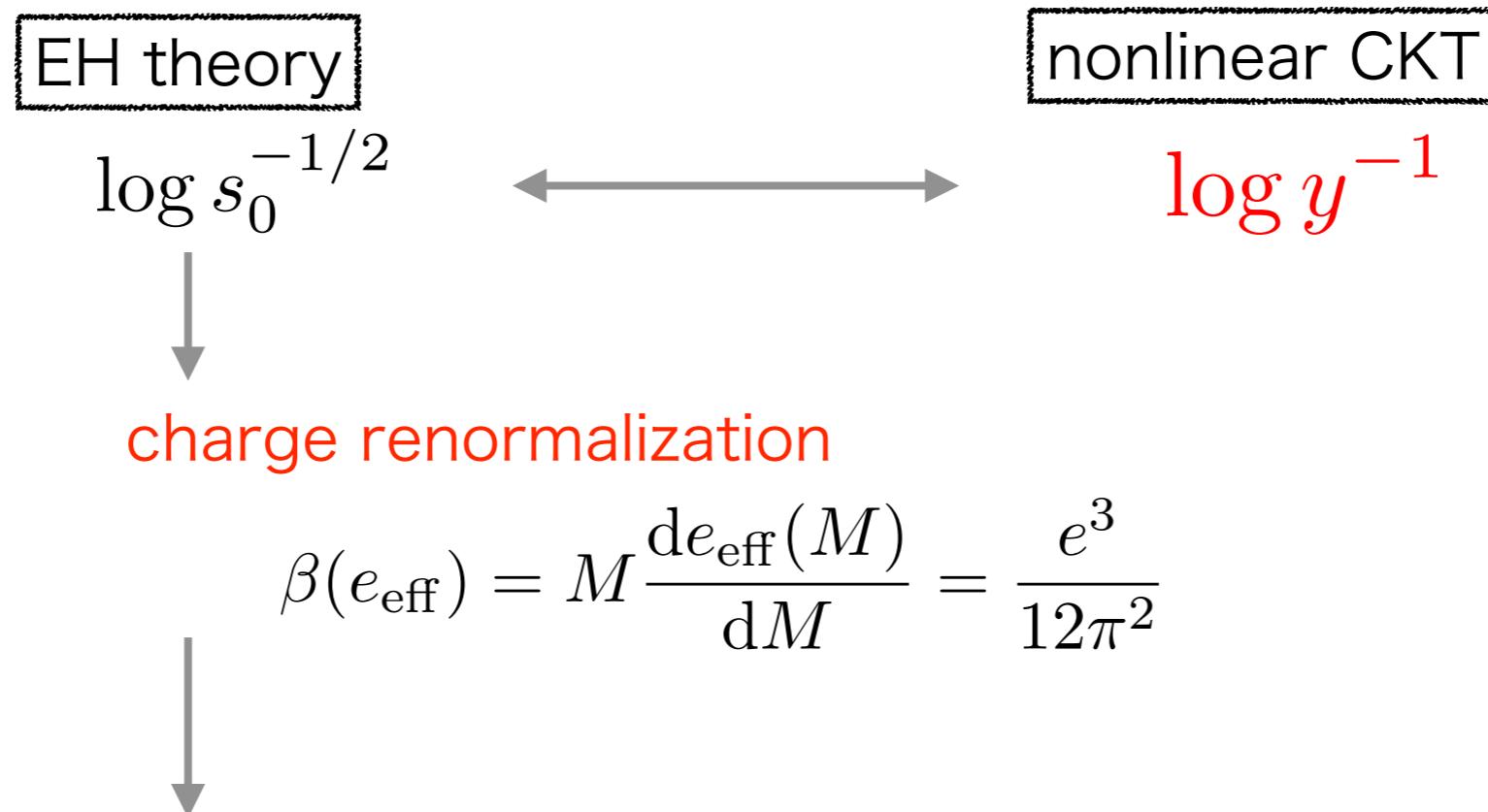
Physics in logarithm

- effective charge

$$\mathcal{L}_{\text{EH}} = -\frac{1}{4}F_{\mu\nu}^2 \left(1 + \frac{e^2}{12\pi^2} \log \frac{s_0^{-1}}{m^2} \right) + \text{const.} + O(F^4)$$

$$= e_{\text{eff}}^2(s_0)/e^2$$

Schwartz (2014)



$$\beta(e_{\text{eff}}) = M \frac{de_{\text{eff}}(M)}{dM} = \frac{e^3}{12\pi^2}$$

trace anomaly

$$T^\mu{}_\mu = \frac{\beta}{2e^2} F_{\mu\nu}^2 = \frac{e^2}{24\pi^2} F_{\mu\nu}^2$$

Summary

- Formulation of **nonlinear** CKT
- Verification for the nondissipativeness of $J^0 \sim \vec{B} \cdot \vec{\omega}$

- Regularization problem

PV : no

DR : no

PS : relatively OK

- Consistency with Euler-Heisenberg theory

$$\log y^{-1} \longleftrightarrow \text{charge renormalization, trace anomaly}$$

- Potential developments

meaning of another frame vector?

nonlinear transport by Berry dipole Sodemann, Fu (2015)

collisional effects Kadanoff, Baym (1962)

chiral plasma instability from dynamical gauge fields

Akamatsu, Yamamoto (2013)