



Spin polarization in a shear flow: Complete contribution

Ziyue Wang

Beijing University of Technology

In collaboration with

Shu Lin

Sun Yat-Sen University

based on

S.Lin and Z.Wang, JHEP 12, 030 (2022)

Z.Wang and S.Lin, to appear

2023.07.17

@ The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Outline

👁 Background

- Spin polarization

- Shear flow: particle redistribution

👁 Methodology

- Quantum transport theory

👁 Complete Contribution

- A. Derivative term is not enough

- B. Collisional effect

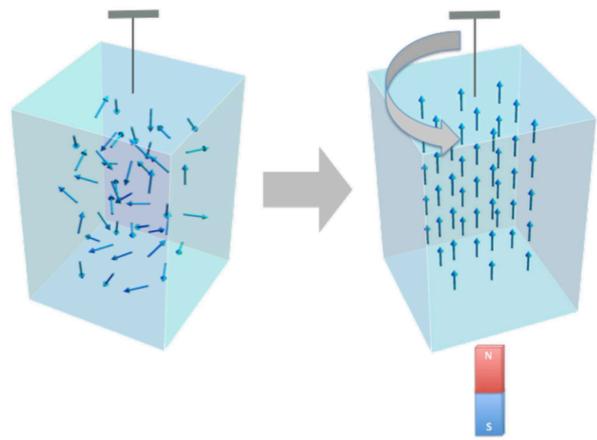
- C. Dynamical part

- D. Complete contribution

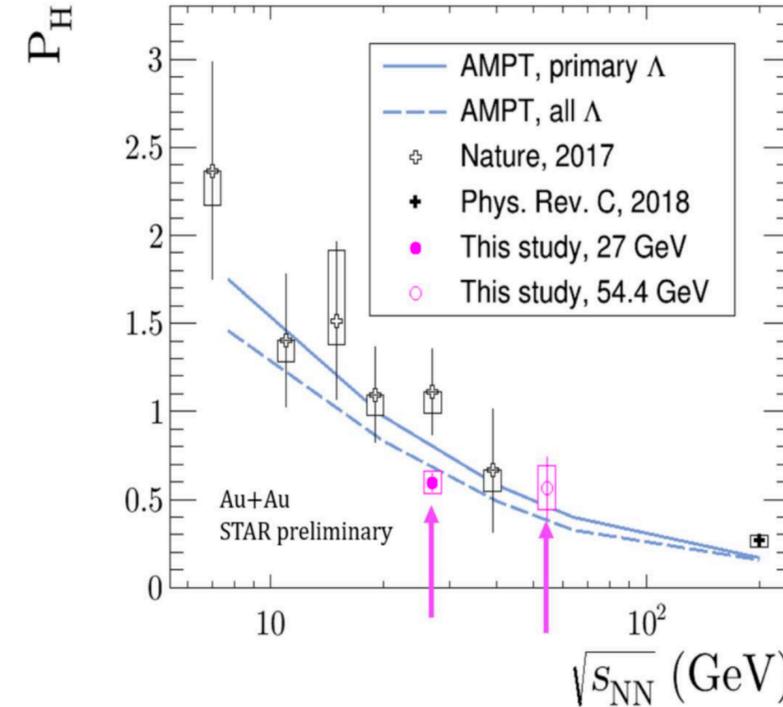
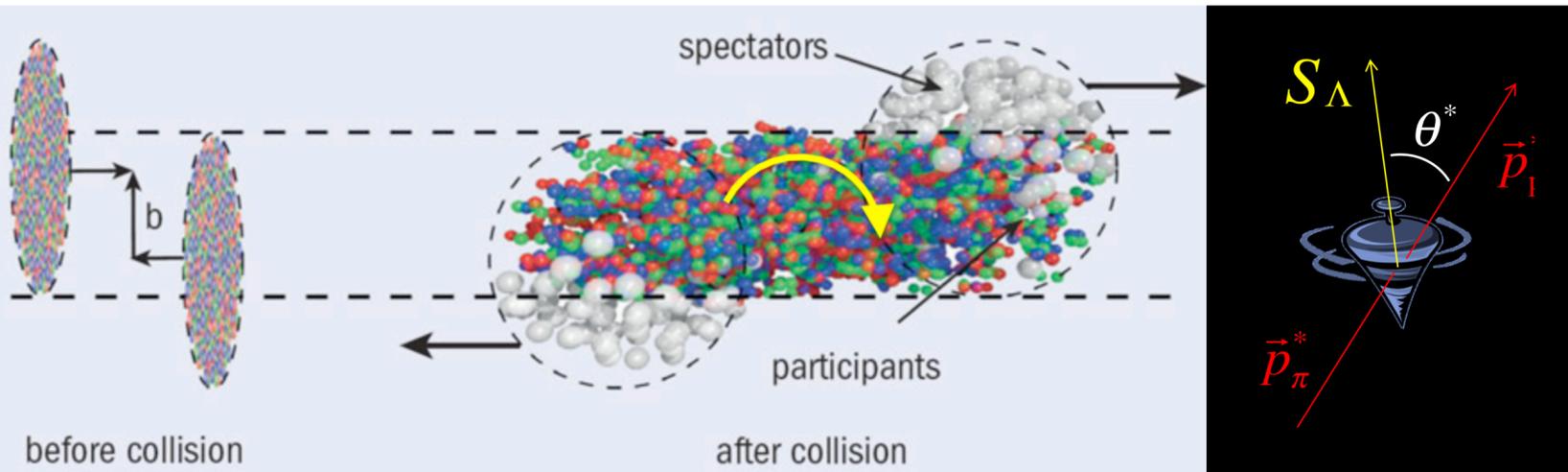
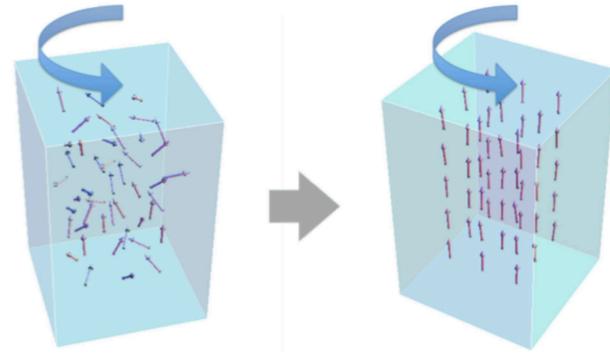
👁 Summary

Spin Polarization

Einstein-de Haas effect



Barnett effect



- ▶ Non central collision creates fireball with large OAM
- ▶ Some part can be transferred from orbital to spin
- ▶ Conservation $J_{ini} = L_{ini} = L_{final} + S_{final} \sim 10^6 \hbar$

- Spin alignment with angular momentum of the emitting system
- Measurable through weak decay of Λ
- Experimentally observed global Λ polarization.

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} \left(1 + \alpha \vec{P} \cdot \hat{p}_p^* \right) = \frac{1}{4\pi} (1 + \alpha P \cos \theta^*)$$

Spin Polarization

Global Polarization

► Spin enslaved by thermal vorticity $\varpi_{\rho\sigma} = \frac{1}{2}(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma)$

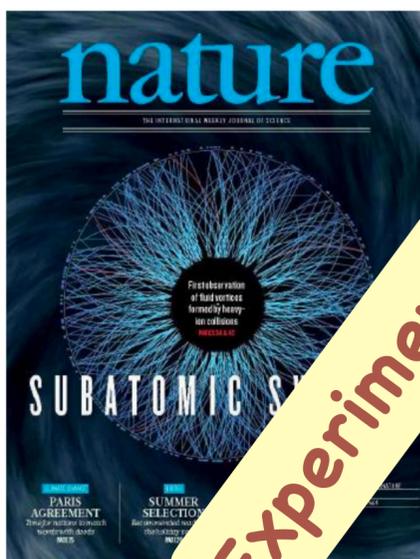
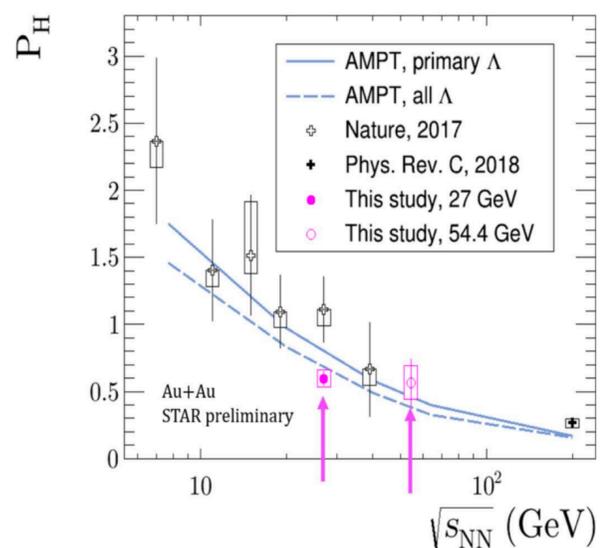
AP. 338 (2013) 32-49 ; PRC94 (2016), 024904

$$P^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x,p) \varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x,p)} + \mathcal{O}(\varpi^2)$$

AMPT, UrQMD, PICR hydro, CKT+collisions

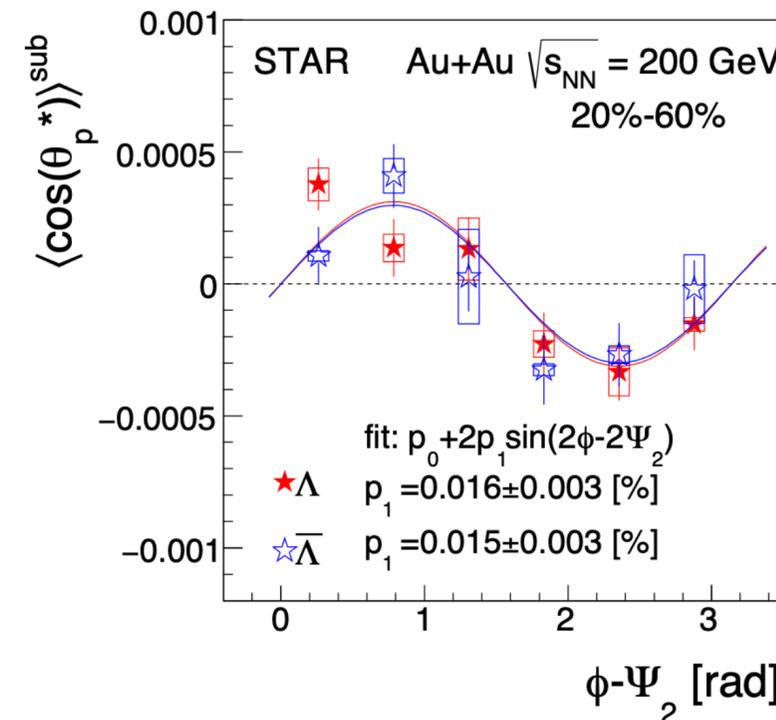
PRC 96, 054908(2017); EPJC 77, 213(2017);

PRC 95, 031901(2017); PRC 96, 024906 (2017)



Experiment = Theory

Local polarization



"Spin sign puzzle"

Experiment \neq Theory

PRL123, 132301 (2019).

Shear-induced polarization : Correct trend

PRL 127, 142301 (2021) ; JHEP 07, 188 (2021);

PRL 127, 272302 (2021) ; PLB 820,136519 (2021);

Eos, initial condition sensitivity

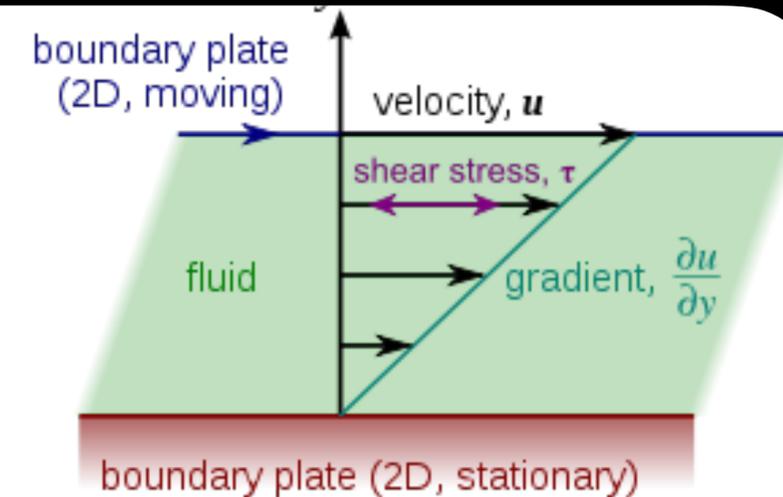
PRC 104, 064901(2021); PRC 105, 064909 (2022)

Non-equilibrium effect : collisional

Shear flow : particle redistribution

- Shear tensor \rightarrow redistribution of particles $f \rightarrow f + \delta f$
 - \rightarrow off-equilibrium contribution to energy-momentum tensor
 - \rightarrow shear viscosity

P. B. Arnold, G. D. Moore and L. G. Yaffe,
 JHEP 01, 030 (2003)
 JHEP 11, 001 (2000)
 JHEP 05, 051 (2003)



LHS: $\sim \partial f$ $\partial_t f + \hat{\mathbf{p}} \cdot \nabla f = C[2 \leftrightarrow 2] \sim e^4 \ln e^{-1} \delta f \sim \frac{1}{\tau} \delta f$

RHS : $\propto \delta f$

$f \rightarrow f + \delta f$

- QED plasma with N_f flavor massless fermions in a shear flow

$$(\partial_t + \hat{\mathbf{p}} \cdot \nabla_x) f_p = C_{\text{Coul},f}[2 \leftrightarrow 2] + C_{\text{Comp},f}[2 \leftrightarrow 2] + C_{\text{anni},f}[2 \leftrightarrow 2]$$

$$(\partial_t + \hat{\mathbf{p}} \cdot \nabla_x) \tilde{f}_p = C_{\text{Comp},\gamma}[2 \leftrightarrow 2] + C_{\text{anni},\gamma}[2 \leftrightarrow 2]$$

redistribution of quark & photon to leading logarithmic(LL) order

$$\sigma_{ij} = \frac{1}{2}(\partial_i \beta_j + \partial_j \beta_i) - \frac{1}{3} \delta_{ij} \partial \cdot \beta$$

$$I_{ij}^p = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$$

$$\delta f_p = f_p(1 - f_p) I_{ij}^p \sigma_{ij} \chi_p$$

$$\delta \tilde{f}_p = \tilde{f}_p(1 + \tilde{f}_p) I_{ij}^p \sigma_{ij} \gamma_p$$

$$\chi_p, \gamma_p \propto \frac{\beta^2 p^2}{e^4 \ln e^{-1}}$$

$$\delta f \propto \frac{1}{e^4 \ln e^{-1}} \propto \tau$$

Outline

👁 Background

- Spin polarization

- Shear flow: particle redistribution

👁 Methodology

- Quantum transport theory

👁 Complete Contribution

- A. Derivative term is not enough

- B. Collisional effect

- C. Dynamical part

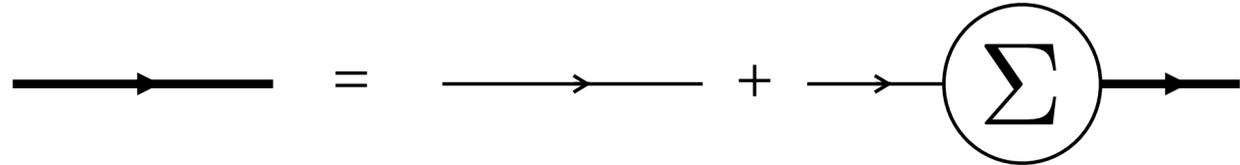
- D. Complete contribution

👁 Summary

Quantum transport theory

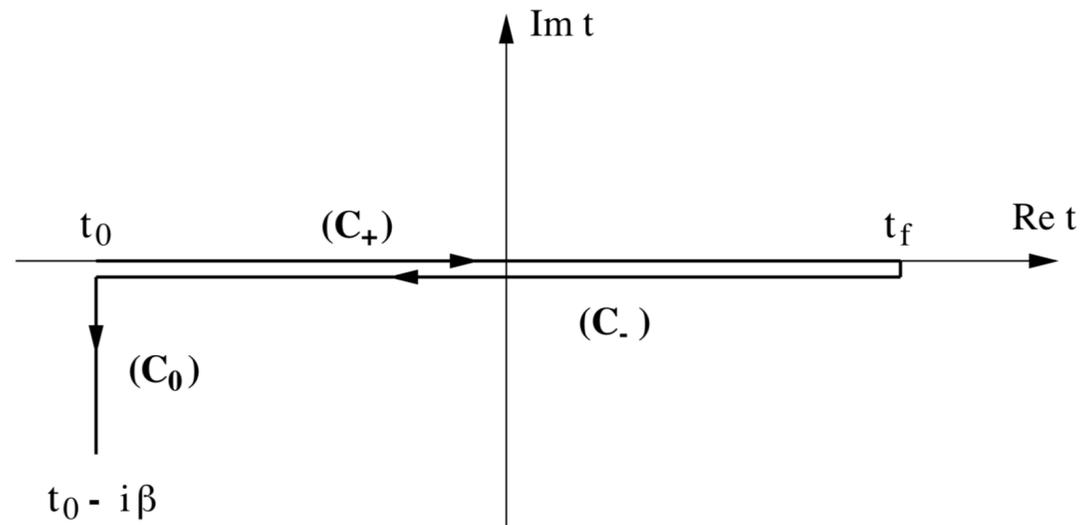
D.L.Yang, K.Hattori and Y.Hidaka, JHEP 07, 070 (2020)
 S.Lin, PRD 105, 076017 (2022)
 Y.Hidaka, S.Pu, Q.Wang and D.L.Yang, Prog.Part.Nucl.Phys127, 103989 (2022)

Dyson Schwinger equation



$$S(x, y) = S^0(x, y) + \int d^4z d^4w S^0(x, w) \Sigma(w, z) S(z, y)$$

Keldysh contour



Wigner transformation

$$S(X, p) = \int d^4u e^{ip \cdot u / \hbar} S \left(X + \frac{u}{2}, X - \frac{u}{2} \right)$$

Kadanoff-Baym equation

$$\left(\gamma^\mu p_\mu - M(X) \right) S^< + \frac{i\hbar}{2} \gamma^\mu \nabla_\mu S^< + \frac{i\hbar}{2} (\nabla_\mu M) (\partial_\mu^p S^<) = -\hbar \Sigma^< \hat{\Lambda} \text{Re} S_R + \frac{i\hbar}{2} \left(\Sigma^< \hat{\Lambda} S^> - \Sigma^> \hat{\Lambda} S^< \right)$$



Eugene Wigner



Leo Kadanoff



Gordon Baym

Development in recent years

Spin 1/2:

CME, Λ polarization, ϕ , K^{*0} alignment

J.Y.Chen, D.T.Son, M.A.Stephanov, H.U.Yee and Y.Yin, Phys.Rev.Lett.113, 182302 (2014)
J.Y.Chen, D.T.Son and M.A.Stephanov, Phys.Rev.Lett.115, 021601 (2015)
Y.Hidaka, S.Pu and D.L.Yang, Phys.Rev.D 95, 091901 (2017)
Y.Hidaka, S.Pu and D.L.Yang, Phys.Rev.D 97, 016004 (2018)
N.Weickgenannt, X.L.Sheng, E.Speranza, Q.Wang and D.H.Rischke, Phys.Rev.D 100, 056018 (2019)
Z.Wang, X.Guo, S.Shi and P.Zhuang, Phys.Rev.D 100, 014015 (2019)
K.Hattori, Y.Hidaka and D.L.Yang, Phys.Rev.D 100, 096011 (2019)
N.Weickgenannt, E.Speranza, X.L.Sheng, Q.Wang and D.H.Rischke, Phys.Rev.Lett 127, 052301 (2021)
X.L.Sheng, N.Weickgenannt, E.Speranza, D.H.Rischke and Q.Wang, Phys.Rev.D 104, 016029 (2021)
N.Weickgenannt, E.Speranza, X.L.Sheng, Q.Wang and D.H.Rischke, Phys.Rev.D 104, 016022 (2021)
D.L.Yang, K.Hattori and Y.Hidaka, JHEP07, 070 (2020)
Z.Wang, X.Guo and P.Zhuang, Eur.Phys.J.C 81, 799 (2021)
D.Hou and S.Lin, Phys.Lett.B 818, 136386 (2021)
X.L.Luo and J.H.Gao, JHEP 11, 115 (2021)
M.Hongo, X.G.Huang, M.Kaminski, M.Stephanov and H.U.Yee, JHEP 08, 263 (2022)
S.Lin, Phys.Rev.D 105, 076017 (2022)
S.Fang, S.Pu and D.L.Yang, Phys.Rev.D 106, 016002 (2022)
Z.Wang, Phys.Rev.D 106, 076011 (2022)
D.L.Yang, JHEP 06, 140 (2022)
S.Lin and Z.Wang, JHEP 12, 030 (2022)
Y.Hidaka, S.Pu, Q.Wang and D.L.Yang, Prog.Part.Nucl.Phys.127, 103989 (2022)
B.Muller and D.L.Yang, Acta Phys.Polon.Supp.16, 37 (2023)
A.Kumar, B.Muller and D.L.Yang, [arXiv:2304.04181 [nucl-th]].

.....

Spin 1:

Zilch effect, ϕ , K^{*0} spin alignment

Photon, Vector meson

X.G.Huang, P.Mitkin, A.V.Sadofyev and E.Speranza, JHEP 10, 117 (2020)
K.Hattori, Y.Hidaka, N.Yamamoto and D.L.Yang, JHEP 02, 001 (2021)
X.L.Sheng, L.Oliva, Z.T.Liang, Q.Wang and X.N.Wang, [arXiv:2206.05868 [hep-ph]].
D.Wagner, N.Weickgenannt and E.Speranza, Acta Phys.Polon.Supp.16, 42 (2023)

.....

Outline

👁️ Background

- Spin polarization

- Shear flow: particle redistribution

👁️ Methodology

- Quantum transport theory

👁️ Complete Contribution

- A. Derivative term is not enough

- B. Collisional effect

- C. Dynamical part

- D. Complete contribution

👁️ Summary

A. Derivative term is not enough

derivative

included in phenomenology

$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} \left(\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V \right) \right)$$

K.Hattori, Y.Hidaka and D.L.Yang, Phys.Rev.D 100, 096011 (2019)

A. Derivative term is not enough

- **Massless** J.Y.Chen, D.T.Son and M.A.Stephanov, Phys.Rev.Lett.115, 021601 (2015)
Y.Hidaka, S.Pu and D.L.Yang, Phys.Rev.D 95, 091901 (2017)

Lorentz covariance + Angular momentum conservation

side-jump + 2-by-2 collision $PK \rightarrow P'K'$

Lorentz covariant current in frame n^μ

$$J_\lambda^\mu = P^\mu f_\lambda + \lambda S_n^{\mu\nu} \partial_\nu f_\lambda + \lambda \int_{KK'P'} C^\lambda \bar{\Delta}^\mu$$

Axial-vector current $A^\mu = J_+^\mu - J_-^\mu$

$$A^\mu = 2\pi\delta(P^2) \left(P^\mu f_A + S_n^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

Helicity λ

Spin tensor $\lambda S_n^{\mu\nu} = \lambda \frac{\epsilon^{\mu\nu\alpha\beta} P_\alpha n_\beta}{2P \cdot n}$

Collision kernel $C^\lambda = |\mathcal{M}|^2 (2\pi)^4 \delta(P + K - P' - K') \times [f_{\bar{n}}^\lambda(P') f_{\bar{n}}^\lambda(K') (1 - f_{\bar{n}}^\lambda(P)) (1 - f_{\bar{n}}^\lambda(K)) - (P, K \leftrightarrow P', K')]$

Side-jump $\lambda \bar{\Delta}^\mu \equiv \lambda \Delta_{nn}^\mu = \lambda \frac{\epsilon^{\mu\alpha\beta\gamma} P_\alpha \bar{n}_\beta n_\gamma}{2(P \cdot n)(P \cdot \bar{n})}$

- **Massive** K.Hattori, Y.Hidaka and D.L.Yang, Phys.Rev.D 100, 096011 (2019)

$$A^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

Dynamical

Pauli-Lubanski vector

Determine from kinetic equation

$$S_{n,m}^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} P_\alpha n_\beta}{2(P \cdot n + m)}$$

Non-dynamical

Magnetization current

Depends on collisional effects

Determine both dynamical and collisional effect

$a^\mu f_A$ dynamical

$S_{n,m}^{\mu\nu} \partial_\nu f_V$ included in phenomenology

$S_{n,m}^{\mu\nu} (\Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V)$ collisional effect

B. Collisional effect

► collisional effect

vanishing for vorticity, non-vanishing for shear due to the redistribution

$$A^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

dissipative

does not depend on relaxation time

$$\Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V \sim e^4 \ln e^{-1} \delta f \sim \frac{1}{\tau} \delta f$$

$$\delta f \propto \frac{1}{e^4 \ln e^{-1}} \propto \tau$$

coupling cancels out

$\partial_\nu f$ and $\Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V$ are comparable

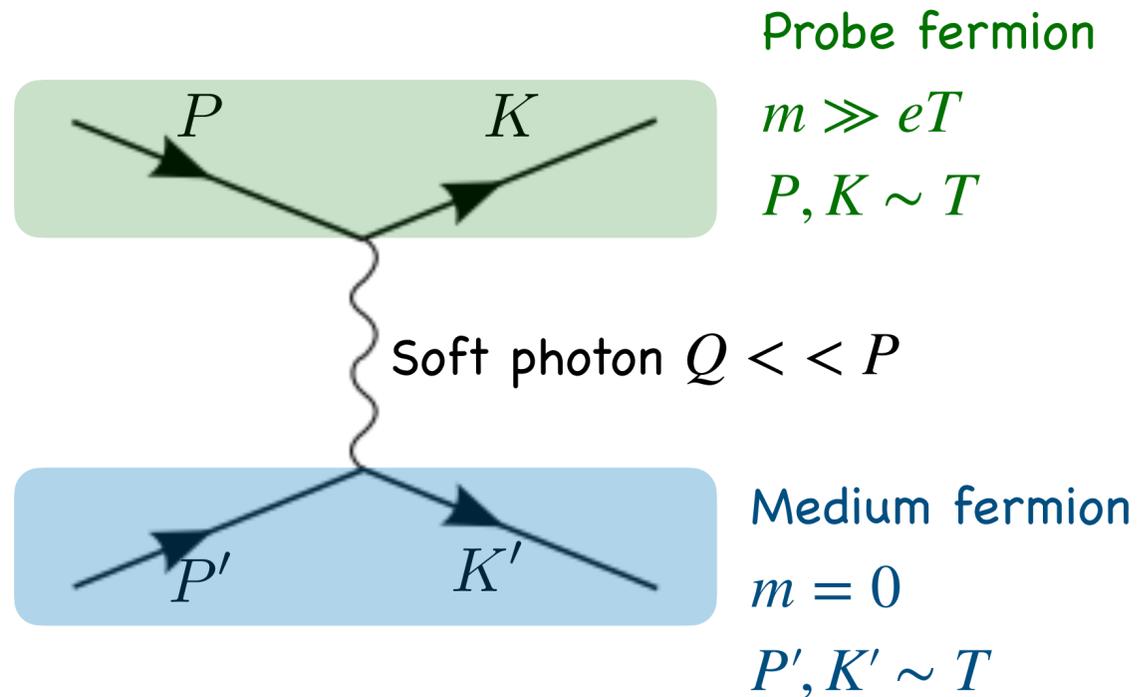
B. Collisional effect

► Evaluate

$$\widehat{\Sigma_{V\mu}^{\text{prob}} f_P} \equiv \Sigma_{V\mu}^> f_P - \Sigma_{V\mu}^< \bar{f}_P$$

Coulomb LL: $e^4 \ln \frac{1}{e}$

Compton suppressed: $e^4 \ln \frac{T}{m}$



- ① Non-eq probe + non-eq Medium
- ② Eq probe + non-eq medium
- ③ Eq probe + eq medium

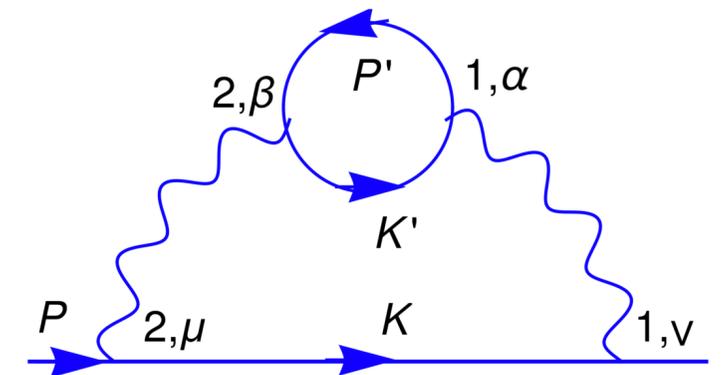
$$\widehat{\Sigma_{V\mu}^{\text{prob}} f_P} = e^4 (2\pi)^3 \int_{K', P', K} \frac{1}{(Q^2)^2} M_\mu^{\text{Coul}} (f_{P'} f_P \bar{f}_K \bar{f}_{K'} - \bar{f}_P \bar{f}_{P'} f_K f_{K'})$$

\propto redistribution δf

$$= e^4 (2\pi)^3 \int_{K', P', K} \frac{1}{(Q^2)^2} M_\mu^{\text{Coul}} \left[2N_f (I_{\alpha\beta}^{\hat{P}'} \chi_{p'} - I_{\alpha\beta}^{\hat{K}'} \chi_{k'}) + (I_{\alpha\beta}^{\hat{P}} \chi_p^{\text{prob}} - I_{\alpha\beta}^{\hat{K}} \chi_k^{\text{prob}}) \right] f_P f_{P'} \bar{f}_K \bar{f}_{K'} \sigma^{\alpha\beta}$$

Non-eq Medium

Non-eq probe



B. Collisional effect

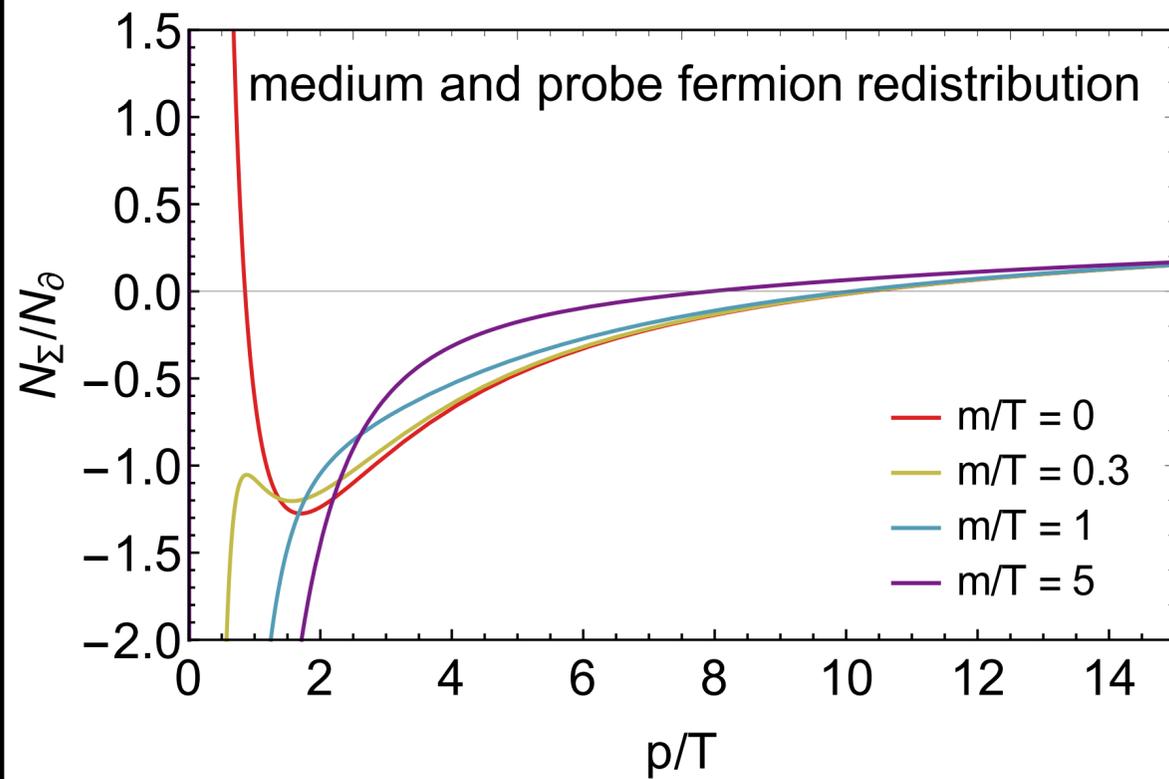
$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

$$\mathcal{A}_\mu^\partial = 2\pi\delta_P S_{\mu\nu}^{m,u} \partial^\nu f_P = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_\partial \quad \text{Derivative}$$

$$\mathcal{A}_\mu^\Sigma = 2\pi\delta_P S_{\mu\nu}^{m,u} \widehat{\Sigma}_{VP}^{\text{prob},\nu} f_P = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_\Sigma \quad \text{Collisional effect}$$

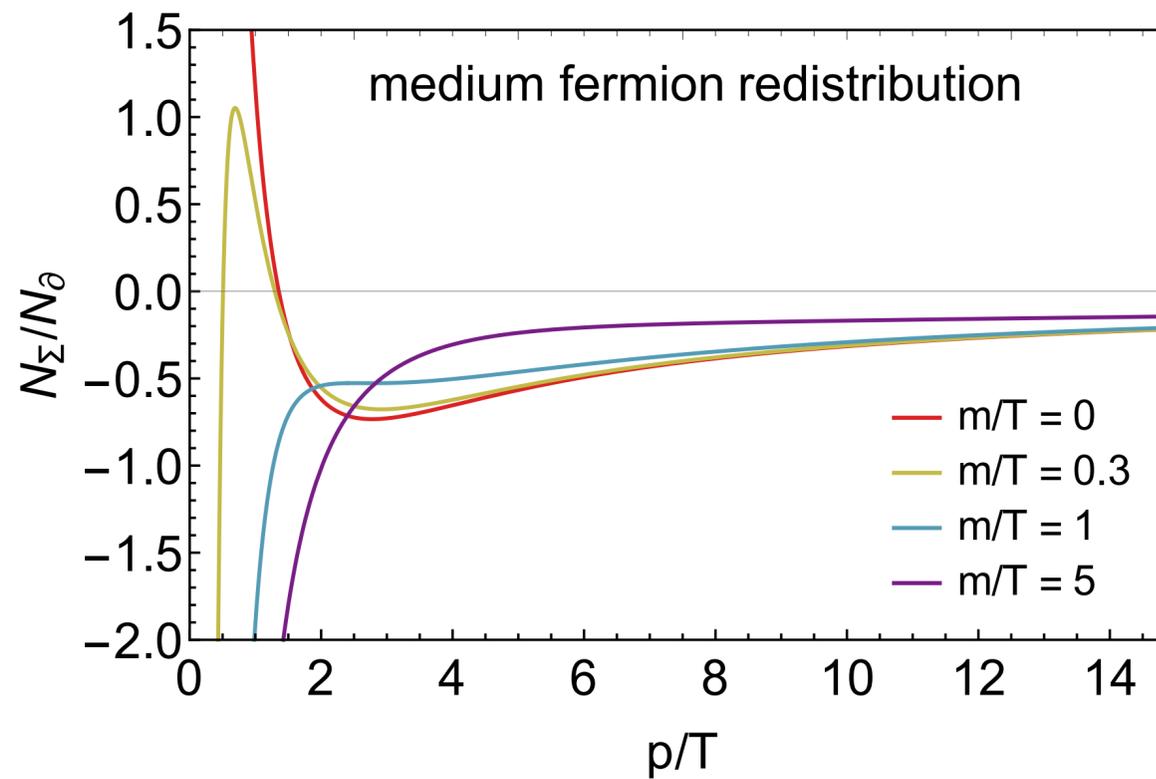
Ratios: N_Σ/N_∂

● Non-eq probe + non-eq medium



$p/T \gg 1, N_\Sigma/N_\partial \rightarrow 1/2$

● Eq probe + non-eq medium



$p/T \gg 1, N_\Sigma/N_\partial \rightarrow 0$

● Eq probe + eq medium

$$\widehat{\Sigma}_{V\mu}^{\text{prob}} f_P \equiv \Sigma_{V\mu}^> f_P - \Sigma_{V\mu}^< \bar{f}_P = 0$$

C. Dynamical Part

collisional effect

From non-equilibrium medium and/or probe fermion

derivative, included in phenomenology

$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} \left(\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V \right) \right)$$

dynamical: from frame dependence & kinetic equation

?

C. Dynamical Part - 1

► Fix the dynamical part from frame dependence \mathcal{A}_μ is frame independent

► **Massless** $\mathcal{A}^\mu = 2\pi\delta(P^2) \left(P^\mu f_A + S_n^{\mu\nu} \mathcal{D}_\nu f_V \right)$

$$\mathcal{D}_\nu f_V = \partial_\nu f_V + \widehat{\Sigma_{V\nu}} f_V$$

Frame dependences in spin and orbital cancel

frame n_μ, n'_μ : $P^\mu (f'_A - f_A) = (S_n^{\mu\nu} - S_{n'}^{\mu\nu}) \mathcal{D}_\nu f_V$

In frame n_μ : $f_A = \frac{\epsilon^{\mu\nu\alpha\beta} u_\nu P_\alpha n_\beta P^\xi \sigma_{\mu\xi}}{2P \cdot n} \frac{A_3}{P \cdot u}$

In frame u_μ : $f_A = 0$

► **Massive** $\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} \mathcal{D}_\nu f_V \right)$
 N_μ

Correct N_μ : f_A has the correct massless limit

a^μ satisfies the constraint $P \cdot a = P^2 - m^2 = 0$

Fix dynamical part in frame u^μ to leading order of m :

$$a^\mu f_A = m \left(1 - \frac{2T_{3p}^c}{p} \right) \frac{\epsilon^{\mu\nu\alpha\beta} P_\alpha u_\beta}{2(P \cdot u)^2} f'_P P^\xi \sigma_{\xi\nu} + \mathcal{O}(m^2)$$

Determine full solution of $a^\mu f_A$ from kinetic equation

C. Dynamical Part - 2

► The static dynamical part — detailed balance

$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} \mathcal{D}_\nu f_V \right)$$

N^μ

$$p \cdot \partial \mathcal{A}_\mu = -p_\mu \widehat{\Sigma_{A\nu} \mathcal{V}^\nu} + p_\nu \widehat{\Sigma_{A\mu} \mathcal{V}^\nu} + m \widehat{\Sigma_S \mathcal{A}_\mu} + p_\rho \widehat{\Sigma_V^\rho \mathcal{A}_\mu} - \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} (\partial^\sigma \widehat{\Sigma_V^\nu}) \mathcal{V}^\rho + \frac{m}{2} \epsilon_{\rho\sigma\lambda\mu} \widehat{\Sigma_T^{\rho\sigma} \mathcal{V}^\lambda} + \frac{1}{2} \epsilon_{\rho\sigma\lambda\mu} (\bar{\Sigma}_V^\rho + \Sigma_V^\rho) \widehat{\Sigma_V^\sigma \mathcal{V}^\lambda}$$

$$\Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V \sim e^4 \ln e^{-1} \delta f \sim \partial f$$

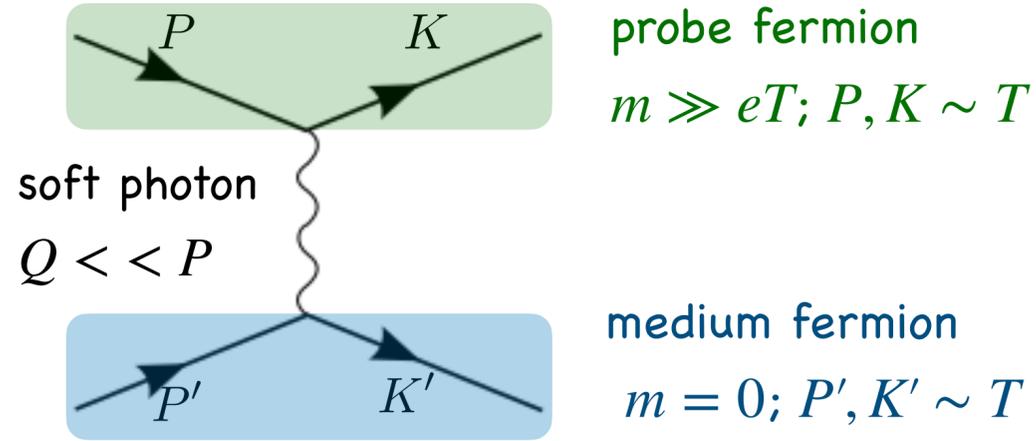
was neglected

same order considering redistribution in shear

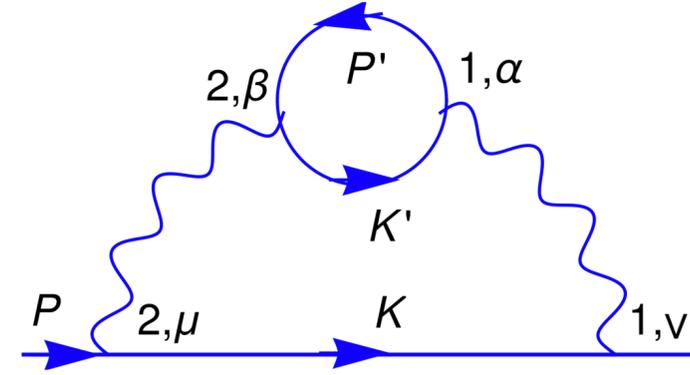
Find the solution for N^μ that eliminates the collision term

subtracting orbital part $S_{n,m}^{\mu\nu} \mathcal{D}_\nu f_V$, giving the dynamical part $a^\mu f_A$

C. Dynamical Part - 3

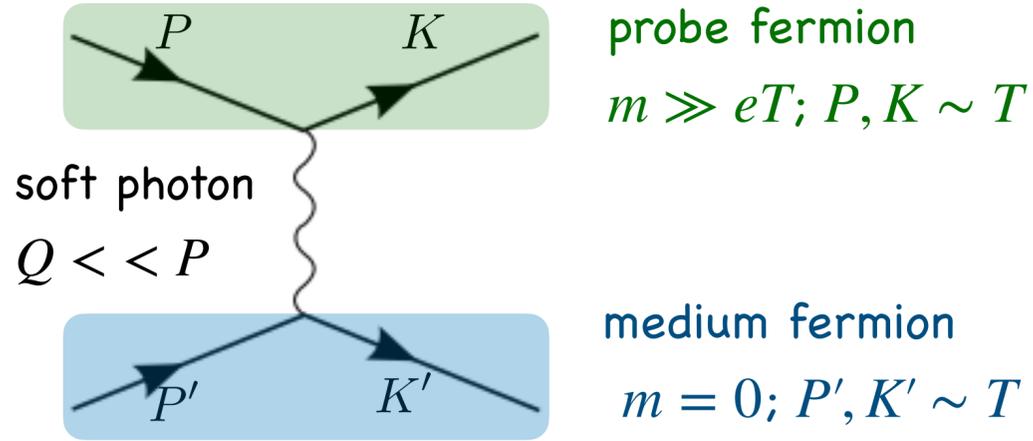


Leading Logarithmic:

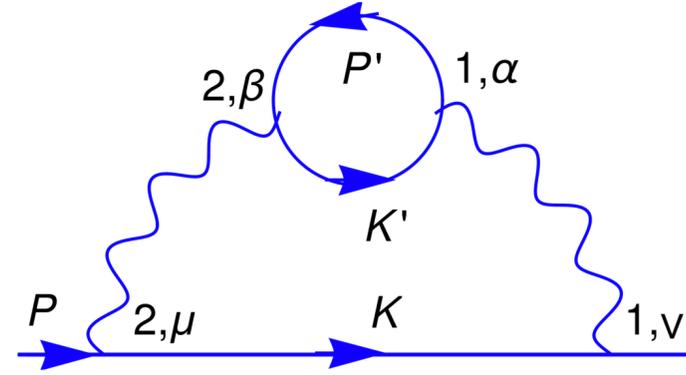


$$P \cdot \partial \mathcal{A}_\mu = - \int_{K, Q, K', P'} \left\{ \begin{aligned}
 & \boxed{M_{\mu\nu}^{A1} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) N_{\text{prob}}^\nu(P) + M_{\mu\nu}^{A2} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) N_{\text{prob}}^\nu(K)} && \text{diffusion} \\
 & + M_{\mu\nu}^{A3} \left(\bar{f}_{K'} f_P \bar{f}_K + f_{K'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(P') + M_{\mu\nu}^{A4} \left(f_{P'} f_P \bar{f}_K + \bar{f}_{P'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(K') && \text{polarization} \\
 & + M_{\mu\nu}^{A5} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) \mathcal{D}^\nu f_P + M_{\mu\nu}^{A6} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) \mathcal{D}^\nu f_K \end{aligned} \right\} && \text{(known)}$$

C. Dynamical Part - 3



Leading Logarithmic:



$$P \cdot \partial \mathcal{A}_\mu = - \int_{K, Q, K', P'} \left\{ \begin{aligned} & \left[M_{\mu\nu}^{A1} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) N_{\text{prob}}^\nu(P) + M_{\mu\nu}^{A2} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) N_{\text{prob}}^\nu(K) \right] && \text{diffusion} \\ & + M_{\mu\nu}^{A3} \left(\bar{f}_{K'} f_P \bar{f}_K + f_{K'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(P') + M_{\mu\nu}^{A4} \left(f_{P'} f_P \bar{f}_K + \bar{f}_{P'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(K') && \text{polarization} \\ & + M_{\mu\nu}^{A5} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) \mathcal{D}^\nu f_P + M_{\mu\nu}^{A6} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) \mathcal{D}^\nu f_K && \text{(known)} \end{aligned} \right.$$

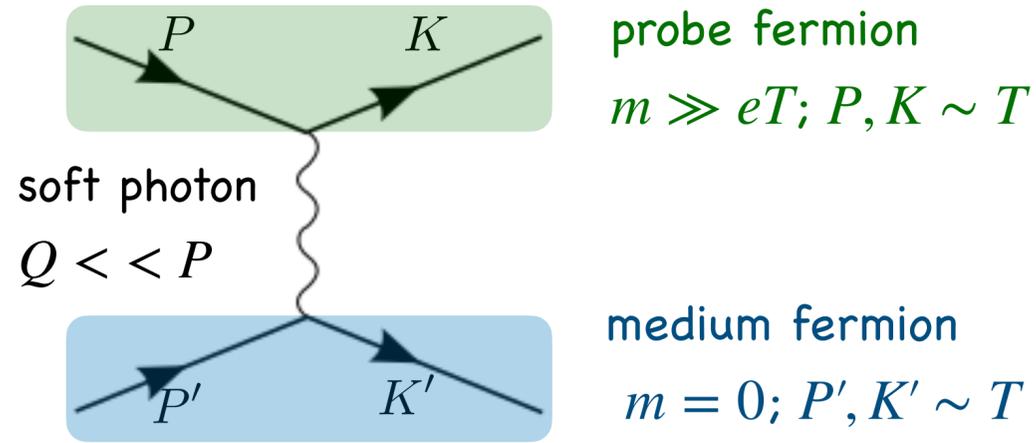
dissipative effects included

$$N_{\text{prob}}^\nu = a^\nu f_A + S_{u,m}^{\nu\rho} \mathcal{D}_\rho f_V$$

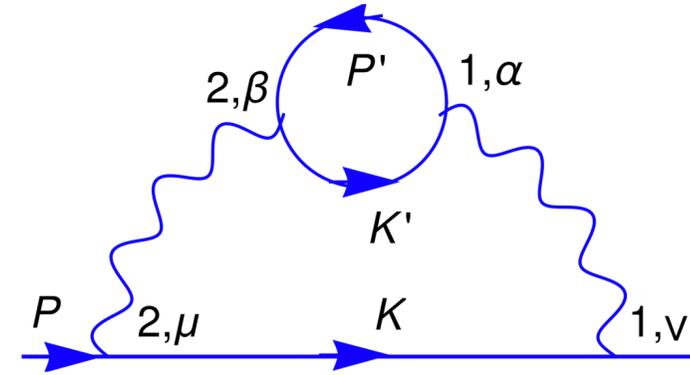
$$N_{\text{med}}^\nu = S_u^{\nu\rho} \mathcal{D}_\rho f_V$$

$$\mathcal{D}_\nu f_V = \partial_\nu f_V + \widehat{\Sigma_{V\nu}} f_V$$

C. Dynamical Part - 3



Leading Logarithmic:



$$P \cdot \partial \mathcal{A}_\mu = - \int_{K, Q, K', P'} \left\{ \begin{aligned} & \boxed{M_{\mu\nu}^{A1} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) N_{\text{prob}}^\nu(P) + M_{\mu\nu}^{A2} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) N_{\text{prob}}^\nu(K)} && \text{diffusion} \\ & + M_{\mu\nu}^{A3} \left(\bar{f}_{K'} f_P \bar{f}_K + f_{K'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(P') + M_{\mu\nu}^{A4} \left(f_{P'} f_P \bar{f}_K + \bar{f}_{P'} \bar{f}_P f_K \right) N_{\text{med}}^\nu(K') && \\ & + M_{\mu\nu}^{A5} \left(\bar{f}_K f_{P'} \bar{f}_{K'} + f_K \bar{f}_{P'} f_{K'} \right) \mathcal{D}^\nu f_P + M_{\mu\nu}^{A6} \left(f_{P'} \bar{f}_{K'} f_P + \bar{f}_{P'} f_{K'} \bar{f}_P \right) \mathcal{D}^\nu f_K \end{aligned} \right\} \text{polarization}$$

Find the solution for N^μ that eliminates the collision term

detailed balance
 soft momentum transfer $K = P - Q$



differential equation of N_P^{prob}

$$0 = G_0 + G_1 N_P^{\text{prob}} + G_2 N_P'^{\text{prob}} + G_3 N_P''^{\text{prob}}$$

C. Dynamical Part - 4

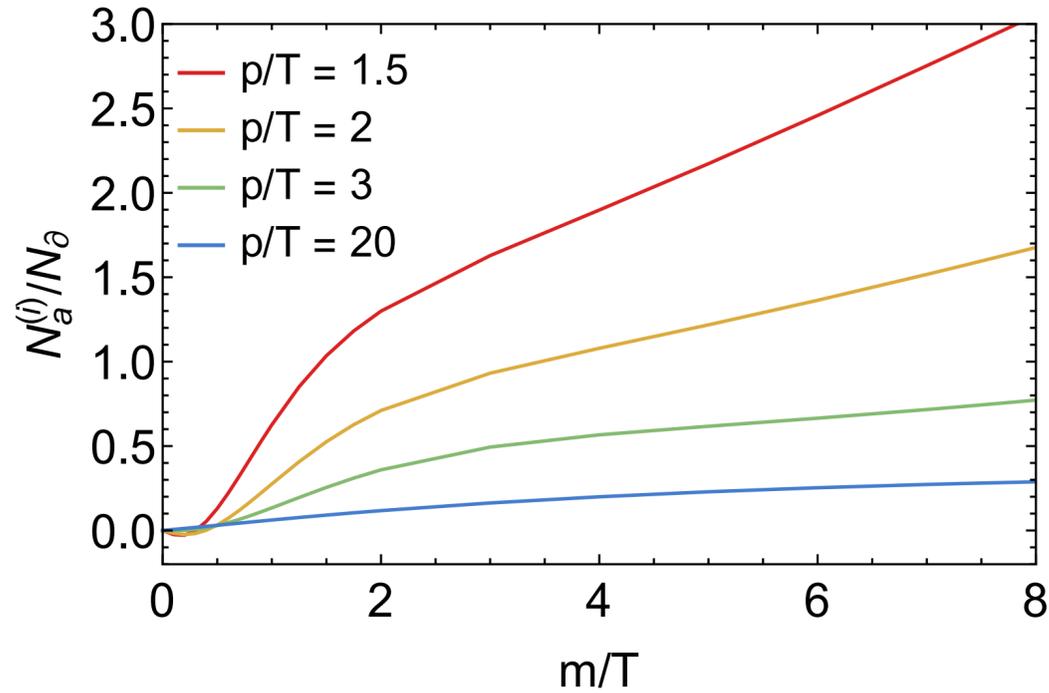
$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

$$\mathcal{A}_\mu^a = 2\pi\delta_P a_\mu f_A = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_a \quad \text{Dynamical part}$$

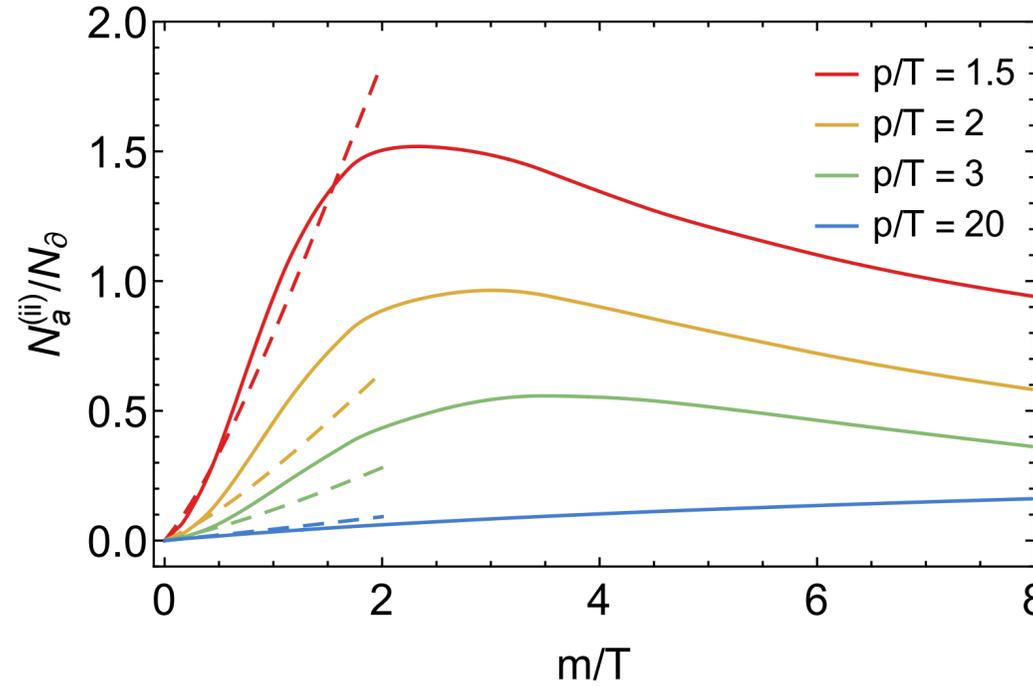
$$\mathcal{A}_\mu^\partial = 2\pi\delta_P S_{\mu\nu}^{m,u} \partial^\nu f_P = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_\partial \quad \text{Derivative}$$

Ratios: N_Σ/N_∂ , N_a/N_∂

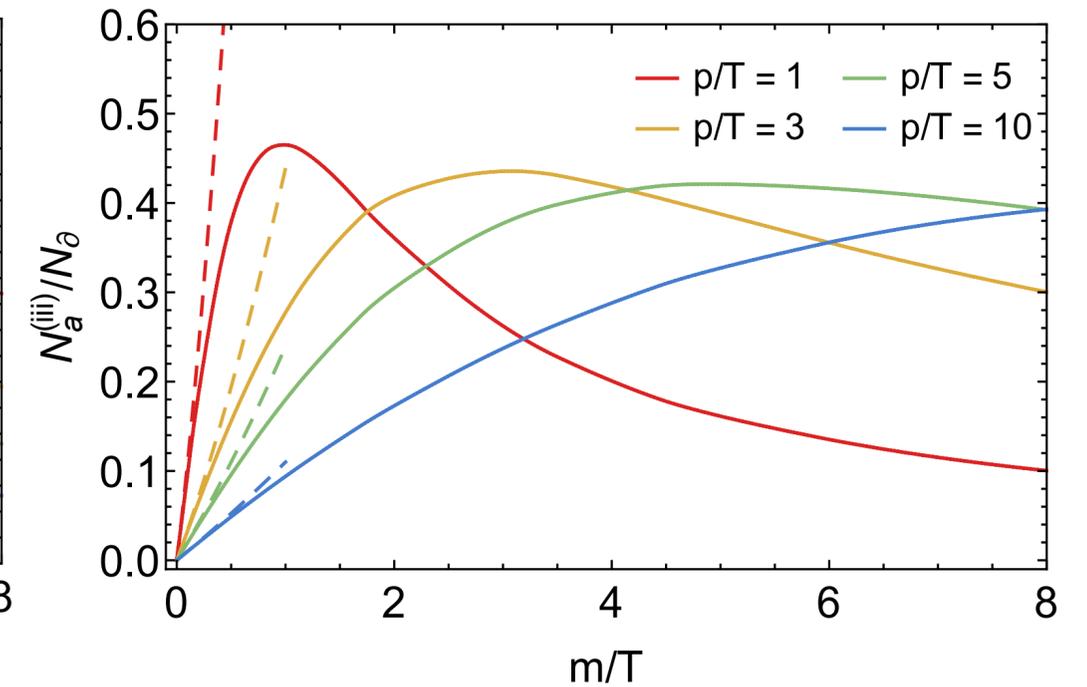
● Non-eq probe + non-eq medium



● Eq probe + non-eq medium



● Eq probe + eq medium



- Static solution
- Generally larger with more dissipative effect included
- Vanishes when massless or large momentum $m \rightarrow 0$, $a_\mu f_A \rightarrow 0$; $p \gg T$, $a_\mu f_A \rightarrow 0$

D. Complete contribution

$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} (\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V) \right)$$

$$\mathcal{A}_\mu = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_P^{\text{prob}} \quad \text{Sum of all}$$

$$\mathcal{A}_\mu^a = 2\pi\delta_P a_\mu f_A = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_a \quad \text{Dynamical part}$$

$$\mathcal{A}_\mu^\partial = 2\pi\delta_P S_{\mu\nu}^{m,u} \partial^\nu f_P = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_\partial \quad \text{Derivative}$$

$$\mathcal{A}_\mu^\Sigma = 2\pi\delta_P S_{\mu\nu}^{m,u} \widehat{\Sigma_{VP}^{\text{prob},\nu}} f_P = 2\pi\delta_P \frac{\epsilon_{\mu\nu\alpha\beta} P_\perp^\alpha u^\beta}{2P \cdot u} P_{\perp\xi} \sigma^{\nu\xi} f'_P N_\Sigma \quad \text{Collisional effect}$$

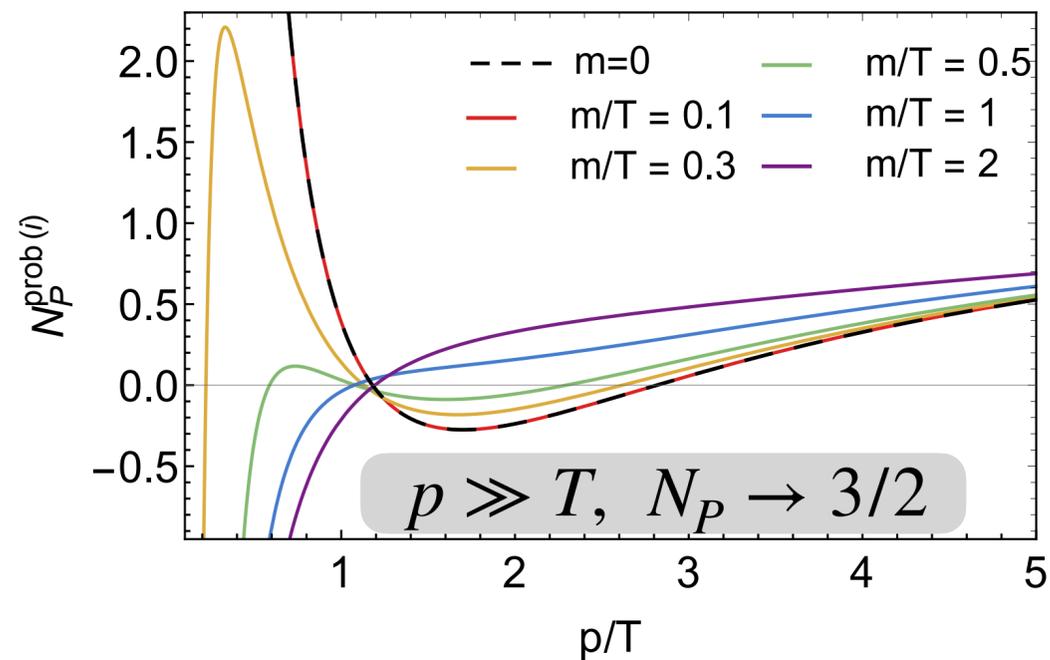
S.Y.F.Liu and Y.Yin, JHEP 07, 188 (2021)

C.Yi, S.Pu and D.L.Yang, Phys.Rev.C 104, 064901 (2021)

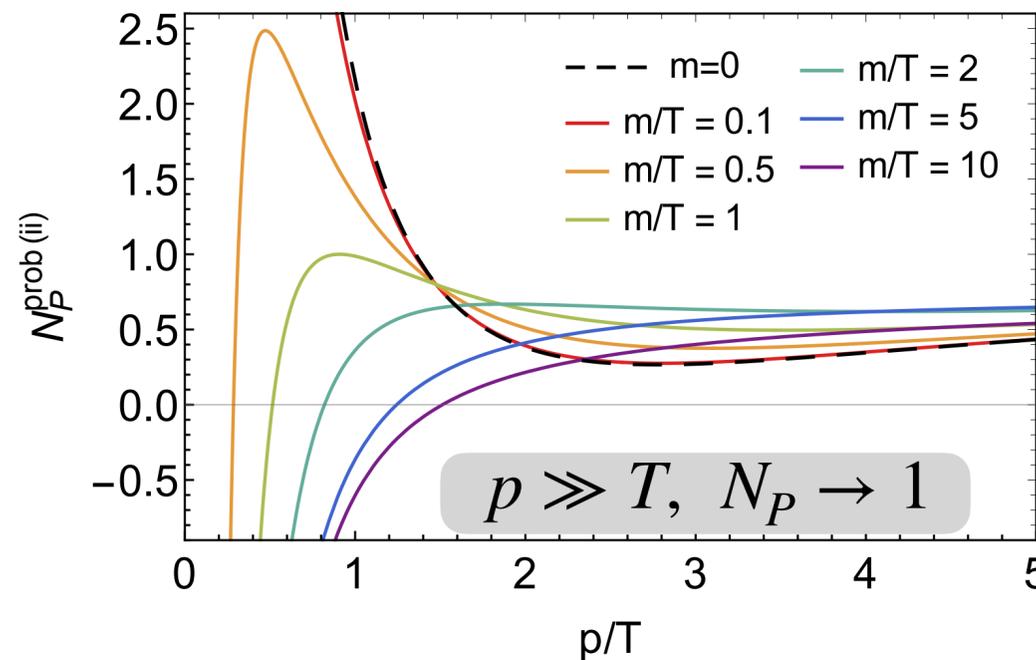
linear response
phenomenology

Previous research corresponds to $N_P = 1$

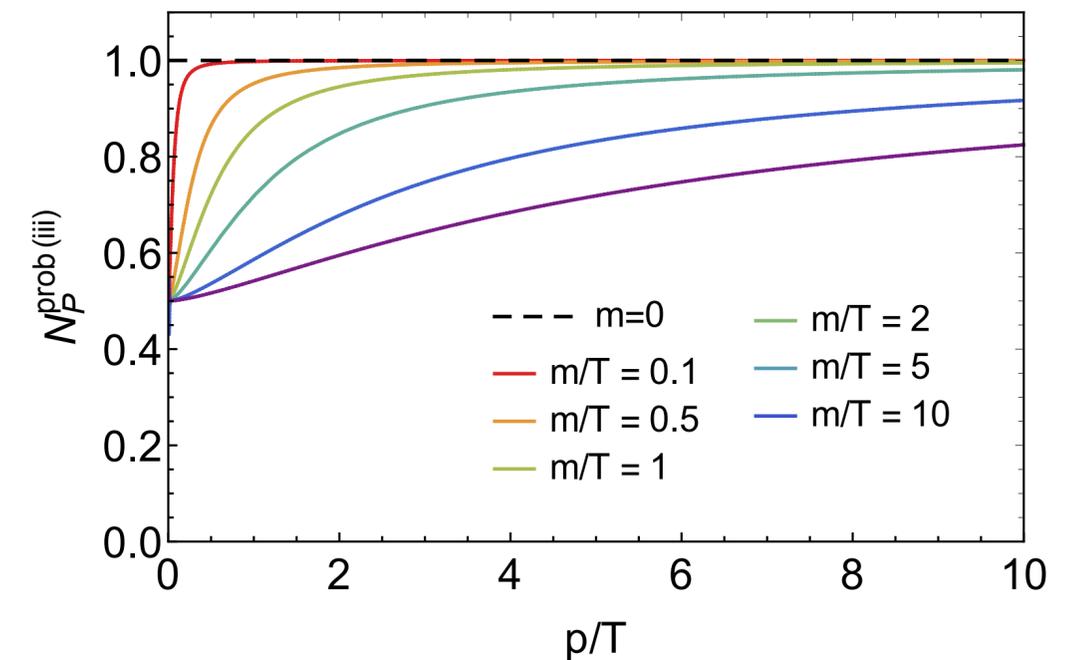
• Non-eq probe + non-eq medium



• Eq probe + non-eq medium



• Eq probe + eq medium



- Without dissipative effect, $N_P = 1$ is the static solution for massless fermion, not for massive case.
- With dissipative effect, N_P is not a constant, could be enhanced.

Outline

👁 Background

👁 Methodology

👁 Complete Contribution

- Spin polarization

- Shear flow: particle redistribution

- Quantum transport theory

- A. Derivative term is not enough

- B. Collisional effect

- C. Dynamical part

- D. Complete contribution

👁 Summary

Summary

- ▶ shear induced polarization gives correct trend of local polarization
- ▶ complete contribution of shear induced polarization includes three parts

derivative:
included in phenomenology

$$A^\mu = 2\pi\delta(P^2 - m^2) \left(a^\mu f_A + S_{n,m}^{\mu\nu} \left(\partial_\nu f_V + \Sigma_{V\nu}^> f_V - \Sigma_{V\nu}^< \bar{f}_V \right) \right)$$

dynamical:
frame dependence & kinetic equation
static: vanishing when $m=0$ or $p \gg T$
numerical solution for finite m, p
comparable with derivative term

collisional effect:
dissipative effect
redistribution in the shear flow
comparable with derivative term

- ▶ go beyond the static solution? gauge dependence?

Thank You

2023.07.17

@ The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions