

Relativistic 2nd-order spin hydrodynamics

Entropy production principle

A.Daher, W.Florkowski, R.Ryblewski, A.Das

arXiv:2202.12609, PhysRevD.107.054043, PhysRevD.107.094022, arXiv:2304.01009



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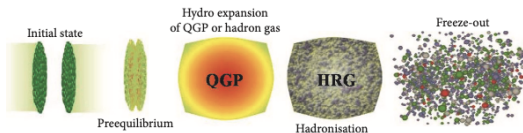


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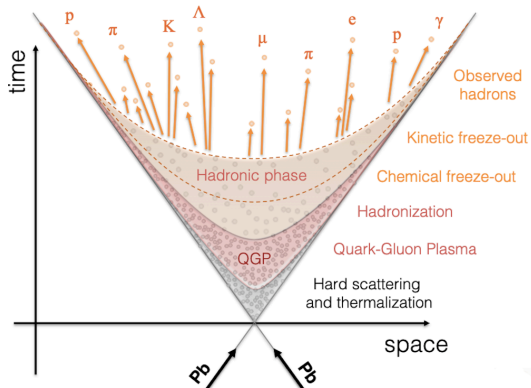
Motivations

■ Successes of relativistic hydrodynamics

[W.Florkowski-Phenomenology of Ultra-relativistic Heavy-ion Collisions-World Scientific]

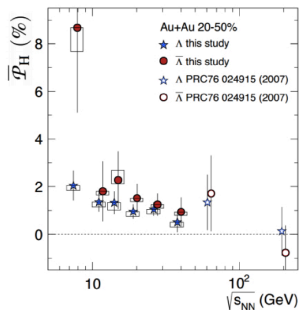


Various stages of relativistic heavy-ion collision
[<http://qgp.phy.duke.edu>]

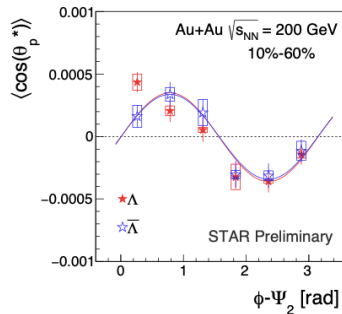


Spacetime diagram of relativistic heavy-ion collision [D.D. Chinellato]

- Growing interest in spin hydrodynamics as a potential theory to describe the polarization data



Average Λ global polarization [STAR, L. Adamczyk et al., Nature 548, 62 (2017)]



Longitudinal polarization [T.Niida, NPA 982 (2019) 511514]

1st-Order Formulation

Relativistic hydrodynamics is based on the conservation of **EMT**,

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \quad \text{s.t} \quad \boxed{T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi}$$

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Up to 1st-order in dissipation, the system is mathematically closed,

$$\left\{ \begin{array}{l} T^{\mu\nu} \rightarrow 4 \text{ independent variables } T, u^\mu \\ \partial_\mu T^{\mu\nu} = 0 \rightarrow 4 \text{ equations,} \\ \pi^{\mu\nu} \& \Pi \text{ can be fixed in terms of } T, u^\mu. \end{array} \right.$$

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The dynamical variables obey the local thermodynamic relation,

$$\boxed{\epsilon + p = Ts}$$

Spin hydrodynamics is formulated based on the conservation of **EMT** as well as **TAM**,

[Hattori et.al j.physletb.2019.05.040; Fukushima, Shi Pu j.physletb.2021.136346]

$$\boxed{\partial_\mu T^{\mu\nu} = 0} \quad \text{s.t} \quad \boxed{T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + T_{1s}^{\mu\nu} + T_{1a}^{\mu\nu}}$$
$$\begin{cases} T_{1s}^{\mu\nu} = h^\mu u^\nu + h^\nu u^\mu + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}, \\ T_{1a}^{\mu\nu} = q^\mu u^\nu - q^\nu u^\mu + \Phi^{\mu\nu}. \end{cases}$$

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$$\boxed{\partial_\mu S^{\mu\alpha\beta} = -2T_{1a}^{\alpha\beta}} \quad \text{s.t} \quad \boxed{S^{\mu\alpha\beta} = u^\mu S^{\alpha\beta} + S_1^{\mu\alpha\beta}}$$

$$\begin{cases} \partial_\mu J^{\mu\alpha\beta} = 0, \\ J^{\mu\alpha\beta} = L^{\mu\alpha\beta} + S^{\mu\alpha\beta}, \text{ with } L^{\mu\alpha\beta} = 2x^{[\alpha} T^{\mu\beta]}. \end{cases}$$

Up to 1st-order in dissipation, the system is mathematically closed,

$$\left\{ \begin{array}{l} T^{\mu\nu} \rightarrow 4 \text{ independent variables } T, u^\mu \\ \partial_\mu T^{\mu\nu} = 0 \rightarrow 4 \text{ equations,} \\ \pi^{\mu\nu}, \Pi, q, \& \phi^{\mu\nu} \text{ can be fixed in terms of independent variables.} \end{array} \right.$$

$$\left\{ \begin{array}{l} S^{\mu\alpha\beta} \rightarrow 6 \text{ indep variables } \omega^{\mu\nu} \equiv \text{spin potential } \sim \mathcal{O}(\partial) \text{ s.t. } \boxed{\epsilon + p = Ts + \omega_{\mu\nu} S^{\mu\nu}}, \\ \partial_\mu S^{\mu\alpha\beta} = -2T_{1a}^{\alpha\beta} \rightarrow 6 \text{ equations,} \\ S_1^{\mu\alpha\beta} \text{ can't be fixed in our scheme at the 1}^{st}\text{-order (as we will see later...).} \end{array} \right.$$

The main goal of spin hydro is to determine the constitutive equations of the antisymmetric part of the energy-momentum tensor q^μ & $\phi^{\mu\nu}$, along with the dissipative part of the spin tensor $S_1^{\mu\alpha\beta}$.

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The machinery of fixing the dissipative currents in terms of T , u^μ , & $\omega^{\mu\nu}$ can be done utilizing positivity of the entropy production rate,

$$\partial_\mu s^\mu = T_{1s}^{\mu\nu} \partial_\mu \beta_\nu + T_{1a}^{\mu\nu} (\partial_\mu \beta_\nu + 2\beta \omega_{\mu\nu}) + \mathcal{O}(\partial^3) \geq 0$$

It is noticeable that fixing $S_1^{\lambda\mu\nu}$ is not possible through entropy-current analysis within the first-order theory considering our gradient counting scheme $\omega_{\mu\nu} \sim \mathcal{O}(\partial)$.

Recently, in collaboration with **F.Beccatini** and **X.L. Sheng**, and as a part of work in progress, we've used a quantum statistical approach to derive a spin hydrodynamic framework s.t,

$$\begin{aligned} \partial_\mu s^\mu = & \partial_\mu \beta_\nu \left[T_s^{\mu\nu} - T_{s(\text{LE})}^{\mu\nu} \right] + (\Omega_{\mu\nu} - \bar{\omega}_{\mu\nu}) \left[T_a^{\mu\nu} - T_{a(\text{LE})}^{\mu\nu} \right] \\ & - \frac{1}{2} \partial_\mu \Omega_{\lambda\nu} \left[S^{\mu\lambda\nu} - S_{(\text{LE})}^{\mu\lambda\nu} \right] \end{aligned}$$

- [8] M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, and H.-U. Yee, “Relativistic spin hydrodynamics with torsion and linear response theory for spin relaxation,” *JHEP* **11** (2021) 150, [arXiv:2107.14231 \[hep-th\]](#).
- [9] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, “Fate of spin polarization in a relativistic fluid: An entropy-current analysis,” *Phys. Lett. B* **795** (2019) 100–106, [arXiv:1901.06615 \[hep-th\]](#).
- [10] K. Fukushima and S. Pu, “Spin hydrodynamics and symmetric energy-momentum tensors – A current induced by the spin vorticity –,” *Phys. Lett. B* **817** (2021) 136346, [arXiv:2010.01608 \[hep-th\]](#).
- [11] A. Daher, A. Das, W. Florkowski, and R. Ryblewski, “Equivalence of canonical and phenomenological formulations of spin hydrodynamics,” [arXiv:2202.12609 \[nucl-th\]](#).
- [12] D. She, A. Huang, D. Hou, and J. Liao, “Relativistic viscous hydrodynamics with angular momentum,” *Sci. Bull.* **67** (2022) 2265–2268, [arXiv:2105.04060 \[nucl-th\]](#).
- [13] A. D. Gallegos, U. Gürsoy, and A. Yarom, “Hydrodynamics of spin currents,” *SciPost Phys.* **11** (2021) 041, [arXiv:2101.04759 \[hep-th\]](#).
- [14] A. D. Gallegos, U. Gursoy, and A. Yarom, “Hydrodynamics, spin currents and torsion,” [arXiv:2203.05044 \[hep-th\]](#).

So please stay tuned...

Based on a quantum field theoretical standpoint of Dirac field, the canonical formalism decomposes the spin tensor in a totally antisymmetric fashion,

[A.D, A.Das, W.Florkowski, & R.Ryblewski 2202.12609]

$$S_{\text{can}}^{\mu\alpha\beta} = u^\mu S^{\alpha\beta} - u^\alpha S^{\mu\beta} + u^\beta S^{\mu\alpha} + S_{\text{can}(1)}^{\mu\alpha\beta}$$

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Solution: $T_{\text{can}}^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + T_{\text{can}(1)}^{\mu\nu} + \partial_\lambda (u^\nu S^{\mu\lambda})$.

Open questions in spin hydrodynamics:

1. Gradient counting scheme,
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In our approach, incorporating physically motivated foundations, we propose:

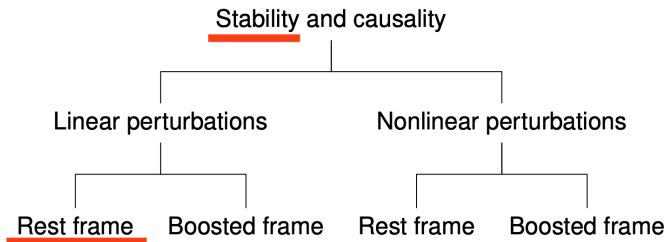
$$S^{\mu\nu} \sim \mathcal{O}(1) \quad \& \quad \omega^{\mu\nu} \sim \mathcal{O}(\partial^1)$$

$$S^{\mu\nu}(T, \omega^{\mu\nu}) = S_0(T) \frac{\omega^{\mu\nu}}{\sqrt{\omega^{\mu\nu} \omega_{\mu\nu}}}$$

Linear Stability Analysis



1) In order to obtain acceptable theories of relativistic dissipative spin hydrodynamics, it is crucial to consider the dynamics of fluid departure from its equilibrium state preserving stability and causality in all boosted frames.



[A.Das, W.Florkowski, & R.Ryblewski PhysRevD.102.031501]

2) We begin by deriving the evolution equations from conservation laws, i.e,

[A.D, A.Das, & R.Ryblewski, PhysRevD.107.054043]

$$\partial_\mu T^{\mu\nu} = 0 \longrightarrow \begin{cases} (\epsilon + p)\partial_\mu u^\mu + u^\mu \partial_\mu \epsilon = -u_\nu \partial_\mu T_1^{\mu\nu}, \\ u^\mu (\epsilon + p)\partial_\mu u^\alpha - \Delta^{\alpha\mu} \partial_\mu p = -\Delta^\alpha_\nu \partial_\mu T_1^{\mu\nu}. \end{cases}$$
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$$\partial_\mu J^{\mu\alpha\beta} = 0 \longrightarrow u^\mu \partial_\mu S^{\alpha\beta} + S^{\alpha\beta} \partial_\mu u^\mu = -2T_{1a}^{\mu\nu}.$$

3) Initial conditions:

$$\begin{cases} \text{Hydrostatic global equilibrium} : \epsilon \rightarrow \epsilon_0 + \delta\epsilon, & \& u^\mu \rightarrow (1, \vec{0}) + (0, \delta\vec{v}^i), \\ \text{Spinless global equilibrium} : S^{\mu\nu} \rightarrow 0 + \delta S^{\mu\nu}. \end{cases}$$

4) Results:

Out of the 10 possible solutions, 3 related to spin evolution are unstable:

$$\omega = \frac{8i\lambda}{X}$$

where
$$\begin{cases} X = \partial S^{i0} / \partial \omega^{i0}, \\ q^\mu = \lambda (\beta \nabla^\mu T + D u^\mu - 4 \omega^{\mu\nu} u_\nu), \\ k = (0, 0, k_z) \text{ is the wave number.} \end{cases}$$

$$\omega = \frac{8i\lambda}{X} - \frac{4i\lambda}{\epsilon_0 + p_0} \frac{\partial p}{\partial \epsilon} k_z^2$$

1st-order spin hydro exhibits instabilities even in rest frame

Boost-Invariant Study



Using the boost-invariant flow ingredients (**Bjorken flow**), the evolution equations of energy and spin densities reduces to,

[R.Biswas, A.D, A.Das, W.Florkowski, & R.Ryblewski, PhysRevD.107.094022]

$$\boxed{\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} - \frac{1}{\tau} \left(\frac{2}{3} \frac{\eta_s}{\tau} + \frac{\zeta}{\tau} \right) = 0} \quad \& \quad \boxed{\frac{\partial S^{\mu\nu}}{\partial \tau} + \frac{S^{\mu\nu}}{\tau} = -2\phi^{\mu\nu}}$$

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By proposing a scaling of the spin density (spin E.O.S) based on the direction of $\omega_{\mu\nu}$,

$$S^{\mu\nu}(T, \omega^{\mu\nu}) = S_0(T) \frac{\omega^{\mu\nu}}{\sqrt{\omega^{\mu\nu} \omega_{\mu\nu}}},$$

the spin equation reduces to an equation for the magnitude of spin potential,

$\sqrt{2}C = \sqrt{\omega^{\mu\nu} \omega_{\mu\nu}}$, such that

$$\boxed{\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + p}{\tau} - \frac{1}{\tau} \left(\frac{2}{3} \frac{\eta_s}{\tau} + \frac{\zeta}{\tau} \right) = 0} \quad \& \quad \boxed{C = -\frac{1}{4\sqrt{2}\gamma\beta} \left(\frac{dS_0}{d\tau} + \frac{S_0}{\tau} \right)}$$

Yet the function S_0 is arbitrary but with known dimensions. Therefore 2 physically motivated scalings are chosen :

- Case:I $S_0(T) \sim n_0(T)$ (number density)
- Case:II $S_0(T) \sim s_0(T)$ (entropy density)

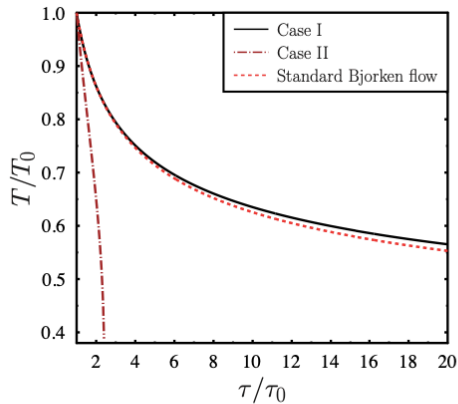


FIG. 1. Proper time evolution of temperature. Black (solid) line represents the temperature evolution for case I. Brown (dashed-dotted) line represents the temperature evolution for case II. Red (dashed) line represents the variation of temperature with the proper time for the standard Bjorken flow without spin. We consider $T_0 = 200$ MeV and $\tau_0 = 0.5$ fm.

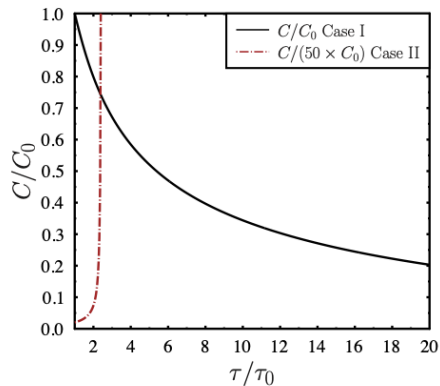


FIG. 2. Proper time evolution of the magnitude of the spin chemical potential C . We consider $C_0 = 50$ MeV and $\tau_0 = 0.5$ fm. For case I the C decreases with proper time. But for the case II the spin chemical potential grows rapidly with proper time.

2^{nd} -Order Formulation

Inspired by the **I-S** theory, which successfully addresses the stability and causality issues of **N-S** relativistic hydrodynamics, we develop a second-order potential theory aimed at resolving these challenges.

	Landau's theory	Israel-Stewart theory
Basic variables	T, μ, u^μ	$T, \mu, u^\mu, \Pi, \pi^{\mu\nu}, n^\mu$
Dissipative flux	$\Pi, \pi^{\mu\nu}, n^\mu$	$\Pi, \pi^{\mu\nu}, n^\mu$
Hydro Equations	$u_\nu \partial_\mu T^{\mu\nu} = 0,$ $\Delta_\nu^\alpha \partial_\mu T^{\mu\nu} = 0,$ $\partial_\mu N^\mu = 0,$	$u_\nu \partial_\mu T^{\mu\nu} = 0,$ $\Delta_\nu^\alpha \partial_\mu T^{\mu\nu} = 0,$ $\partial_\mu N^\mu = 0,$
$\partial_\mu S^\mu \geq 0$	$\Pi = -\zeta \partial_\mu u^\mu,$ $n^\mu = \kappa \nabla^\mu (\mu/T),$ $\pi^{\mu\nu} = 2\eta \sigma^{\mu\nu}.$	$\tau_\Pi \dot{\Pi} + \Pi = f_\Pi(\Pi, \pi^{\mu\nu}, n^\mu),$ $\tau_n \dot{n}^{(\mu} + n^{\mu)} = f_n(\Pi, \pi^{\mu\nu}, n^\mu),$ $\tau_\pi \dot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} = f_\pi(\Pi, \pi^{\mu\nu}, n^\mu).$
Causality and stability	No	Yes (linear level)

[A.Das, W.Florkowski, & R.Ryblewski PhysRevD.102.031501]

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- New spin transport coefficients related to $S_1^{\lambda\mu\nu}$
- Solve the stability problem (Future work)
- Numerically solve the system and compare with the data (Future Work)

Starting from local thermodynamic relation, and allowing for 2nd-order contributions, we get

$$s_{IS}^{\mu} = s_{NS}^{\mu} - \beta \omega_{\alpha\beta} S_1^{\mu\alpha\beta} + Q^{\mu}$$

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Comments on the above expression are in order here:

- Unlike the 1^{st} -order theory, the term $S_1^{\mu\alpha\beta}$ which appears in the entropy current s_{IS}^{μ} can be fixed.

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- The most general decomposition a rank-3 tensor antisymmetric in last 2 indices in terms of irreducible tensors takes the form [F. Becattini, L. Tinti, PhysRevD.84.025013],

$$S^{\mu\alpha\beta} = u^{\mu} S^{\alpha\beta} + S_1^{\mu\alpha\beta} = u^{\mu} S^{\alpha\beta} + 2u^{[\alpha} \Delta^{\mu\beta]} \Phi + 2u^{[\alpha} \tau_s^{\mu\beta]} + 2u^{[\alpha} \tau_a^{\mu\beta]} + \Theta^{\mu\alpha\beta}$$

- The term Q^μ contains all possible combinations of 2^{nd} -order dissipations,

$$\boxed{h^\mu, \pi^{\mu\nu}, \Pi, q^\mu, \phi^{\mu\nu}} \quad \& \quad \boxed{\Phi, \tau_s^{\mu\nu}, \tau_a^{\mu\nu}, \theta^{\lambda\mu\nu}}$$

Yet constrained by the condition that entropy is maximum in the equilibrium state, i.e.,

$$\boxed{s_{IS} - s = u_\mu Q^\mu \text{ or } u_\mu Q^\mu \leq 0}$$

$$\begin{aligned}
Q^\mu = & u^\mu \left(a_1 \Pi^2 + a_2 \pi^{\lambda\nu} \pi_{\lambda\nu} + a_3 h^\lambda h_\lambda + a_4 q^\lambda q_\lambda + a_5 \phi^{\lambda\nu} \phi_{\lambda\nu} \right) \\
& + u^\mu \left(\tilde{a}_1 \Phi^2 + \tilde{a}_2 \tau_{(s)}^{\lambda\nu} \tau_{(s)\lambda\nu} + \tilde{a}_3 \tau_{(a)}^{\lambda\nu} \tau_{(a)\lambda\nu} + \tilde{a}_4 \Theta^{\lambda\alpha\beta} \Theta_{\lambda\alpha\beta} \right) \\
& + \left(b_1 \Pi h^\mu + b_2 \pi^{\mu\nu} h_\nu + b_3 \phi^{\mu\nu} h_\nu + b_4 \Pi q^\mu + b_5 \pi^{\mu\nu} q_\nu + b_6 \phi^{\mu\nu} q_\nu \right) \\
& + \left(\tilde{b}_1 \Phi h^\mu + \tilde{b}_2 \tau_{(s)}^{\mu\nu} h_\nu + \tilde{b}_3 \tau_{(a)}^{\mu\nu} h_\nu + \tilde{b}_4 \Phi q^\mu + \tilde{b}_5 \tau_{(s)}^{\mu\nu} q_\nu + \tilde{b}_6 \tau_{(a)}^{\mu\nu} q_\nu \right) \\
& + \left(c_1 \Theta^{\mu\alpha\beta} \phi_{\alpha\beta} + c_2 \Theta^{\mu\alpha\beta} \tau_{(a)\alpha\beta} \right) \\
& + \left(c_3 \Theta^{\alpha\beta\mu} \Delta_{\alpha\beta} \Pi + c_4 \Theta^{\alpha\beta\mu} \pi_{\alpha\beta} + c_5 \Theta^{\alpha\beta\mu} \Delta_{\alpha\beta} \Phi + c_6 \Theta^{\alpha\beta\mu} \tau_{(s)\alpha\beta} \right) \\
& + \left(c_7 \Theta^{\alpha\beta\mu} \phi_{\alpha\beta} + c_8 \Theta^{\alpha\beta\mu} \tau_{(a)\alpha\beta} \right) .
\end{aligned}$$

Using the positivity of entropy production, we can obtain relaxation-type dynamical equations for various dissipative currents,

$$\partial_\mu S_{IS}^\mu = (\partial_\mu \beta_\nu + 2\omega_{\mu\nu}\beta) T_{1a}^{\mu\nu} + T_{1s}^{\mu\nu} \partial_\mu \beta_\nu - S_1^{\mu\alpha\beta} \partial_\mu (\beta \omega_{\alpha\beta}) + \partial_\mu Q^\mu \geq 0$$

Each relaxation-type equation will be associated with a kinetic coefficient, that cannot be determined at the entropy current level.

Using the positivity of entropy production, we can obtain relaxation-type dynamical equations for various dissipative currents,

$$\partial_\mu S_{IS}^\mu = (\partial_\mu \beta_\nu + 2\omega_{\mu\nu}\beta) T_{1a}^{\mu\nu} + T_{1s}^{\mu\nu} \partial_\mu \beta_\nu - S_1^{\mu\alpha\beta} \partial_\mu (\beta\omega_{\alpha\beta}) + \partial_\mu Q^\mu \geq 0$$

Each relaxation-type equation will be associated with a kinetic coefficient, that cannot be determined at the entropy current level.

We expect that these relation-type equations will solve the stability and the boost-invariant studies.

Summary and Outlook

1. In rest frame, the 1st-order spin hydrodynamic theory displays instabilities and reveals nonphysical evaluations of T and $\sqrt{\omega^{\mu\nu}\omega_{\mu\nu}}$ within a boost-invariant analysis.

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1. In rest frame, the 1st-order spin hydrodynamic theory displays instabilities and reveals nonphysical evaluations of T and $\sqrt{\omega^{\mu\nu}\omega_{\mu\nu}}$ within a boost-invariant analysis.
2. We construct a second-order theory based on the inspiration drawn from the success of the “IS” theory in addressing similar challenges in relativistic hydrodynamics without spin.
3. The subsequent task involves conducting a linear stability analysis for the second-order spin hydrodynamic theory and later numerically solving to compare with the data.



Thank You !