Spin transport (and hydrodynamics) with nonlocal collisions

David Wagner

in collaboration with

Nora Weickgenannt, Enrico Speranza, and Dirk Rischke

based mainly on

NW, ES, X.-L. Sheng, Q. Wang, DHR, Phys.Rev.D 104 (2021) 1, 016022

NW, DW, ES, DHR, Phys.Rev.D 106 (2022) 9, 096014

DW, NW, ES, Phys.Rev.Res. 5 (2023) 1, 013187

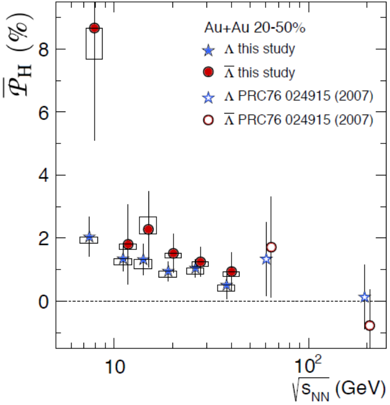
DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

DW, NW, ES, 2306.05936 (2023)

Chirality, Vorticity & Magnetic Field in HIC | 17.07.2023

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction

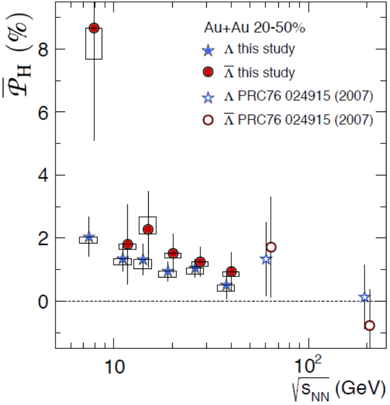


L. Adamczyk et al. (STAR), Nature 548 (2017) 62

Global Λ-polarization

- Global polarization: polarization of Λ-hyperons along angular-momentum direction
  - Can be well explained by considering **local equilibrium** on freeze-out hypersurface

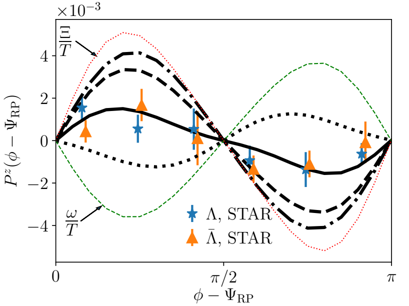
$$S^\mu_\varpi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\varpi_{\alpha\beta}}{8m \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



L. Adamczyk et al. (STAR), Nature 548 (2017) 62

$$\varpi_{\mu\nu} := -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu),\ \beta^\mu := u^\mu/T,\ f_0 = [\exp(u^\mu k_\mu/T) + 1]^{-1}$$

- Local polarization:
  Angle-dependent polarization of Λ-hyperons along beam-direction



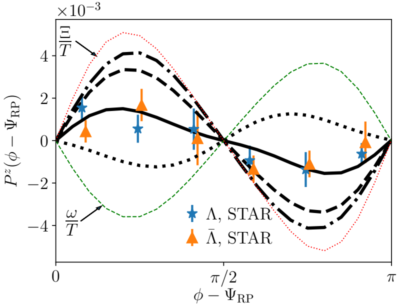
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, PRL 127 (2021) 142301

Local Λ-polarization

- Local polarization: Angle-dependent polarization of Λ-hyperons along beam-direction
  - Could only be explained recently by incorporating shear effects (neglecting temperature gradients)

$$S^\mu_\xi = -\epsilon^{\mu\nu\alpha\beta} k_\nu \frac{\int \mathrm{d}\Sigma_\lambda k^\lambda f_0(1-f_0)\hat{t}_\alpha \frac{k^\gamma}{k^0} \Xi_{\gamma\beta}}{4mT \int \mathrm{d}\Sigma_\lambda k^\lambda f_0}$$



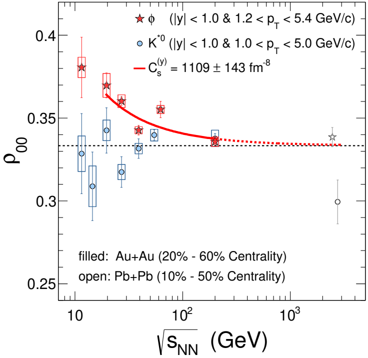
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127 (2021) 272302

B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, PRL 127 (2021) 142301

$$\omega_{\mu\nu} := \tfrac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu),\ \Xi_{\mu\nu} := \tfrac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu),\ \Delta^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$$

Alignment of ϕ-mesons

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)

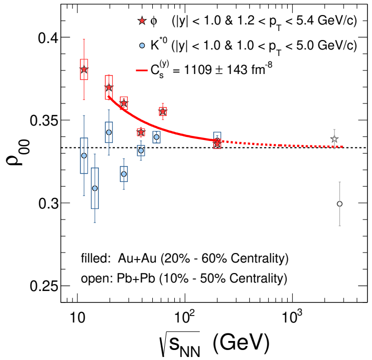


STAR collaboration, arXiv:2204.02302 (2022)

Alignment of ϕ-mesons

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)
  - Larger than expected
  - Some theoretical developments, but no definitive answer yet

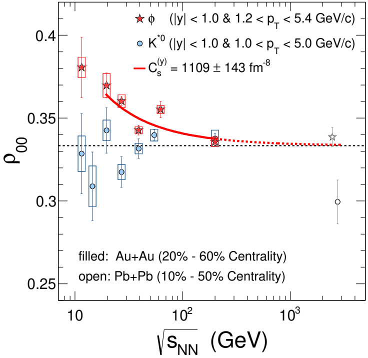
X.-L. Xia, H. Li, X-G. Huang, H.-Z. Huang, PLB 817 (2021) 136325
X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, arXiv:2206.05868 (2022)
F. Li, S. Y. F. Liu, arXiv:2206.11890 (2022)
DW, NW, ES, 2207.01111 (2022)



STAR collaboration, arXiv:2204.02302 (2022)

Alignment of ϕ-mesons

- Spin-1 particles feature tensor polarization ($\hat{=}$ alignment)
  - Larger than expected
  - Some theoretical developments, but no definitive answer yet
    X.-L. Xia, H. Li, X-G. Huang, H.-Z. Huang, PLB 817 (2021) 136325
    X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, arXiv:2206.05868 (2022)
    F. Li, S. Y. F. Liu, arXiv:2206.11890 (2022)
    DW, NW, ES, 2207.01111 (2022)
  - Can **spin-1 hydrodynamics** help explain this?



STAR collaboration, arXiv:2204.02302 (2022)

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

- **10** equations for **16**+**24** quantities

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

- **10** equations for **16+24** quantities
- Additional information about dissipative quantities has to be provided
  - $\rightarrow$ Use **kinetic theory** as effective microscopic model

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - → Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

- **10** equations for **16**+**24** quantities
- Additional information about dissipative quantities has to be provided
  - → Use **kinetic theory with spin** as effective microscopic model

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Spin Hydrodynamics: Basics

- Hydrodynamics is based on conservation laws
  - Consider a system of uncharged fields
    - $\rightarrow$ Should conserve energy-momentum and total angular momentum

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \tag{1a}$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0 \tag{1b}$$

- **10** equations for **16**+**24** quantities
- Additional information about dissipative quantities has to be provided
  - $\rightarrow$ Use **kinetic theory** **with spin** as effective microscopic model
- Rest of the presentation:
  - Construct such a kinetic theory
  - Perform hydrodynamic limit
  - Obtain expressions for observables

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

How to: Quantum kinetic theory

- Spin is a quantum property
  - $\rightarrow$ Start from quantum field theory
  - $\rightarrow$ Use **Wigner-function formalism**

How to: Quantum kinetic theory

- Spin is a quantum property
  - → Start from quantum field theory
  - → Use **Wigner-function formalism**

Wigner function (Spin 1)

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int \mathrm{d}^4 v e^{-ik\cdot y/\hbar}\left\langle : V^{\dagger\mu}(x+y/2)V^{\nu}(x-y/2) : \right\rangle$$

How to: Quantum kinetic theory

- Spin is a quantum property
  - $\rightarrow$ Start from quantum field theory
  - $\rightarrow$ Use **Wigner-function formalism**

Wigner function (Spin 1)

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int \mathrm{d}^4v e^{-ik\cdot y/\hbar}\left\langle : V^{\dagger\mu}(x+y/2)V^{\nu}(x-y/2) : \right\rangle$$

- Determines a **quantum phase-space distribution function**
- Equations of motion follow from field equations
  - Determined by Lagrangian $\mathcal{L}_0 + \mathcal{L}_{\text{int}}$

How to: Quantum kinetic theory

- Spin is a quantum property
  - $\rightarrow$ Start from quantum field theory
  - $\rightarrow$ Use **Wigner-function formalism**

Wigner function (Spin 1)

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int \mathrm{d}^4 v e^{-ik\cdot y/\hbar}\left\langle : V^{\dagger\mu}(x+y/2)V^{\nu}(x-y/2) : \right\rangle$$

- Determines a **quantum phase-space distribution function**
- Equations of motion follow from field equations
  - Determined by Lagrangian $\mathcal{L}_0 + \mathcal{L}_{\text{int}}$
- Independent components: scalar f_K, axial vector G^μ and traceless symmetric tensor $F_K^{\mu\nu}$

$f_K := (1/3)K_{\mu\nu}W^{\mu\nu}$, $G^\mu := -(i/2m)\epsilon^{\mu\nu\alpha\beta}k_\nu W_{\alpha\beta}$, $F_K^{\mu\nu} := K^{\mu\nu}_{\alpha\beta}W^{\alpha\beta}$
$K^{\mu\nu} := g^{\mu\nu} - k^\mu k^\nu/m^2$, $K^{\mu\nu}_{\alpha\beta} := (K^\mu_\alpha K^\nu_\beta + K^\mu_\beta K^\nu_\alpha)/2 - 1/3 K^{\mu\nu}K_{\alpha\beta}$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x, k)-phase space

$$k \cdot \partial f_K(x, k) = \mathcal{C}_K \ , \quad k \cdot \partial G^\mu(x, k) = \mathcal{C}_G^\mu \ , \quad k \cdot \partial F_K^{\mu\nu}(x, k) = \mathcal{C}_K^{\mu\nu}$$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x, k)-phase space

$$k \cdot \partial f_K(x,k) = \mathcal{C}_K \ , \quad k \cdot \partial G^\mu(x,k) = \mathcal{C}_G^\mu \ , \quad k \cdot \partial F_K^{\mu\nu}(x,k) = \mathcal{C}_K^{\mu\nu}$$

- Way to compactify this: **Enlarge** phase space from (x, k) to $(x, k, \mathfrak{s})$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x, k)-phase space

$$k \cdot \partial f_K(x,k) = \mathcal{C}_K \ , \quad k \cdot \partial G^\mu(x,k) = \mathcal{C}_G^\mu \ , \quad k \cdot \partial F_K^{\mu\nu}(x,k) = \mathcal{C}_K^{\mu\nu}$$

- Way to compactify this: **Enlarge** phase space from (x, k) to $(x, k, \mathfrak{s})$
- Measure $\mathrm{d}S := \frac{3m}{2\sigma\pi}\mathrm{d}^4\mathfrak{s}\,\delta[\mathfrak{s}^2 + \sigma^2]\delta(k \cdot \mathfrak{s})$

Extending phase space

Boltzmann equations

- Not one, but nine equations in (x,k)-phase space

$$k\cdot\partial f_K(x,k)=\mathcal{C}_K\ ,\quad k\cdot\partial G^\mu(x,k)=\mathcal{C}_G^\mu\ ,\quad k\cdot\partial F_K^{\mu\nu}(x,k)=\mathcal{C}_K^{\mu\nu}$$

- Way to compactify this: **Enlarge** phase space from (x,k) to $(x,k,\mathfrak{s})$
- Measure $\mathrm{d}S:=\frac{3m}{2\sigma\pi}\mathrm{d}^4\mathfrak{s}\,\delta[\mathfrak{s}^2+\sigma^2]\delta(k\cdot\mathfrak{s})$

Boltzmann equation in extended phase space

$$\mathfrak{f}(x,k,\mathfrak{s}):=f_K-\mathfrak{s}_\mu G^\mu+\frac{5}{4}\mathfrak{s}_\mu\mathfrak{s}_\nu F_K^{\mu\nu} \tag{2}$$

- Only on-shell parts $\mathfrak{f}(x,k,\mathfrak{s})=\delta(k^2-m^2)f(x,k,\mathfrak{s})$ contribute

$$k\cdot\partial f(x,k,\mathfrak{s})=\mathfrak{C}[f] \tag{3}$$

$\mathfrak{C}:=\mathcal{C}_K-\mathfrak{s}_\mu\mathcal{C}_G^\mu+\frac{5}{4}\mathfrak{s}_\mu\mathfrak{s}_\nu\mathcal{C}_K^{\mu\nu}$

Collision term

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Collision kernel

$$\mathfrak{C}[f] = \frac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\delta^{(4)}(k_1+k_2-k-k')\mathcal{W}$$
$$\times\left[f(x+\Delta_1-\Delta, k_1, \mathfrak{s}_1)f(x+\Delta_2-\Delta, k_2, \mathfrak{s}_2)\right.$$
$$\left.-f(x,k,\bar{\mathfrak{s}})f(x+\Delta'-\Delta, k', \mathfrak{s}')\right] \tag{4}$$

$\mathrm{d}\Gamma := 2\mathrm{d}^4k\delta(k^2-m^2)\mathrm{d}S(k)$

Collision term

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Collision kernel

$$\mathfrak{C}[f] = \frac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\delta^{(4)}(k_1+k_2-k-k')\mathcal{W}$$
$$\times\left[f(x+\Delta_1-\Delta, k_1, \mathfrak{s}_1)f(x+\Delta_2-\Delta, k_2, \mathfrak{s}_2)\right.$$
$$\left.-f(x,k,\bar{\mathfrak{s}})f(x+\Delta'-\Delta, k', \mathfrak{s}')\right] \quad (4)$$

- Contributions inside the collision term have gradient corrections

$$f(x,k,\mathfrak{s}) + \Delta^\mu \partial_\mu f(x,k,\mathfrak{s}) \approx f(x+\Delta, k, \mathfrak{s}) \quad (5)$$

- A (momentum- and spin-dependent) **spacetime shift** Δ^μ enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!

$\mathrm{d}\Gamma := 2\mathrm{d}^4k\delta(k^2-m^2)\mathrm{d}S(k)$

Collision term

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Collision kernel

$$\mathfrak{C}[f] = \frac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \mathrm{d}\bar{S}(k)\delta^{(4)}(k_1+k_2-k-k')\mathcal{W}$$
$$\times\left[f(x+\Delta_1-\Delta, k_1, \mathfrak{s}_1)f(x+\Delta_2-\Delta, k_2, \mathfrak{s}_2)\right.$$
$$\left.-f(x,k,\bar{\mathfrak{s}})f(x+\Delta'-\Delta, k', \mathfrak{s}')\right] \quad (4)$$

- Contributions inside the collision term have gradient corrections

$$f(x,k,\mathfrak{s}) + \Delta^\mu \partial_\mu f(x,k,\mathfrak{s}) \approx f(x+\Delta, k, \mathfrak{s}) \quad (5)$$

- A (momentum- and spin-dependent) **spacetime shift Δ^μ** enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!
- This enables a conversion of orbital and spin angular momenta

$\mathrm{d}\Gamma := 2\mathrm{d}^4 k\,\delta(k^2-m^2)\mathrm{d}S(k)$

Nonlocal collisions

DW, NW, ES, 2306.05936 (2023)

Spacetime shifts

$$\Delta^\mu := \frac{1}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{32}\frac{i\hbar}{m^2} M^{\gamma_1\gamma_2\delta_1\delta_2} M^{\zeta_1\zeta_2\eta_1\eta_2} h_{1,\gamma_1\eta_1} h_{2,\gamma_2\eta_2} h'_{\zeta_2\delta_2} \times \left(H^\mu{}_{\delta_1} k_{\zeta_1} - k_{\delta_1} H_{\zeta_1}{}^\mu\right) \tag{6}$$

- Depend on the transfer-matrix elements

$$\langle 11'|\,\widehat{t}\,|22'\rangle = \epsilon^*_{1,\alpha}\epsilon^*_{1',\beta}\epsilon_{2,\gamma}\epsilon_{2',\delta} M^{\alpha\beta\gamma\delta} \tag{7}$$

- Manifestly covariant
  - $\rightarrow$ no "no-jump" frame

$$h^{\mu\nu} := \tfrac{1}{3}K^{\mu\nu} + \tfrac{i}{2m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta + K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta,$$
$$H^{\mu\nu} := \tfrac{1}{3}K^{\mu\nu} + \tfrac{i}{2m}\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta + \tfrac{5}{8}K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta$$

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\mathsf{eq}}] = 0$

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\text{eq}}] = 0$
- Has to depend on the **collisional invariants**
  - $\rightarrow$ Charge, four-momentum and total angular momentum

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\mathsf{eq}}] = 0$
- Has to depend on the **collisional invariants**
  - $\rightarrow$ Charge, four-momentum and total angular momentum

Local-equilibrium distribution function

$$f_{\mathsf{eq}}(x, k, \mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \tag{8}$$

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta,\ E_{\mathbf{k}} := k \cdot u$

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\text{eq}}] = 0$
- Has to depend on the **collisional invariants**
  $\rightarrow$ Charge, four-momentum and total angular momentum

Local-equilibrium distribution function

$$f_{\text{eq}}(x, k, \mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \tag{8}$$

- Necessary conditions on Lagrange multipliers $\beta_0 u^\mu$, $\Omega^{\mu\nu}$ for a vanishing collision term: $\partial^{(\mu}(\beta_0 u^{\nu)}) = 0$, $\Omega^{\mu\nu} = -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$
- Same conditions as for **global** equilibrium, where $k \cdot \partial f_{\text{eq}} = 0$

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$

Equilibrium

- Local equilibrium distribution function fulfills $\mathfrak{C}[f_{\text{eq}}] = 0$
- Has to depend on the **collisional invariants**
  - $\rightarrow$ Charge, four-momentum and total angular momentum

Local-equilibrium distribution function

$$f_{\text{eq}}(x,k,\mathfrak{s}) = \exp\left(-\beta_0 E_{\mathbf{k}} + \frac{\hbar}{2}\Omega_{\mu\nu}\Sigma_{\mathfrak{s}}^{\mu\nu}\right) \quad (8)$$

- Necessary conditions on Lagrange multipliers $\beta_0 u^\mu$, $\Omega^{\mu\nu}$ for a vanishing collision term: $\partial^{(\mu}(\beta_0 u^{\nu)}) = 0$, $\Omega^{\mu\nu} = -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$
- Same conditions as for **global** equilibrium, where $k \cdot \partial f_{\text{eq}} = 0$
- **However**, we can relax these constraints if we only demand that the **local** part of the collision term vanishes!

$\Sigma_{\mathfrak{s}}^{\mu\nu} := -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}k_\alpha \mathfrak{s}_\beta$, $E_{\mathbf{k}} := k \cdot u$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) \quad := \quad \int \mathrm{d}\Gamma\, E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9a}$$

$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9a}$$

$$\tau_r^{\mu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9b}$$

$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9a}$$

$$\tau_r^{\mu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9b}$$

$$\psi_r^{\mu\nu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma K_{\alpha\beta}^{\mu\nu} \mathfrak{s}^\alpha \mathfrak{s}^\beta E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s}) \tag{9c}$$

$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Moment expansion

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1 \cdots \mu_\ell}(x) := \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x, k, \mathfrak{s}) \quad (9a)$$

$$\tau_r^{\mu, \mu_1 \cdots \mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x, k, \mathfrak{s}) \quad (9b)$$

$$\psi_r^{\mu\nu, \mu_1 \cdots \mu_\ell}(x) := \int \mathrm{d}\Gamma K_{\alpha\beta}^{\mu\nu} \mathfrak{s}^\alpha \mathfrak{s}^\beta E_{\mathbf{k}}^r k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} \delta f(x, k, \mathfrak{s}) \quad (9c)$$

- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

---

$k^{\langle \mu_1} \cdots k^{\mu_\ell \rangle} := \Delta^{\mu_1 \cdots \mu_\ell}_{\nu_1 \cdots \nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Polarization observables in kinetic theory

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) := \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{10}$$

$N(k) := \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu := -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

Polarization observables in kinetic theory

CRC–TR 211

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) := \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{10}$$

Tensor Polarization

$$\rho_{00}(k) = \frac{1}{3} - \sqrt{\frac{2}{3}}\epsilon^{(0)}_\mu(k)\epsilon^{(0)}_\nu(k)\Theta^{\mu\nu}(k) \tag{11a}$$

$N(k) := \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu := -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

Polarization observables in kinetic theory

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) \coloneqq \mathrm{Tr}\left[\hat{S}^\mu\, \hat{\rho}(k)\right] = \frac{1}{N(k)} \int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) \mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{10}$$

Tensor Polarization

$$\rho_{00}(k) = \frac{1}{3} - \sqrt{\frac{2}{3}} \epsilon^{(0)}_\mu(k) \epsilon^{(0)}_\nu(k) \Theta^{\mu\nu}(k) \tag{11a}$$

$$\begin{aligned}
\Theta^{\mu\nu}(k) &\coloneqq \frac{1}{2}\sqrt{\frac{3}{2}} \mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3}K^{\mu\nu}\right) \hat{\rho}(k)\right] \\
&= \frac{1}{2}\sqrt{\frac{3}{2}} \frac{1}{N(k)} \int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) K^{\mu\nu}_{\alpha\beta} \mathfrak{s}^\alpha \mathfrak{s}^\beta f(x,k,\mathfrak{s})
\end{aligned} \tag{11b}$$

$N(k) \coloneqq \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu \coloneqq -(1/2m)\epsilon^{\mu\nu\alpha\beta} \hat{J}_{\nu\alpha} \hat{P}_\beta$

Truncation

- Goal: Express polarization observables through **fluid-dynamical** quantities

Truncation

- Goal: Express polarization observables through **fluid-dynamical** quantities
  - $\leftrightarrow$ **Truncate** moment expansion

Truncation

- Goal: Express polarization observables through **fluid-dynamical** quantities
  - $\leftrightarrow$ **Truncate** moment expansion
- Which moments are contained in **conserved quantities** and **polarization observables**?

Truncation

- Goal: Express polarization observables through **fluid-dynamical** quantities
  $\leftrightarrow$ **Truncate** moment expansion
- Which moments are contained in **conserved quantities** and **polarization observables**?

Needed moments

$$\Pi := -\frac{m^2}{3}\rho_0 \,, \quad \pi^{\mu\nu} := \rho_0^{\mu\nu} \quad (T^{\mu\nu}) \tag{12a}$$

$$\mathfrak{p}^{\mu} := \tau_0^{\langle\mu\rangle} \,, \quad \mathfrak{z}^{\mu\nu} := \tau_1^{(\langle\mu\rangle,\langle\nu\rangle)} \,, \quad \mathfrak{q}^{\lambda\mu\nu} := \tau_0^{\langle\lambda\rangle,\mu\nu} \quad (J^{\lambda\mu\nu}) \tag{12b}$$

$$\psi_1^{\mu\nu} \,, \quad \psi_0^{\mu\nu,\lambda} \quad (\Theta^{\mu\nu}) \tag{12c}$$

Results I: Dissipative Spin Hydro

Dissipative Hydro: Evolution equations

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \text{h.o.t.} \tag{13a}$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \text{h.o.t.} \tag{13b}$$

$$\tau_{\mathfrak{p}} \dot{\mathfrak{p}}^{\langle\mu\rangle} + \mathfrak{p}^{\langle\mu\rangle} = \mathfrak{e}^{(0)}(\tilde{\Omega}^{\mu\nu} - \tilde{\varpi}^{\mu\nu})u_\nu + \text{h.o.t.} \tag{13c}$$

$$\tau_{\mathfrak{z}} \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\langle\mu\rangle\langle\nu\rangle} = \text{h.o.t.} \tag{13d}$$

$$\tau_{\mathfrak{q}} \dot{\mathfrak{q}}^{\langle\lambda\rangle\langle\mu\nu\rangle} + \mathfrak{q}^{\langle\lambda\rangle\langle\mu\nu\rangle} = \mathfrak{d}^{(2)}\beta_0\sigma_\alpha{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\beta + \text{h.o.t.} \tag{13e}$$

$$\tau_{\psi_1} \dot{\psi}_1^{\langle\mu\nu\rangle} + \psi_1^{\langle\mu\nu\rangle} = \xi\beta_0\pi^{\mu\nu} + \text{h.o.t.} \tag{13f}$$

$$\tau_{\psi_0} \dot{\psi}_0^{\langle\mu\nu\rangle,\lambda} + \psi_0^{\langle\mu\nu\rangle,\lambda} = \text{h.o.t.} \tag{13g}$$

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

CRC-TR 211

Dissipative Hydro: Evolution equations

$$\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta\theta + \text{h.o.t.} && (13a)\\
\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + \text{h.o.t.} && (13b)\\
\tau_{\mathfrak{p}} \dot{\mathfrak{p}}^{\langle\mu\rangle} + \mathfrak{p}^{\langle\mu\rangle} &= \mathfrak{e}^{(0)}(\tilde{\Omega}^{\mu\nu} - \tilde{\varpi}^{\mu\nu})u_\nu + \text{h.o.t.} && (13c)\\
\tau_{\mathfrak{z}} \dot{\mathfrak{z}}^{\langle\mu\rangle\langle\nu\rangle} + \mathfrak{z}^{\langle\mu\rangle\langle\nu\rangle} &= \text{h.o.t.} && (13d)\\
\tau_{\mathfrak{q}} \dot{\mathfrak{q}}^{\langle\lambda\rangle\langle\mu\nu\rangle} + \mathfrak{q}^{\langle\lambda\rangle\langle\mu\nu\rangle} &= \mathfrak{d}^{(2)}\beta_0 \sigma_\alpha{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\beta + \text{h.o.t.} && (13e)\\
\tau_{\psi_1} \dot{\psi}_1^{\langle\mu\nu\rangle} + \psi_1^{\langle\mu\nu\rangle} &= \xi\beta_0\pi^{\mu\nu} + \text{h.o.t.} && (13f)\\
\tau_{\psi_0} \dot{\psi}_0^{\langle\mu\nu\rangle,\lambda} + \psi_0^{\langle\mu\nu\rangle,\lambda} &= \text{h.o.t.} && (13g)
\end{aligned}$$

- Evaluate polarization and alignment in the **Navier-Stokes limit**

$\varpi^{\mu\nu} := -\frac{1}{2}\partial^{[\mu}(\beta_0 u^{\nu]})$, $\tilde{A}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

Results II: Alignment

- Moments of spin-rank 2:

$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi\beta_0\pi^{\mu\nu} \,, \quad \psi_0^{\langle\mu\nu\rangle,\lambda} \simeq 0 \tag{14}$$

Results II: Alignment

- Moments of spin-rank 2:

$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi\beta_0\pi^{\mu\nu} \,, \quad \psi_0^{\langle\mu\nu\rangle,\lambda} \simeq 0 \tag{14}$$

- For an uncharged fluid in the Navier-Stokes limit, tensor polarization is induced by the **shear-stress tensor** $\pi^{\mu\nu}$

Results II: Alignment

- Moments of spin-rank 2:

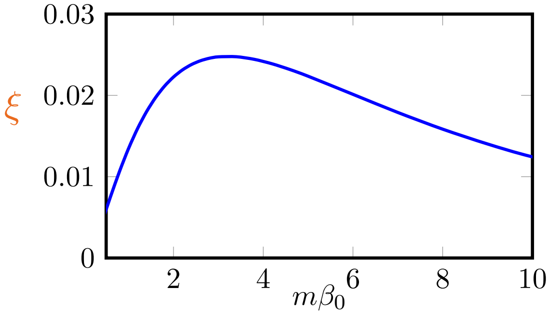
$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi\beta_0\pi^{\mu\nu} \,, \quad \psi_0^{\langle\mu\nu\rangle,\lambda} \simeq 0 \tag{14}$$

- For an uncharged fluid in the Navier-Stokes limit, tensor polarization is induced by the **shear-stress tensor** $\pi^{\mu\nu}$
- We can estimate the coefficient ξ for a four-point interaction $\mathcal{L}_{\text{int}} = (V^\dagger \cdot V)^2/2$

- Moments of spin-rank 2:

$$\psi_1^{\langle\mu\nu\rangle} \simeq \xi \beta_0 \pi^{\mu\nu} \,, \quad \psi_0^{\langle\mu\nu\rangle,\lambda} \simeq 0 \tag{14}$$

- For an uncharged fluid in the Navier-Stokes limit, tensor polarization is induced by the **shear-stress tensor** $\pi^{\mu\nu}$
- We can estimate the coefficient ξ for a four-point interaction $\mathcal{L}_{\text{int}} = (V^\dagger \cdot V)^2/2$



Results II: Alignment

Alignment: Explicit expression

$$
\begin{aligned}
\rho_{00}(k) &= \frac{1}{3} \\
&- \frac{4}{15}\left[\int \mathrm{d}\Sigma_\lambda k^\lambda f_{0\mathbf{k}}\left(1 - 3\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi/m^2 + \mathcal{H}^{(0,2)}_{\mathbf{k}0}\pi^{\mu\nu}k_\mu k_\nu\right)\right]^{-1} \\
&\times \int \mathrm{d}\Sigma_\lambda k^\lambda \mathcal{H}^{(2,0)}_{\mathbf{k}1}\xi\beta_0 f_{0\mathbf{k}}\epsilon^{(0)}_\mu \epsilon^{(0)}_\nu K^{\mu\nu}_{\alpha\beta}\Xi^{\alpha\beta}_{\gamma\delta}\pi^{\gamma\delta} \qquad (15)
\end{aligned}
$$

$f_{0\mathbf{k}} \coloneqq \exp(-\beta_0 E_\mathbf{k})$
$\Xi^{\mu\nu}_{\alpha\beta} \coloneqq \frac{1}{2}\Xi^{(\mu}_\alpha \Xi^{\nu)}_\beta - \frac{1}{\Xi^2}\Xi^{\mu\gamma}\Xi^\nu_\gamma \Xi_{\alpha\delta}\Xi^\delta_\beta$
$\Xi^{\mu\nu} \coloneqq \Delta^{\mu\nu} + k^{\langle\mu\rangle}k^{\langle\nu\rangle}/E^2_\mathbf{k}$

Results III: Polarization

- Polarization is determined by the Pauli-Lubanski (pseudo)vector

Results III: Polarization

- Polarization is determined by the Pauli-Lubanski (pseudo)vector

Pauli-Lubanski pseudovector (spin 1/2)

$$
\begin{aligned}
S^\mu(k) &= \frac{1}{2\mathcal{N}} \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) \mathfrak{s}^\mu f(x,k,\mathfrak{s}) && \text{(16a)} \\
&\simeq \int \mathrm{d}\Sigma_\lambda k^\lambda \frac{f_0}{2\mathcal{N}} \Bigg\{ -\frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} k_\nu + \left(\delta^\mu_\nu - \frac{u^\mu k_{\langle\nu\rangle}}{E_{\mathbf{k}}} \right) \\
&\quad \times \left[\mathfrak{e}\chi_{\mathfrak{p}} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_\rho - \chi_{\mathfrak{q}} \mathfrak{d} \beta_0 \sigma_\rho{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\sigma\rho} u_\sigma k_{\langle\alpha} k_{\beta\rangle} \right] \Bigg\} && \text{(16b)}
\end{aligned}
$$

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

- Polarization is determined by the Pauli-Lubanski (pseudo)vector

Pauli-Lubanski pseudovector (spin 1/2)

$$S^\mu(k) = \frac{1}{2\mathcal{N}} \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) \mathfrak{s}^\mu f(x,k,\mathfrak{s}) \tag{16a}$$

$$\simeq \int \mathrm{d}\Sigma_\lambda k^\lambda \frac{f_0}{2\mathcal{N}} \Bigg\{ -\frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} k_\nu + \left(\delta^\mu_\nu - \frac{u^\mu k_{\langle\nu\rangle}}{E_\mathbf{k}} \right) \times \left[\mathfrak{e}\chi_\mathfrak{p} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_\rho - \chi_\mathfrak{q} \mathfrak{d} \beta_0 \sigma_\rho{}^{\langle\alpha} \epsilon^{\beta\rangle\nu\sigma\rho} u_\sigma k_{\langle\alpha} k_{\beta\rangle} \right] \Bigg\} \tag{16b}$$

- Contains novel contributions from fluid shear
  - Only sourced by **nonlocal** collisions

$\mathcal{N} := \int \mathrm{d}\Sigma_\lambda k^\lambda \mathrm{d}S(k) f(x,k,\mathfrak{s})$

Summary

- Developed quantum kinetic theory and dissipative spin hydrodynamics

Summary

- Developed quantum kinetic theory and dissipative spin hydrodynamics
  - Kinetic formulation can be derived rigorously from QFT via the Wigner-function formalism
  - Quantum effects result in nonlocal collisions

Summary

- Developed quantum kinetic theory and dissipative spin hydrodynamics
  - Kinetic formulation can be derived rigorously from QFT via the Wigner-function formalism
  - Quantum effects result in nonlocal collisions
  - Employed method of moments to extract hydrodynamic limit
    - $\rightarrow$ Introduce multiple sets of moments dependent on spin
    - $\rightarrow$ Follow standard procedure to obtain equations of motion
    - $\rightarrow$ Truncate such that the evolution of $S^{\lambda\mu\nu}$ can be described

Summary

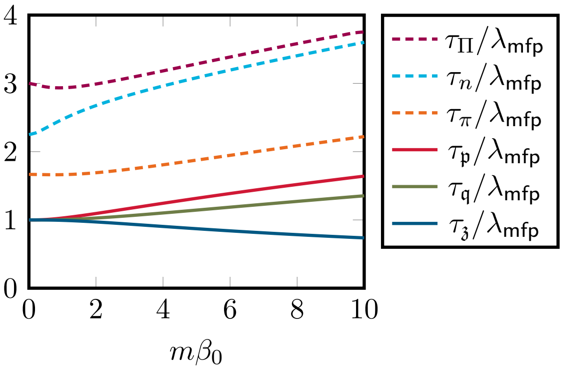
- Developed quantum kinetic theory and dissipative spin hydrodynamics
  - Kinetic formulation can be derived rigorously from QFT via the Wigner-function formalism
  - Quantum effects result in nonlocal collisions
  - Employed method of moments to extract hydrodynamic limit
    - $\rightarrow$ Introduce multiple sets of moments dependent on spin
    - $\rightarrow$ Follow standard procedure to obtain equations of motion
    - $\rightarrow$ Truncate such that the evolution of $S^{\lambda\mu\nu}$ can be described
- Connected polarization and alignment to fluid quantities in the Navier-Stokes limit

Future perspectives

- Evaluate expressions for polarization and alignment with hydrodynamic simulations
- Implement full spin hydrodynamics numerically
- Include electric and magnetic fields

Appendix

Relevant time scales: An estimation



- Simplest interaction: constant cross section
- Spin-related relaxation times shorter than standard dissipative time scales, but not much

Moment equations: Spin-rank 0

- Moments follow relaxation-type equations

Moment equation for $\ell = 0$

$$\begin{aligned} \dot{\rho}_r - \mathfrak{C}_{r-1} \quad = \quad & [(1-r)I_{r1} - I_{r0}]\theta - I_{r0}\dot{\alpha}_0 + I_{r+1,0}\dot{\beta}_0 \\ & +(r-1)\rho^{\mu\nu}_{r-2}\sigma_{\mu\nu} + r\rho^{\mu}_{r-1}\dot{u}_\mu - \nabla_\mu \rho^\mu_{r-1} \\ & -\frac{1}{3}\left[(r+2)\rho_r - (r-1)m^2\rho_{r-2}\right]\theta \end{aligned} \tag{17}$$

- Depend both on equilibrium and dissipative quantities
- Not a closed system
- Blue terms will become Navier-Stokes values

$\dot{A} := u \cdot \partial A$, $\nabla^\mu := \Delta^{\mu\nu}\partial_\nu$
$\theta := \nabla \cdot u$, $\sigma^{\mu\nu} := \nabla^{\langle\mu}u^{\nu\rangle}$, $E_k := k \cdot u$
$I_{nq} := [(2q+1)!!]^{-1} \int \mathrm{d}\Gamma E_k^{n-2q}(-k^{\langle\alpha\rangle}k_\alpha)^q$

Moment equations: Spin-rank 1

- Same procedure as for the moments of spin-rank 0

Moment equation for $\ell = 0$

$$
\begin{aligned}
\dot{\tau}_r^{\langle\mu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\rangle} \quad = \quad & \frac{\hbar}{2m}\left\{[I_{r+1,0} + rI_{r+1,1}]\theta + I_{r+1,0}\dot{\alpha}_0 - I_{r+2,0}\dot{\beta}_0\right\}\omega_0^\mu \\
& - \frac{\hbar}{4m} I_{r+1,1}\Delta^\mu_\lambda \nabla_\nu \tilde{\Omega}^{\lambda\nu} - \frac{\hbar}{4m} I_{r+1,0}\epsilon^{\mu\nu\alpha\beta} u_\nu \dot{\Omega}_{\alpha\beta} \\
& - \frac{\hbar}{4m}\tilde{\Omega}^{\langle\mu\rangle\nu}\left[I_{r+1,1}\nabla_\nu\alpha_0 - I_{r+2,1}\left(\nabla_\nu\beta_0 + \beta_0\dot{u}_\nu\right)\right] \\
& + r\,\dot{u}_\nu \tau_{r-1}^{\langle\mu\rangle,\nu} + (r-1)\sigma_{\alpha\beta}\tau_{r-2}^{\langle\mu\rangle,\alpha\beta} - \Delta^\mu_\lambda\nabla_\nu\tau_{r-1}^{\lambda,\nu} \\
& - \frac{1}{3}\left[(r+2)\tau_r^{\langle\mu\rangle} - (r-1)m^2\tau_{r-2}^{\langle\mu\rangle}\right]\theta \qquad (18)
\end{aligned}
$$

- Determine the **(vector) polarization** of particles

$\tilde{\Omega}^{\mu\nu} := \epsilon^{\mu\nu\alpha\beta}\Omega_{\alpha\beta}$, $\Omega^{\mu\nu} = u^{[\mu}\kappa_0^{\nu]} + \epsilon^{\mu\nu\alpha\beta}u_\alpha\omega_{0,\beta}$

Moment equations: Spin-rank 2

Moment equation for $\ell = 0$

$$\dot{\psi}_r^{\langle\mu\nu\rangle} - \mathfrak{C}_{r-1}^{\langle\mu\nu\rangle} = -\frac{\theta}{3}\left[(r+2)\psi_r^{\langle\mu\nu\rangle} - (r-1)m^2\psi_{r-2}^{\langle\mu\nu\rangle}\right] + r\psi_{r-1}^{\langle\mu\nu\rangle,\alpha}\dot{u}_\alpha - \Delta^{\mu\nu}_{\alpha\beta}\nabla_\gamma\psi_{r-1}^{\alpha\beta,\gamma} + (r-1)\psi_{r-2}^{\langle\mu\nu\rangle,\alpha\beta}\sigma_{\alpha\beta} \tag{19}$$

- No dependence on equilibrium quantities appears because moments of spin-rank 2 do not appear in any conserved current
- Nonetheless, they determine the **tensor polarization** of spin-1 particles

$\Delta^{\mu\nu}_{\alpha\beta} \coloneqq (\Delta^{(\mu}_{\alpha}\Delta^{\nu)}_{\beta})/2 - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$

Truncation

- Spin-1: Which moments are contained in the *total* tensor polarization?

Truncation

- Spin-1: Which moments are contained in the *total* tensor polarization?

Total tensor polarization

$$\bar{\Theta}^{\mu\nu} \coloneqq \int \mathrm{d}K\, N(k)\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}} \int \mathrm{d}\Sigma_\lambda \left(u^\lambda \psi_1^{\mu\nu} + \psi_0^{\mu\nu,\lambda} \right) \tag{20}$$

$\mathrm{d}K \coloneqq \mathrm{d}^3\mathbf{k}/[(2\pi\hbar)^3 2k^0]$

Truncation

- Spin-1: Which moments are contained in the *total* tensor polarization?

Total tensor polarization

$$\bar{\Theta}^{\mu\nu} := \int \mathrm{d}K\, N(k)\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}}\int \mathrm{d}\Sigma_\lambda \left(u^\lambda \psi_1^{\mu\nu} + \psi_0^{\mu\nu,\lambda}\right) \tag{20}$$

- Lowest-order approximation: Keep only these moments in the employed basis, i.e.,

$$\delta f(x,k) \mathrel{\hat{=}} \delta f\left(\Pi\,,\ \pi^{\mu\nu}\,,\ \mathfrak{p}^{\mu}\,,\ \mathfrak{z}^{\mu\nu}\,,\ \mathfrak{q}^{\lambda\mu\nu}\,,\ \psi_1^{\mu\nu}\,,\ \psi_0^{\mu\nu,\lambda}, k\right) \tag{21}$$

$\mathrm{d}K := \mathrm{d}^3\mathbf{k}/[(2\pi\hbar)^3 2k^0]$