# Tensor Polarization and Spectral Properties of Vector Meson in QCD Medium


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In collaboration: Feng Li

Special thanks to Yi Yin


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**Based on work, Li and Liu, arXiv: 2206.11890 and Liu & Rapp series research 2018-2022**

***Postdoc position at Hunan University, please contact: lshphy@hnu.edu.cn***

# Outline

1) Spin observables in heavy-ion collisions
2) Theory for tensor polarization and spin alignment
3) Spectral properties of in-medium mesonic resonance
4) Phenomenology implications
5) Summary and outlooks

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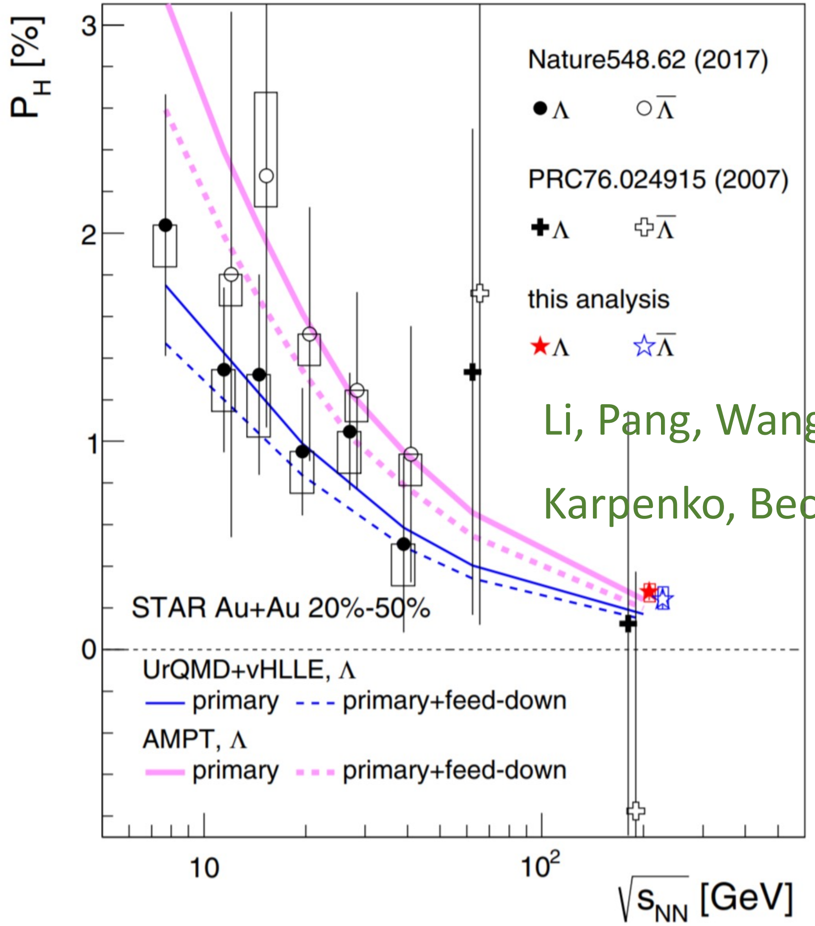
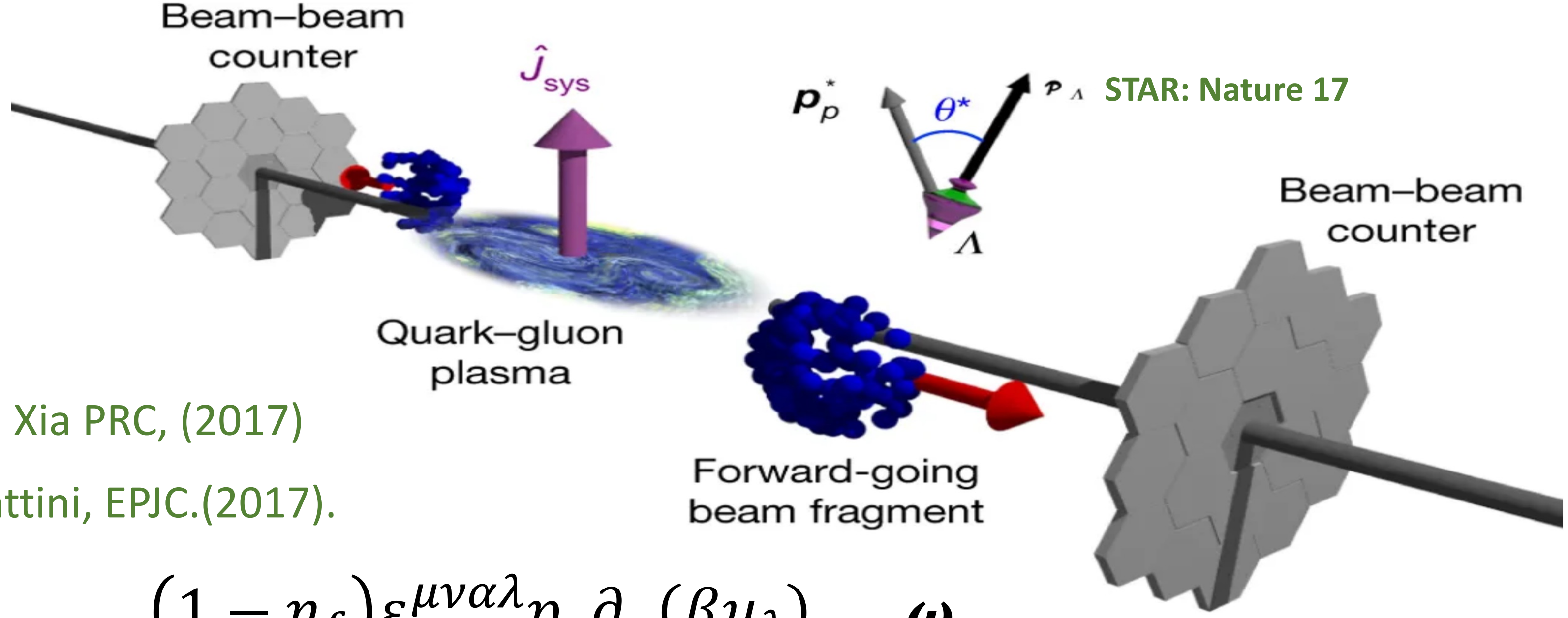
# Vorticity Induced Polarization

STAR PRC 2018



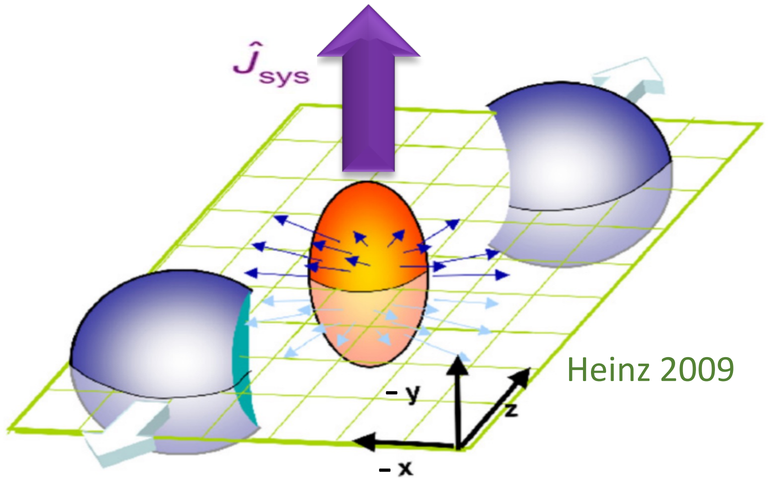

Li, Pang, Wang, Xia PRC, (2017)

Karpenko, Becattini, EPJC.(2017).




STAR: Nature 17

$$\mathcal{P} = \frac{(1-n_f)\varepsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m} \approx \frac{\boldsymbol{\omega}}{2T} \sim 1\%$$

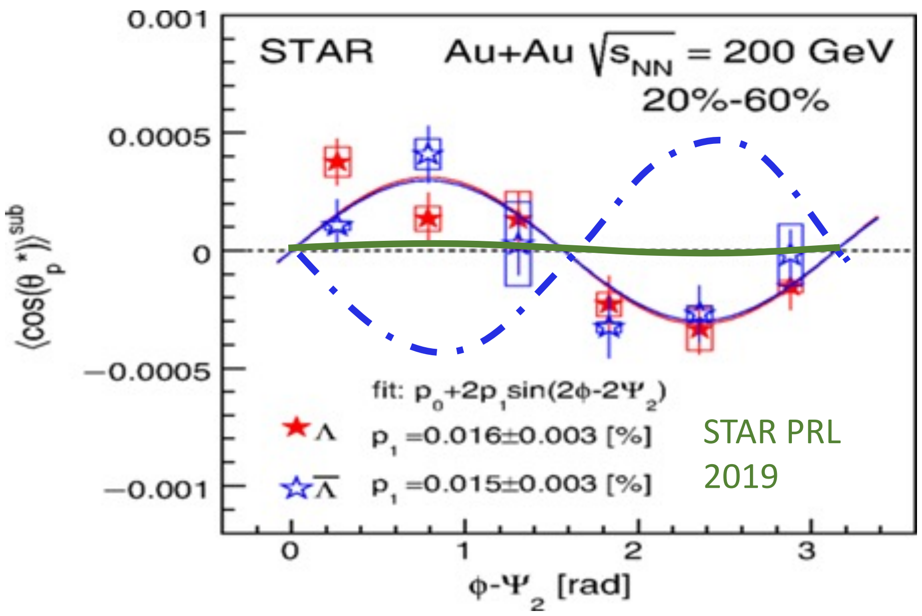



Heinz 2009

- ❖Rotation, vorticity, most vortical fluid
- ❖Vorticities can lead to polarization

# Local Polarization and Shear Induced Polarization

## Local polarization sign puzzles




Sketches of theoretical curves

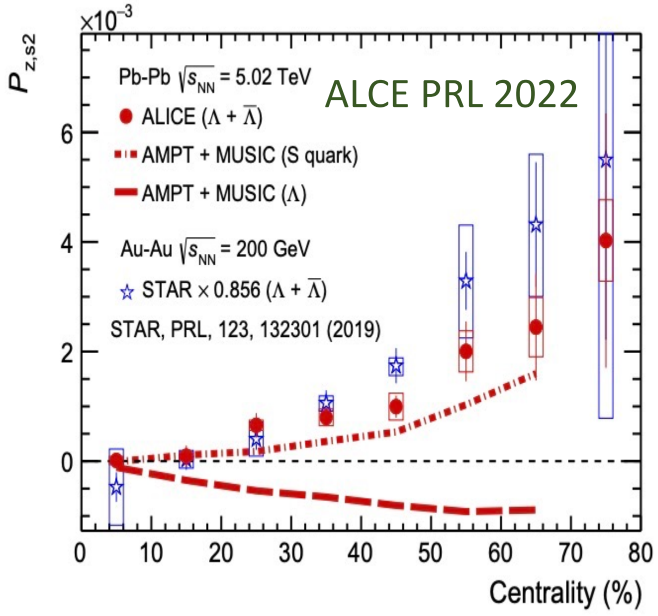
Voloshin, EPJ 2018
Becattini, Karpenko, PRL 2018

❖Previous theory

$$\mathcal{P} = \frac{(1-n_f)\epsilon^{\mu\nu\alpha\lambda}p_\nu\partial_\alpha(\beta u_\lambda)}{4m}$$

❖Sign is opposite!

## Progress




Liu, Yin, JHEP, 2021;Becatinni, Buzzegoli, Palermo, PLB, 2021

❖Shear-Induced Polarization

$$-n_0(1-n_0)\frac{1}{\varepsilon_0}\epsilon^{\mu\nu\alpha\rho}u_\nu\, p_\rho p^\lambda\partial_{(\alpha}(\beta u)_{\lambda)}$$

❖Right sign, large phenomenologically

Fu, Liu, Pang, Song, Yin, PRL, 2021;
Becatinni, Buzzegoli, Palermo, Inghirami,Karpenko PRL, 2021

# Local Polarization and Shear Induced Polarization

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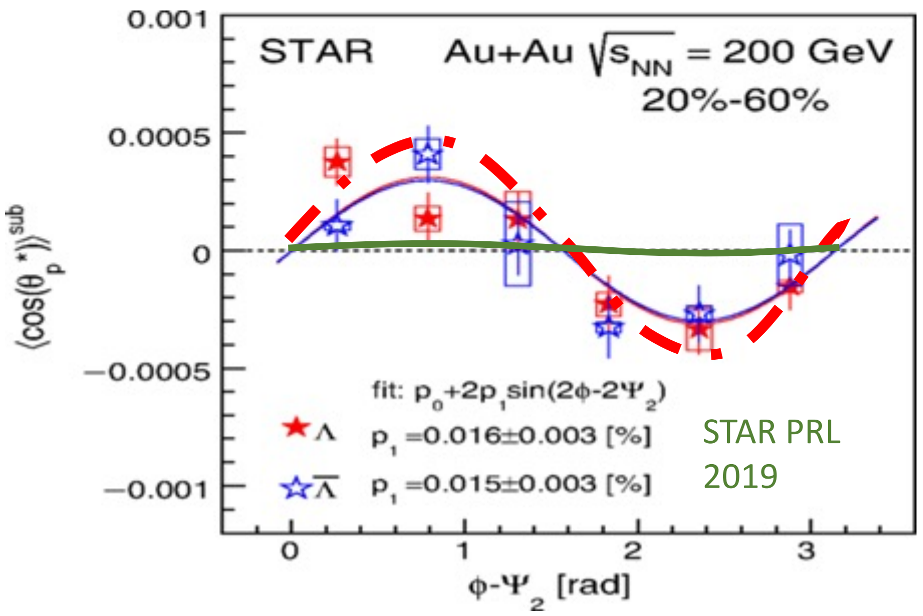


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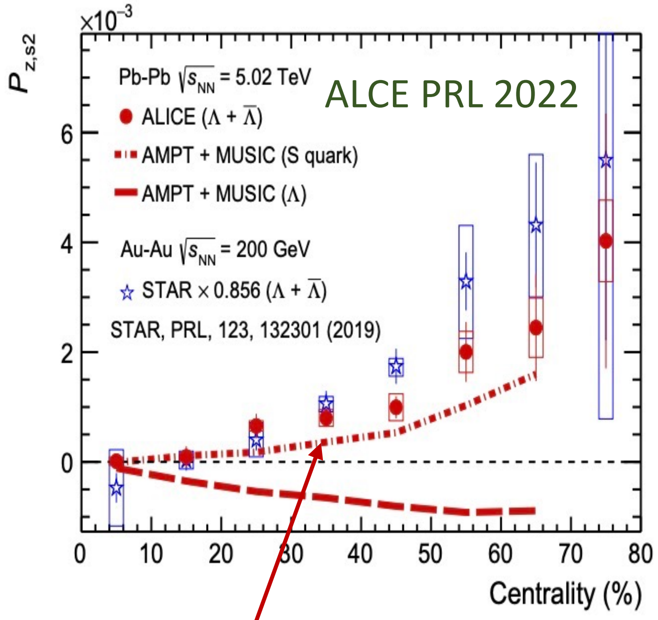


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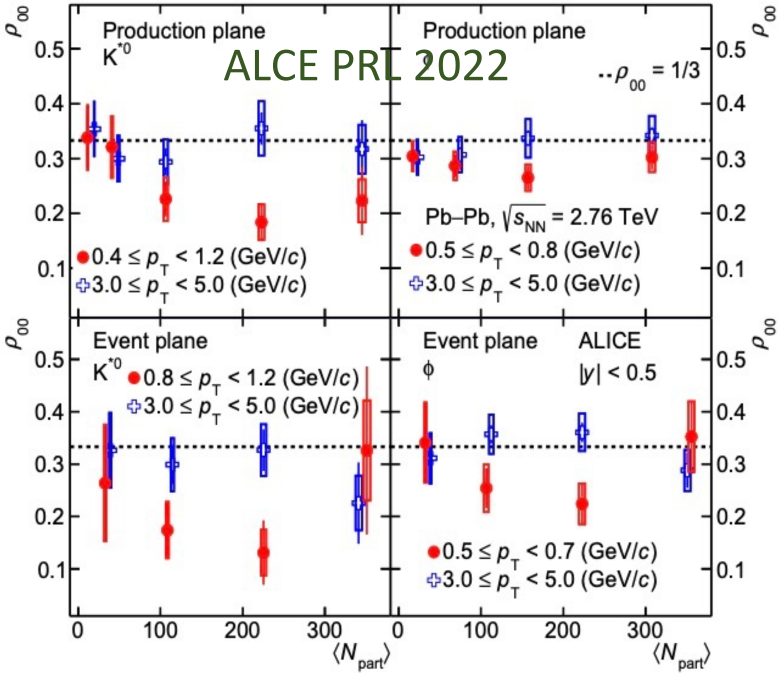
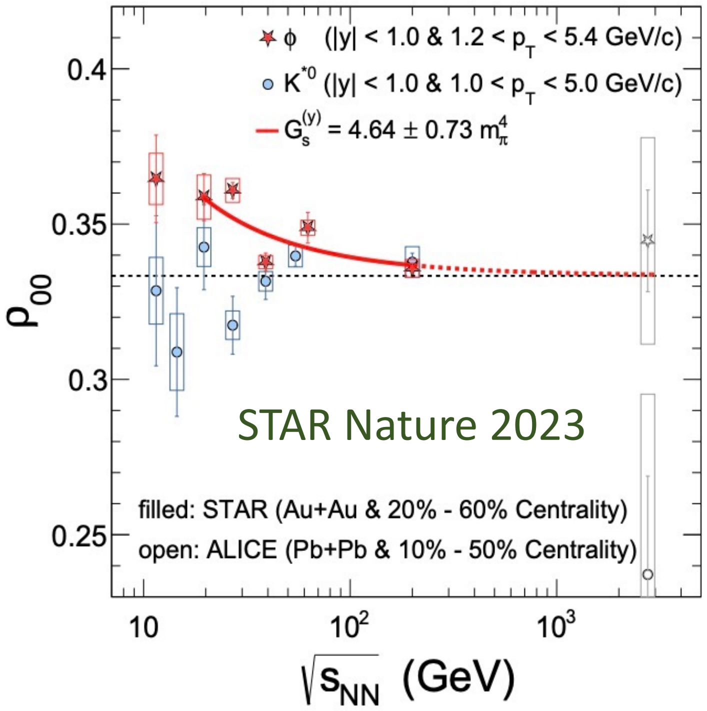
# Challenges on Spin Alignment

## Experiments




## Early Theories

❖Coalescence picture

Liang & Wang PLB 2005

$$\frac{1 + P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \propto \left(\frac{\omega}{T}\right)^2$$

❖Thermal picture

Becattini,Piccinini,Rizzo,PRC, 2008

$$\frac{1 - (\hat{p} \cdot \hat{\omega})^2}{18}\left(\frac{\omega}{T}\right)^2 \propto \left(\frac{\omega}{T}\right)^2$$

❖$(\omega/T)^2 \sim 1/10^4$ , 2nd order in gradient, small

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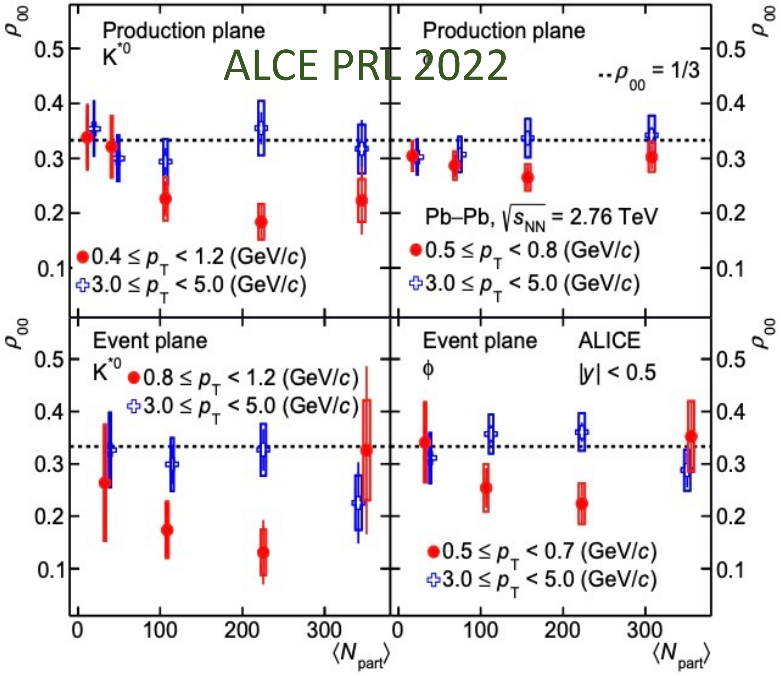


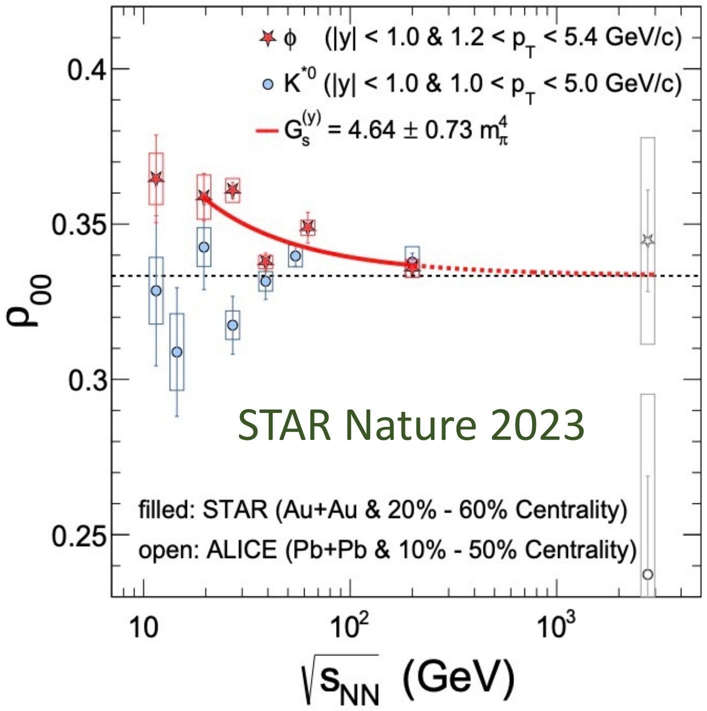


❖Features：

- Magnitude are large
- Physics are rich (beam energy, pT, species…) and highly nontrivial behaviors

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❖Problems：

- Magnitude too small
- Physics are less abundant.

**How we progress?**

# New ideas

- ❖New external field — Sheng, Oliva, Liang, Wang, Wang, 2206.05868
- ❖Initial stage physics, such as Glasma — *Kumar, Müller, Yang, PRD* 2023
- ❖Others attempts — Wei and Huang arxiv:2303.01897
- ❖Study with thermal field theory carefully, discover missing effects, such as Shear Induced Tensor Polarization（SITP）

Li, Liu, 2022, arXiv:2206.11890

**Attempt to convince you in this talk that these new effects are:**

- **Natural**, appeared naturally once included more realistic physics in theory
- **Universal**, applying to all interacting medium with massive vector boson
- **Large and Rich**, magnitude can be large & containing rich physics

Similar physics also discussed later in Wagner, Weickgenannt, Speranza , PRR, 2022

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# Structure of the density matrix

❖The spin-1 boson has (8 degrees of freedom)

$$\rho_{ss'} = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k(J_k)_{ss'} - \mathcal{T}_{ij}(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1})_{ss'}$$

$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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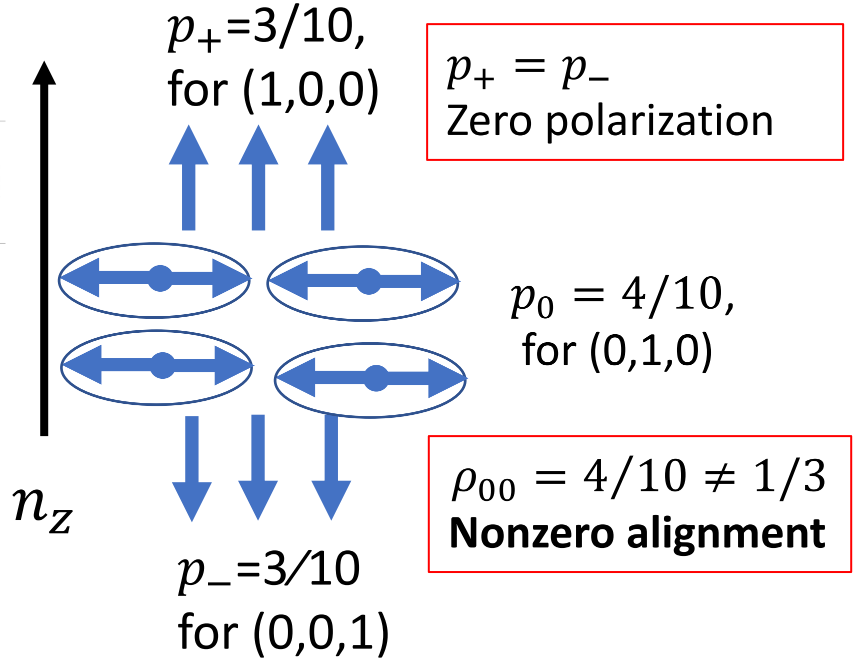
❖For a density matrix (quantize along z direction)

$$\rho = \begin{pmatrix} p_+ & & \\ & p_0 & \\ & & p_- \end{pmatrix}$$




❖Trivial case $p_+ = p_- = p_0 = 1/3$, no polarization

❖Vector Polarization exist when $\mathcal{P} = p_+ - p_- \neq 0$

❖Tensor Polarization & alignment exist if $p_0 \neq 1/3$, $T_{zz} = \rho_{00} - \frac{1}{3}$ (Even $\mathcal{P} = 0$, when $p_+ = p_-$, )

❖**Only** the tensor polarization part contribute to $\rho_{00} - 1/3$

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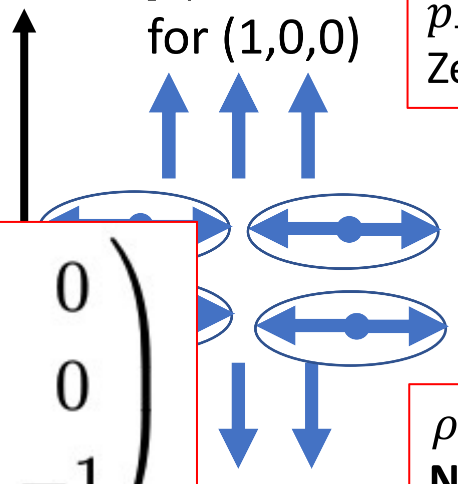
$p_+$=3/10, for (1,0,0)



$p_0 = 4/10$, for (0,1,0)

$_-$=3/10 for (0,0,1)

$p_+ = p_-$
Zero polarization

$\rho_{00} = 4/10 \neq 1/3$
**Nonzero alignment**

- Trivial case $p_+ = p_- = p_0 = 1/3$, no polarization
- Vector Polarization exist when $\mathcal{P} = p_+ - p_- \neq 0$
- Tensor Polarization & alignment exist if $p_0 \neq 1/3$, $T_{zz} = \rho_{00} - \frac{1}{3}$ (Even $\mathcal{P} = 0$, when $p_+ = p_-$, )
- **Only** the tensor polarization part contribute to $\rho_{00} - 1/3$

# Relation to Wigner function

- Full Wigner function

$$W^{\mu\nu}(x,\boldsymbol{p}) \equiv \varepsilon_{\boldsymbol{p}} \int dp^0 \int d^4y e^{ip\cdot y} \langle V^\mu(x_-) V^\nu(x_+)\rangle \;= W_+^{\mu\nu}(x,\boldsymbol{p}) + W_-^{\mu\nu}(x,\boldsymbol{p})$$

- Positive mode $W_+^{\mu\nu}$, with projections, and normalization

$$\mathcal{W}^{\mu\nu}(x,\boldsymbol{p}) \equiv 2\tilde{\Delta}^\mu_\alpha \tilde{\Delta}^\nu_\beta W_+^{\alpha\beta}(x,\boldsymbol{p})$$

$\tilde{\Delta} = -\eta^{\mu\nu} + \tilde{p}^\mu \tilde{p}^\nu / \tilde{p}^2$

$\tilde{p}$ is on-shell 4 momentum

- The density matrices related to it as

$$\varrho_{ss'}(x,\boldsymbol{p}) = \epsilon^\mu_{s'}(\boldsymbol{p}) \epsilon^{\nu *}_s(\boldsymbol{p}) \mathcal{W}_{\mu\nu}(x,\boldsymbol{p})$$

- Decomposition of $\mathcal{W}^{\mu\nu}$

$$\mathcal{W}^{\mu\nu} = \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} + \mathcal{W}^{[\mu\nu]} + \mathcal{T}^{\mu\nu}$$

$$\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma} W_+^{(\lambda\gamma)}$$

# Related to Wigner function

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$$\mathcal{W}^{\mu\nu} = \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} + \mathcal{W}^{[\mu\nu]} + \mathcal{T}^{\mu\nu} \quad \Longleftrightarrow \quad = \frac{1}{3}\delta_{ss'} + \frac{1}{2}\mathcal{P}_k(J_k)_{ss'} - \mathcal{T}_{ij}(J_{(i}J_{j)} - \frac{2}{3}\delta_{ij}\mathbf{1})_{ss'}$$

❖Tensor polarization

$$\boxed{\mathcal{T}^{\mu\nu} \equiv \mathcal{W}^{\langle\mu\nu\rangle} \equiv \mathcal{W}^{(\mu\nu)} - \frac{1}{3}\tilde{\Delta}^{\mu\nu}\mathcal{S} = 2\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}W_+^{(\lambda\gamma)}}$$

# Gradient Expansion and Symmetry analysis

❖ Expansion up to 1st order gradient expansion

$$T^{\mu\nu} = \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \boxed{\kappa_{\rm sh}\sigma^{\lambda\gamma}} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta$$
$$+\kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]$$

T-even

T-even, 0th order

T-odd, Shear Again!

Early theories include terms such as $(\omega/T)^2 \sim (1/100)^2$, 2nd order in gradient

**Many Missing BUT Naturally Allowed Contribution at Lower Orders!**

❖ Why missed before?

- In-medium spectral properties/interactions required, **not been well studied before**
- Shear Induced Tensor Polarization(SITP) with $\kappa_{sh}$ to be T-odd and indicate the nature of the dissipative physics, **not been studied before**

Could we find these terms in a concrete calculation?
Yes, see later

# $0^{\text{th}}$ order—a compact non-perturbative result

❖ The tensor polarization related to spectral function as

$$\mathcal{T}^{\mu\nu}_{(0)} = 2\tilde{\Delta}^{\langle\mu}_{\alpha}\tilde{\Delta}^{\nu\rangle}_{\beta}\int_0^\infty dp^0 \int d^4y e^{ip\cdot y}\langle V^\alpha(x_-)V^\beta(x_+)\rangle = 2\tilde{\Delta}^{\langle\mu}_{\alpha}\tilde{\Delta}^{\nu\rangle}_{\beta}\int_0^\infty dp^0 n(p^0)A^{\alpha\beta}(p)$$

❖ The spectral function $\Delta_L^{\mu\nu} = v^\mu v^\nu/(-v^2)$, $\Delta_T^{\mu\nu} = \Delta^{\mu\nu} - \Delta_L^{\mu\nu}$, $\tilde{v}^\mu = \tilde{\Delta}^{\mu\nu}u_\nu$

$$A^{\mu\nu} = \sum_{a=L,T}\Delta_a^{\mu\nu}A_a,\ A_a = \frac{1}{\pi}\mathrm{Im}\,\frac{-1}{p^2 - m^2 - \Pi_a}.$$

❖ The result

$$\mathcal{T}^{\mu\nu}_{(0)} = \alpha_0 n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}_L^{\langle\mu\nu\rangle} = \frac{\alpha_0}{-\tilde{v}^2}n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}u^\lambda u^\gamma,$$

$$\alpha_0 = 2\varepsilon_{\boldsymbol{p}}\int_0^\infty d\omega\,\frac{n(\omega)}{n(\varepsilon_{\boldsymbol{p}})}\left[(A_L - A_T) - \frac{\Delta\omega^2\tilde{v}^2}{p^2}A_L\right]$$

$\propto \frac{\varepsilon_{\boldsymbol{p}}^T - \varepsilon_{\boldsymbol{p}}^L}{T}$ $\qquad \propto \Gamma^2$

T-even

# 1st order–linear response theorem

$$\xi_{\lambda\gamma} \equiv \beta^{-1}\partial_{(\lambda}(\beta u)_{\gamma)}$$
$$\approx \sigma_{\lambda\gamma} + \left[\frac{1}{3}\bar{\Delta}_{\lambda\gamma} + c_s^2 u_\lambda u_\gamma\right]\theta$$

❖Linear response (like those for calculate $\eta/s$)

$$W^{\mu\nu}_{+(1)} = \varepsilon_{\boldsymbol{p}} \lim_{\nu,q\to 0} \frac{\partial}{\partial\nu}[-\mathrm{Im}G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p})]\xi_{\lambda\gamma}$$

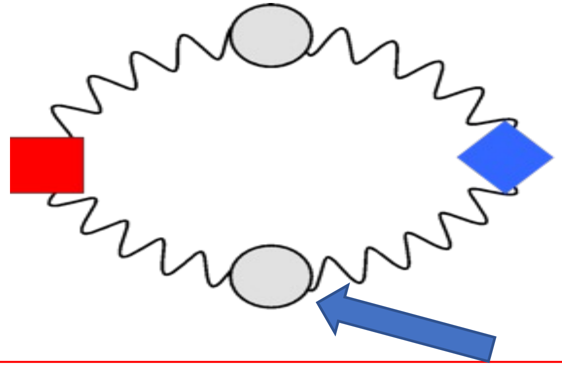
❖The green function is connected to energy momentum tensor

$$G^{\mu\nu\lambda\gamma}_{R}(\nu,\boldsymbol{q},\boldsymbol{p}) \xleftrightarrow{\text{Wigner Trans}} (-i)\Theta(t-t')\langle[V^\mu(t,\boldsymbol{x}^-)V^\nu(t,\boldsymbol{x}^+),T^{\lambda\gamma}(t',\boldsymbol{z})]\rangle,$$

$$T^{\mu\nu} \equiv -F^{\mu}{}_{\alpha}F^{\nu\alpha} + m^2V^\mu V^\nu - \eta^{\mu\nu}\left(-F^2/4 + m^2V^2/2\right)$$

❖One skeleton/dressed loop calculation with spectral functions

$$G^{\mu\nu\lambda\gamma}_{R+}(\nu,\boldsymbol{q},\boldsymbol{p}) = -\int_0^\infty dk_0 \int_0^\infty dk_0' \frac{n(k_0')-n(k_0)}{\nu+k_0'-k_0+i0^+} \times \sum_{a,b=L,T} A_a(k)A_b(k')I^{\mu\nu\lambda\gamma}_{ab}(k,k')\,.$$



**In-medium interactions are implicitly included in self-energies!**

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+ higher order terms

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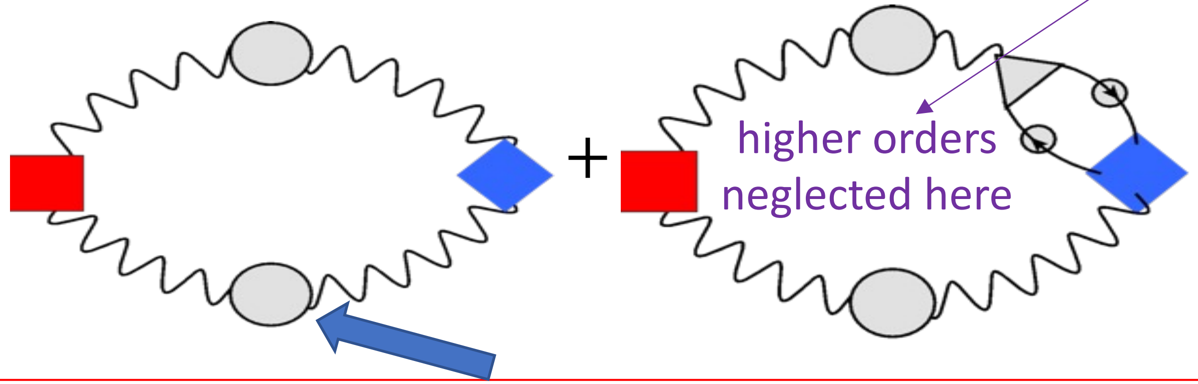


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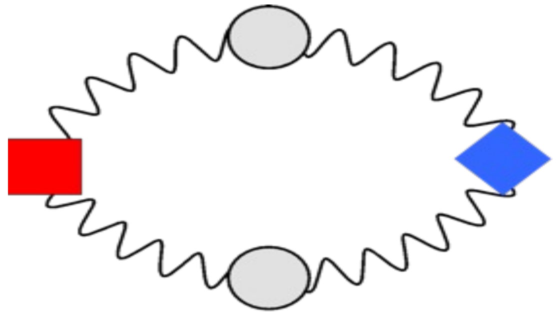
❖With some calculation:

$$\begin{aligned}I^{\mu\nu\lambda\gamma}_{ab}(k,k') &= [k^\lambda k'^\gamma + k^\gamma k'^\lambda]\Delta^{\nu\alpha}_a(k)\Delta^{\mu}_{b,\alpha}(k') \\ &- [k_\alpha k'^\gamma\Delta^{\nu\lambda}_a(k)\Delta^{\mu\alpha}_b(k') + k^\gamma k'_\alpha\Delta^{\nu\alpha}_a(k)\Delta^{\mu\lambda}_b(k')] \\ &- [k^\lambda k'_\alpha\Delta^{\nu\alpha}_a(k)\Delta^{\mu\gamma}_b(k') + k_\alpha k'^\lambda\Delta^{\nu\gamma}_a(k)\Delta^{\mu\alpha}_b(k')] \\ &+ (k_\alpha k'^\alpha - m^2)[\Delta^{\nu\lambda}_a(k)\Delta^{\mu\gamma}_b(k') + \Delta^{\nu\gamma}_a(k)\Delta^{\mu\lambda}_b(k')] \\ &- \eta^{\gamma\lambda}[(k^\zeta k'_\zeta - m^2)\eta_{\alpha\beta} - k_\beta k'_\alpha]\Delta^{\nu\alpha}_a(k)\Delta^{\mu\beta}_b(k')]\end{aligned}$$



$$\boxed{\begin{aligned}k &= (k_0,\boldsymbol{p}+\boldsymbol{q}/2), \\ k' &= (k_0',\boldsymbol{p}-\boldsymbol{q}/2).\end{aligned}}$$

# Compact results for 1st order

- A one-line formula for tensor polarization

$$\boxed{\mathcal{T}^{\mu\nu}_{(1)} = \beta n(\varepsilon_{\boldsymbol{p}})\tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\left[\alpha_{\rm sh}\xi^{\gamma\lambda} + \alpha_{\rm sp}\xi_p\frac{u^\lambda u^\gamma}{-\tilde{v}^2}\right]}$$

with coefficient

$$\alpha_{\rm sh} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega(\omega^2 - \varepsilon_{\boldsymbol{p}}^2)A^2_{T/L}(\omega,\boldsymbol{p})$$

$$\alpha_{\rm sp} = \frac{4\varepsilon_{\boldsymbol{p}}\pi}{\beta n(\varepsilon_{\boldsymbol{p}})}\int_0^\infty \frac{\partial n(\omega)}{\partial\omega}d\omega\varepsilon_{\boldsymbol{p}}^2(A^2_T(\omega,\boldsymbol{p}) - A^2_L(\omega,\boldsymbol{p}))$$

in quasi-particle spectral function $A_a(\omega,\boldsymbol{p}) \approx \frac{1}{2\varepsilon_{\boldsymbol{p}}}\frac{1}{\pi}\mathrm{Im}\frac{-1}{\omega - \omega^a_{\boldsymbol{p}} + i\Gamma^a_{\boldsymbol{p}}/2}$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + 2\frac{\Delta\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\frac{\Delta\varepsilon_{\boldsymbol{p}}}{T} + \frac{\Gamma_{\boldsymbol{p}}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\left(\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} - \frac{\Delta\varepsilon_{\boldsymbol{p}}}{T}\frac{\Gamma^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}} + \frac{\Gamma_{\boldsymbol{p}}}{T}\frac{\omega^\Delta_{\boldsymbol{p}}}{\Gamma_{\boldsymbol{p}}}\right) \sim \mathcal{O}(\delta_{\rm qp}^{-1}\delta_{\rm sp}).$$

T-odd, dissipative

- Width $\Gamma_p$, energy/mass-shift $\Delta\varepsilon_p$, split of width $\Gamma^\Delta_p$ and energy $\omega^\Delta_{\boldsymbol{p}}$

**Well-defined old players in thermal field theory, no extra new players are required**

# Total Theory Results

❖ Total results:

$$\boxed{\begin{aligned} T^{\mu\nu} =& \tilde{\Delta}^{\langle\mu}_{\lambda}\tilde{\Delta}^{\nu\rangle}_{\gamma}\Big[\kappa_0^u u^\lambda u^\gamma + \kappa_1^u u^\lambda u^\gamma + \kappa_{\rm sh}\sigma^{\lambda\gamma} + \kappa_T u^{(\lambda}\partial_\perp^{\gamma)}\beta \\ & + \kappa_{\rm su} u^{(\lambda}\sigma^{\gamma)\alpha}\tilde{p}_\alpha + \kappa_{\rm ou} u^{(\lambda}\Omega^{\gamma)\alpha}\tilde{p}_\alpha + \cdots\Big]\end{aligned}}$$

$$\kappa_0^u = \frac{\alpha_0}{-\tilde{v}^2}n_0,\ \kappa_1^u = \left[\alpha_{\rm sh}\left(c_s^2 - \frac{1}{3}\right)\theta + \frac{\alpha_{\rm sp}\xi_p}{-\tilde{v}^2}\right]\beta n_0$$

$$\kappa_{\rm sh} = \alpha_{\rm sh}\beta n_0, \quad \kappa_T = 0, \quad \kappa_{\rm su} = 0, \quad \kappa_{\rm ou} = 0$$

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}} + 2\frac{\Delta\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}}\frac{\Delta\varepsilon_{\boldsymbol p}}{T} + \frac{\Gamma_{\boldsymbol p}}{2T} \sim \mathcal{O}(1)$$

$$\alpha_{\rm sp} \approx -\frac{\varepsilon_{\boldsymbol p}}{\Gamma_{\boldsymbol p}}\left(\frac{\Gamma_{\boldsymbol p}^\Delta}{\Gamma_{\boldsymbol p}} - \frac{\Delta\varepsilon_{\boldsymbol p}}{T}\frac{\Gamma_{\boldsymbol p}^\Delta}{\Gamma_{\boldsymbol p}} + \frac{\Gamma_{\boldsymbol p}}{T}\frac{\omega_{\boldsymbol p}^\Delta}{\Gamma_{\boldsymbol p}}\right) \sim \mathcal{O}(\delta_{\rm qp}^{-1}\delta_{\rm sp}).$$

$$\alpha_0 \approx (\omega_{\boldsymbol p}^T - \omega_{\boldsymbol p}^L)/T$$

Nonanalytical energy-shift more subtle

❖ Features of the result

- **Natural**, suggested by symmetry, verified in concrete thermal field theory calculation, all have been done is a more careful theory study with more realistic spectral functions
- **Universal**, SITP exist in all spin-1 particles including heavy quarkonium, in relativistic or non-relativistic (SITP has a coefficient have no mass suppression)

# Outline

1) Spin Observables in Heavy-ion Collisions
2) Theory for Tensor Polarization and Spin Alignment
3) Spectral properties of in-medium mesonic resonance
4) Phenomenology implications
5) Summary and outlooks

# Spectral properties of in-medium degree of freedom

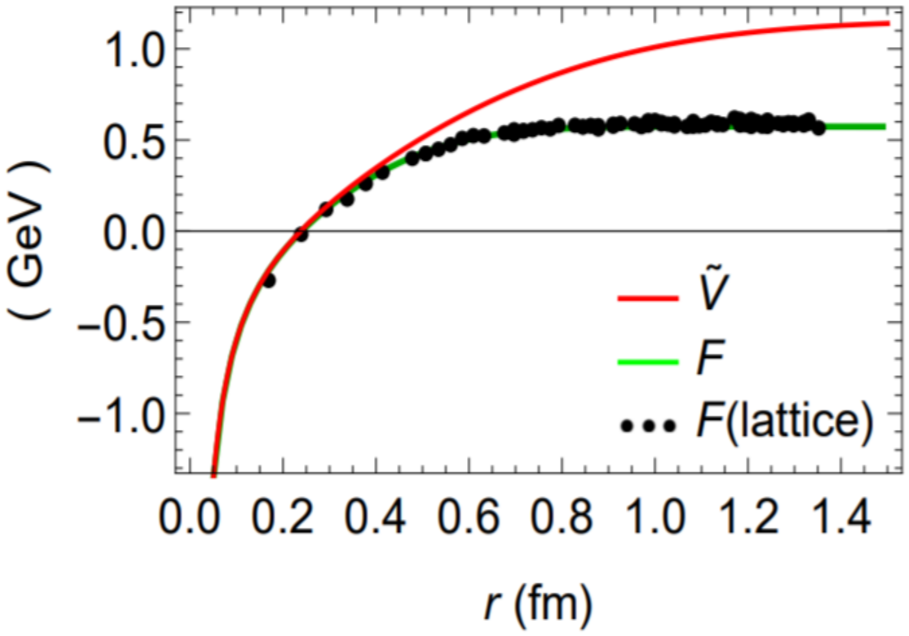
- Where the mesons of freedom forms?
  - Chemical freezeout
  - Late stage of QGP?

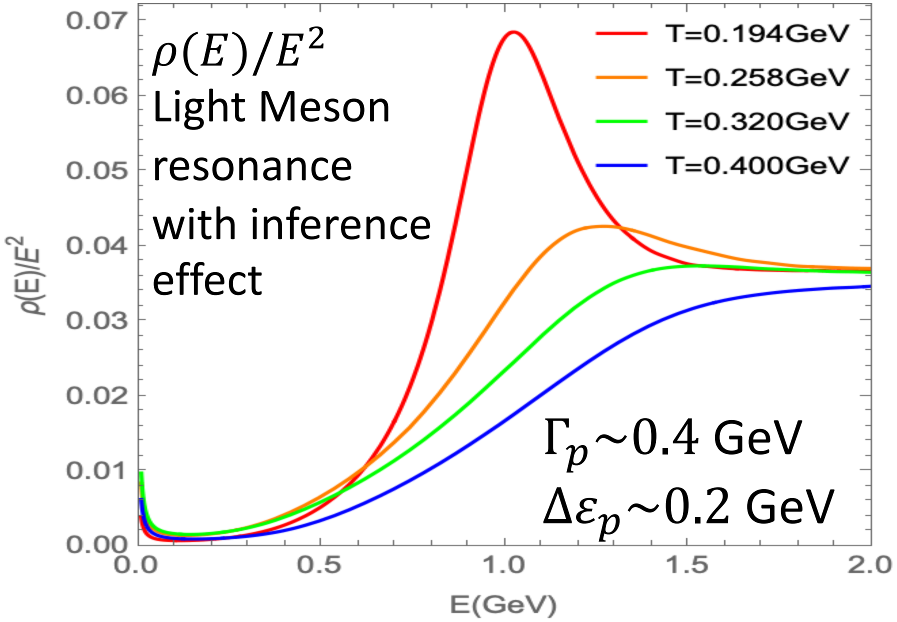
**Towards the Theory of Binary Bound States in Quark-Gluon Plasma**

Edward V.Shuryak and Ismail Zahed

Shuryak, Zahed, PRD 2004

- Large confining potential ⟺ mesonic resonance at $T\sim 0.2$ GeV




Liu, Rapp, PRC 2018

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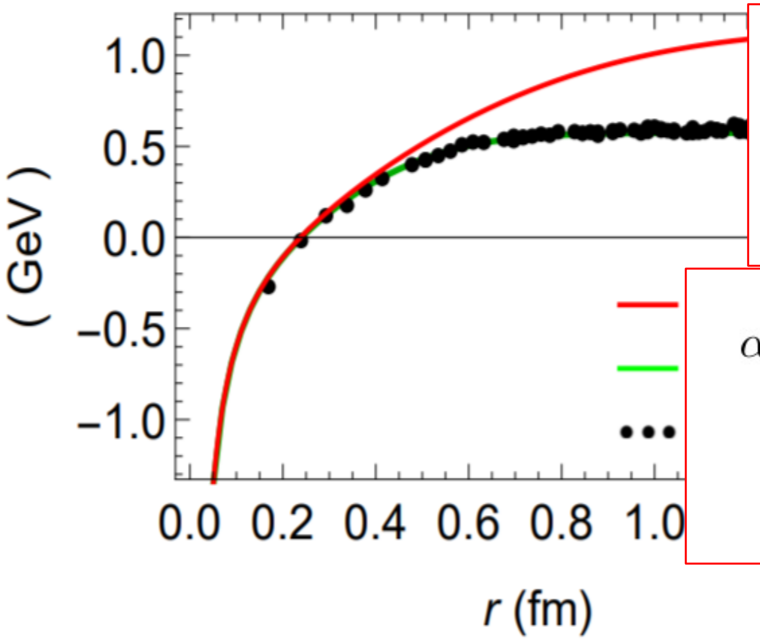
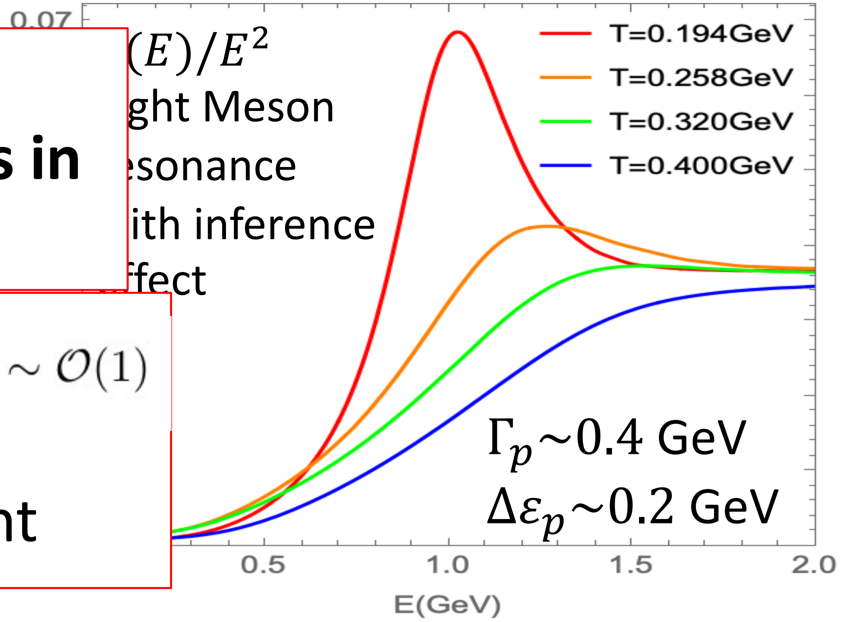



**Large width can be generated by collisions in strong coupling**

$$\alpha_{\rm sh} \approx -\frac{2\Delta\varepsilon_p}{\Gamma_p} + 2\frac{\Delta\varepsilon_p}{\Gamma_p}\frac{\Delta\varepsilon_p}{T} + \frac{\Gamma_p}{2T} \sim \mathcal{O}(1)$$

$\alpha_{\rm sh} \sim 1$, Large Coefficient




$\Gamma_p \sim 0.4$ GeV

$\Delta\varepsilon_p \sim 0.2$ GeV

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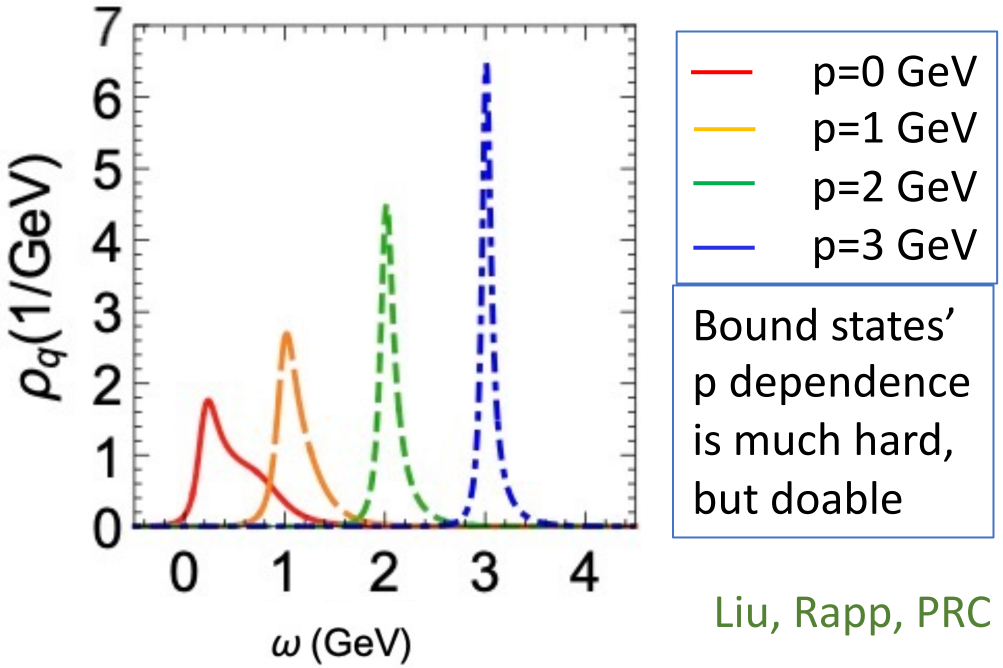
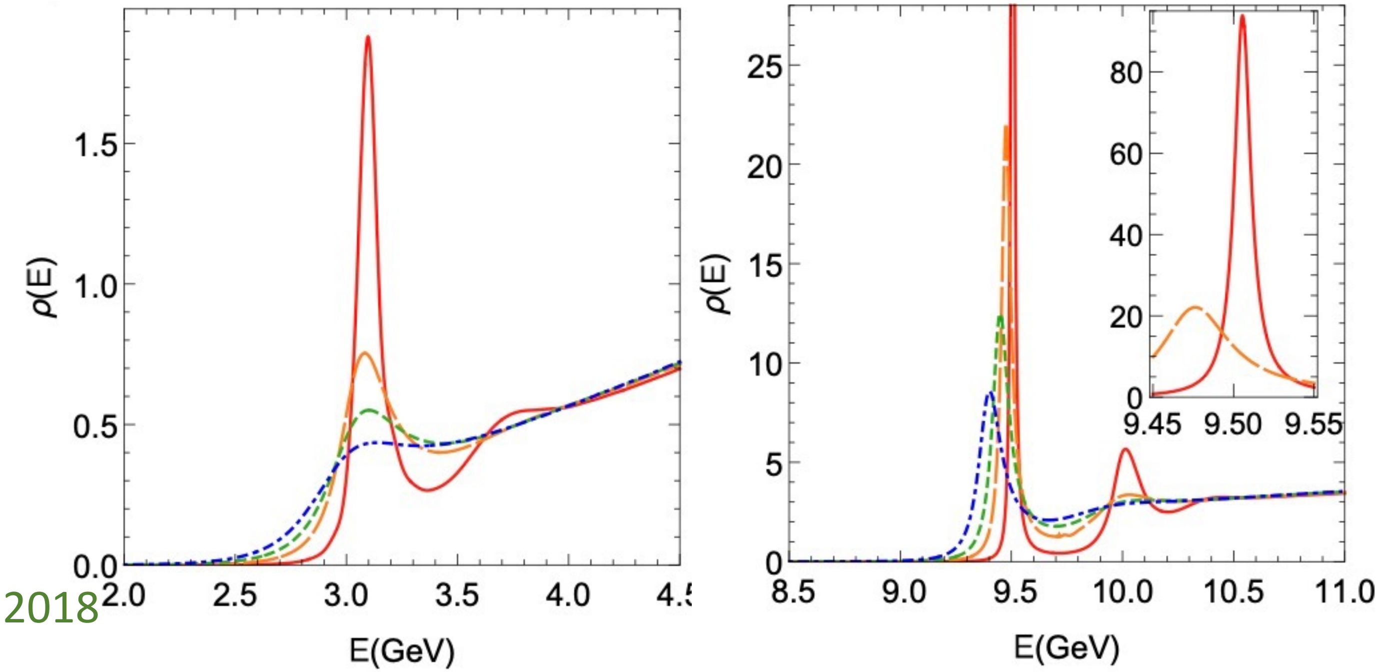
# Rich physics in spectral functions

❖Spectral function (quark) as a function of momentum




Bound states' p dependence is much hard, but doable

❖Different particles, different formation T, different mass shift

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Spectral properties are manifestations of in-medium QCD interactions

Rich QCD interactions → Rich spectral properties → Rich behavior in spin alignment

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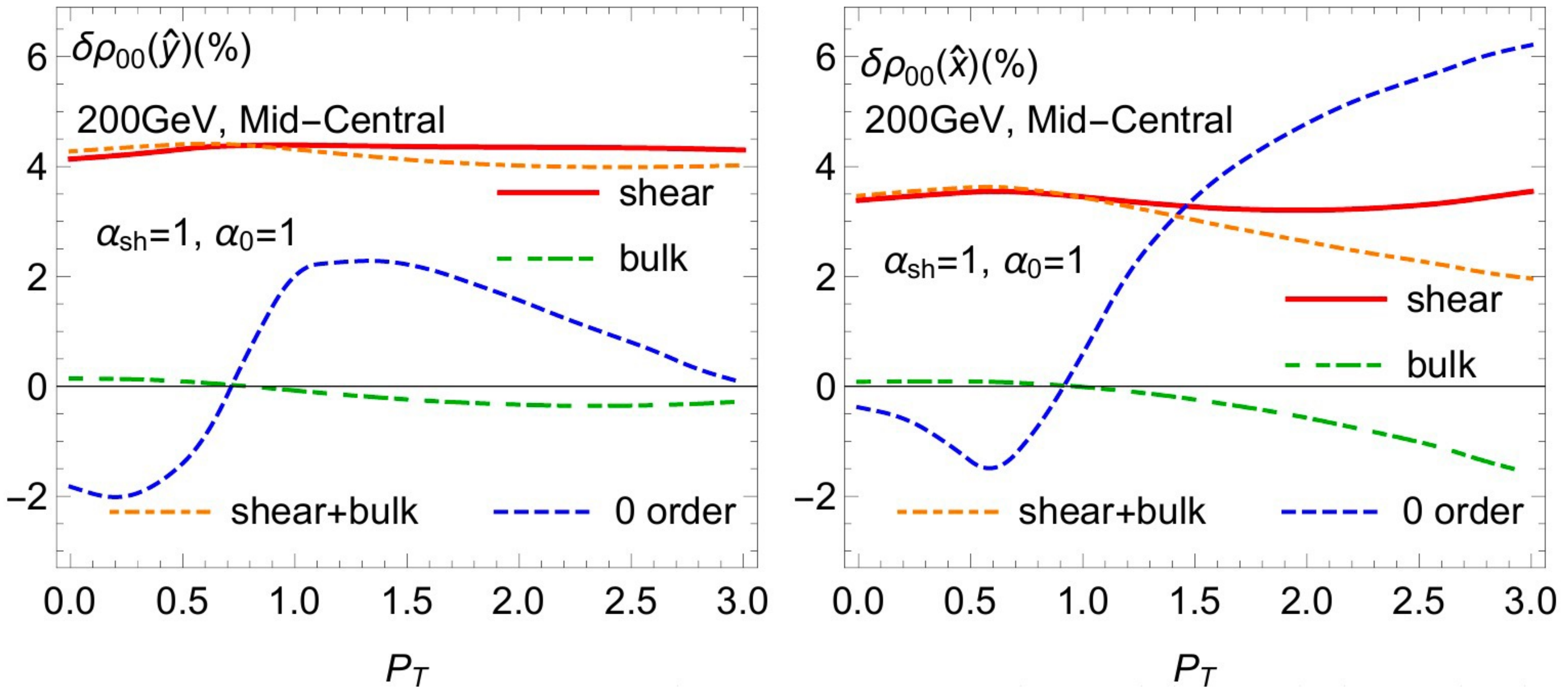
# Phenomenology implication

❖ The connection to $\delta\rho_{00} \equiv \rho_{00} - 1/3$

$$\delta\rho_{00}(\hat{n}_{\rm pr}, \boldsymbol{p}) = \frac{\int d\Sigma^\lambda p_\lambda \mathcal{T}^{\mu\nu}(x,\boldsymbol{p})\hat{n}_\mu(\boldsymbol{p})\hat{n}_\nu(\boldsymbol{p})}{d\Sigma^\lambda p_\lambda \mathcal{S}(x,\boldsymbol{p})}$$




$\alpha_{\rm sh}\sim 1$, estimate from spectral of light heavy meson in previous slide (no p dependence)

$\alpha_0\sim 1$, just a value for convenience (need better spectral function to estimate)

❖ **Large** phenomenologically, especially **SITP** can generate ~1% level spin alignment at the relatively late stage of QGP phase

❖ Using spectral functions with $p$, T(many others) species dependence, can include **Rich** physics that may help in understand rich structures observed in experiment

# Summary

- ❖Discovered a **Shear-Induced Tensor Polarization(SITP)**, together with other new $0^{th}$ and $1^{st}$ order effects
- ❖**Natural**, allowed by the symmetry and verified in calculation
- ❖**Universal**, SITP exist in all interacting many-body system with spin-1 particle, in relativistic/non-relativistic scenarios.
- ❖**Large and Rich,** effects especially SITP can contribute to spin alignment at the order of ~1% level, promising to include rich physics with more realistic spectral functions.

Standard many-body interactions (such as collisions) can lead to large spin alignment with the discovery of the missing new effects, such as SITP!

Many works need to be done to make quantitative predictions, due to the rich physics and complexity in strongly coupled many-body system