Photon radiation by rotating fermions in magnetic field

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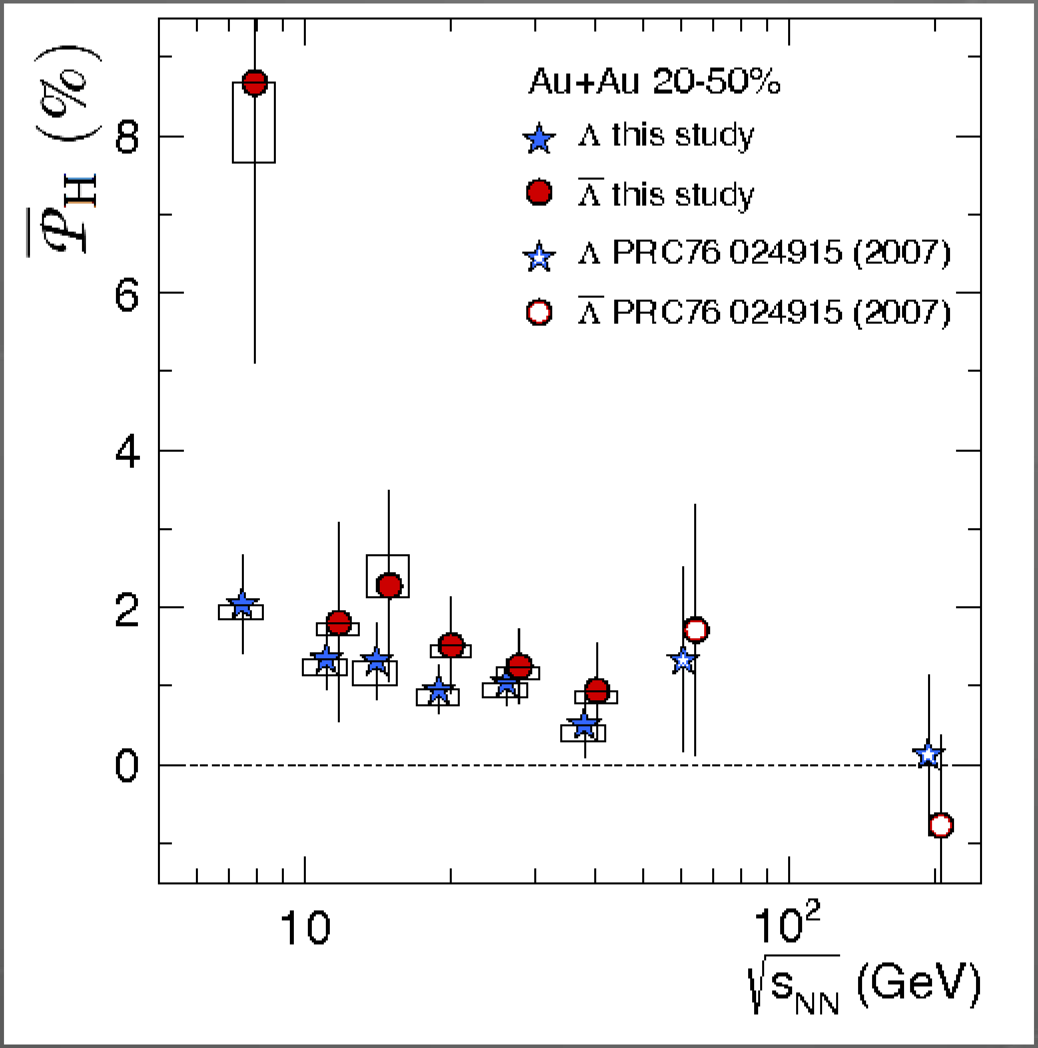
Matteo Buzzegoli

IOWA STATE UNIVERSITY

and J.D. Kroth, K. Tuchin and N. Vijayakumar

2209.02597, 2209.03991, 2306.03863

Spin polarization by Rotation



STAR Collaboration, Nature 548 6265, (2017)

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\varpi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

$$\beta^\mu = \frac{1}{T}u^\mu \qquad n_F = \left(\mathrm{e}^{\beta\cdot p-\zeta}+1\right)^{-1}$$

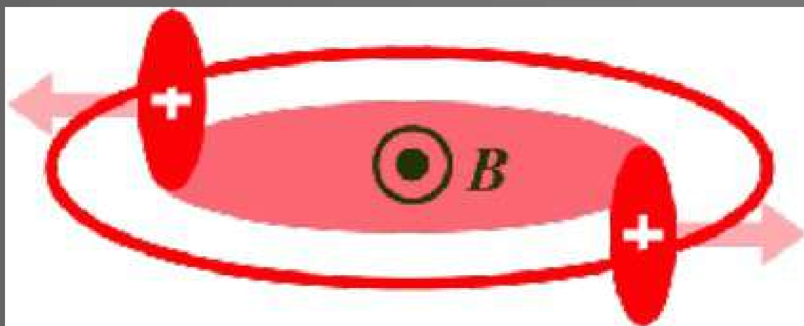
$$\varpi^{\mu\nu} = -\frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$$

Consider only rotation for simplicity Ω^μ

$$S^\mu_\Omega = \frac{1}{4m}(1-n_F)\beta E\left(\Omega^\mu - u^\mu\frac{\Omega\cdot p}{E}\right)$$

$$E = u\cdot p$$

Magnetic field

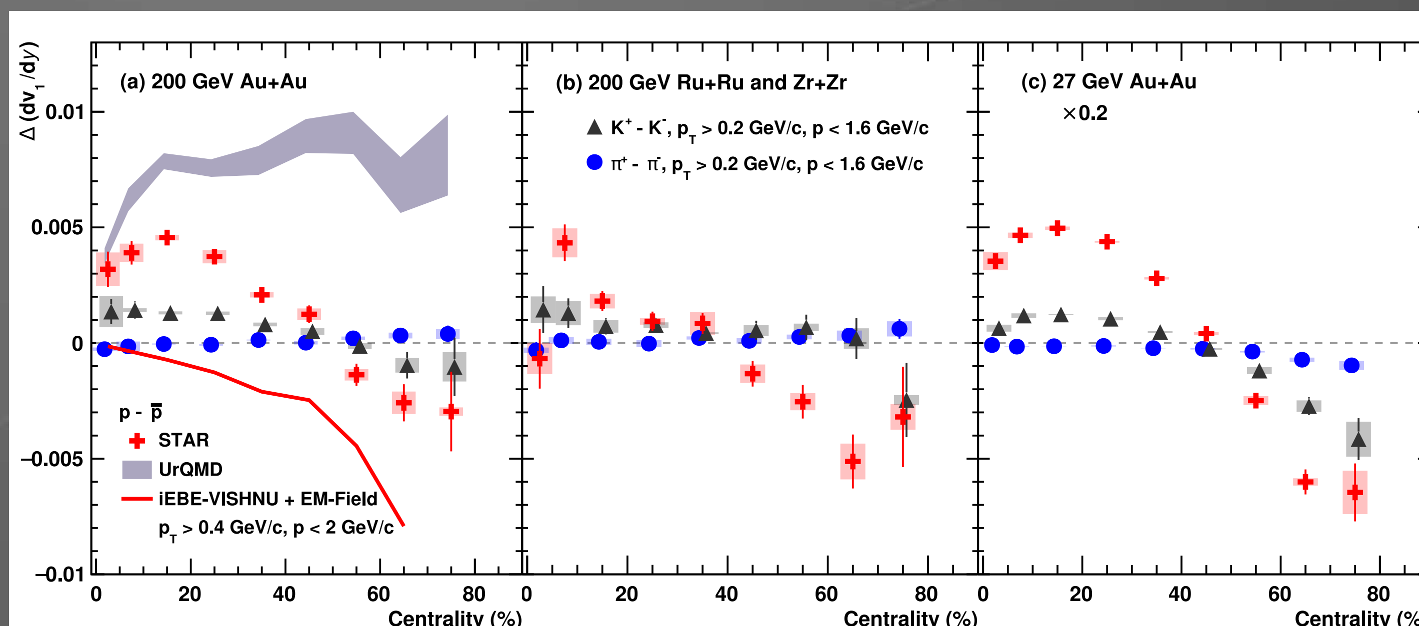


$$eB \sim \gamma \alpha_{\rm em} \frac{Z}{b^2} \simeq \frac{1}{\rm fm^2} \simeq 10^{18} \text{ Gauss}$$

[V. Skokov, A. Y. Illarionov and V. Toneev, Internat. J. Modern Phys. A 24 (2009) 5925]

Difference of direct flow between positive and negative electrical charges

[See A. Tang talk]



Faraday induction + Coulomb effect

[STAR, 2304.03430]

Analogy between Rotation Ω and Magnetic field B

[MB, Nucl. Phys. A 1036, 122674 (2023)]

Magnetization

Barnett Effect (1915)

$$\frac{qB^{\mu}_{\mathrm{Eff}}}{E} = \Omega^{\mu}$$

Spin polarization

$$S^{\mu}_{\Omega} = \frac{1}{4m}(1-n_F)\beta E\left(\Omega^{\mu} - u^{\mu}\frac{\Omega\cdot p}{E}\right) \qquad S^{\mu}_{B} = \frac{1}{4m}(1-n_F)\beta\left(qB^{\mu} - u^{\mu}\frac{qB\cdot p}{E}\right)$$

[F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, PRC 95 (2017)]

Electromagnetic radiation

Classical total radiation intensity from circular motion [Schott (1912)]

$$W = \frac{q^2\Omega^2}{c}\sum_{\nu=1}^{\infty}\int_0^{\pi}\sin\theta d\theta\left[\cot^2\theta J_{\nu}(\nu\beta\sin\theta) + \beta^2 J_{\nu}'^{\,2}(\nu\beta\sin\theta)\right]$$

In magnetic field: Classical synchrotron radiation

$$\Omega \to \omega_B = \frac{qB}{E}$$

Gravitational anomaly?

The Axial Vortical Effect (AVE) conductivity

Massless AVE $\langle j_A^z \rangle \propto \mathcal{N} T^2 \Omega$

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107 (2011)
K. Jensen, R. Loganayagam, A. Yarom, JHEP 02 (2013)
M. Stone, J. Kim, PRD 98 (2) (2018)
G.Y. Prokhorov, O.V. Teryaev, V.I. Zakharov, PRL 129 (2022)

Barnett effect: Classical model for spin polarization

Angular momentum-rotation coupling

Classical inertial effect

$$H \to H - \boldsymbol{J} \cdot \boldsymbol{\Omega}$$

[MB, Nucl. Phys. A 1036, 122674 (2023)]

$$\langle \boldsymbol{S} \cdot \hat{\boldsymbol{\Omega}} \rangle = \langle \boldsymbol{\mu} \cdot \hat{\boldsymbol{\Omega}} \rangle \simeq \frac{1}{3} \frac{\mu}{\gamma} \frac{\Omega}{T}$$

γ gyromagnetic ratio

μ magnetic moment

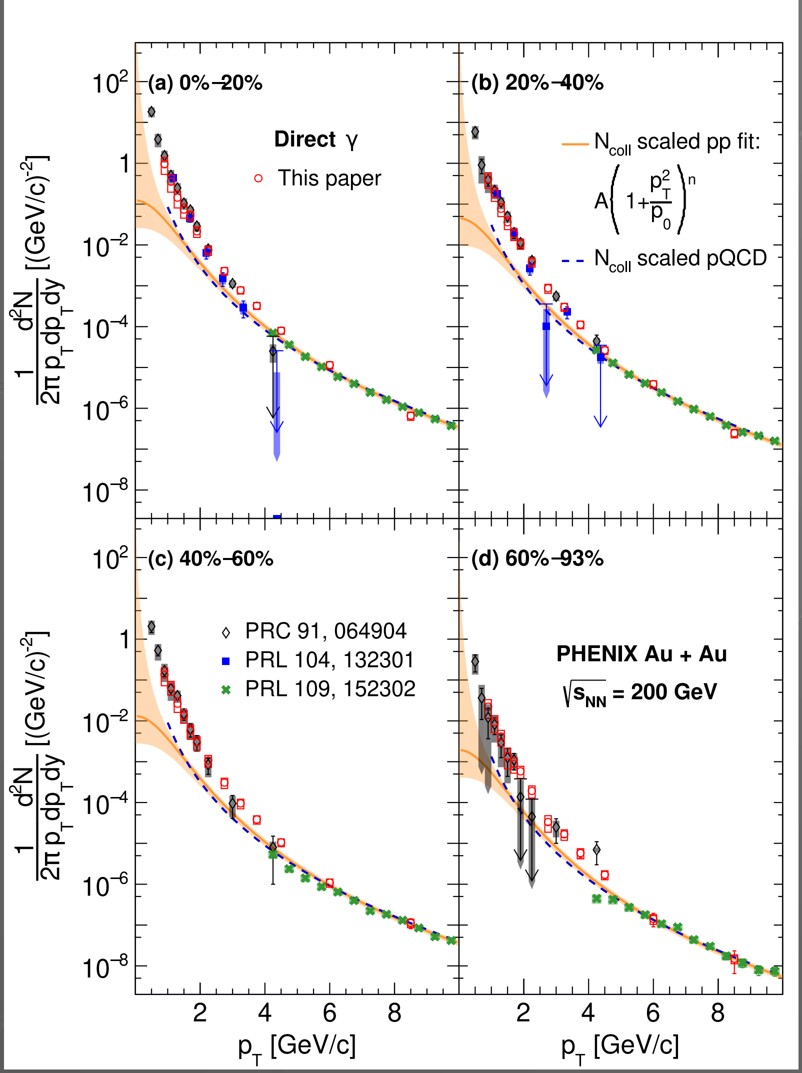
Furthermore:
Unlike the CVE/CME the (massive) AVE is allowed at the actual equilibrium (no chiral imbalance)
No connection between the massive AVE and the gravitational anomaly

$$\langle j_A^z \rangle \simeq \frac{\Omega}{T} \left(1 + 2\frac{T}{m} \right) \frac{(mT)^{3/2}}{\sqrt{2}\,\pi^{3/2}} e^{|\beta|(\mu - m)} \qquad T \ll m$$

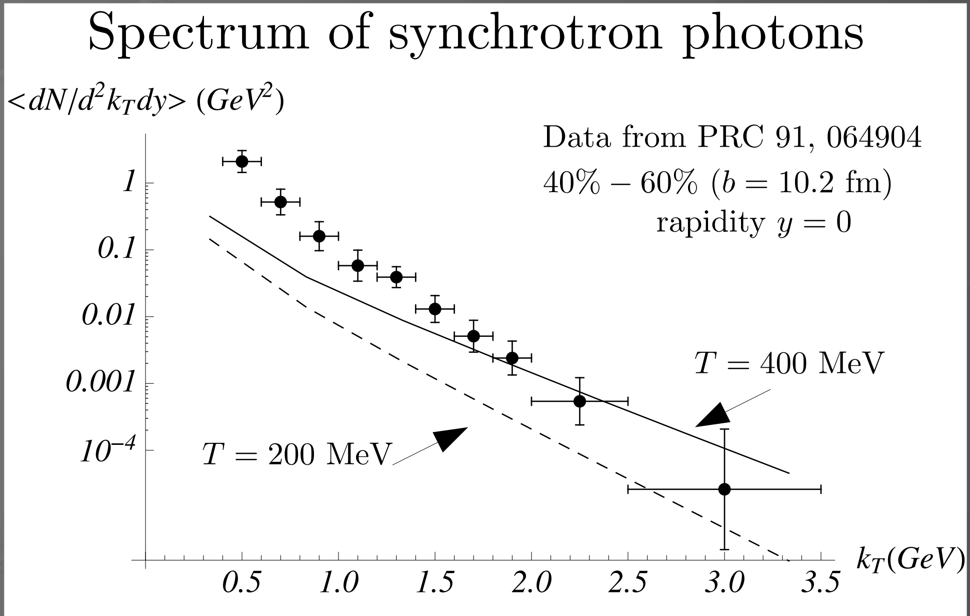
[MB, Lect. Notes Phys. 987 (2021)]

Effect of the magnetic field?
Direct photons puzzle

Direct Photons @ PHENIX



[2203.17187]



[K. Tuchin, PRC 87 (2013), PRC 92 (2015)]

Similar approach
[X. Wang, I. A. Shovkovy, PRD 104 (2021) and PRD 106 (2022)]

Synchrotron radiation is not sufficient.

v_2 of photons in magnetic field
[X. Wang, I. A. Shovkovy, L. Yu, M. Huang, PRD 102 (2020)]

Effect of the magnetic field?
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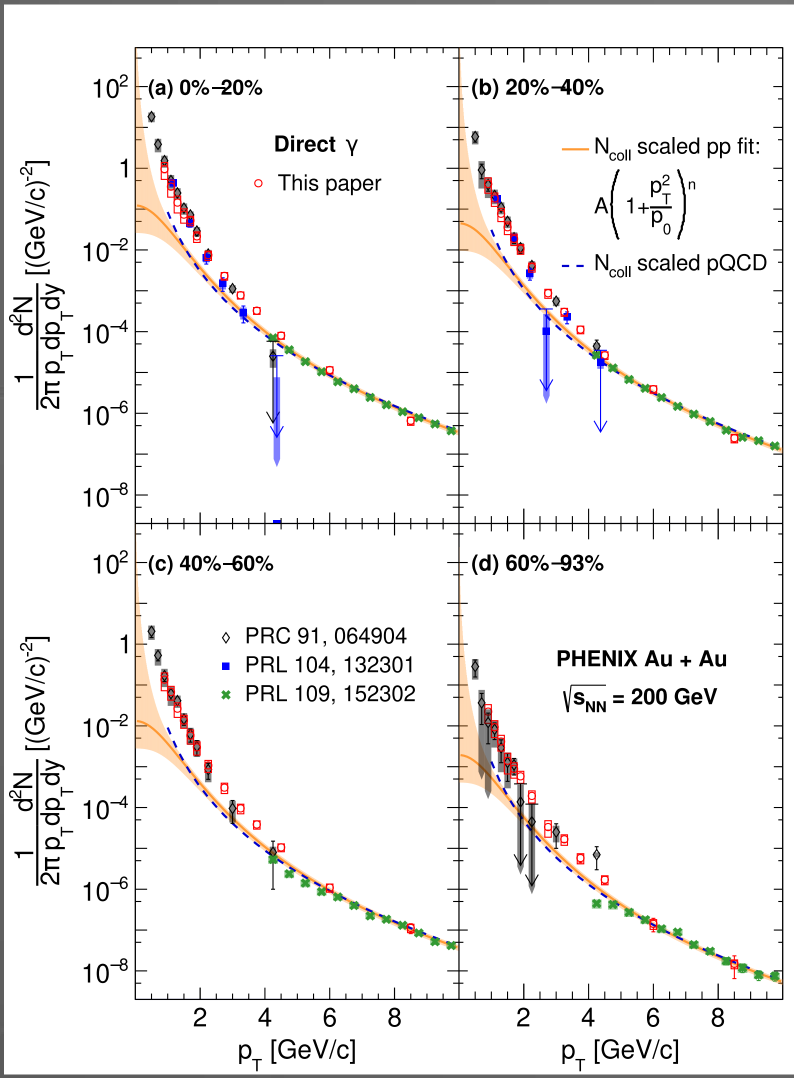
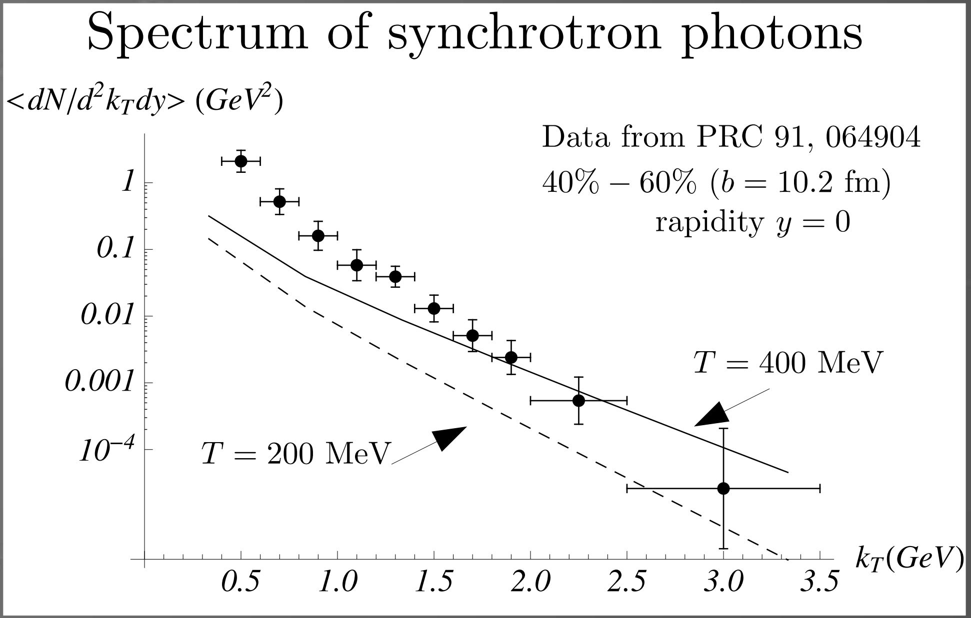


[2203.17187]



[K. Tuchin, PRC 87 (2013), PRC 92 (2015)]

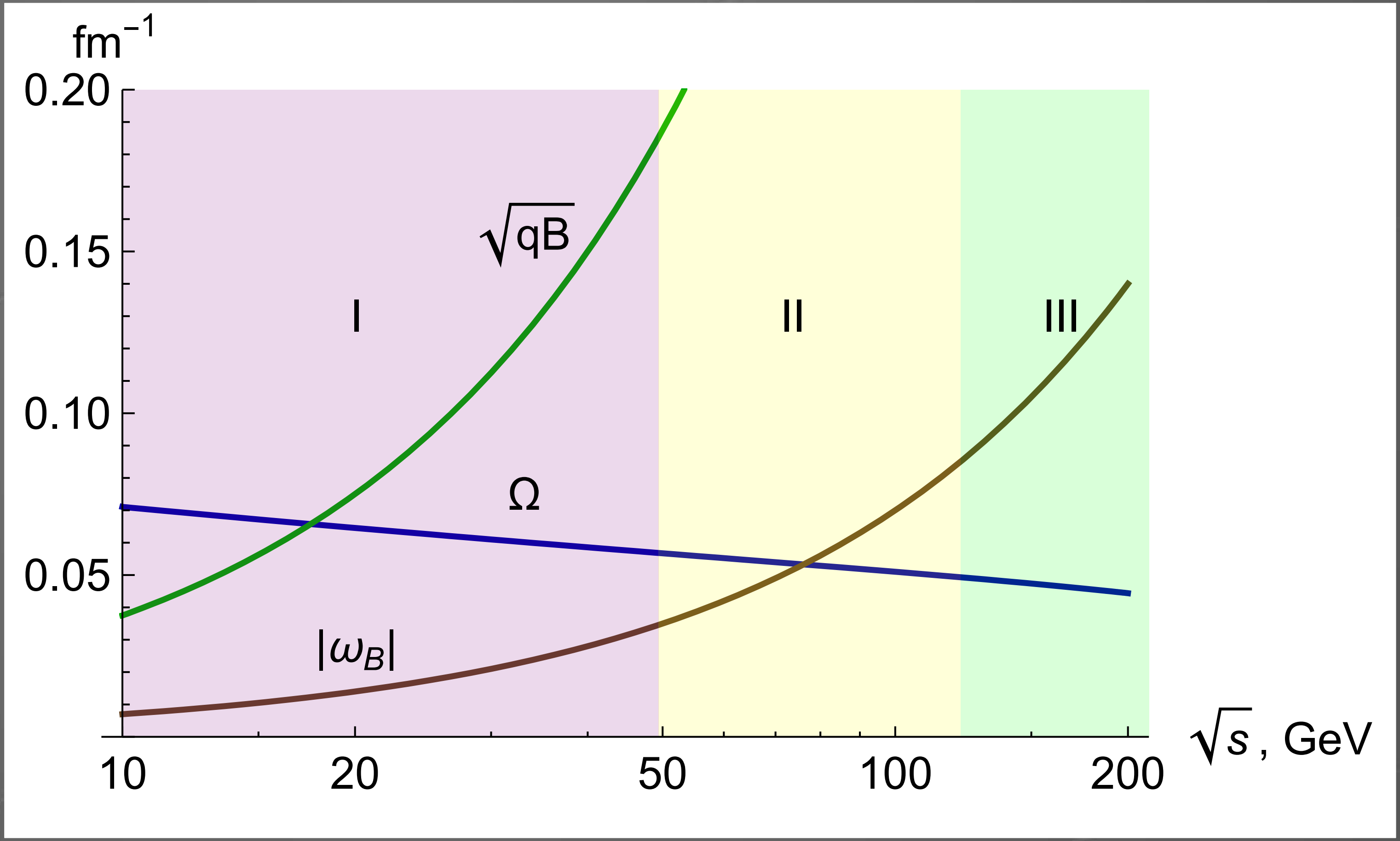
Similar approach
[X. Wang, I. A. Shovkovy, PRD 104 (2021) and PRD 106 (2022)]

Synchrotron radiation is not sufficient.

Rotating particles also radiate!
How much can rotation help?

v_2 of photons in magnetic field
[X. Wang, I. A. Shovkovy, L. Yu, M. Huang, PRD 102 (2020)]

Synchrotron Radiation (SR) in a rotating medium



Synchrotron frequency: $\omega_B = \dfrac{|qB|}{E}$

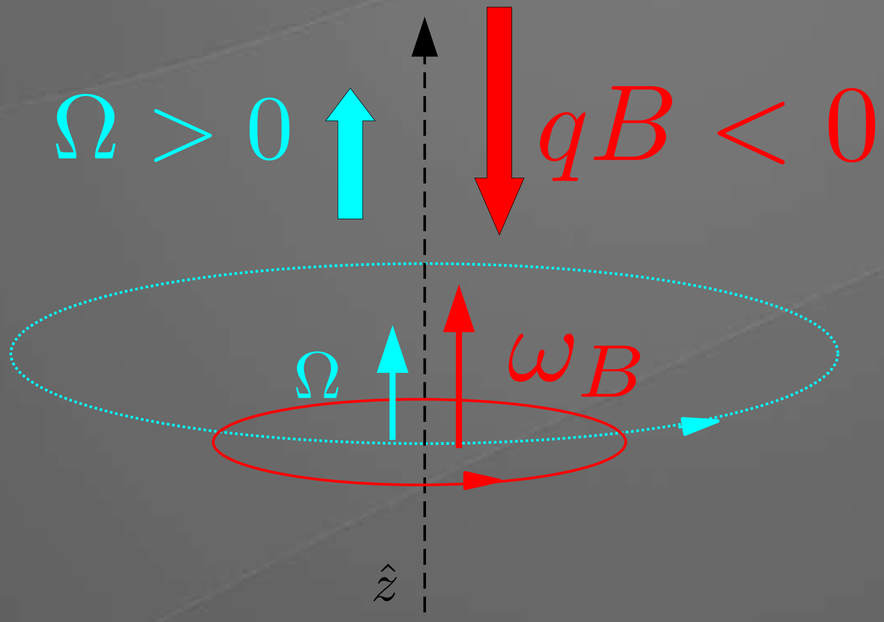
Angular velocity (vorticity): Ω

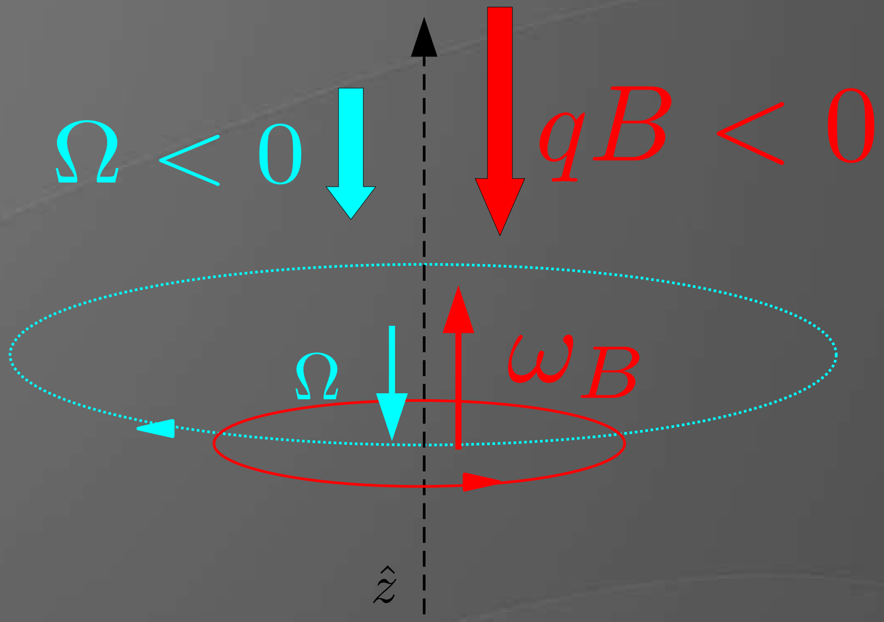
Developing intuition (non-relativistic)

Lorentz Force: $\boldsymbol{F}_L = q(\boldsymbol{v} \times \boldsymbol{B})$ Centrifugal Force: $\boldsymbol{F}_\Omega = -\boldsymbol{p} \times \boldsymbol{\Omega}$

$q < 0, \quad B > 0$

Relative angular velocity as seen by an external stationary observer:





Increase in angular velocity

→ Larger radiation Intensity

Decrease in angular velocity

→ Smaller radiation Intensity

Synchrotron Radiation (SR) in a rotating medium

OUR SET-UP:

The medium provides a global rotation Ω.
No other effect of the medium is considered yet.
Constant homogeneous magnetic field and rotation.
Cylindrical symmetry.

- Our system is kept together like a rotating fluid in a cup.

An element of fluid rotates on a trajectory confined by the cup walls.

- Consider a (non-relativistic) particle in a rotating frame:

$$m\frac{\mathrm{dv}}{\mathrm{dt}} = 2m\boldsymbol{\Omega}\times\boldsymbol{v} + q\boldsymbol{v}\times\boldsymbol{B} - \left[\left(\frac{1}{2}q\boldsymbol{B} - m\boldsymbol{\Omega}\right)\times\boldsymbol{r}\right]\times\boldsymbol{\Omega}$$

The centrifugal force pushes this particle to infinity $r\to\infty$. This is **not** the system we study!

"Slow" rotation

- Effects of rotation can be conveniently studied in the rotating frame:

$$ds^2 = (1 - r^2\Omega^2)dt^2 - 2r^2\Omega dt d\phi - dr^2 - r^2 d\phi^2 - dz^2$$

The spacetime is restricted to the light cylinder $r\Omega < 1$. *(Think of it as a mug wall)*

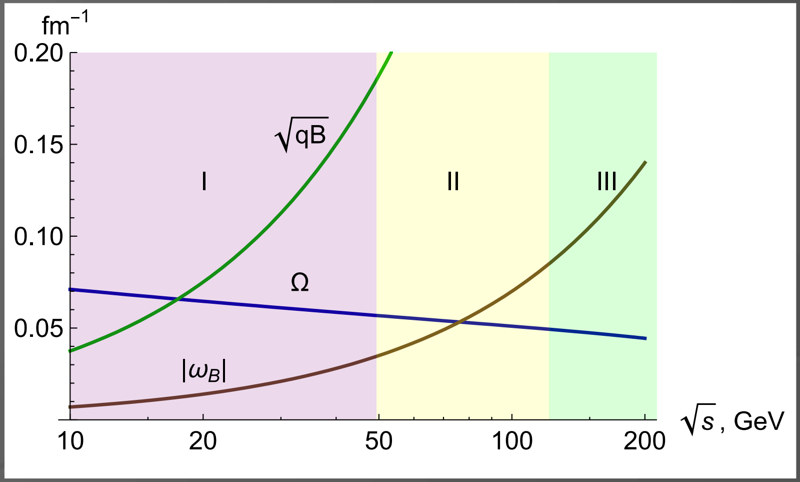
[Ambrus, Chernodub, Fukushima, Mameda]

- If rotation is slow enough, one can ignore the space boundary.

Example: in magnetic field, the extent of the wave functions of a particle in the radial direction is given by the magnetic length

$$l_B = 1/\sqrt{|qB|}$$

"Slow" rotation: $\Omega \ll \sqrt{|qB|}$



Note: "slow" rotation may in fact be extremely fast! It is only slow as compared to the magnetic field scale.

Relativistic and quantum model

details in [MB, J. Kroth, K. Tuchin, N. Vijayakumar 2306.03863]

- Consider the Dirac equation with respect to the frame rotating with -Ω

$$(i\gamma \cdot D - M)\psi = 0, \quad D_\mu = \partial_\mu + \Gamma_\mu + qA_\mu$$

[Ambrus, Chernodub, Fukushima, Mameda]

Symmetric gauge: $A^\mu = (0, -By/2, Bx/2, 0)$

Christoffel symbols: $\Gamma_0 = -\Omega[\gamma^x, \gamma^y]/4$

$$i\partial_t \psi = \widehat{H}\psi = \left(\widehat{H}_0 + \Omega \widehat{J}_z\right)\psi$$

Hamiltonian in stationary frame

Landau levels

Total angular momentum

Energy shift: $+m\Omega$

Photon emission amplitude:

$$\mathcal{S} = (2\pi)\delta(E' + \omega - E)\frac{(-iq)}{\sqrt{2\omega V}}\int \bar{\psi}_{n', a', p'_z, \zeta'}(\boldsymbol{x})\boldsymbol{\Phi}^*_{h,l,k_\perp,k_z}(\underline{\mathrm{x}})\cdot\boldsymbol{\gamma}\psi_{n, a, p_z, \zeta}(\boldsymbol{x})\,\mathrm{d}^3\mathrm{x}$$

Final — Photon wave function — Initial

Radiation intensity:

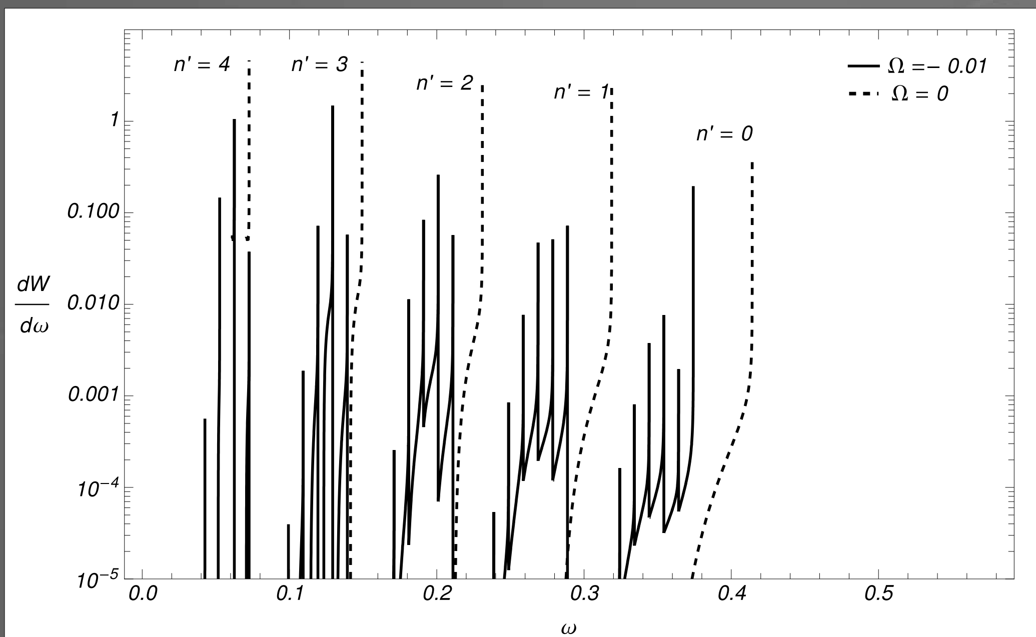
$$W = \sum_{n',a',\zeta'}\sum_{l,h}\int \frac{\mathrm{dp}'_z}{2\pi}\frac{\mathrm{d}^3\mathrm{k}}{(2\pi)^3}\omega|\mathcal{S}|^2$$

Photon frequency

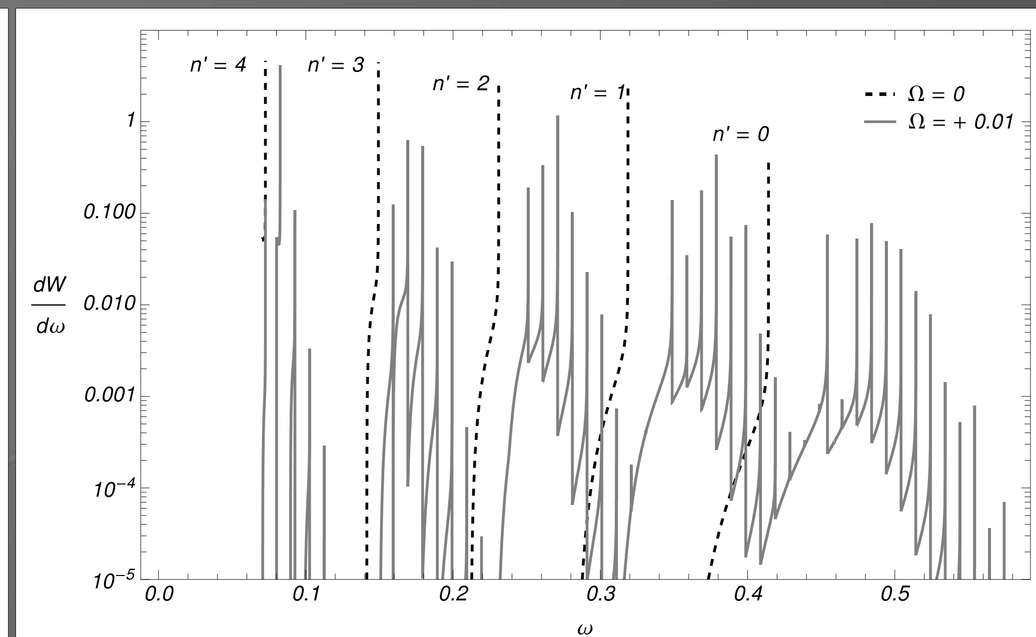
Spectrum

$$\hbar = c = M = 1 \quad qB = -0.1 \quad n = 5, m = 7/2 \quad p_z = 0, E \simeq 1.4$$

$\Omega < 0$



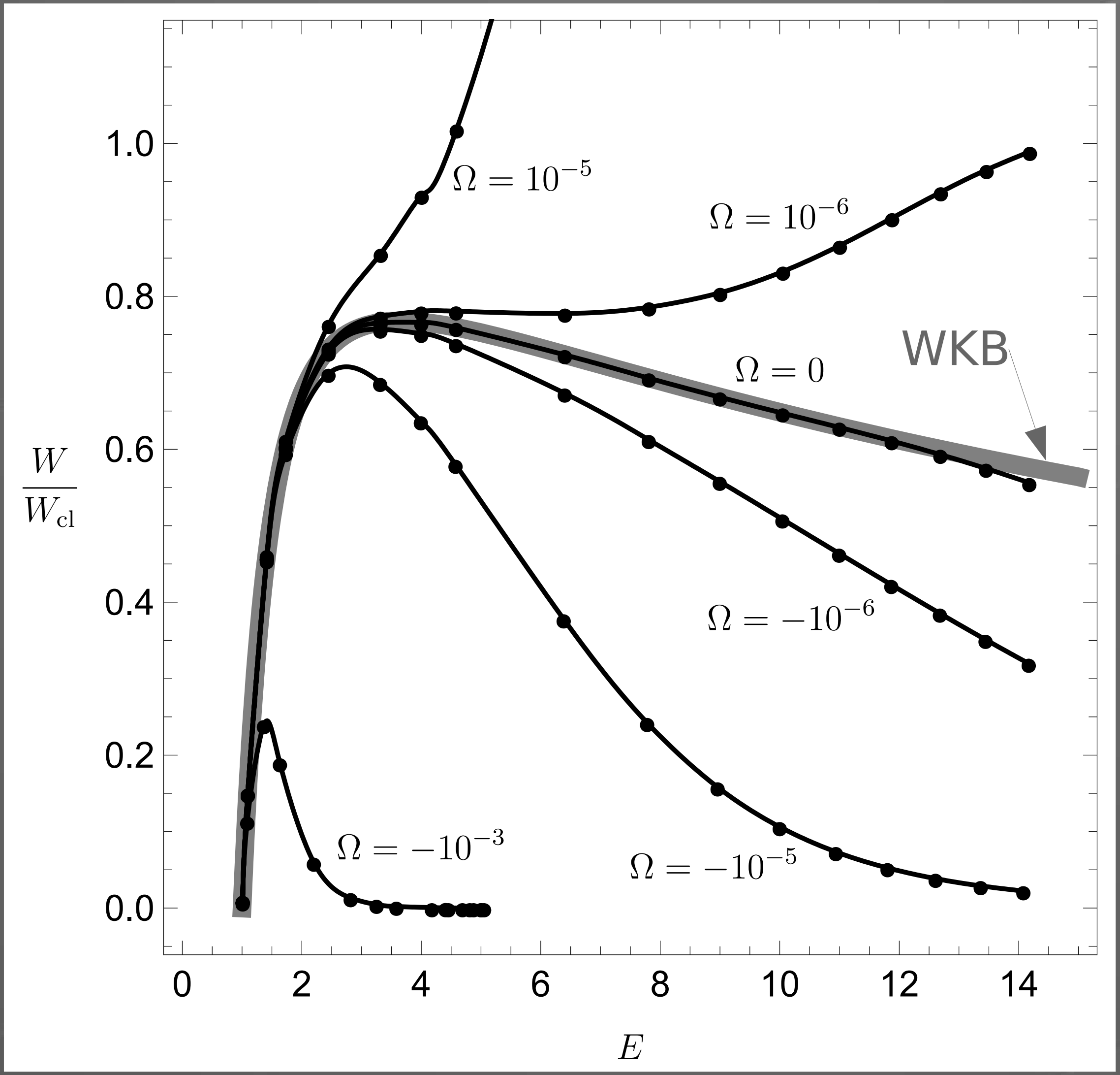
$\Omega > 0$



- Rotation breaks the degeneracy in total angular momentum number m
- Emission frequencies are shifted down by ~mΩ and peak intensities reduced
- Only certain m give significant contribution

Total Intensity

$$\hbar = c = M = 1 \quad qB = -0.01$$



- Intensity increases or decreases depending on the sign of Ω.
- Deviation from Ω=0 is large at high E.

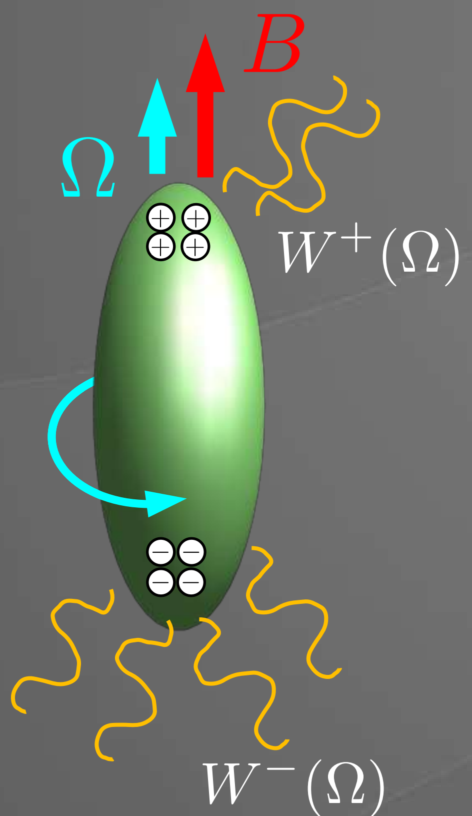
$$\omega_B = \frac{|qB|}{E}$$ decrease with E

while Ω=constant.

$$W_{cl} = \frac{2\alpha}{3}(qB)^2\left(1 - \frac{1}{E^2}\right)E^2$$

Qualitatively in the QGP

When Ω and B are aligned, the radiation from positive charges is suppressed



$$W^+(\Omega) = rW^+(\Omega = 0) \qquad r \ll 1$$

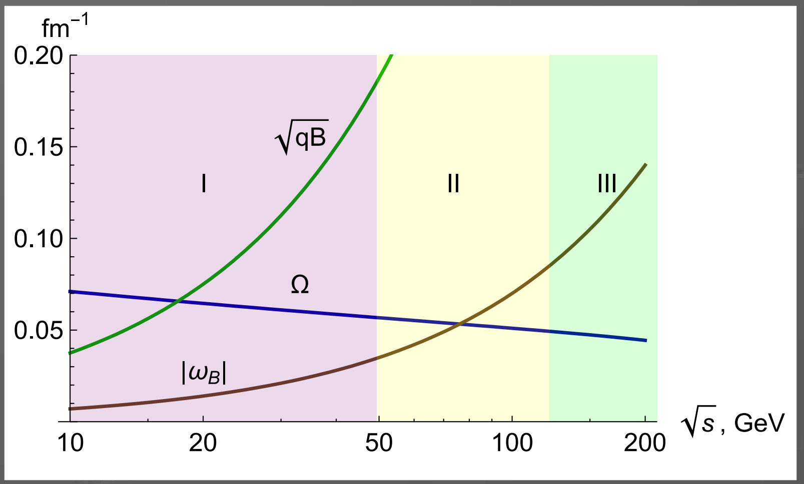
Suppose strong suppression
Approximately, radiation from negative charges is enhanced by the same factor:

$$W^-(\Omega) \simeq W^+(\Omega = 0)/r$$

$$W^-(\Omega) \gg W^+(\Omega)$$

Then, the total radiation overall is strongly enhanced:

$$\frac{W_{Tot}(\Omega)}{W_{Tot}(\Omega = 0)} = \frac{W^+(\Omega) + W^-(\Omega)}{W^+(\Omega = 0) + W^-(\Omega = 0)} \simeq \frac{1}{2r} \gg 1$$



Fast rotation and boundary conditions

High vorticity or high energy: $\Omega \gtrsim \omega_B$

Size of the system: R

Causal problems if: $R > \frac{c}{\Omega}$ → Energy spectrum **not** bounded from below; → Divergences

See for instance [V.E. Ambrus and E. Winstanley, PRD 93 (2016)]

Boundary conditions must be taken into account.

→ Energy spectrum **is** bounded from below → New ground state ⇒ Many possible phenomenological consequences

Suppression of the CME in finite systems [MB and K. Tuchin, 2305.13149]

Fast rotation and boundary conditions

Proof of principle: Toy model, rotating cylindrical Coulomb potential $U = -\frac{\alpha}{r}$

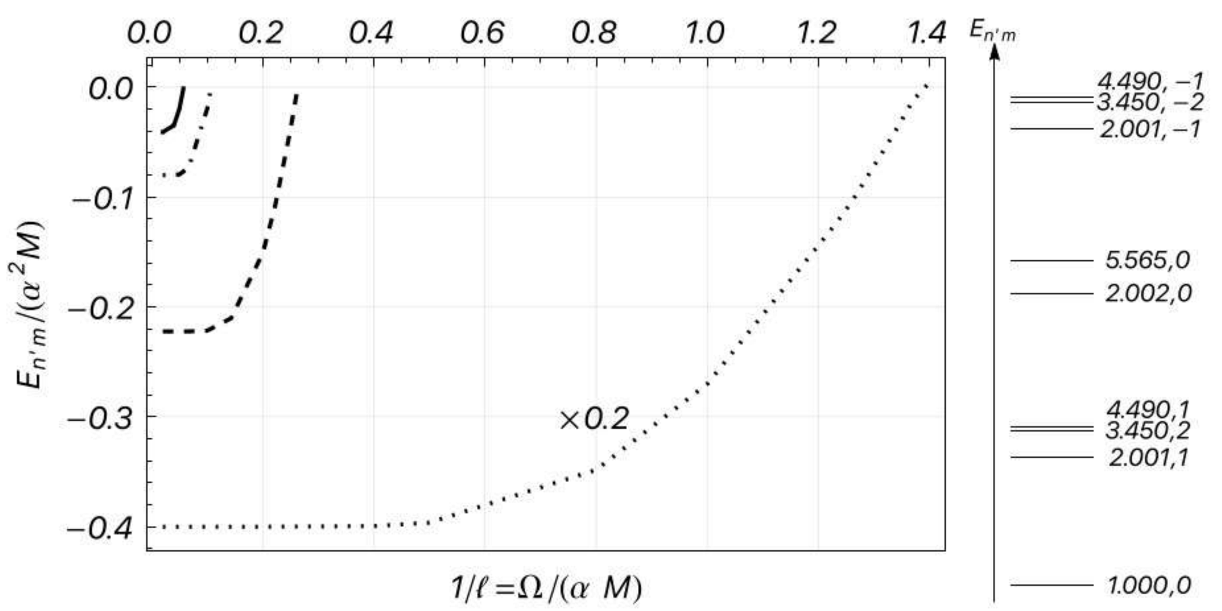


Fig. 3. Left panel: Energy levels $m = 0$, $n' = 1, 2, 3, 4$ (dotted, dashed, dashed-dotted and solid lines respectively) in the two-dimensional potential $U = -\alpha/r$ at different values of $1/\ell$. Right panel: energy levels with given n' (first number) and m (second number) at $\ell = 10$ and $\alpha\ell = 1$.

The binding energy decrease and cross into the continuum: dissociation of the bound state.
(A realistic model needs 3D potential)

[MB and K. Tuchin, Nucl. Phys. A 1030 (2023), 2209.03991]

Summary and outlook

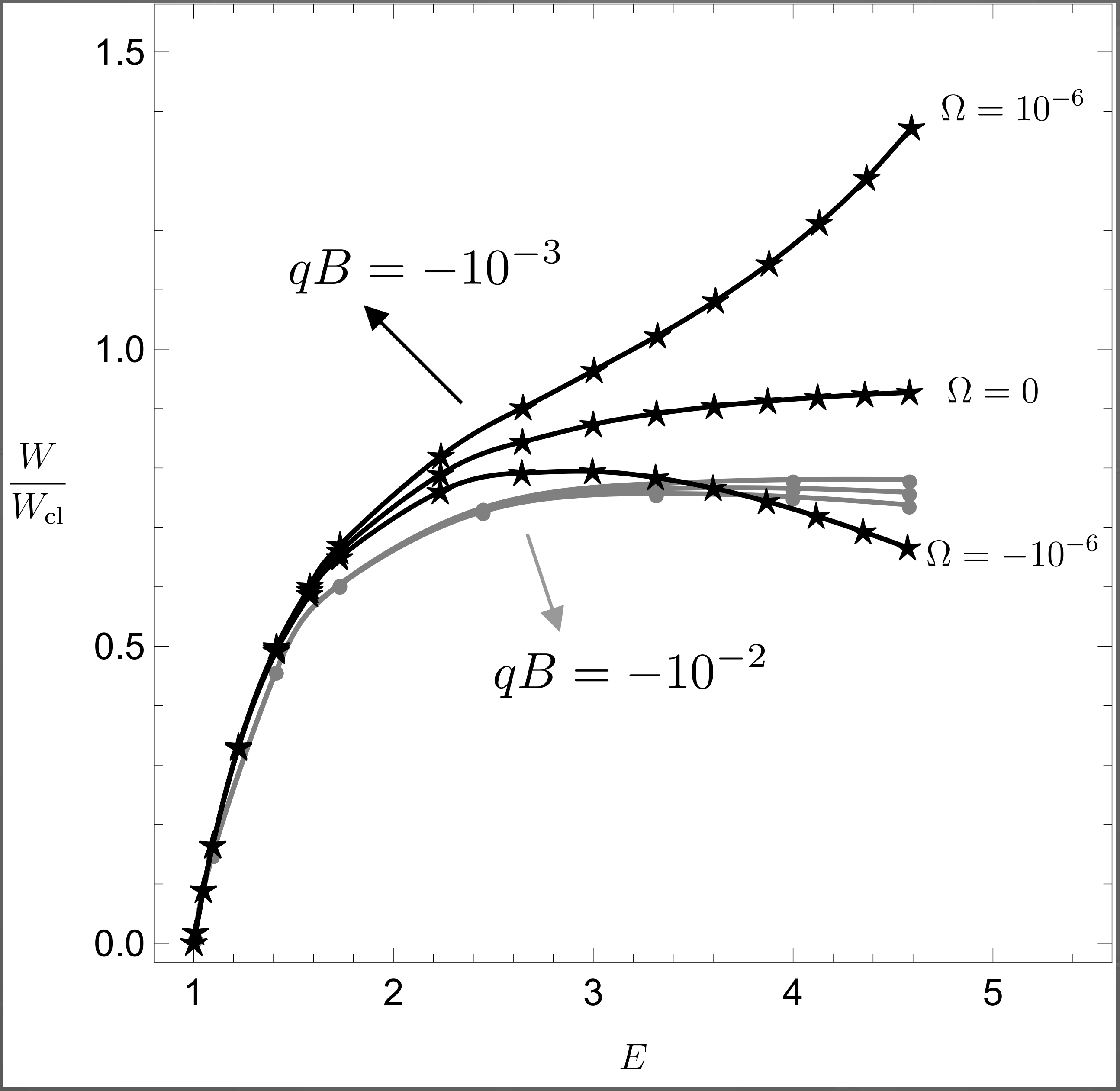
- A charge embedded into uniformly rotating medium in magnetic field radiates synchrotron radiation that depends on B and Ω.
- Radiation intensity increases for Ω>0 and decreases for Ω<0 (assuming qB<0)
- At any Ω the effect emerges for large enough energy

  → Thus even if $\Omega \ll \sqrt{|qB|}$, there is an effect at high E.

- Fast rotation and boundary conditions?
- Synchrotron photons in the QGP?
- Not only Heavy-ion collisions. Astrophysics (Rotating black holes, compact stars)

Thank you!

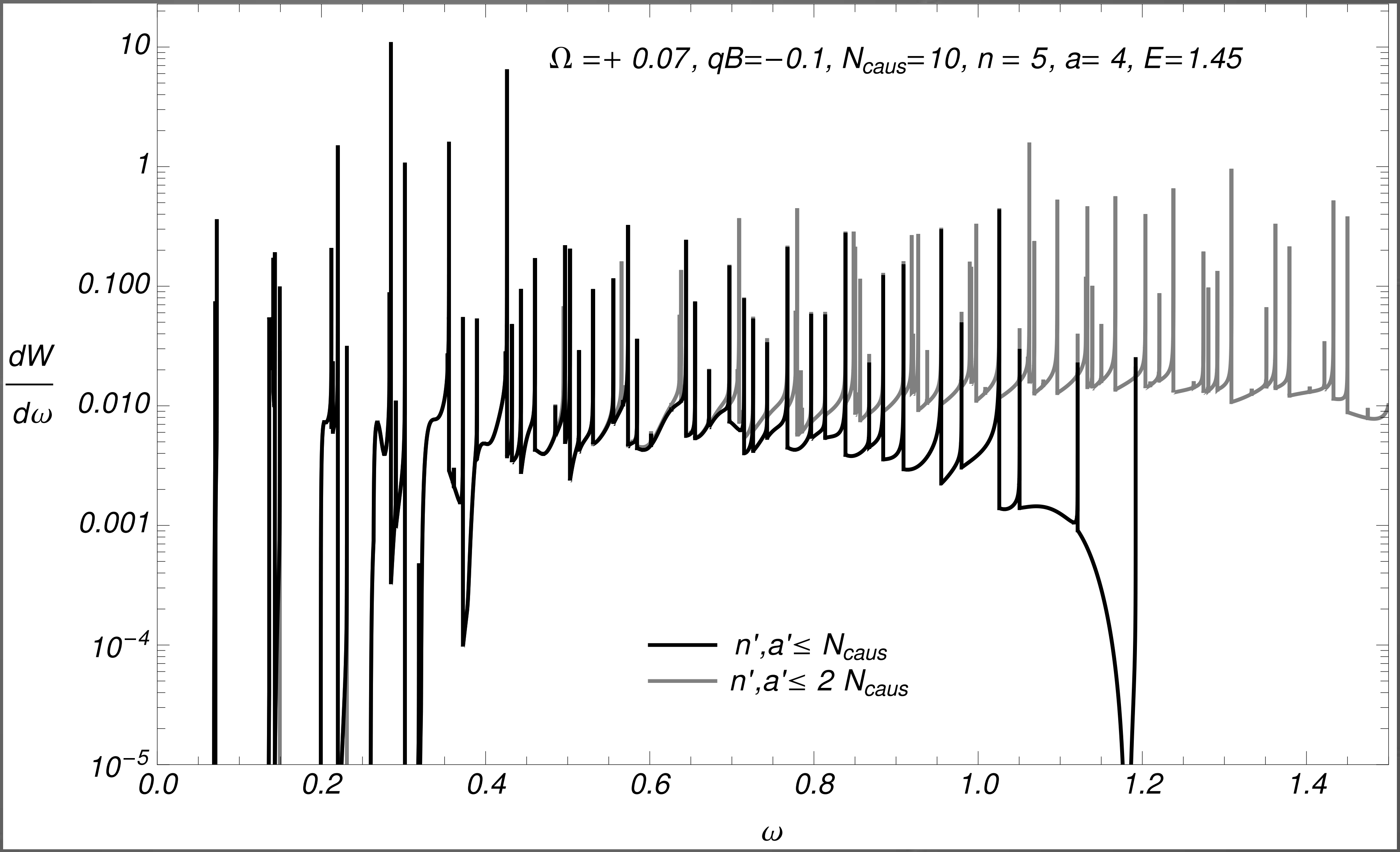
Other values of qB



$\hbar = c = M = 1$

$$W_{cl} = \frac{2\alpha}{3}(qB)^2\left(1 - \frac{1}{E^2}\right)E^2$$

Effect of the boundary

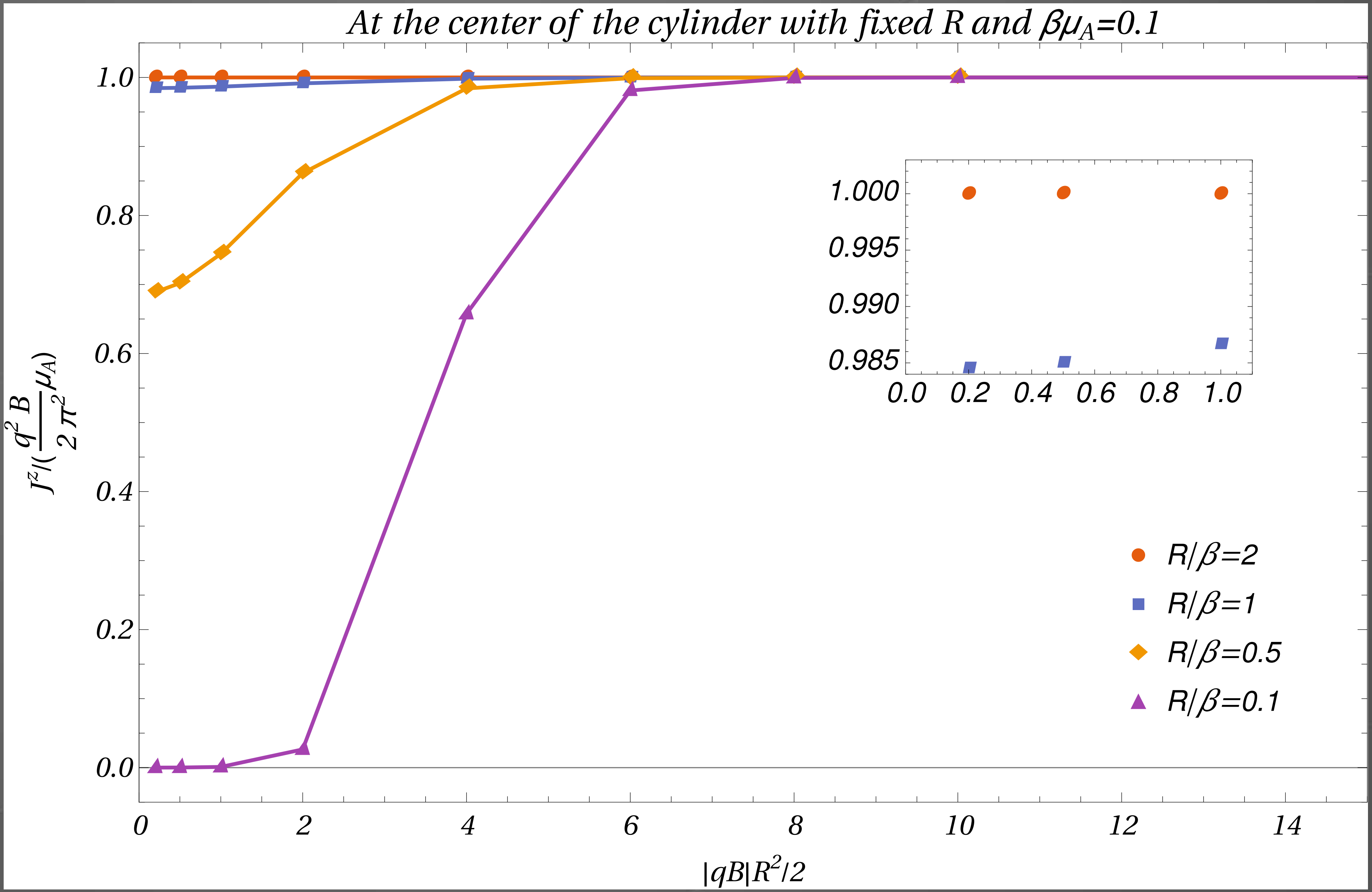


$\hbar = c = M = 1$

$$N_{\text{caus}} = \frac{|qB|}{2\Omega^2}$$

CME with MIT BC

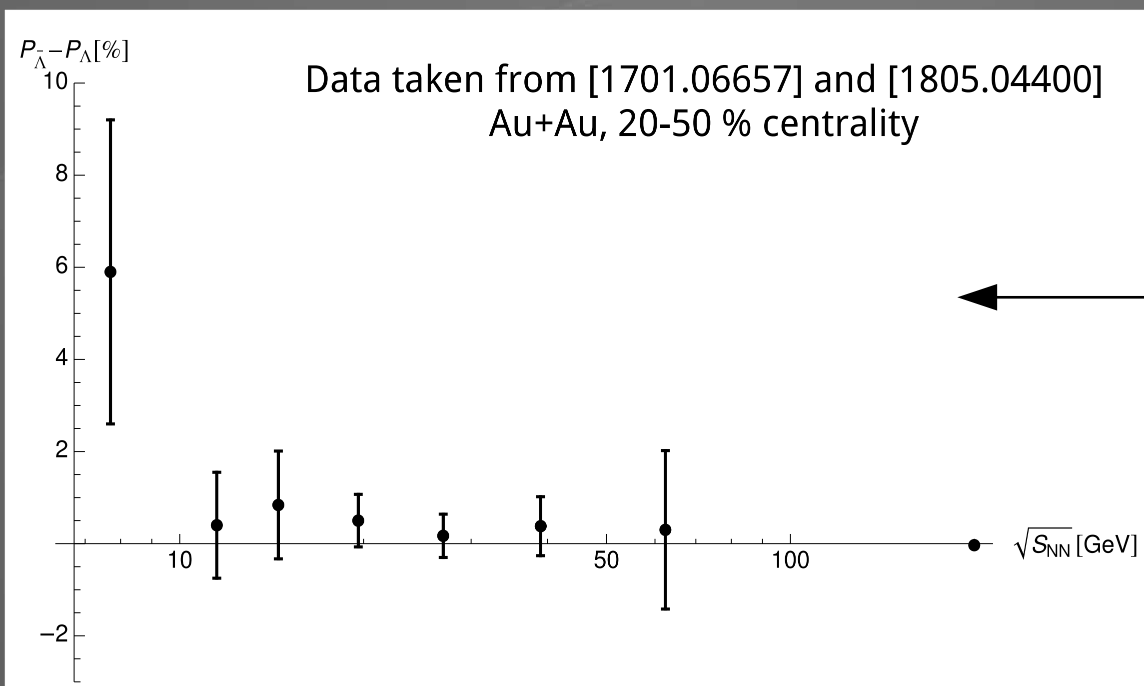
[MB and K. Tuchin, 2305.13149]



R: radius of the cylinder; β=1/T: inverse temperature; μ_A: axial imbalance

Spin polarization induced by Magnetic field

$$S^\mu = \frac{1}{4m}(1 - n_F)\beta\left(qB^\mu - u^\mu \frac{qB \cdot p}{\varepsilon_p}\right)$$



Magnetic contribution in spin polarization?

B. Müller, A. Schäfer, PRD 98 (2018)
S. Ryu, V. Jupic, C. Shen, PRC 104 (2021)
X.-Y. Wu, C. Yi, G.-Y. Qin, S. Pu, PRC 105 (2022)
O. Vitiuk, L.V. Bravina, E.E. Zabrodin, Phys. Lett. B 803 (2020)

Late time magnetic field is too small
[H. Li, X. Xia, X-G. Huang, H-Z. Huang, 2306.02829]