Rotation-induced Perturbative Realization of Nonperturbative QCD Phenomena

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— 7th Chirality, Vorticity and Magnetic Field in HIC —

Physics Backgrounds

Rotation Effects

Suppose that $\Omega \sim 10^{22}\,\mathrm{s}^{-1}$ and we know 1fm $\sim 3\times10^{-24}\,\mathrm{s}$.

Therefore, at $\tau = 1$ fm/c the rotation angle is only a few %.

If the thermalization time $\sim \tau = 0.1$ fm/c, it makes sense to think of the phase diagram with the surface velocity: $v/c \sim 0.1$

No hair theorm of Black Holes → Mass/Charge/Ang. Mom.

QGP as a holographic dual of black hole should be characterized by temperature (mass), density (charge), and angular momentum.

Maximum neutron star mass depends on angular mom.

Black widow (PSR J0952-0607) $\sim 2.35 M_\odot$ with $\Omega \sim 5\times10^{3}\,\mathrm{s}^{-1}$.

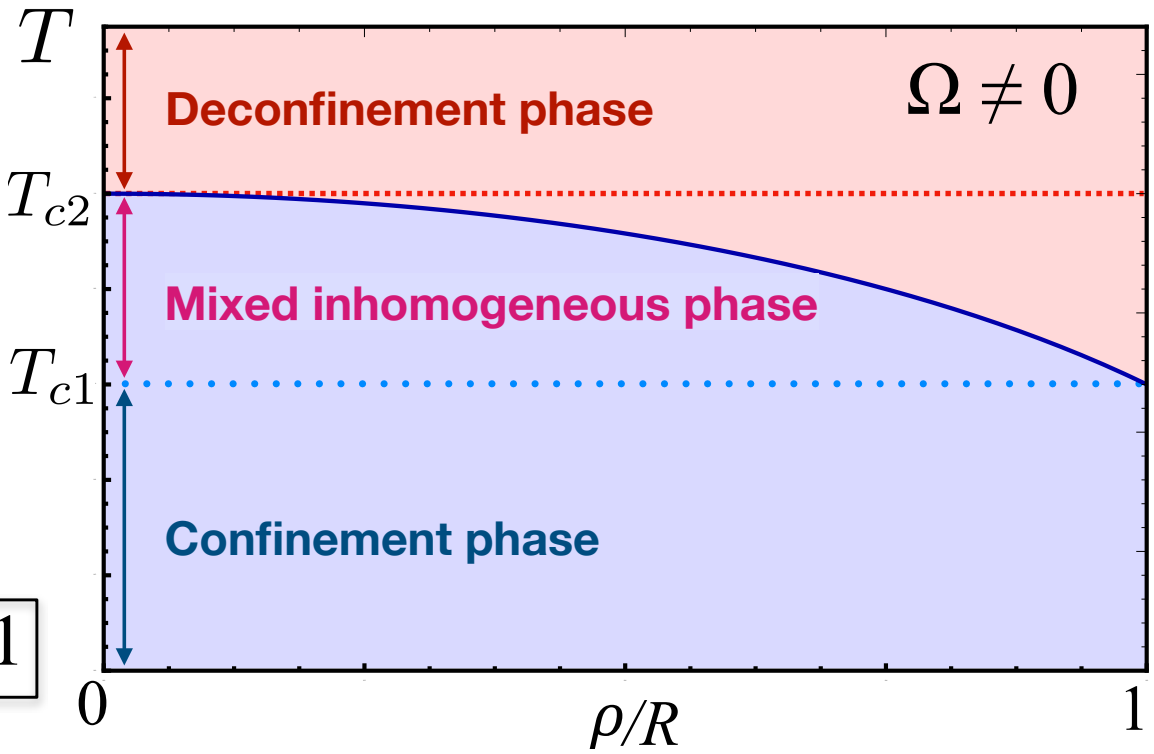
The surface velocity: $v/c \sim 0.2$

Controversies

Tolman-Ehrenfest Effect

Position dependent T ?

Chernodub PRD (2021), (2022)



$g_{00} = 1$

$g_{00} = 1 - \rho^2\Omega^2$

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$$

Landau-Lifshitz gives the full explanation for this formula.

Tolman-Ehrenfest Effect

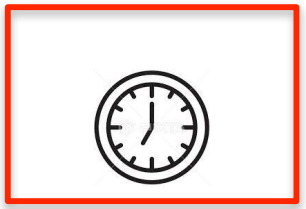
Good analogy — Time delay under gravity

$\Delta\tau_{\mathrm{I}} = \Delta\tau_{\mathrm{F}}$

Height I

g

Free Falling



$\Delta\tau_{\mathrm{F}}$

$$m\Delta\phi = \frac{1}{2}mv^2$$

$\Delta\tau_{\mathrm{II}} < \Delta\tau_{\mathrm{F}}$

Height II

Reference observer in the frame to be inertial (free falling)

$$\Delta\tau_{\mathrm{II}} = \sqrt{1 - \frac{2\Delta\phi}{c^2}}\,\Delta\tau_{\mathrm{I}} \;\rightarrow\; \Delta\tau_{\mathrm{II}} = \sqrt{g_{00}}\,\Delta\tau_{\mathrm{I}}$$

(No twin paradox in this case.)

Tolman-Ehrenfest Effect

Conserved energy and the local energy

Local time would become smaller under gravity: $\tau = \sqrt{g_{00}}\,\tau_0$

Local energy would become larger under gravity: $E = E_0/\sqrt{g_{00}}$

Entropy counts the degrees of freedom of the system, which should not be affected by gravity. $\rightarrow S$ **unchanged**

Local temperature $\partial E/\partial S \sim T$ **would become larger:** $T = T_0/\sqrt{g_{00}}$

Here, the conserved energy is E_0 **and the Lagrange multiplier in the distribution function should be** T_0 **.**

The local observer sees $E(\boldsymbol{x})$ **and** $T(\boldsymbol{x})$ **but** $E/T = E_0/T_0$ **.**

Equilibrium condition = Constant T_0 **and constant** μ_0

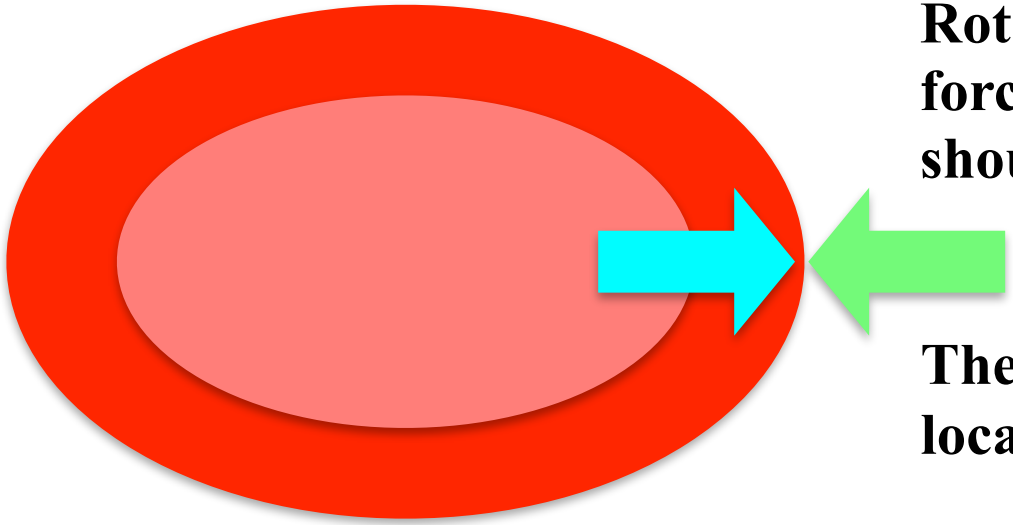
Inhomogeneous Pressure

Constant temperature T_0 does NOT mean that the system is simply uniform. → Pressure has gradient!

In the rotating frame: $dF' = -SdT - \boldsymbol{J} \cdot d\boldsymbol{\Omega}$

The energy in the rotating and the static frames: $E' = E - \boldsymbol{J} \cdot \boldsymbol{\Omega}$

This means: $dF = -SdT + \boldsymbol{\Omega} \cdot d\boldsymbol{J}$



Rotating observer sees the centrifugal force and the pressure gradient which should be balanced in equilibrium.

The larger pressure implies the larger local temperature for constant T_0 .

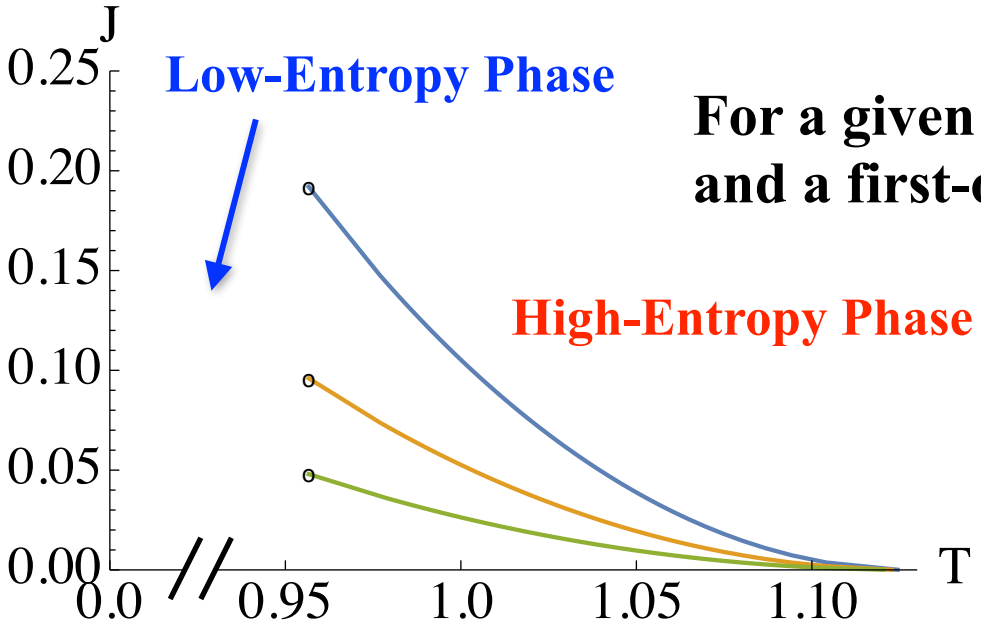
Canonical vs. Grand Canonical

Ahmed-Cong-Kubiznak-Mann-Visser (2023)

$$\text{fixed} \quad (J, \mathcal{V}, C): \qquad F \equiv E - TS \qquad \textbf{Canonical}$$

$$\text{fixed} \quad (\tilde{\Omega}, \mathcal{V}, C): \qquad W \equiv E - TS - \tilde{\Omega} J \qquad \textbf{Grand Canonical}$$

$$\text{fixed} \quad (J, \mathcal{V}, \mu): \qquad G \equiv E - TS - \mu C$$



For a given *J*, there are multiple solutions and a first-order phase transition occurs.

Also, the "end-point" appears that is of second-order.

Canonical vs. Grand Canonical

Ahmed-Cong-Kubiznak-Mann-Visser (2023)

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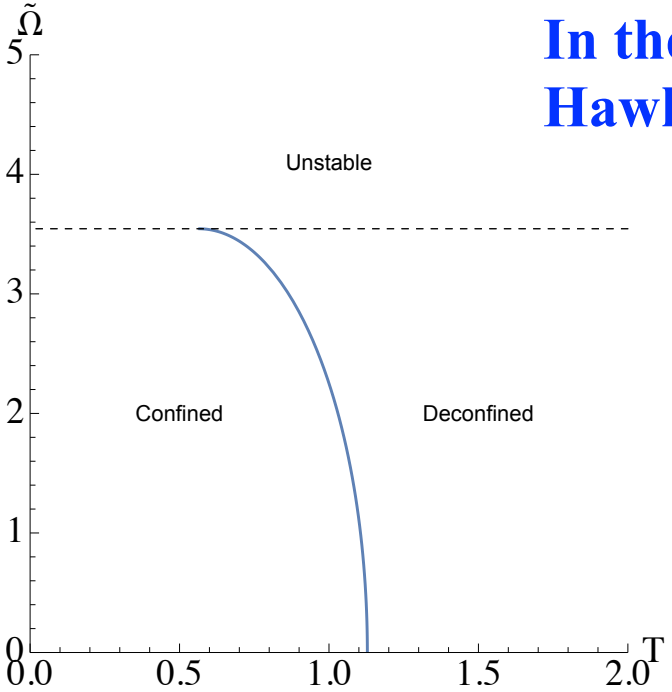
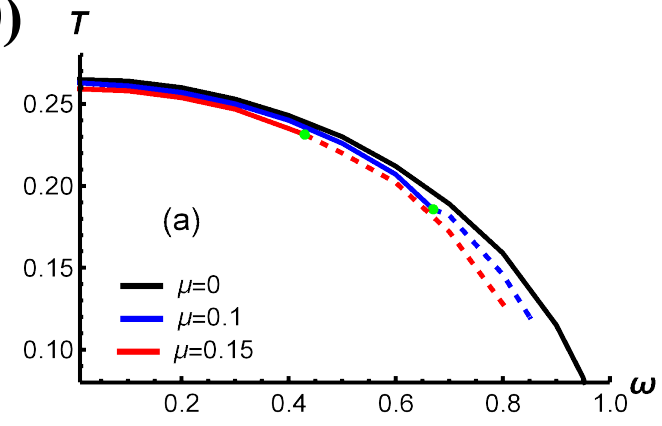


In the grand canonical case, a first-order Hawking-Page transition is seen.

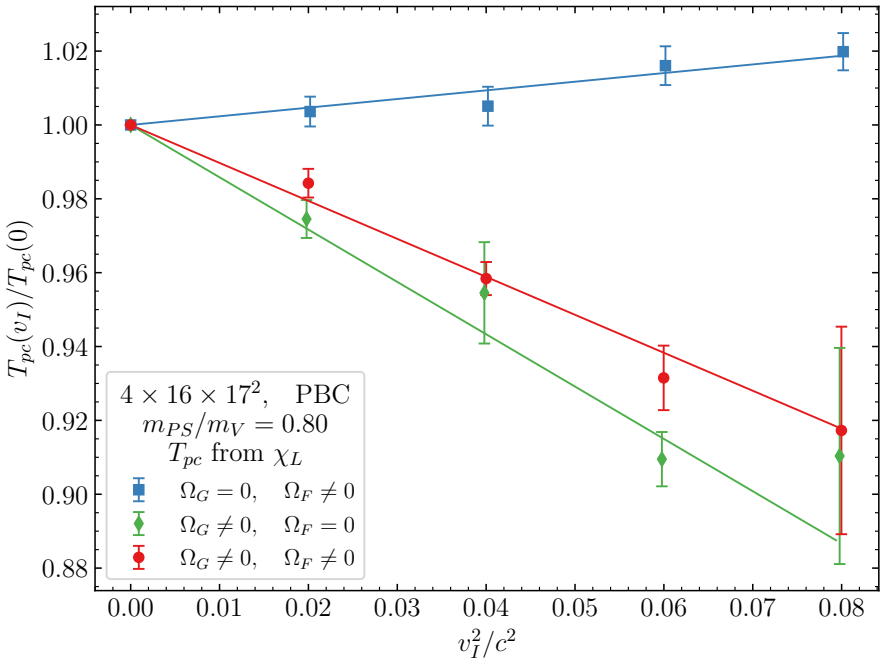
Consistent with Chen-Zhang-Li-Hou-Huang (2020)



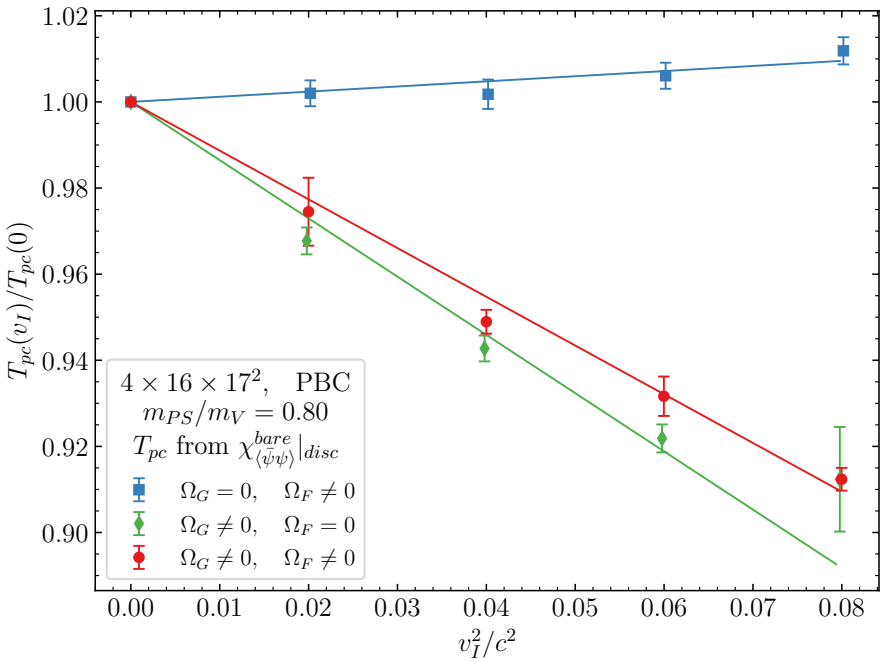
Lattice QCD

Braguta-Kotov-Roenko-Sychev (2023)

[Deconfinement]



[Chiral Restoration]



Imaginary Velocity ⟶

Sizable effects already at $v/c \sim 0.1$

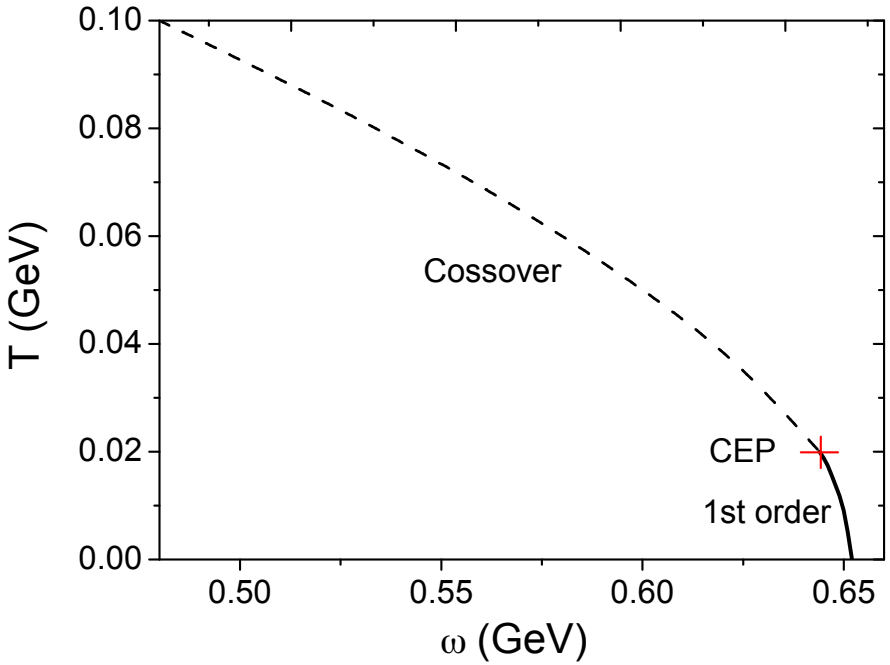
Critical temperautre increases with more rapid rotation!?

Lattice QCD

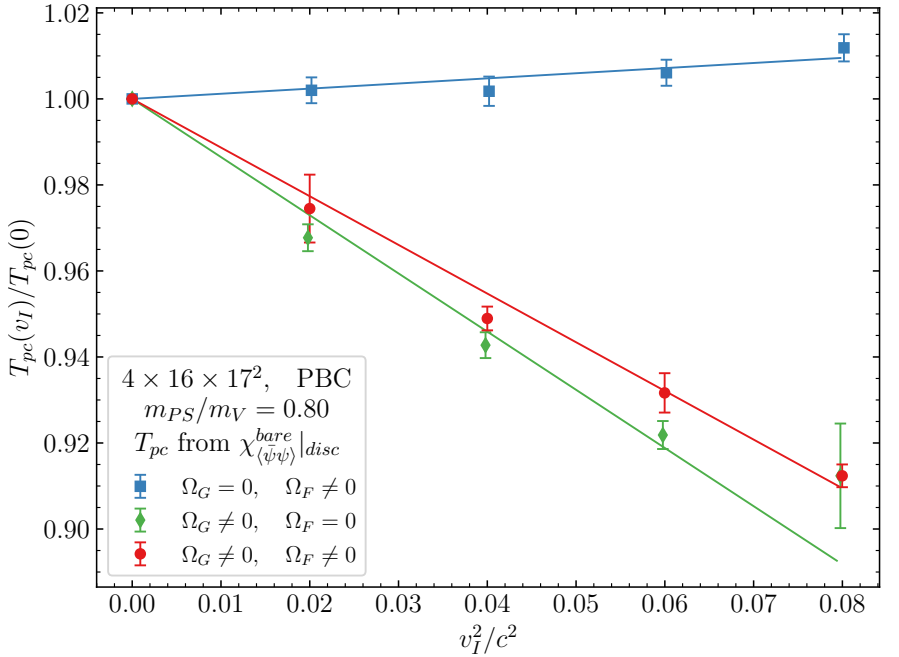
Braguta-Kotov-Roenko-Sychev (2023)

[Chiral Restoration]

Chiral sector is not contradicting with the chiral model studies.



Jiang-Liao (2016)



Imaginary Velocity →

Perturbative QCD

Quantities in the Rotating Frame

Theoretical treatments in the rotating frame

Chen-Fukushima-Shimada (2022/2023)

Two seemingly different but EQUIVALENT approaches:

Writing down the EoM with the metric —

$$g_{\mu\nu} = \begin{pmatrix} 1-\Omega_{\rm I}\rho^2 & -i\Omega_{\rm I}\rho^2 & 0 & 0 \\ -i\Omega_{\rm I}\rho^2 & \rho^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Writing down the EoM with the boundary condition —

$(\tau,\, \theta,\, r,\, z) \sim (\tau+\beta,\, \theta-\tilde{\Omega}_{\rm I},\, r,\, z)$ **with the flat metric**

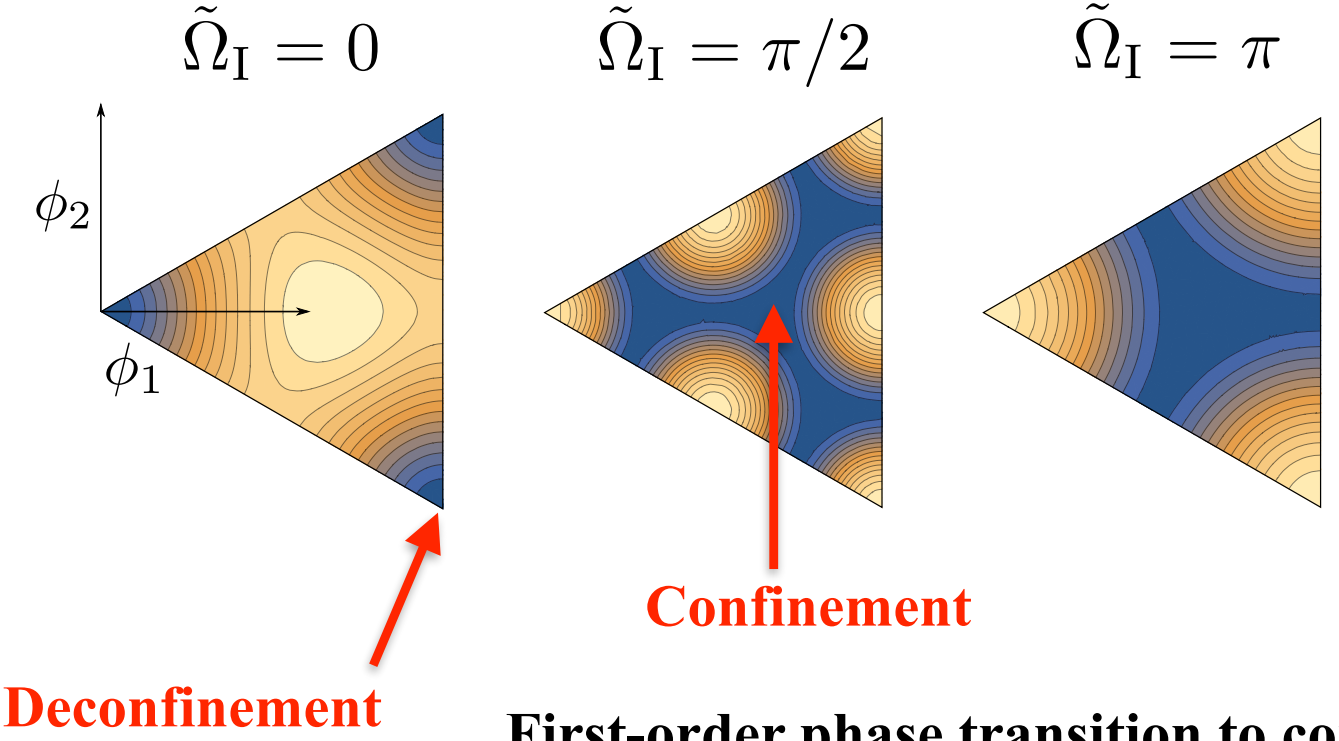
Temporal derivative has a mixture with the angular momentum

They lead to the same energy spectra! **(Another controversy…)**

Confinement

Chen-Fukushima-Shimada (2022/2023)

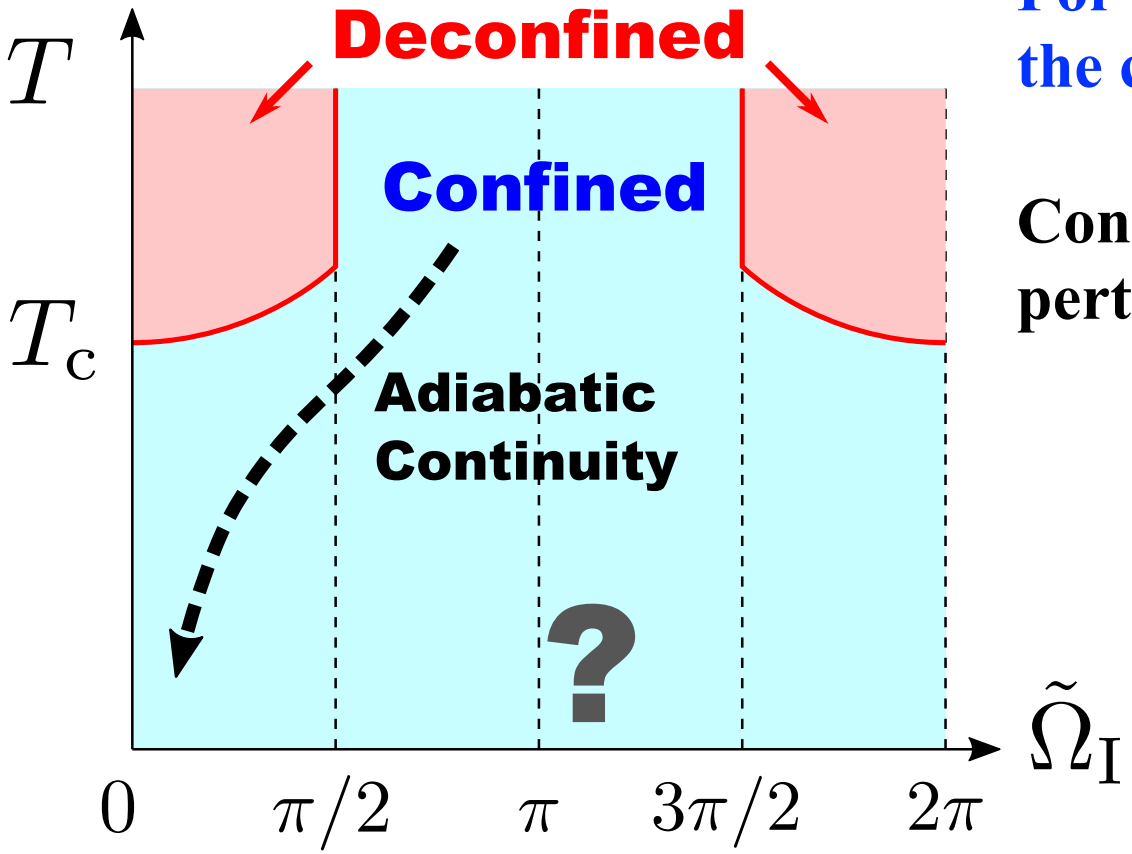
Real-time rotation is dynamical, while imaginary-time rotation is geometrical (like the chemical potential).



First-order phase transition to confinement!

Confinement

Chen-Fukushima-Shimada (2022/2023)



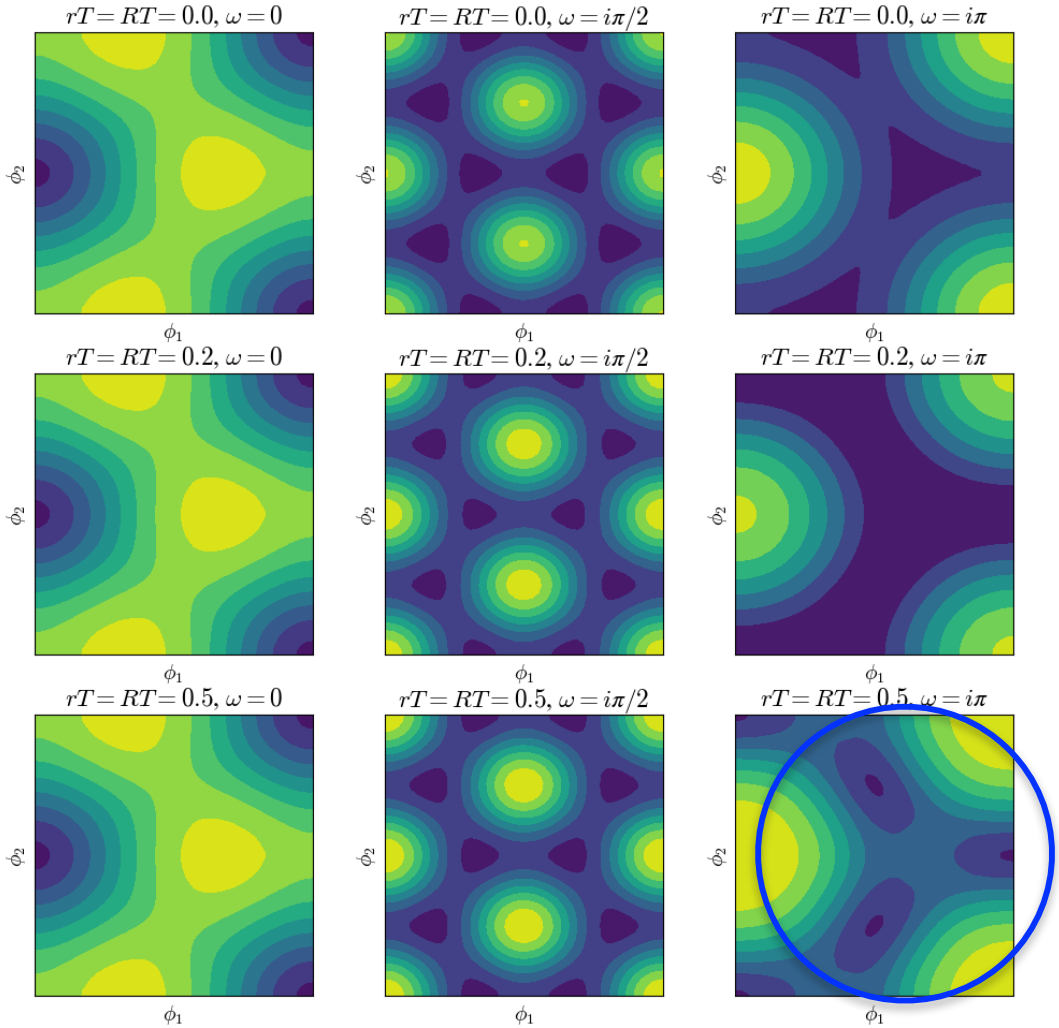
For whatever high temperature the confined phase is favored!

Confinement can be studied by perturbative calculations only.

Chiral symmetry!?

Emergent Symmetry

Chen-Fukushima-Shimada (2022/2023)



Even without fermions pure gluonic theory shows a very exotic phase pattern with increasing distance.

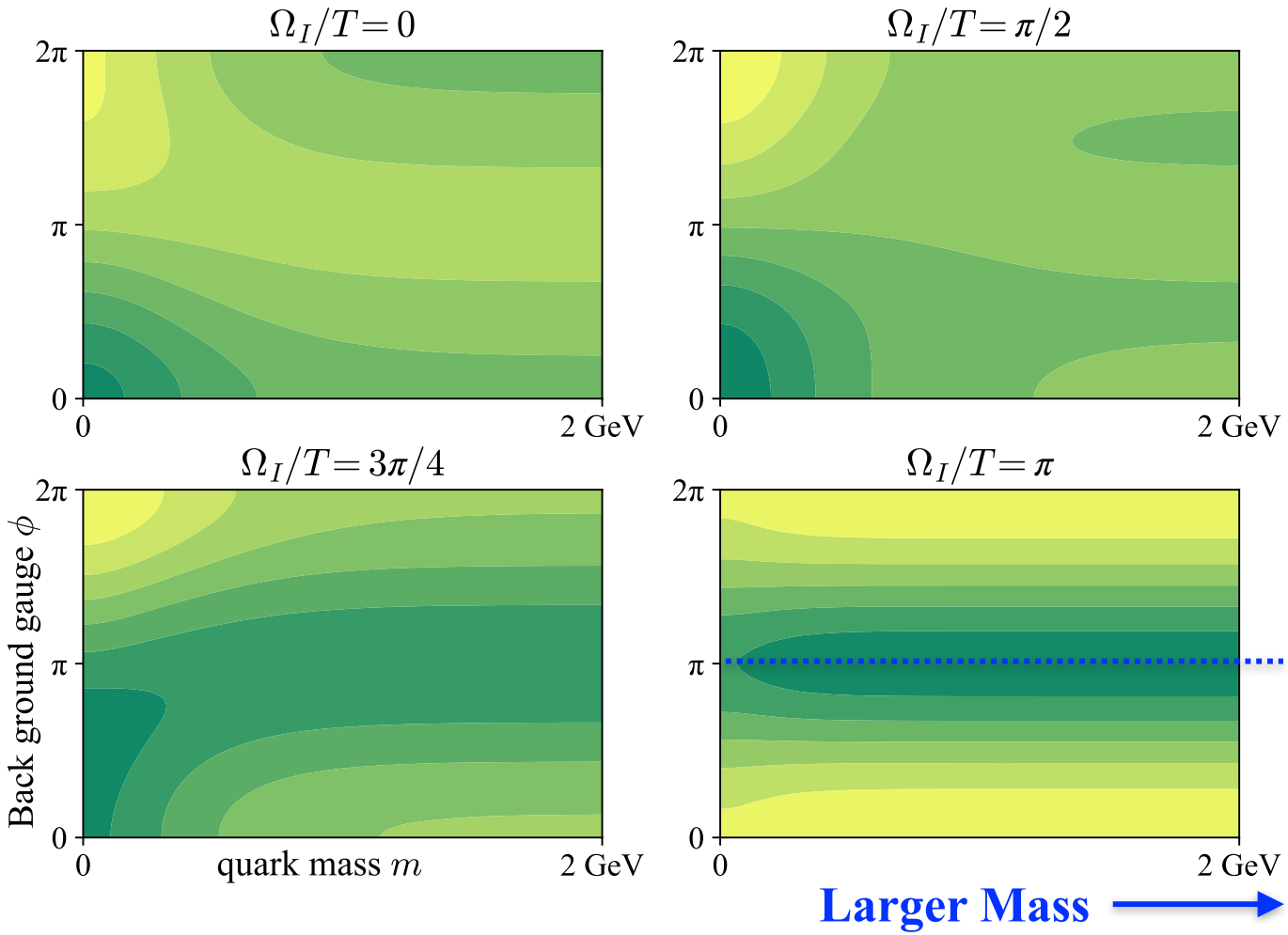
T_0 is constant, but still, the pressure (and the potential) is position dependent.

Secondary phase transition at $\rho \neq 0$.

Spontaneous breaking of "emergent Z_3 symmetry"

Chiral Symmetry

Chen-Fukushima-Shimada (2022/2023)



Potential minimum is shifted toward the larger mass as the imaginary-rotation increases.

Dynamical mass generation concluded!

Confined Phase

Experimental Observable

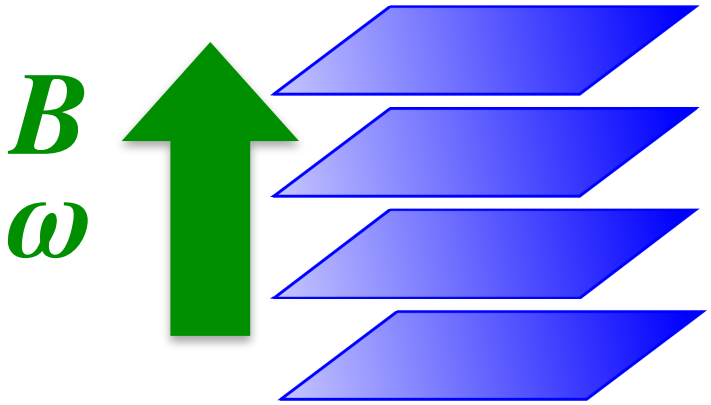
Inhomogenous States

Rotation may induce:

Inhomogeneity with spontaneous breaking of emergent symmetry

if it survives in the regime with real rotation velocity.

Chiral Soliton Lattice — Huang-Nishimura-Yamamoto (2017)



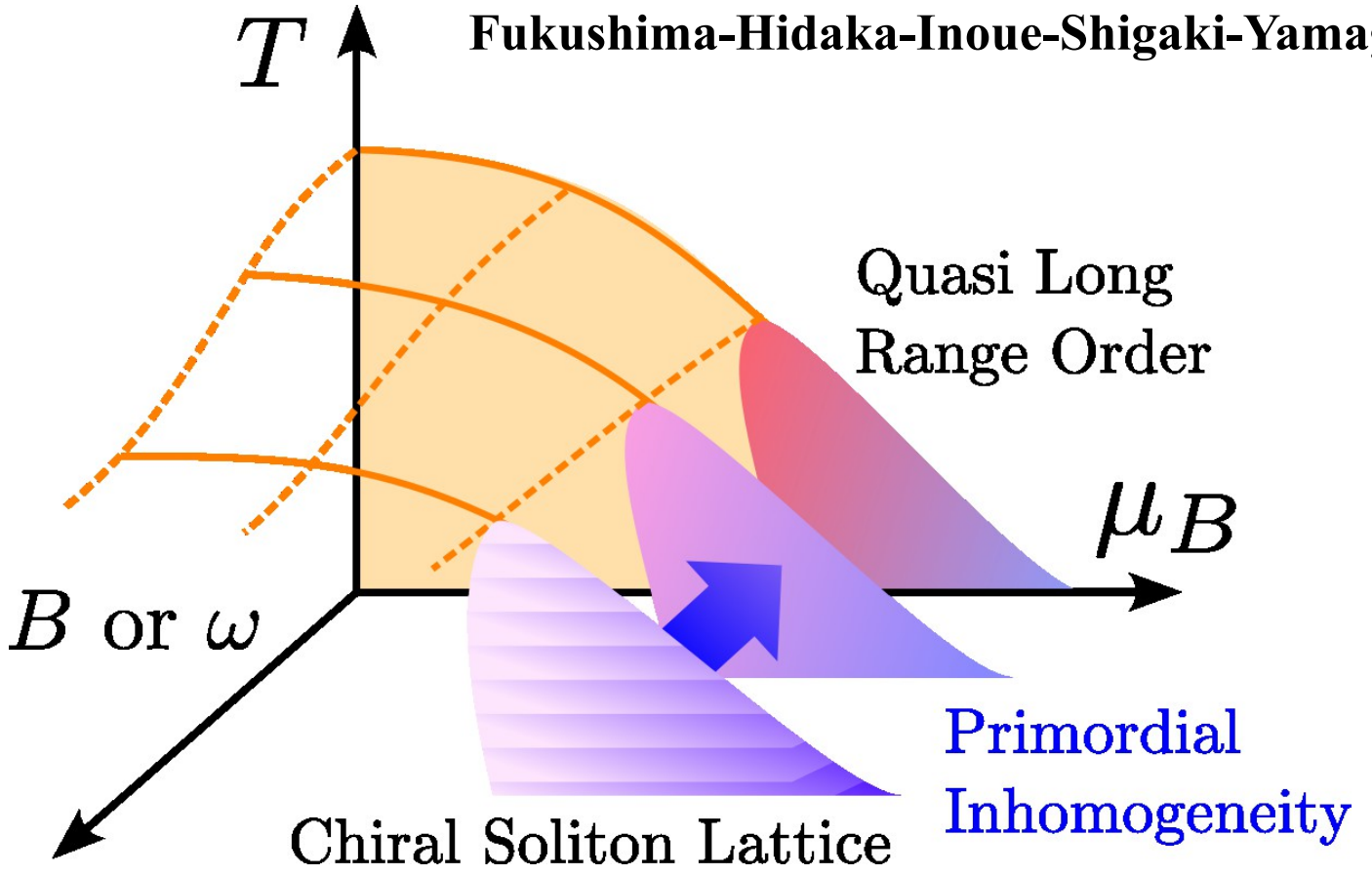
~ Layered pion domain walls

Inhomogeneous states are fragile against fluctuations in rotationary symmetric systems, but the rotation could be the "perturbation" to induce inhomogeneity.

cf. Moat Regime by Rennecke-Pisarski (2021)

Primordial Inhomogeneity

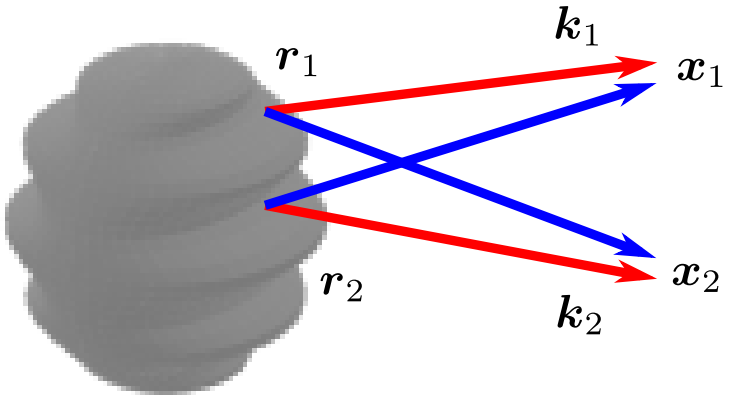
Rotation as a perturbation to induce inhomogeneity



Primordial Inhomogeneity

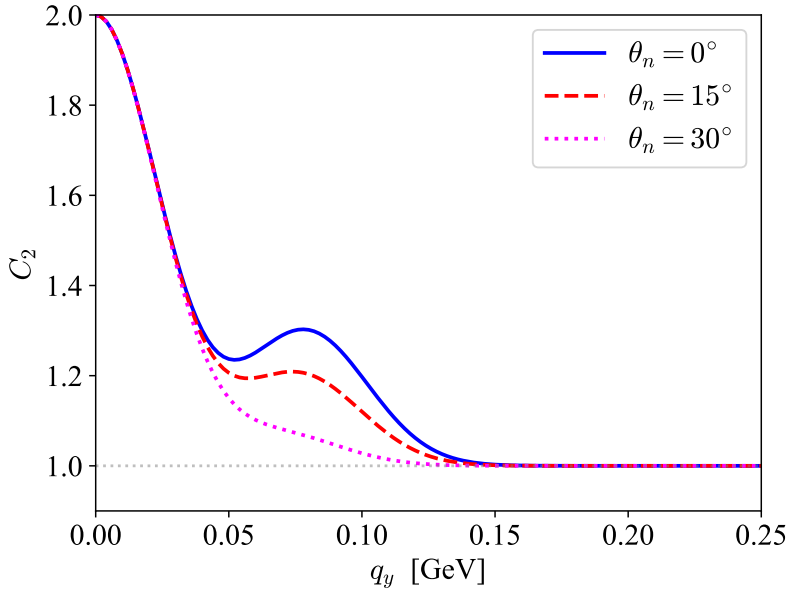
HBT sensitve to the size of the "cluster"

Fukushima-Hidaka-Inoue-Shigaki-Yamaguchi (2023)



Inhomogeneous particle distribution assumed in the source function.

HBT with the Gaussian formalism



Precise identification of the 1D modulation axis is very important.

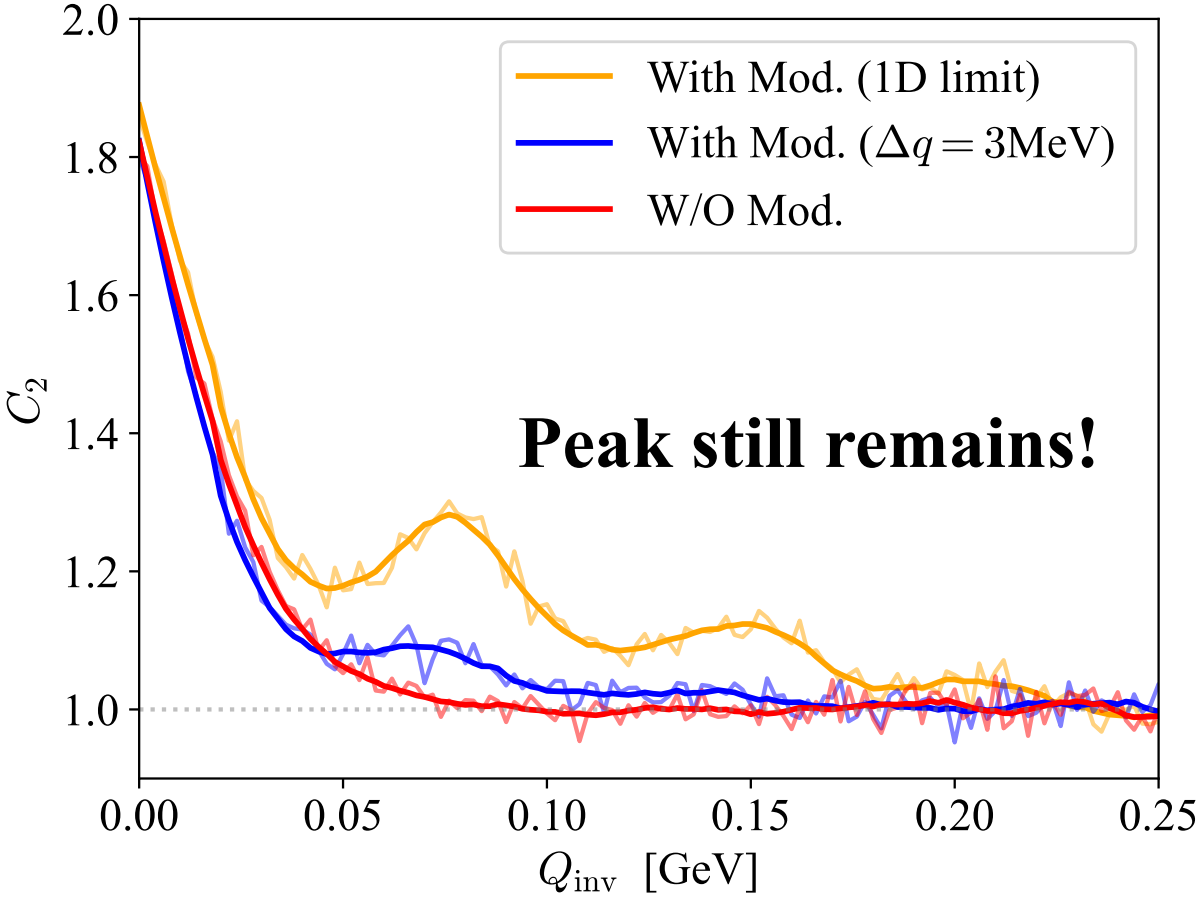
Peak stands at the modulation scale.

cf. Rennecke-Pisarski-Rischke (2023)

Primordial Inhomogeneity

More realistic accessment using AMPT

Fukushima-Hidaka-Inoue-Shigaki-Yamaguchi (2023)



Summary

- **Rotation involving various controversies.**
  - □ Interpretation of the Tolman-Ehrenfest effects
  - □ Tension between lattice-QCD and others
- **Perturbative realization with imaginary rotation**
  - □ Confinement at whatever high temperature
  - □ Dynamical mass at whatever high temperature
  - □ Spontaneous breaking of emergent symmetry???
- **HBT signature for the primordial inhomogeneity**
  - □ Magnetic field and/or rotation perturbing the system
  - □ Peak may survive at the scale of spatial modulation.
  - □ Axis dependence is extremely large… why???