Improving the ZPC parton cascade with an exact solution of the relativistic Boltzmann equation

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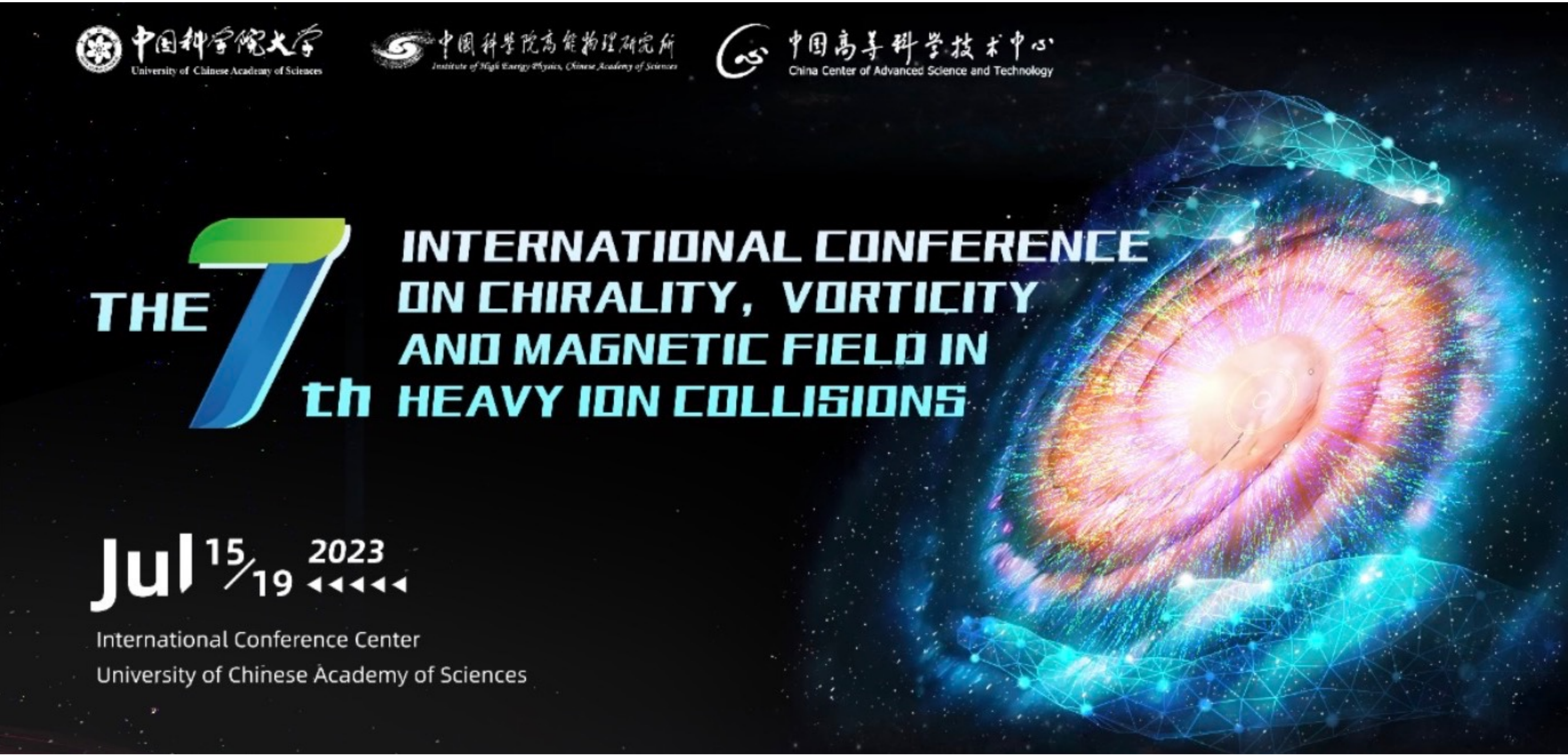


Outline

- Introduction
- Earlier test and improvement of the ZPC parton cascade
- Recent improvement with an exact solution of RBE
- Outlook and summary

Based on works with Todd Mendenhall,
Xin-Li Zhao, Guo-Liang Ma, and Yu-Gang Ma, …

National Science Foundation

Introduction: transport models for non-equilibrium

- **For large systems at very high energies:**
  transport models are similar to hydrodynamics,
  transport models (using microscopic particles & scatterings)
  are complementary to
  hydrodynamics-based models (using $T_{\mu\upsilon}$, EoS & transport coefficients).

- **For finite/small systems at finite energies:**
  non-equilibrium effects are expected to be important.
  One example is the escape mechanism for flow:
  interaction-induced response from kinetic theory
  to the anisotropic spatial geometry (*without collective flow*).

  Liang He et al., PLB (2016);
  ZWL et al., NPA (2016);
  Hanlin Li et al., PRC (2019)

- **Recent small system data also seem to show collective flow signals:**
  are they real signals from collectivity?
  do they require formation of a parton matter?
  is the small system far from or close to equilibrium?
  To answer these questions and study properties of parton matter/QGP,
  transport models/kinetic theory are crucial as they address non-equilibrium dynamics.

Heiselberg & Levy, PRC (1999),
Borghini et al., EPJC (2018),
Kurkela et al., PLB (2018) & EPJC (2019), …

Introduction: the ZPC parton cascade

Currently, ZPC solves the Boltzmann equation for 2-body scatterings:

$$\partial_t f + \frac{\partial \boldsymbol{x}}{\partial t} \cdot \nabla_x f = C[|M^2|f_1 f_2] \propto \sigma f_1 f_2$$

- $gg \rightarrow gg$ cross section in leading-order pQCD is used

$$\frac{d\sigma_{gg}}{dt} = \frac{9\pi\alpha_s^2}{2s^2}\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)$$
$$\simeq \frac{9\pi\alpha_s^2}{2}\left(\frac{1}{t^2} + \frac{1}{u^2}\right) \simeq \frac{9\pi\alpha_s^2}{2t^2}$$

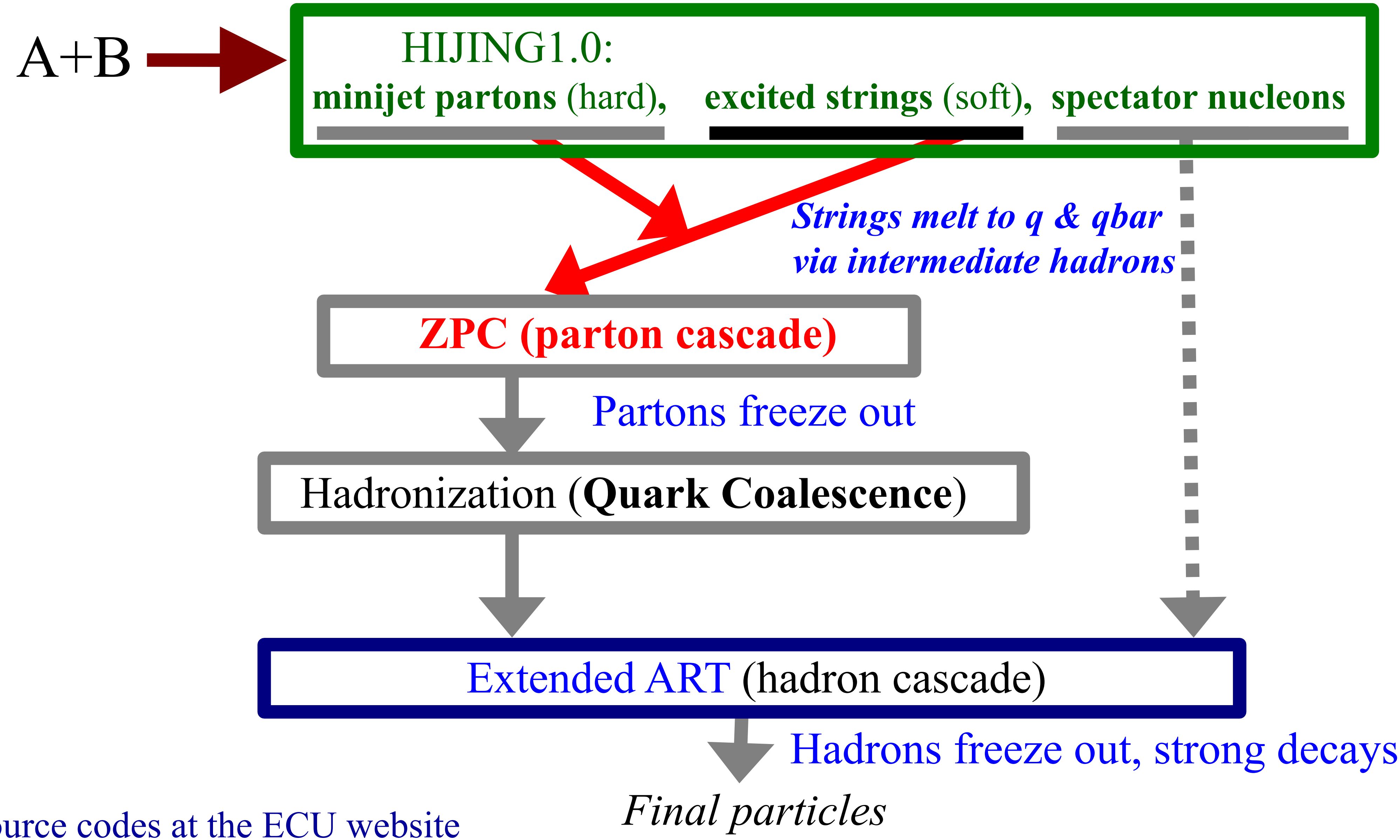
Bin Zhang, Comp Phys Comm (1998);
ZWL, Ko, Li, Zhang & Pal, PRC (2005)

- σ is divergent for massless g, so a Debye screening mass μ is applied:

$$\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{2} \frac{1+a}{(t-\mu^2)^2}$$

$a \equiv \frac{\mu^2}{s}$ is added to obtain an s-independent cross section: $\sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$

Introduction: a multi-phase transport (AMPT) model

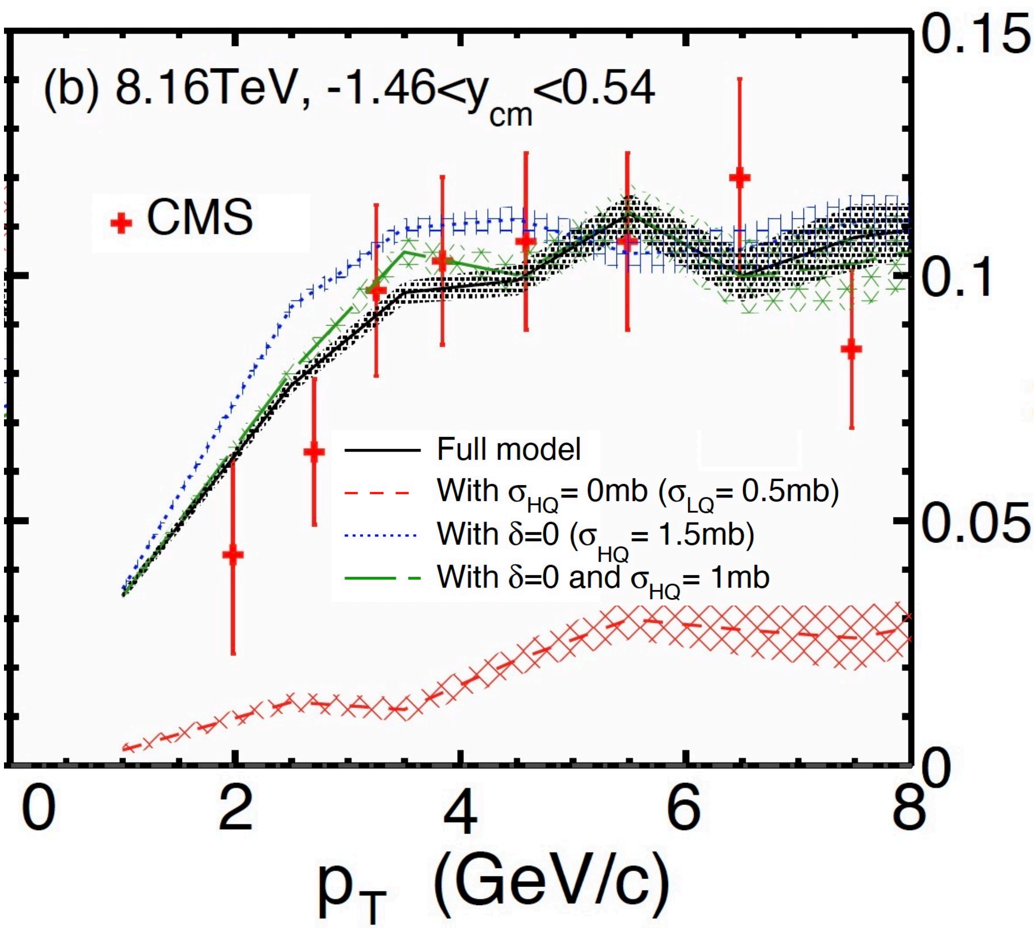


Source codes at the ECU website
https://myweb.ecu.edu/linz/ampt/
ZWL, Ko, Li, Zhang & Pal, PRC (2005);
ZWL & Liang Zheng, Nucl Sci Tech (2021)

Introduction: the ZPC parton cascade

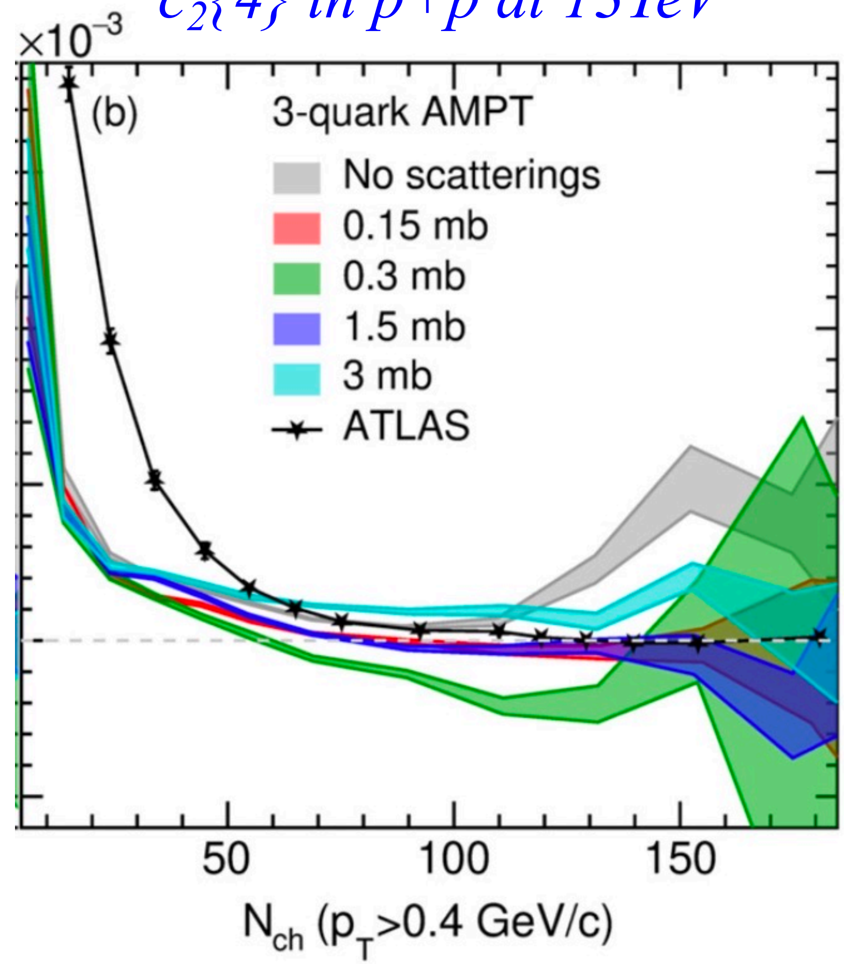
In the AMPT model, parton interactions in ZPC are responsible for generating flows in big systems & heavy flavor flows in small systems, they also significantly affect $c_2\{4\}$ in p+p collisions:

D^0 v_2 in p-Pb



Chao Zhang et al., 2210.07767

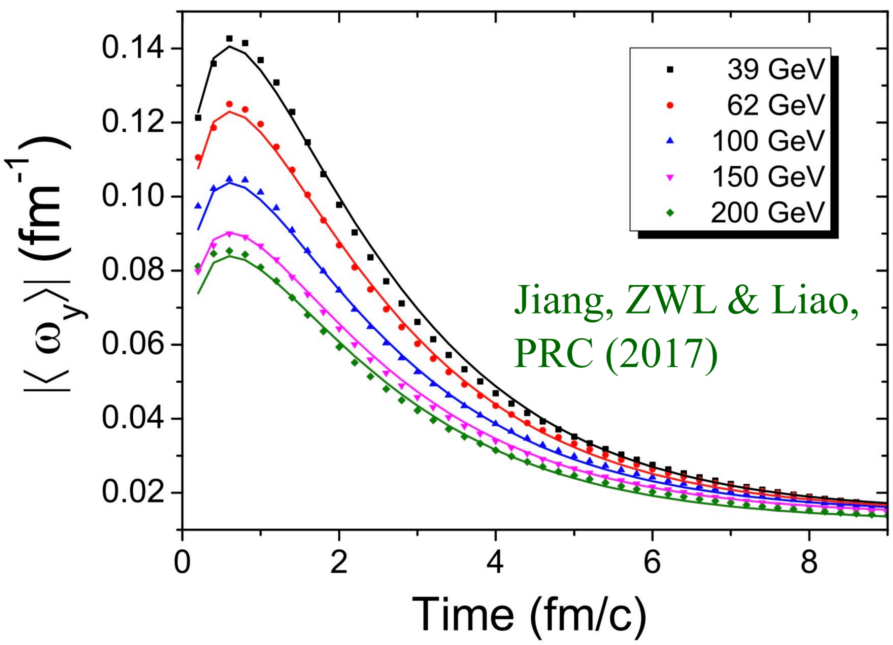
$c_2\{4\}$ in p+p at 13TeV

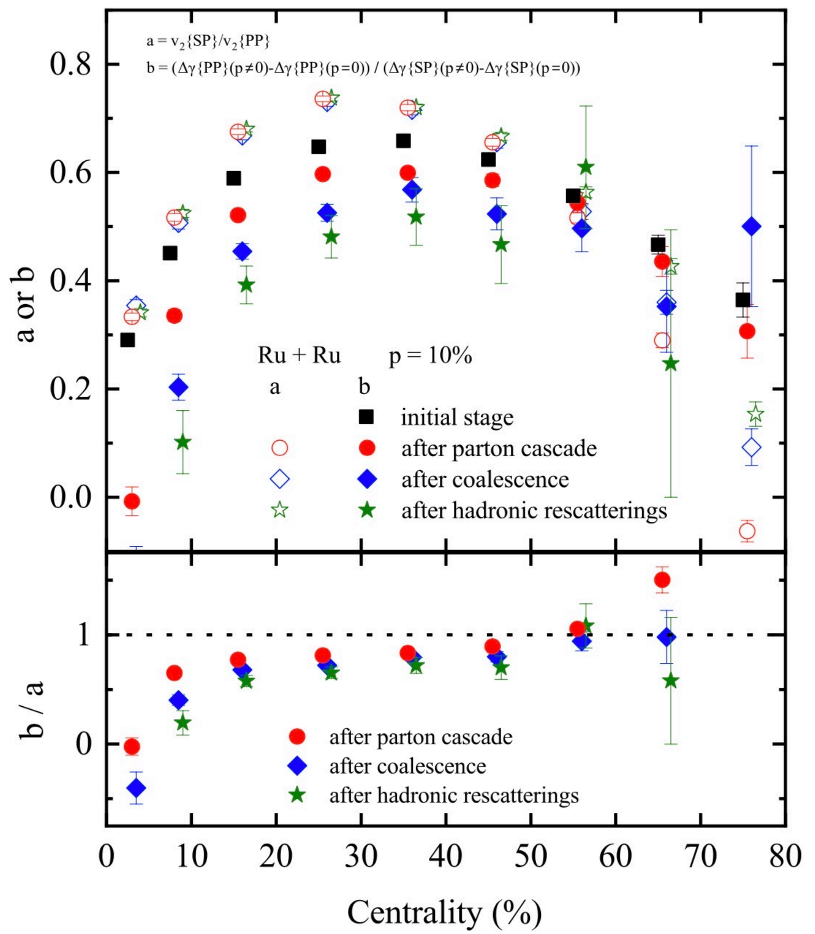


Xin-Li Zhao et al., PLB (2023)

Introduction: the ZPC parton cascade

Parton interactions modify the evolution of vorticity fields (e.g. through flows), E&M fields & CME signals:





Zilin Yuan's talk on 7/19:
ZPC is extended to perform chiral anomaly transport under the influence of magnetic fields

Guo-Liang Ma's talk on 7/15;
Chen, Zhao & Ma, 2301.12076

So we need to check: *is the parton cascade accurate?*
if not, how to improve its accuracy?

Earlier test and improvement of ZPC

Currently, ZPC solves the Boltzmann equation for 2-body scatterings:

$$\partial_t f + \frac{\partial \boldsymbol{x}}{\partial t} \cdot \nabla_x f = C[|M^2|f_1 f_2] \propto \sigma f_1 f_2$$

But ZPC/MPC cascade solution of the relativistic Boltzmann equation (RBE) at large densities n and/or cross sections σ is well known to suffer from causality violation.

Zhang, Comp Phys Comm (1998); Monlar & Gyulassy, PRC (2000); Cheng et al., PRC (2002); …

Naively, the cascade solution using geometric cross sections is only accurate in the dilute limit when the opacity parameter χ is small:

$$\chi \equiv \frac{r}{\lambda} = \frac{\sigma^{3/2} n}{\sqrt{\pi}} < 1,$$

Zhang, Gyulassy & Pang, PRC (1998)

i.e., when the range of particle interaction $\boldsymbol{r}$ < mean free path $\boldsymbol{\lambda}$

$$r \equiv \sqrt{\frac{\sigma}{\pi}} \qquad \lambda = \frac{1}{\sigma n}$$

Earlier test and improvement of ZPC

Particle subdivision (or the test particle method) reduces/removes causality violation:

Pang, CU-TP-815 (1996)
Gyulassy, Zhang, Pang, PRC (1998)

$$\partial_t f + \frac{\partial \boldsymbol{x}}{\partial t} \cdot \nabla_x f = C[|M^2| f_1 f_2] \propto \sigma f_1 f_2$$

This is because the above Boltzmann equation is invariant under transformation:

$f \longrightarrow f * l$ and $\sigma \longrightarrow \frac{\sigma}{l}$ ($\frac{d\sigma}{dt} \longrightarrow \frac{d\sigma}{dt}/l$ *to be exact*)

Xin-Li Zhao, Ma, Ma & ZWL, PRC (2020)

which reduces the opacity χ to approach the dilute limit:

$$\chi \equiv \frac{\sigma^{3/2} n}{\sqrt{\pi}} \longrightarrow \frac{\chi}{\sqrt{l}}$$

l: subdivision factor

However, subdivision method is very CPU-consuming;
more importantly, it drastically changes event-by-event fluctuations & correlations.

→ We test then improve the accuracy of ZPC (*without using subdivision*)

Earlier test and improvement of ZPC

We have tested ZPC for partons in a box:

Xin-Li Zhao, Ma, Ma & ZWL, PRC (2020)

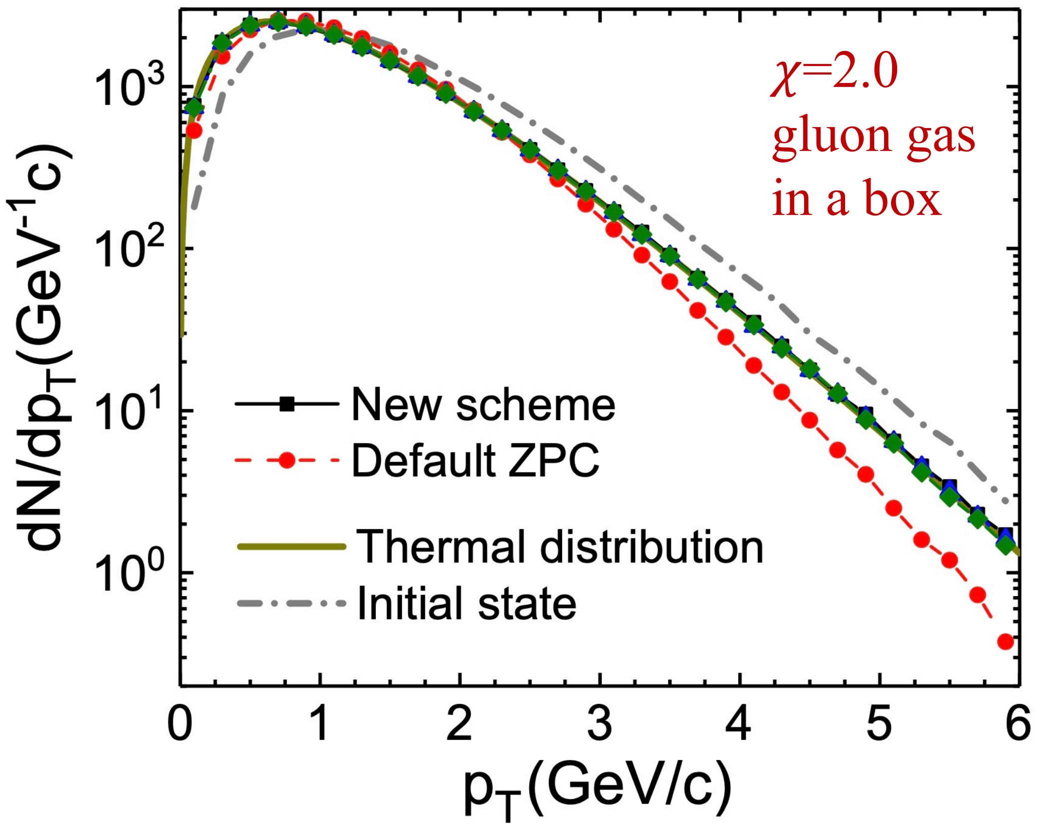
Ordering time \ Collision time	ct_1 & ct_2	$min(ct_1, ct_2)$	$(ct_1 + ct_2)/2$	$max(ct_1, ct_2)$
$min(ct_1, ct_2)$	A	B (new scheme)	C	D
$(ct_1 + ct_2)/2$	E	F	G (default ZPC scheme)	H
$max(ct_1, ct_2)$	I	J	K	L

ct_1 & ct_2: collision times of the two partons after the boost to the global frame.

- Parton cascade has freedom in choosing the collision time (ct) and/or collision ordering time in global frame

- Default ZPC (*t-avg scheme*) fails to maintain thermal equilibrium at high opacities

- **A new choice** (*t-min scheme*) gives the expected thermal distribution

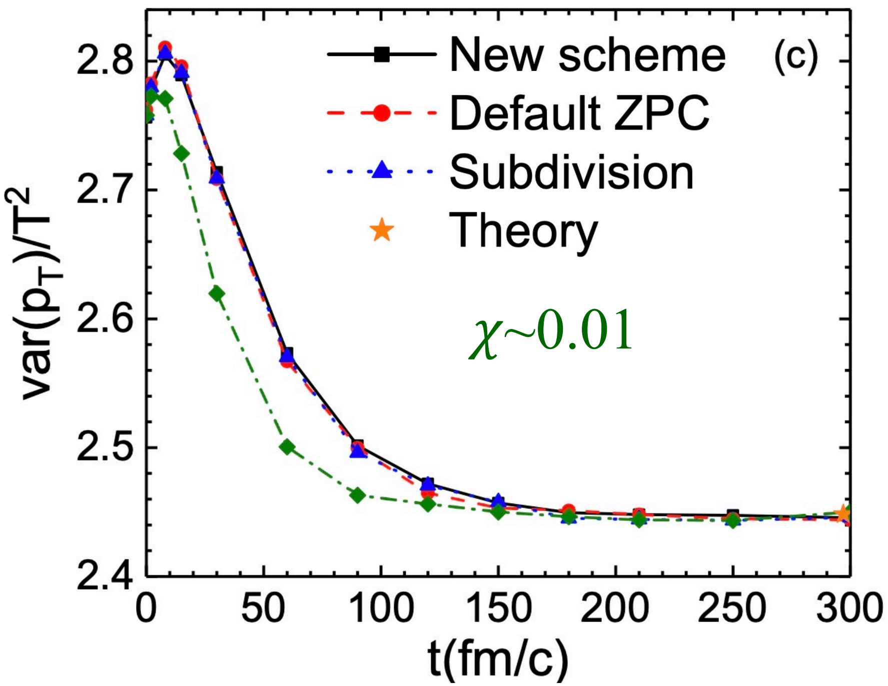


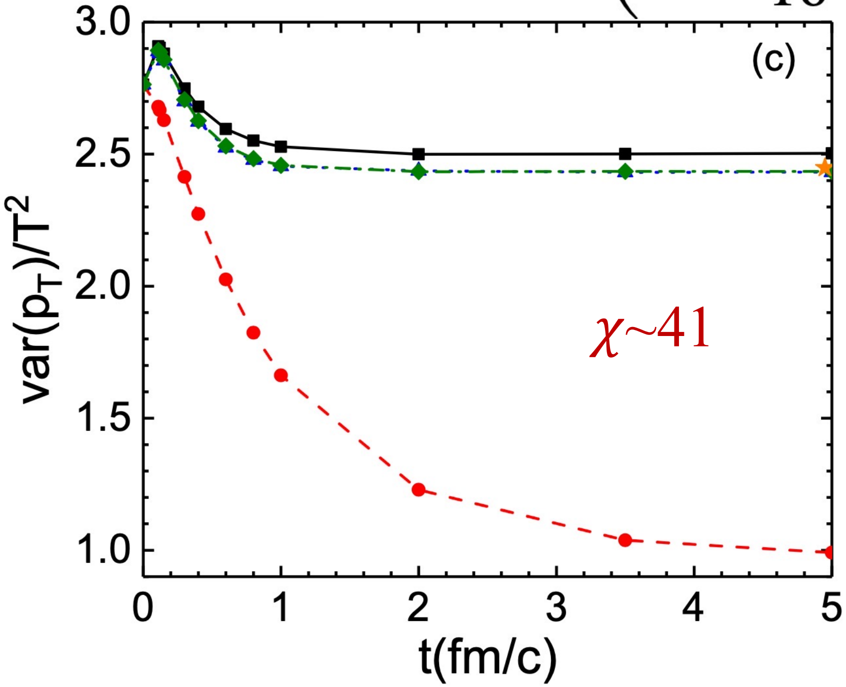
Earlier test and improvement of ZPC

Time evolution of $\mathrm{var}(p_\mathrm{T}) = \langle p_\mathrm{T}^2 \rangle - \langle p_\mathrm{T} \rangle^2$:

In equilibrium,

$$\mathrm{var}(p_\mathrm{T}) = \left(8 - \frac{9\pi^2}{16}\right) T^2$$





New time evolution of spectrum agrees well with subdivision results at small or large opacities

Xin-Li Zhao, Ma, Ma & ZWL, PRC (2020)

For parton cascade in a box,
we found a new parton subdivision method:
to realize $f \longrightarrow f * l$, instead of $N \longrightarrow N * l$ & V unchanged,
we do N unchanged & $V \longrightarrow V/l$

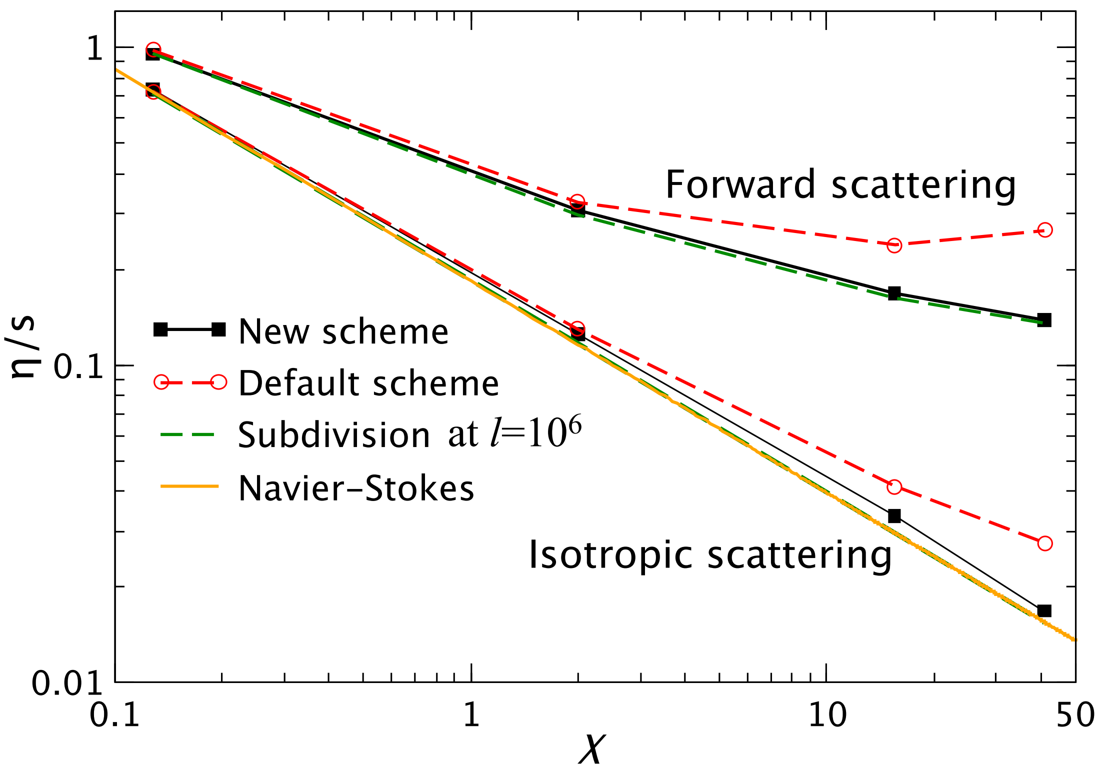
This subdivision method does not increase the computation time much
& allows us to use a huge ***l*=10^6** to reach the dilute limit.

Earlier test and improvement of ZPC

Shear viscosity η and η/s:

- the new t-avg scheme agrees well with Navier-Stokes result for isotropic scatterings $\eta^{NS}=1.265\,\frac{T}{\sigma}$, even at very high opacities up to $\chi\sim41$

- Default ZPC scheme fails at high χ.



De Groot, Van Leeuwen & Van Weert, Relativistic Kinetic Theory (1980);
Huovinen & Molnar, PRC (2009);
Plumari, Puglisi, Scardina & Greco, PRC (2012);
MacKay and ZWL, EPJC (2022)

Xin-Li Zhao, Ma, Ma & ZWL, PRC (2020); ZWL & Liang Zheng, NST (2021)

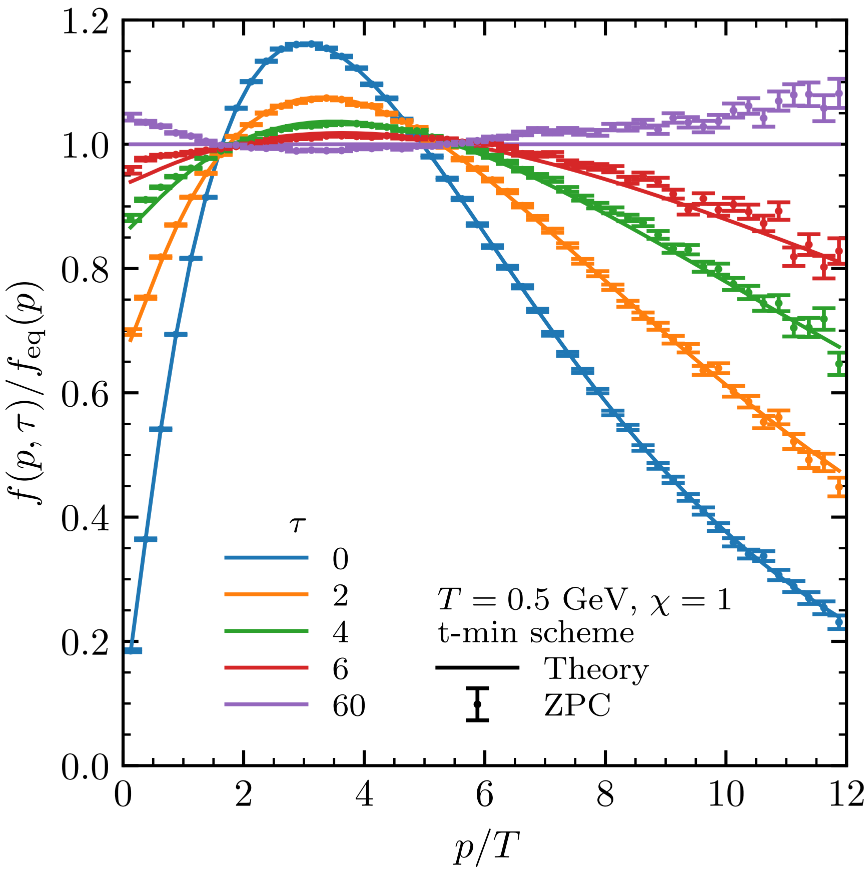
Recent improvement with an exact solution of RBE

An exact solution of the relativistic Boltzmann equation has been found for a massless homogeneous gas under 2-body isotropic scatterings.

→ We test the full time-evolution of momentum spectra & improve parton transport.

For non-expanding spacetime, the solution is

$$f_{\text{theory}}(p,\tau) = \exp\left(-\frac{p}{T\kappa(\tau)}\right)\left[\frac{4\kappa(\tau)-3}{\kappa^4(\tau)} + \frac{p}{T}\frac{1-\kappa(\tau)}{\kappa^5(\tau)}\right].$$



Bazow, Denicol, Heinz, Martinez & Noronha, PRL (2016) & PRD (2016)

$\tau \propto t$ is a scaled time,

$$\kappa(\tau) = 1 - \exp(-\tau/6)/4$$

- Spectra evolves from highly off-equilibrium to a thermal distribution $f_{eq}(p)$
- ZPC with *t-min scheme* performs quite well.

Mendenhall & ZWL, in preparation

Recent improvement with an exact solution of RBE

We then use a more general collision scheme for parton collision time:

$$ct = \min(ct_1, ct_2) + r\,|ct_1 - ct_2|$$

Default ZPC (t-avg scheme): r =1/2

The t-min scheme: r =0

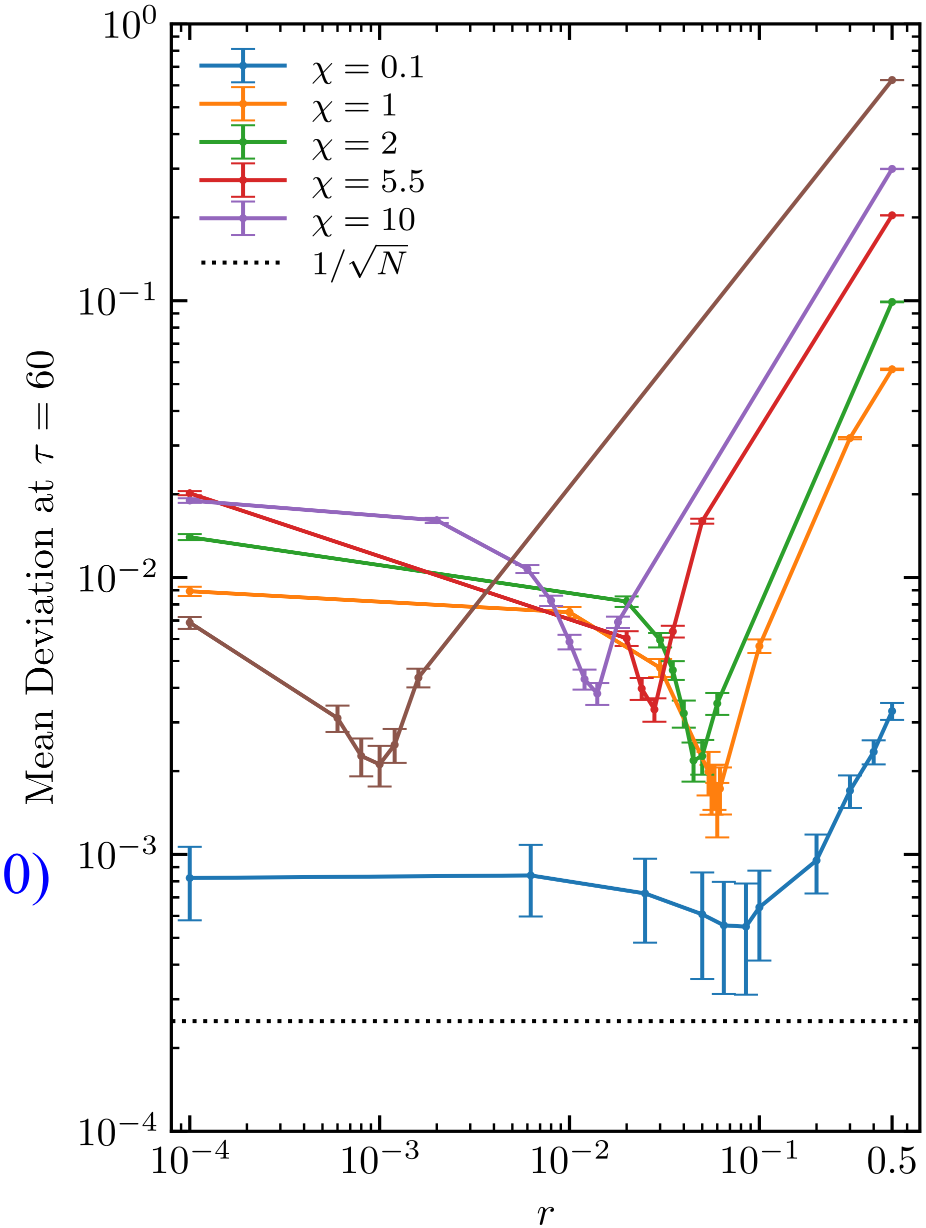
General scheme: any r between 0 & 1

We use Mean Deviation to measure the ZPC deviation from the exact solution:

$$\mathrm{MD}(\tau) = \sqrt{\frac{\sum_i \left[N_{\mathrm{ZPC}}(p_i,\tau) - N_{\mathrm{Theory}}(p_i,\tau)\right]^2}{\sum_i \left[N_{\mathrm{Theory}}(p_i,\tau)\right]^2}}$$

→ certain r value minimizes MD(τ=60) & it depends on χ

→ we parameterize $r(\chi)$



Mendenhall & ZWL, in preparation

Recent improvement with an exact solution of RBE

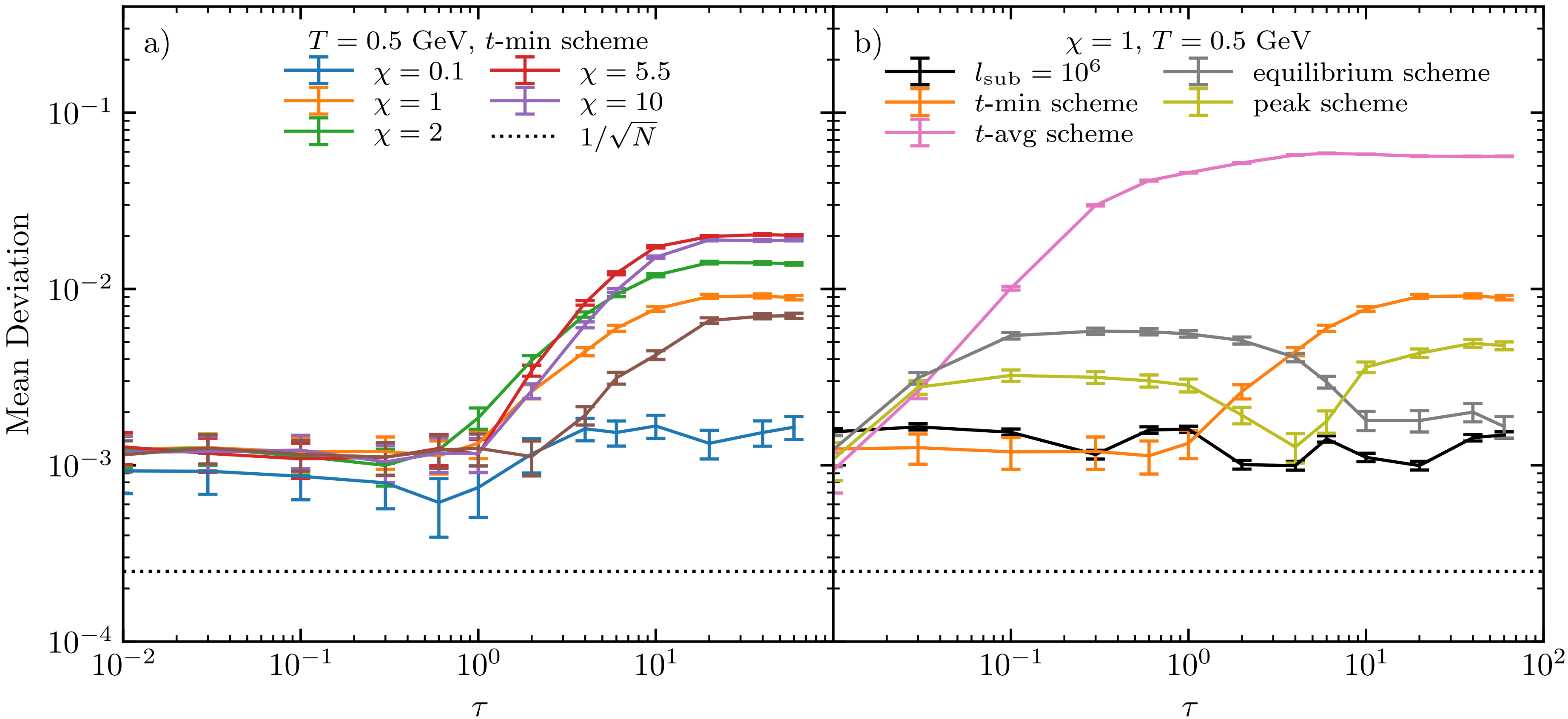
General collision scheme: $ct = \min(ct_1, ct_2) + r\,|ct_1 - ct_2|$

We parameterize $r(\chi)$ to minimize

- the Mean Deviation in equilibrium (*equilibrium scheme*)

  or

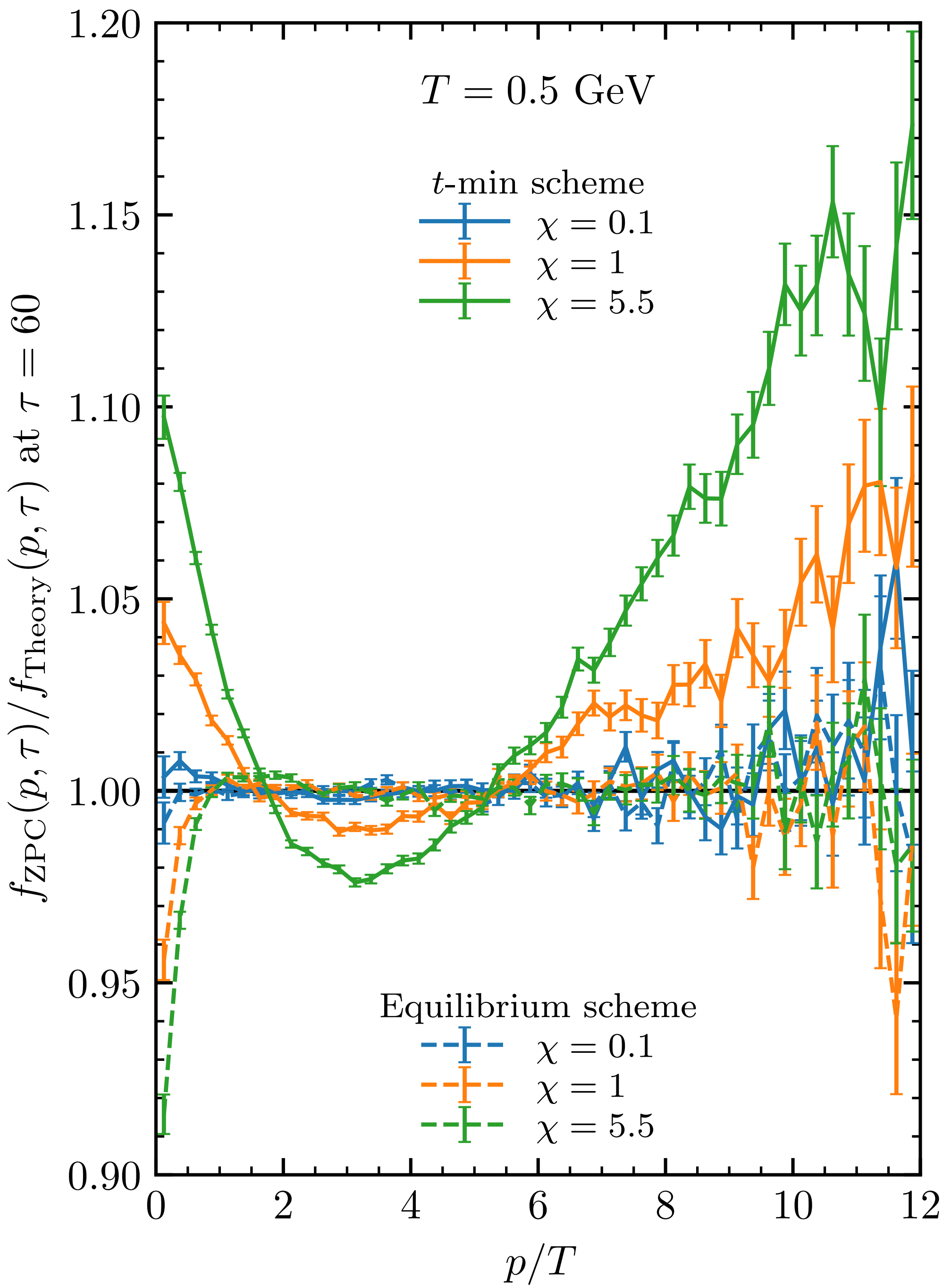
- the peak Mean Deviation during the time evolution (*peak scheme*)



Recent improvement with an exact solution of RBE

Momentum spectrum in equilibrium (over the theory spectrum):

equilibrium scheme
gives much better overall spectra and smaller deviation from the exact solution
than t-min scheme



Mendenhall & ZWL, in preparation

Recent improvement with an exact solution of RBE

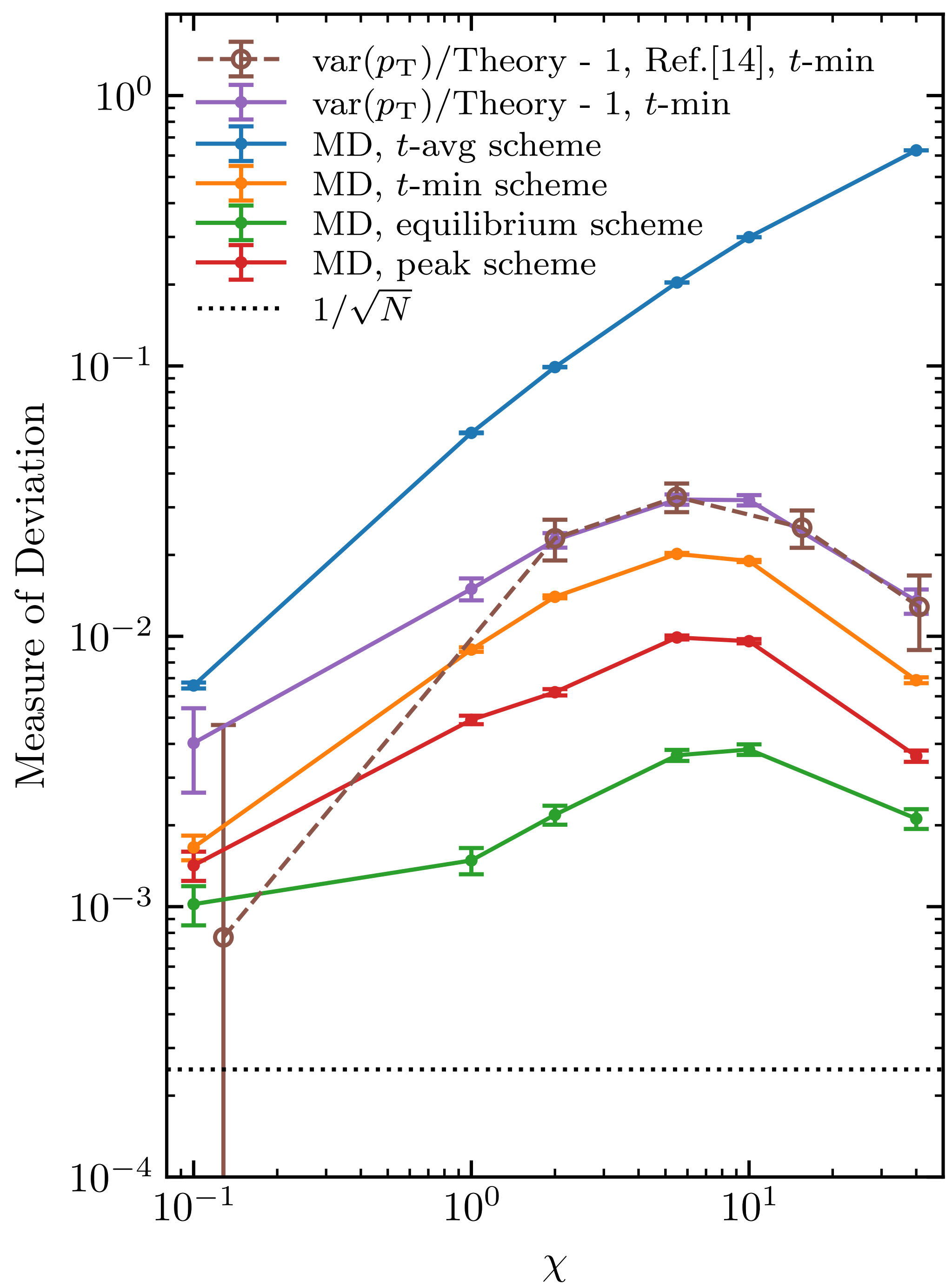
Deviation vs opacity χ:

- *general collision schemes* are even better than *t-min scheme*

- they reduce mean deviation by a factor up to ~2 (*peak scheme*) or ~7 (*equilibrium scheme*): *general scheme* MD < 1% at all χ.

- Deviation vs χ is nonmonotonous, just like previous var(p_T) results

[14] Xin-Li Zhao, Ma, Ma & ZWL, PRC (2020)



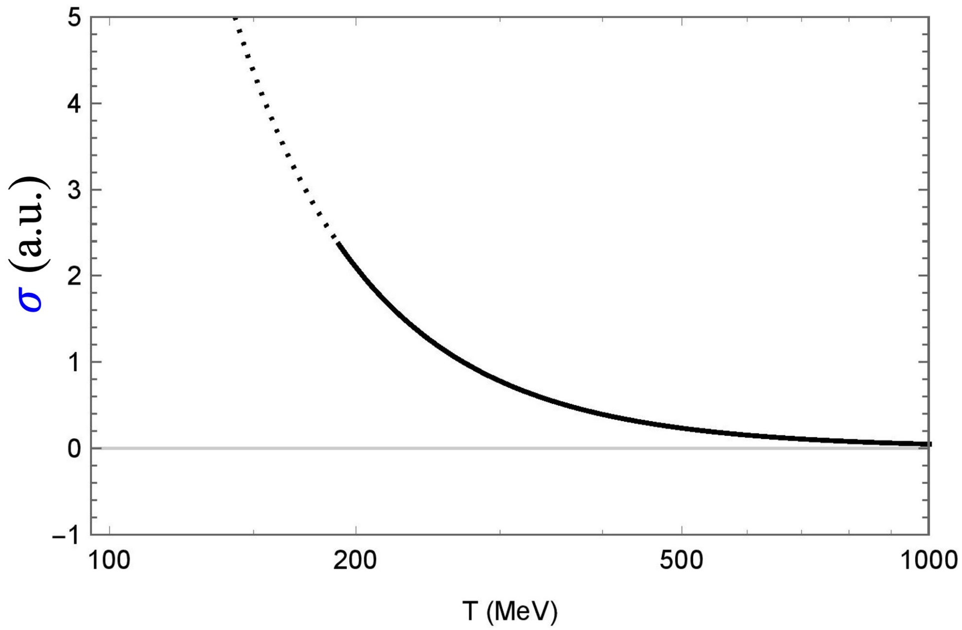
Mendenhall & ZWL, in preparation

Outlook

- Causality violation in current AMPT is small due to small σ (<=3mb) Molnar 1906.12313

But finite-temperature pQCD → $\mu \propto gT$

Arnold, Moore & Yaffe, JHEP (2003);
Csernai, Kapusta & McLerran, PRL (2006)



So far, AMPT always uses constant σ & μ.

With $\mu \propto gT$

→ $\sigma \propto 1/\mu^2$ will be larger at lower T

Improved ZPC here would still be accurate.

→ $\eta \propto T/\sigma$, $\eta/s \propto \frac{1}{T^2\sigma}$

will have the expected T- & t-dependences

MacKay and ZWL, EPJC (2022)

→ improve ZPC/AMPT as a dynamical model of finite-T QCD kinetic theory

- Recently we have modified ZPC to make its structure compatible with parton transport under E&M fields Mendenhall & ZWL, in preparation

→ next: include E&M fields to ZPC & study their evolution and effect on CME signals

Summary

- Transport models including ZPC and AMPT
  are especially suitable for studies of non-equilibrium dynamics

- We have tested and improved ZPC for massless partons in a box

- The default ZPC collision scheme is not accurate at large opacities

- New collision schemes can drastically improve the ZPC accuracy
  at large opacities, to the level of <1% mean deviation at all χ

- This lays the foundation to
  extend the test to parton systems with 3-d expansion,
  extend to parton transport under evolving E&M fields,
  and generalize ZPC with finite-T pQCD

Thank you for your attention!