

Relativistic Viscous Hydro & de Hass-Aphen Effect with Angular Momentum



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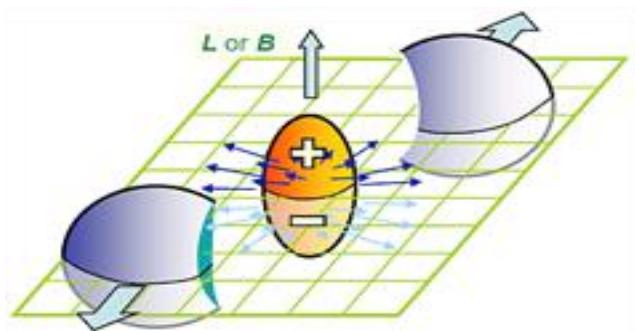
Duan She, Anping Huang, DF Hou, Jinfeng Liao, Sci.Bull. 67 (2022) 2265-2268,

Shu-Yun Yang, Ren-Da Dong, DF Hou, Hai-Cang Ren, PRD 107, 076020 (2023)

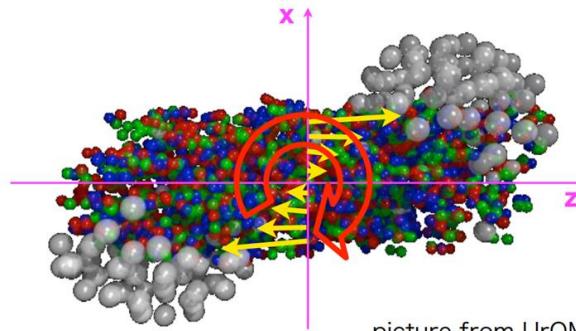
Outlines

- **Introduction and motivation**
- **Relativistic viscous hydrodynamics with angular moment**
- **De Hass-van Alphen effect with rotation**
- **Summary**

QCD under new extreme conditions



$$E, B \sim \gamma \frac{Z\alpha_{EM}}{R_A^2} \sim 3m_\pi^2$$



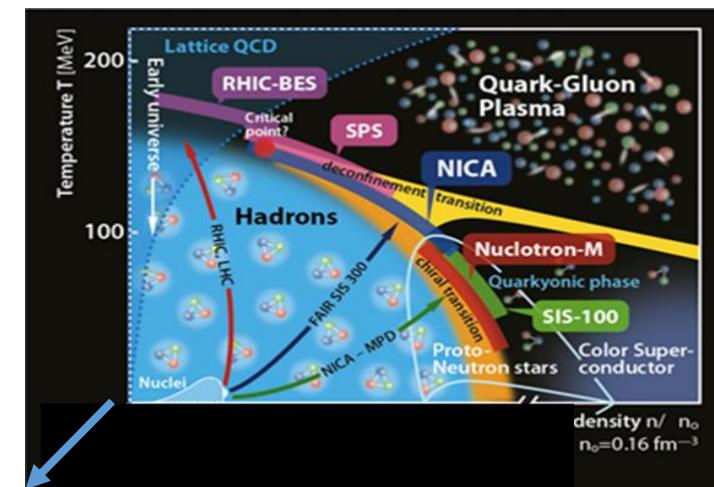
picture from UrQMD



$$L \approx \frac{A\sqrt{s_{NN}}}{2} b \hbar \sim 10^6 \hbar$$

Explore the new dimensions of the QCD phase diagram

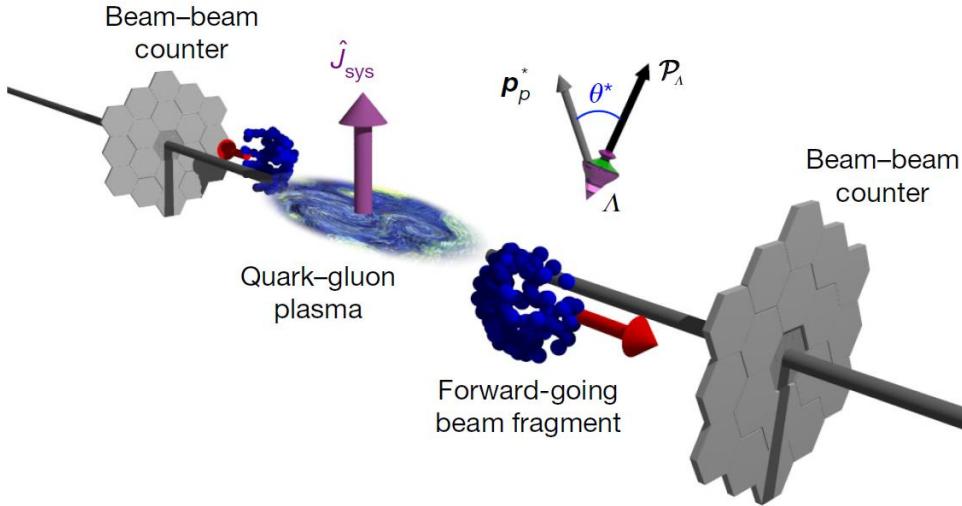
Talks in this WS: Edroedi, Fukushima, He, Li, Yang



Khazeev, Liao Nature 2021; Becattini- Karpenko et al 2015, 2016; Jiang-Lin-Liao 2016; Deng-XGH-Ma-Zhang 2020; Deng-XGH 2016, Xie-Csnerai et al 2014; Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017;)

Global spin polarization: Experiments

- First measurement of Λ polarization by STAR@ RHIC



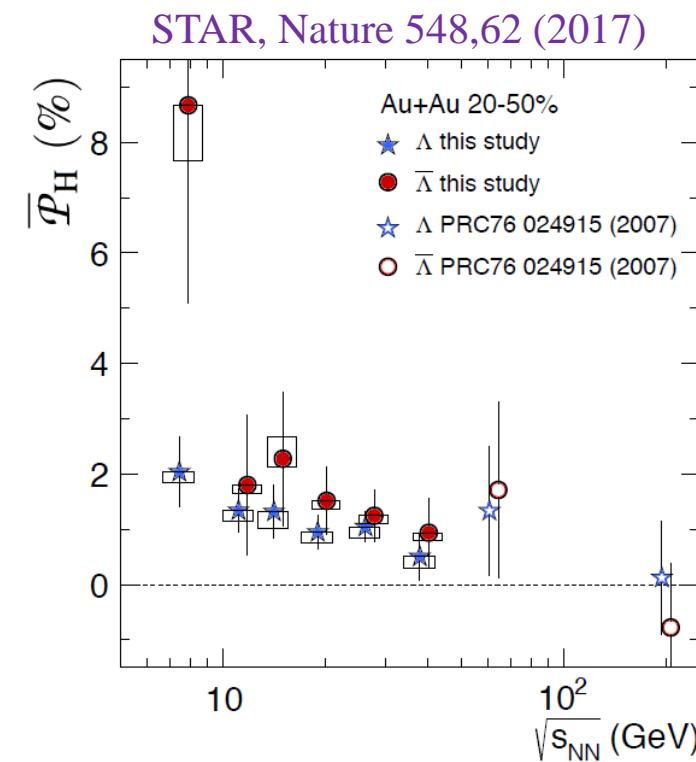
Liang, Wang, PRL(2005)

Betz, Gyulassy, Torrieri, PRC (2007)

Becattini, Piccinini, Rizzo, PRC (2008)

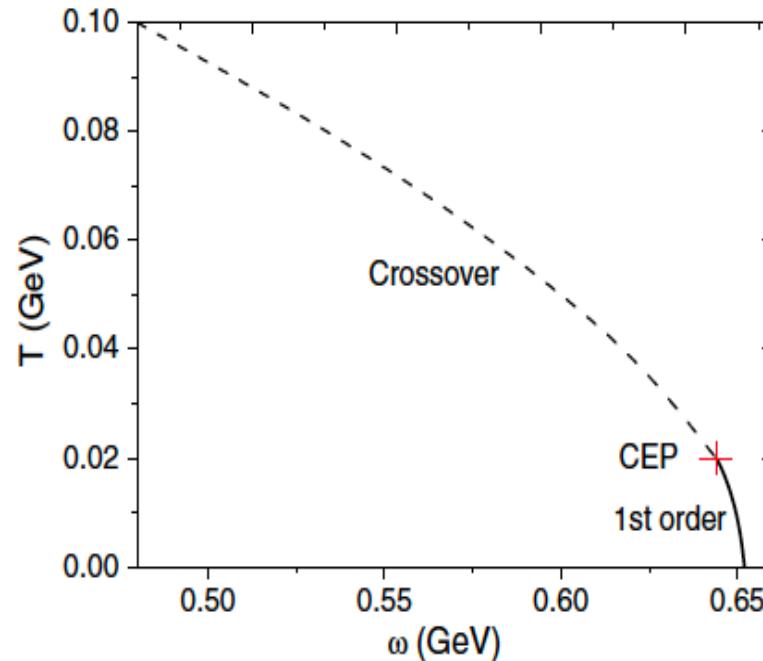
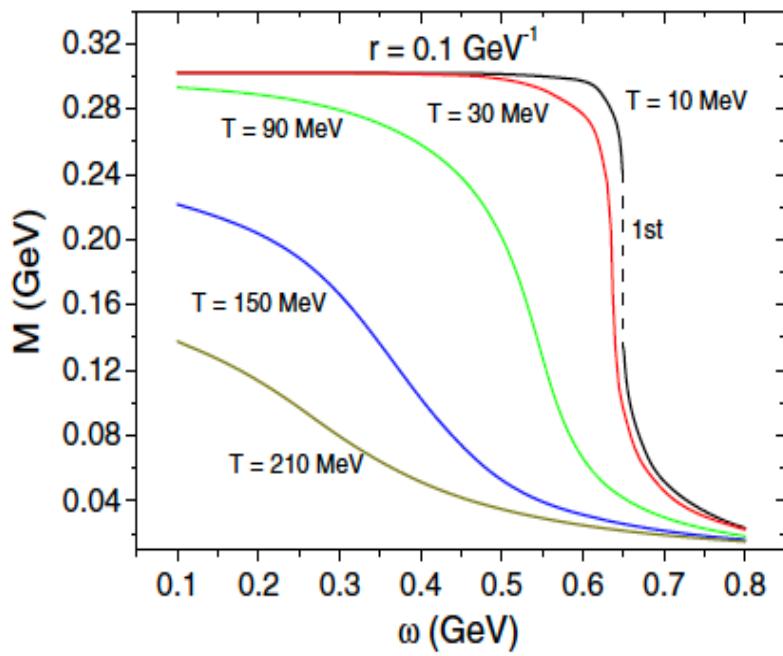
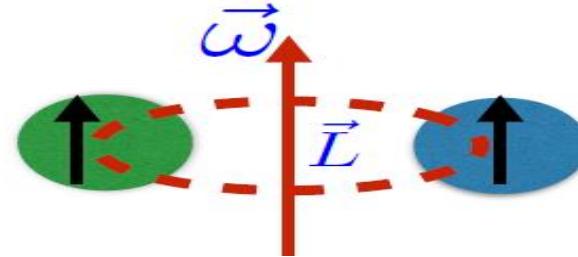
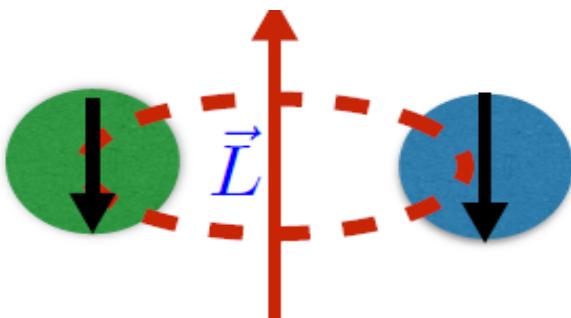
Becattini et al, Annals Phys (2013)

Becattini, Karpenko, Lisa, Uppsala, Voloshin, PRC (2017)



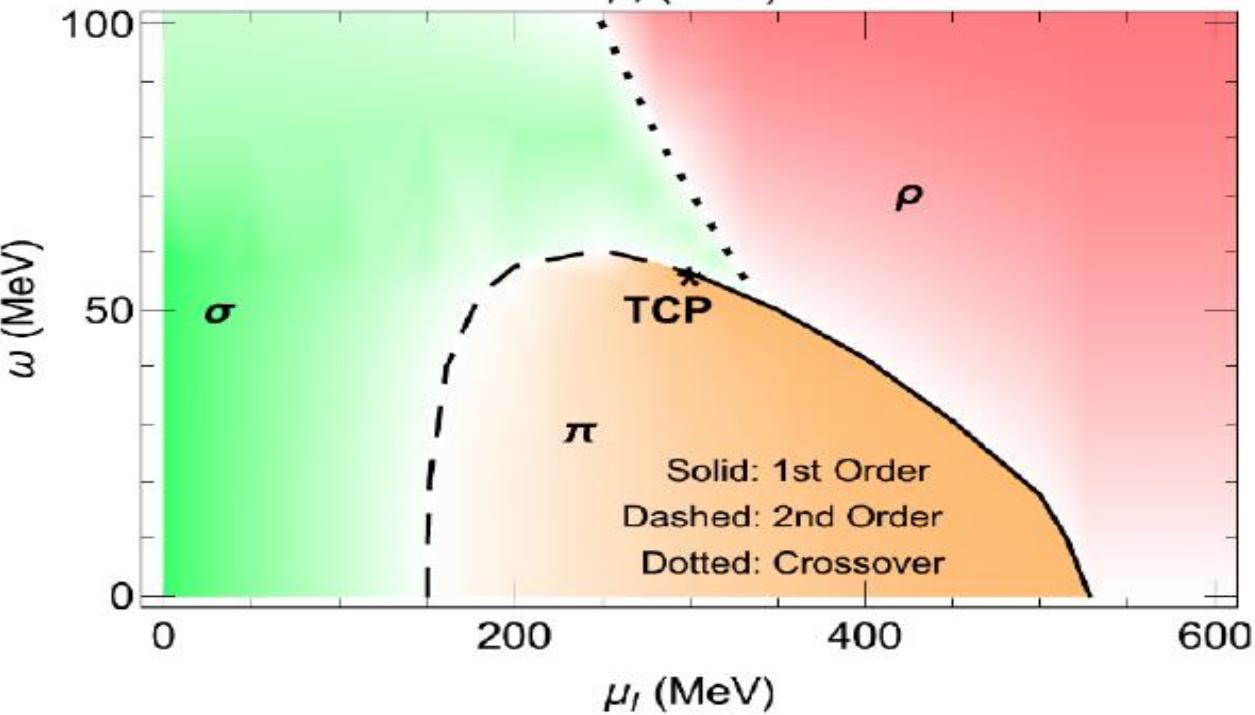
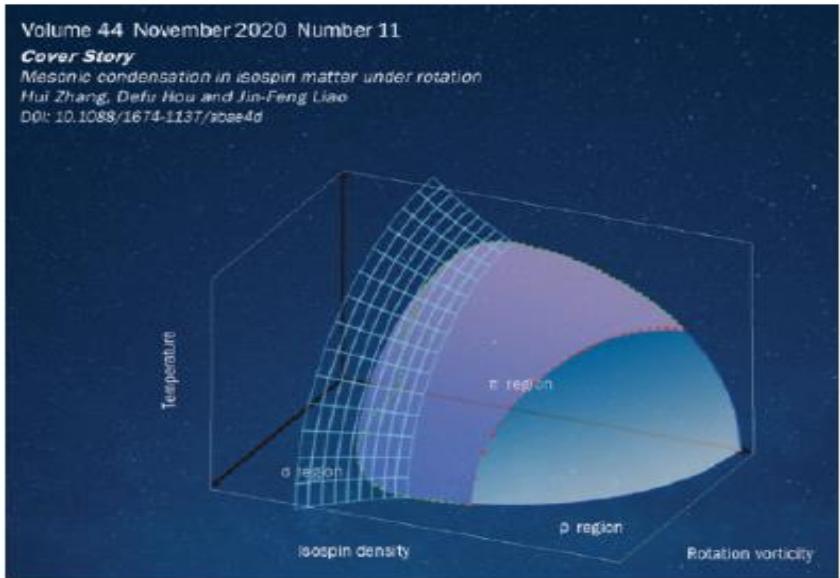
Vorticity interpretation of global Λ polarization works well!

Phase structure under rotation



Isospin Matter under Rotation

*Vacuum: sigma condensate;
Static isospin matter: pion superfluidity;
Isospin matter under rotation: emergence of rho condensate!*



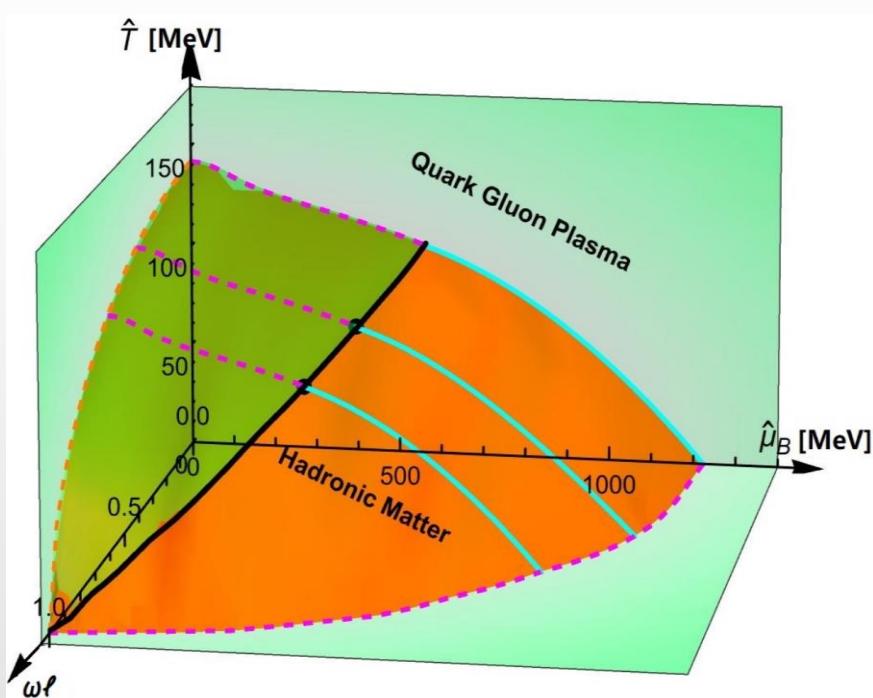
*Rich phase structures found;
Could be relevant to low energy HIC
or neutron star matter*



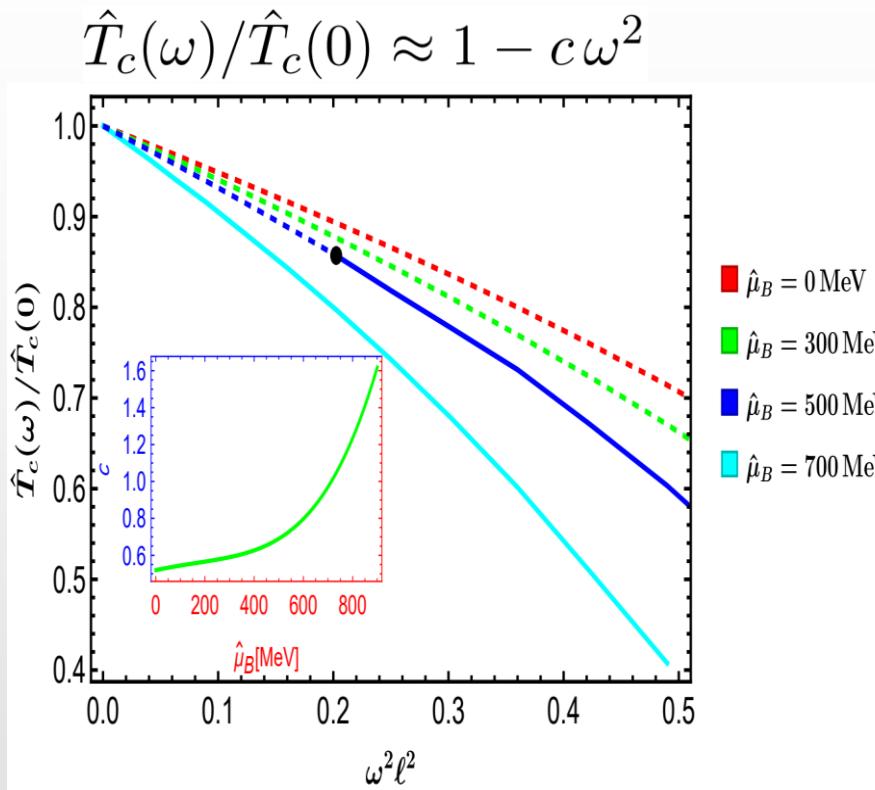
[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

Phase structure with rotation with hQCD

➤ 2+1 flavor:



Talks in this WS: Song He, DanLin Li, Jichong Yang

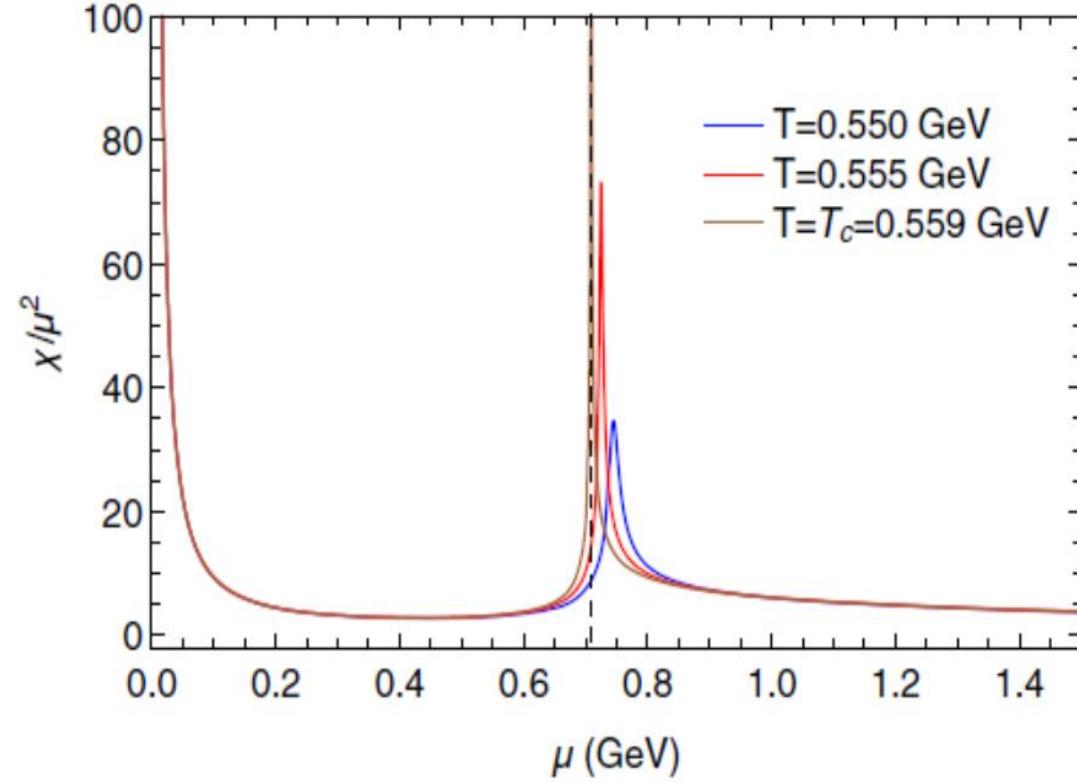
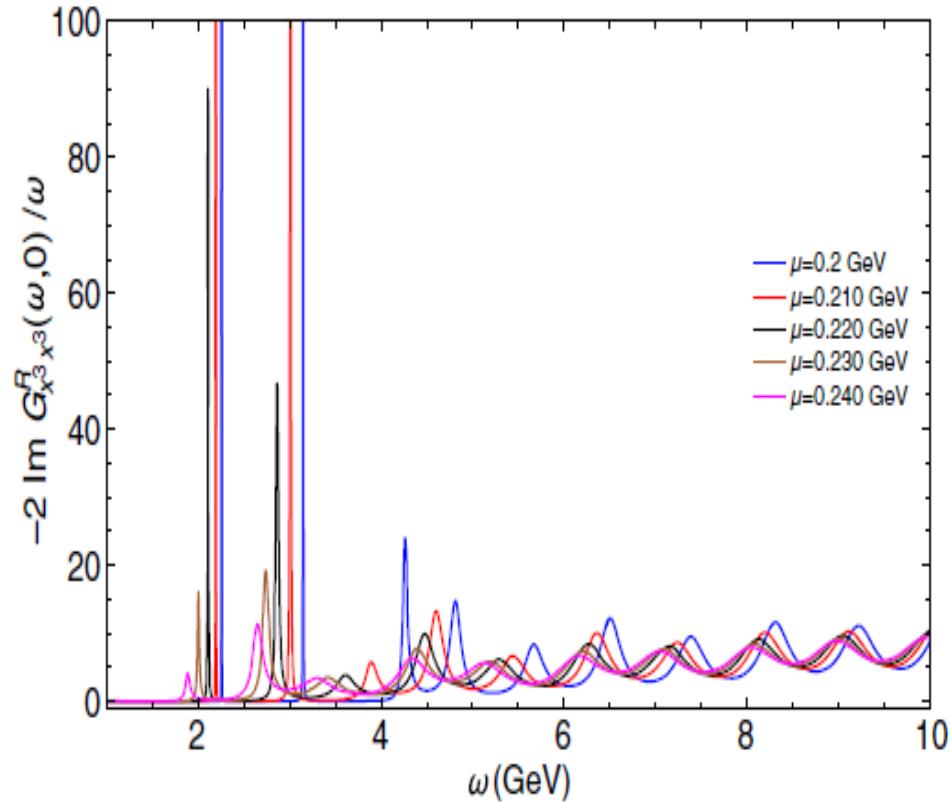


ID: Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, *JHEP* 04 (2023) 115 • e-Print: 2212.14662

X.Chen, L. Zhang, D.N. Li, D. Hou , M.Huang ,*JHEP* 07 (2021) 132

The spectral function of heavy vector mesons

Mamani , Hou, Braga, PRD 105, 126020 (2022)



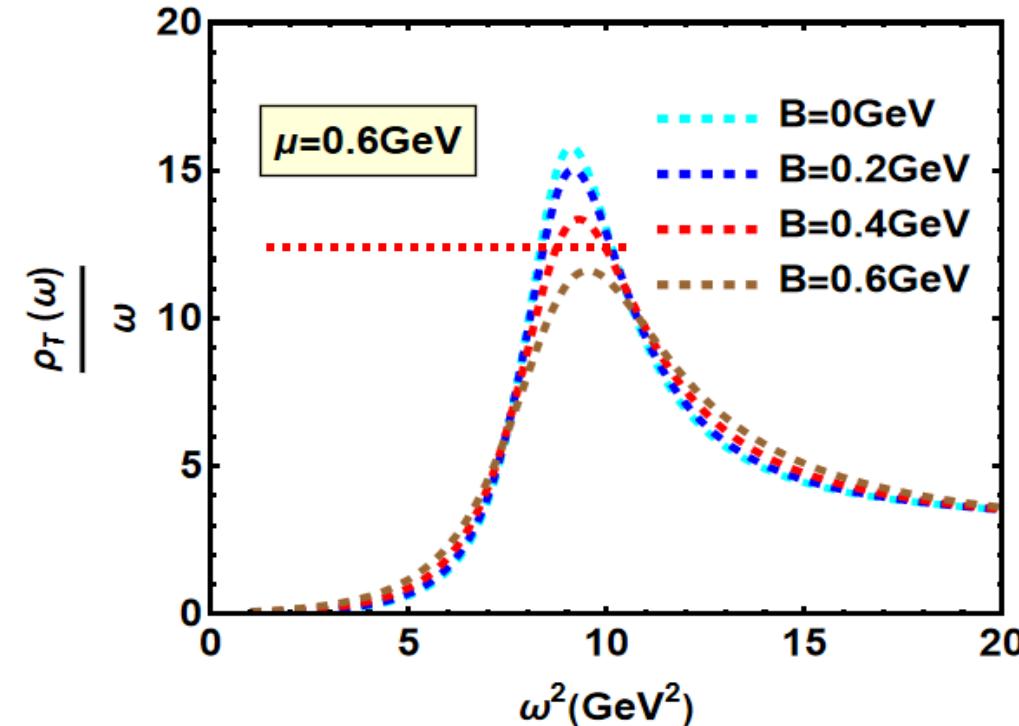
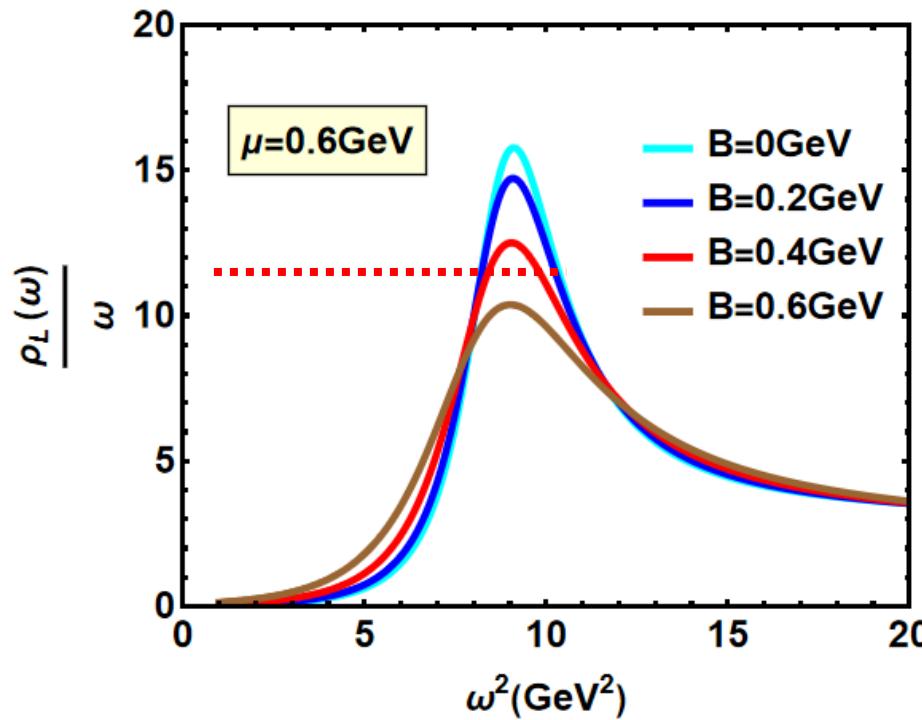
Spectral function of Heavy quarkonium with B

Talk in this WS: YQ Zhao , JX Chen

● Spectral function:

Yan-Qing Zhao, Defu Hou, Eur.Phys.J.C 82 (2022) 12, 1102 • e-Print: 2108.08479

As increasing magnetic field, the dissociation effect increases and it is stronger for the parallel case.



Hydrodynamics with Angular Momentum

Phenomenological issues?

How to incorporate the angular momentum into the hydrodynamic framework?

In particular, how to include spin degrees of freedom?

Florkowski, Ryblewski, Kumar, ...;

Becattini, Tinti, Buzzegoli, ...;

Hattori, Hongo, Huang, ...;

Fukushima, Pu;

Shi, Gale, Jeon;

Weickgenannt, Speranza, Sheng, Wang, Rischke;

Liu, Yin, ...;

Gallegos, Gursoy, Yarom;

Li, Stephanov, Yee;

.....

Talks in this WS:

Bacattini, Rischke, Wang,
Hattori, Pu, Daher, Wagner
+...

[See the ref list of arXiv:2105.04060]



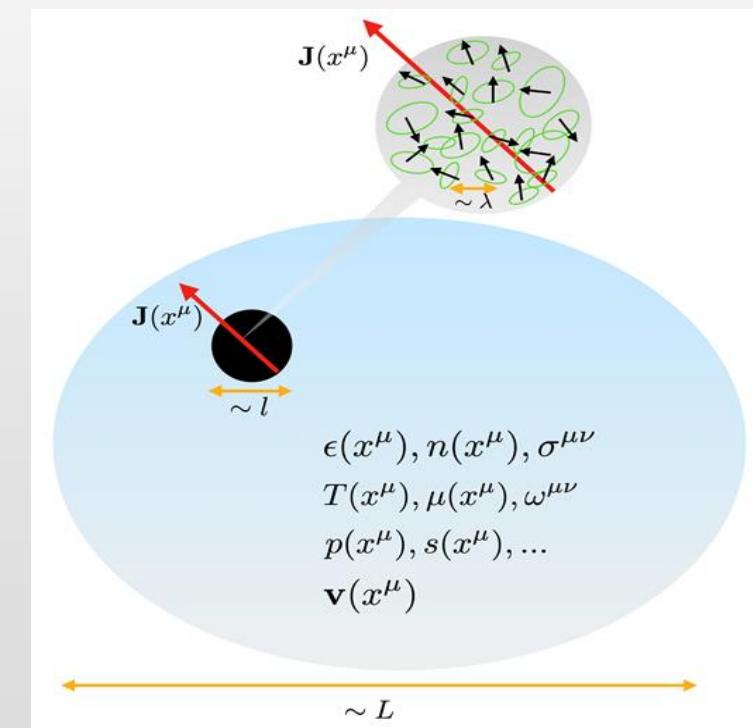
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journal homepage: www.elsevier.com/locate/scib

Perspective

Relativistic viscous hydrodynamics with angular momentumDuan She ^{a,b,c,1}, Anping Huang ^{b,d,1}, Defu Hou ^{a,*}, Jinfeng Liao ^{b,*}

Ideal Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0,$$

$$T_{(0)}^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu}, \quad N_{(0)}^\mu = n u^\mu.$$

$$\epsilon = -p + Ts + \mu n,$$

$$S_{(0)}^\mu = s u^\mu,$$

$$\partial_\mu S_{(0)}^\mu = \partial_\mu (s u^\mu) = 0.$$

Microscopic physics enters via thermodynamic relations (i.e. EOS)

See e.g. Landau and Lifshitz

Navier-Stokes Viscous Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0,$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} + \tilde{T}^{\mu\nu},$$

$$N^\mu = n u^\mu + \tilde{N}^\mu, \quad S^\mu = s u^\mu + \tilde{S}^\mu.$$

$$\partial_\mu S^\mu \geq 0.$$

See e.g. Landau and Lifshitz

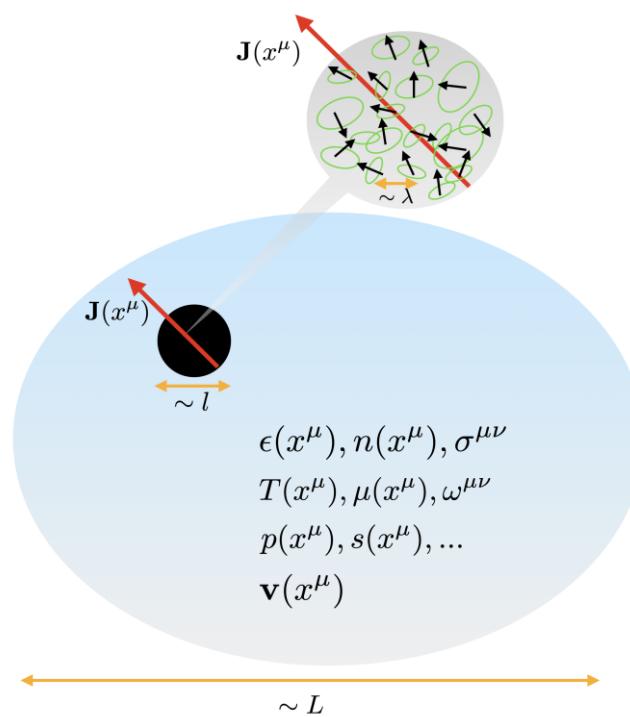
$$\Pi = -\zeta \theta,$$

$$\pi^{\mu\nu} = 2\eta \nabla^{\langle\mu} u^{\nu\rangle},$$

$$q^\mu = \lambda T \left(\frac{\nabla^\mu T}{T} - D u^\mu \right)$$

Microscopic physics also enters via transport coefficients in viscous terms.

Goal: Navier-Stokes Program for Ang. Mom.



$\lambda \ll l \ll L$, a coarse-graining process

Local angular momentum current

Local angular momentum density

Local angular momentum chemical potential

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^{\mu\nu\alpha} = 0$$

$$J^{\mu\nu\alpha} = x^\nu T^{\mu\alpha} - x^\alpha T^{\mu\nu} + \Sigma^{\mu\nu\alpha}$$

$$\implies \partial_\mu \Sigma^{\mu\nu\alpha} = T^{\alpha\nu} - T^{\nu\alpha}$$

We choose to deal with only the conserved quantities, i.e. angular momentum.

$$\Sigma^{\mu\alpha\beta},$$

$$\sigma^{\alpha\beta}(x^\mu),$$

$$\omega_{\alpha\beta}(x^\mu).$$

Ideal Hydro with Ang. Mom.

$$T_{(0)}^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu}, N_{(0)}^\mu = n u^\mu.$$

$$\Sigma_{(0)}^{\mu\alpha\beta} = \sigma^{\alpha\beta} u^\mu. \quad \partial_\mu J_{(0)}^{\mu\alpha\beta} = \sigma^{\alpha\beta} \theta + D\sigma^{\alpha\beta} = 0$$

Generalized thermodynamics:

$$\epsilon + p = T s + \mu n + \omega_{\alpha\beta} \sigma^{\alpha\beta}$$

$$dp = s dT + n d\mu + \sigma^{\alpha\beta} d\omega_{\alpha\beta}$$

$$d\epsilon = T ds + \mu dn + \omega_{\alpha\beta} d\sigma^{\alpha\beta}$$

It is straightforward to verify: no entropy generation

$$\partial_\mu S_{(0)}^\mu = \partial_\mu (s u^\mu) = 0$$

Viscous Hydro with Angular Momentum

$$\begin{aligned}T^{\mu\nu} &= \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} + \tilde{T}^{\mu\nu} \\N^\mu &= n u^\mu + \tilde{N}^\mu \\\Sigma^{\mu\alpha\beta} &= u^\mu \sigma^{\alpha\beta} + \tilde{\Sigma}^{\mu\alpha\beta} \\S^\mu &= s u^\mu + \tilde{S}^\mu.\end{aligned}$$

Let us first focus on entropy current: W. Israel, J. Stewart, Annals Phys. 118, 341 (1979)

$$S^\mu = p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}$$

Leading order:

$$\partial_\mu S_{(0)}^\mu = 2\beta\omega_{\alpha\beta}\tilde{T}_{(a)}^{\alpha\beta} \implies \tilde{T}_{(a)}^{\alpha\beta} = 0.$$

Next order (2nd-gradient):

$$\begin{aligned}\partial_\mu S^\mu &= \partial_\mu(p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta}\Sigma^{\mu\alpha\beta}) \\&= \tilde{T}^{\mu\nu}\partial_\mu\beta_\nu - \tilde{N}^\mu\partial_\mu\alpha - \tilde{\Sigma}^{\mu\alpha\beta}\partial_\mu(\beta\omega_{\alpha\beta}) \geq 0\end{aligned}$$

Viscous Hydro with Angular Momentum: Eckart Frame

$$u_E^\mu = \frac{N^\mu}{\sqrt{N^\nu N_\nu}},$$

**Write down all allowed Lorentz structures
to the correct order of gradient expansion**

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + 2u^{(\mu} q^{\nu)} + \pi^{\mu\nu}$$

$$N^\mu = n u^\mu$$

$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha} \Delta^{\mu\beta]} \Phi + 2u^{[\alpha} \tau_{(s)}^{\mu\beta]} + 2u^{[\alpha} \tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}$$

**Plug these into the entropy current divergence and
look for conditions of positivity:**

$$\partial_\mu S^\mu \geq 0$$

$$\Pi = -\zeta \theta$$

$$\pi^{\mu\nu} = 2\eta \nabla^{\langle \mu} u^{\nu \rangle}$$

$$q^\mu = \lambda (\nabla^\mu - T D u^\mu)$$

$$= -\frac{\lambda n T^2}{\epsilon + p} \left[\nabla^\mu \left(\frac{\mu}{T} \right) + \frac{S^{\alpha\beta}}{n} \nabla^\mu \left(\frac{\omega_{\alpha\beta}}{T} \right) \right].$$

**Five new positive angular momentum
transport coefficients:**

$\chi_1, \chi_2, \chi_3, \chi_4$ and χ_5

$$\begin{aligned}\Phi &= -\chi_1 u^\alpha \nabla^\beta \left(\frac{\omega_{\alpha\beta}}{T} \right), \\ \tau_{(s)}^{\mu\beta} &= -\chi_2 u^\alpha \left[(\Delta^{\beta\rho} \Delta^{\mu\gamma} + \Delta^{\mu\rho} \Delta^{\beta\gamma}) \right. \\ &\quad \left. - \frac{2}{3} \Delta^{\mu\beta} \Delta^{\rho\gamma} \right] \nabla_\gamma \left(\frac{\omega_{\alpha\rho}}{T} \right), \\ \tau_{(a)}^{\mu\beta} &= -\chi_3 u^\alpha (\Delta^{\beta\rho} \Delta^{\mu\gamma} - \Delta^{\mu\rho} \Delta^{\beta\gamma}) \nabla_\gamma \left(\frac{\omega_{\alpha\rho}}{T} \right), \\ \Theta^{\mu\alpha\beta} &= -\chi_4 (u^\beta u^\rho \Delta^{\alpha\delta} - u^\alpha u^\rho \Delta^{\beta\delta}) \Delta^{\mu\gamma} \nabla_\gamma \left(\frac{\omega_{\delta\rho}}{T} \right) \\ &\quad + \chi_5 \Delta^{\alpha\delta} \Delta^{\beta\rho} \Delta^{\mu\gamma} \nabla_\gamma \left(\frac{\omega_{\delta\rho}}{T} \right).\end{aligned}$$

Viscous Hydro with Angular Mom. : Landau Frame

$$u_L^\mu = \frac{T_\nu^\mu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta_L^{\mu\nu} + \pi^{\mu\nu},$$

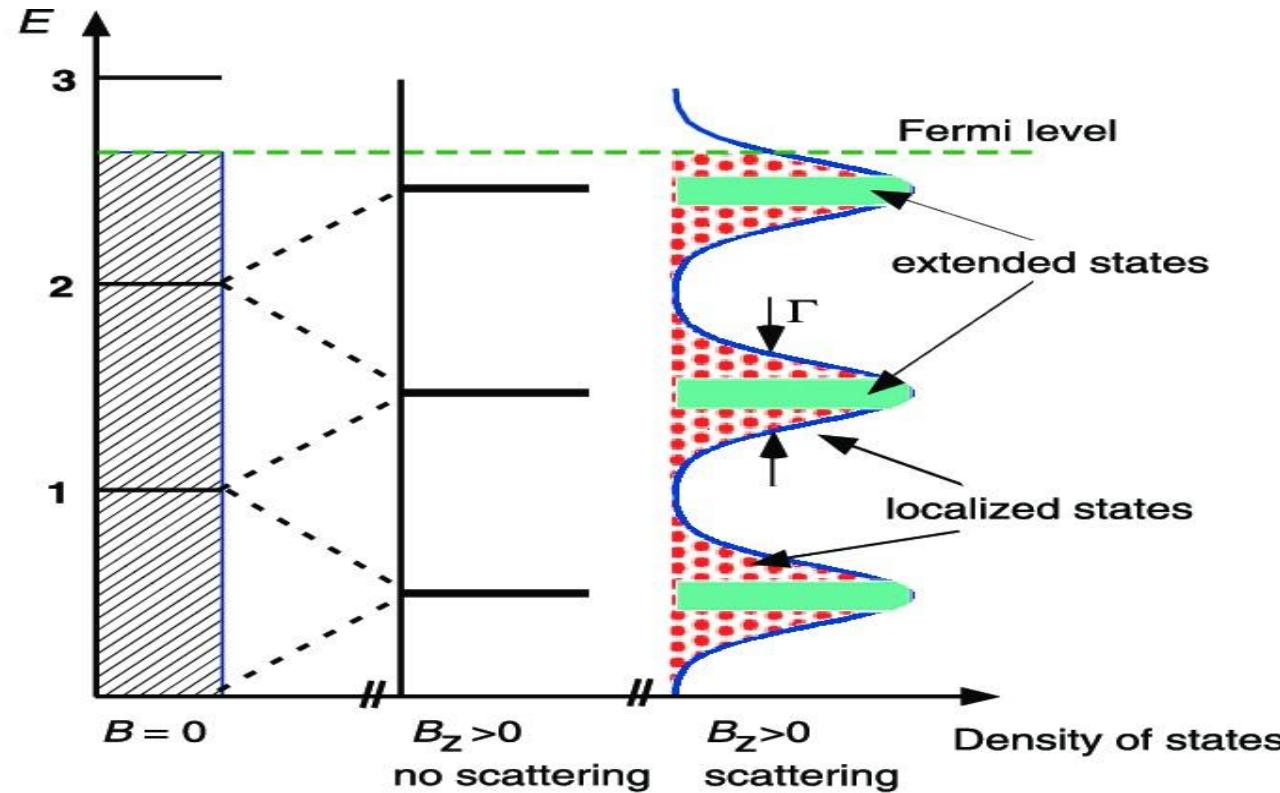
$$N^\mu = n u^\mu - \frac{n}{\epsilon + p} q^\mu,$$

$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} - \frac{q^\mu}{\epsilon + p} \sigma^{\alpha\beta} + 2u^{[\alpha} \Delta^{\mu\beta]} \Phi + 2u^{[\alpha} \tau_{(s)}^{\mu\beta]} + 2u^{[\alpha} \tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.$$

**Following the same procedure, one obtains
essentially the same consistent results.**

De Hass-van Alphen Effect

W. J. De Haas and P. M. Van Alphen, in Comm. NO. 212a from the Phys. Lab, Leiden (1930).



The dHvA effect in a degenerated system of charged fermions is the consequence of filling discrete but highly degenerate Landau levels in a magnetic field.

De Hass-van Alphen Effect

Without rotation (in Ultra-relativistic) :

$$P_{\text{dHvA}} = -\frac{T(eB)^{\frac{3}{2}}}{4\pi^2} \sum_{l=1}^{\infty} \frac{1}{l^{3/2}} \frac{\cos \left[\frac{l\pi}{eB} \mu^2 - \frac{\pi}{4} \right]}{\sinh \frac{2l\pi^2 T \mu}{eB}} \xrightarrow{T \rightarrow 0} \frac{(eB)^{\frac{5}{2}}}{8\pi^4 \mu} \sum_{l=1}^{\infty} \frac{1}{l^{5/2}} \cos \left[\frac{l\pi}{eB} \mu^2 - \frac{\pi}{4} \right]$$

In static, all degenerate states within a LL are equally probable to be occupied. Varying the strength of the magnetic field will cause a large number of fermions to jump between different discrete LLs, resulting in oscillatory dependence of P

Ultra-relativistic gas in cylindrical coordinate

$$H = -i\vec{\sigma} \cdot (\vec{\nabla} - ieA)$$

Eigen-Value Eq.

$$H\chi = E\chi$$

$$\chi(\vec{r}) = \begin{pmatrix} f(\rho)e^{i(M-\frac{1}{2})\phi} \\ g(\rho)e^{i(M+\frac{1}{2})\phi} \end{pmatrix} e^{iqz},$$

Eqs. For radial WFs $f(\rho)$ and $g(\rho)$

$$\begin{cases} qf(\rho) - i \left(\frac{d}{d\rho} + \frac{M+\frac{1}{2}}{\rho} - \frac{1}{2}eB\rho \right) g(\rho) = Ef(\rho) \\ -i \left(\frac{d}{d\rho} - \frac{M-\frac{1}{2}}{\rho} + \frac{1}{2}eB\rho \right) f(\rho) - qg(\rho) = Eg(\rho) \end{cases}$$

$M = \pm 1/2, \pm 3/2, \dots$ Eigen value
of angular momentum

Normalized WFs

$$\chi_{nMs}(\vec{r}) = \frac{1}{2\pi} \sqrt{\frac{n!}{(n+m)!}} e^{-\frac{\zeta}{2}} \begin{pmatrix} \sqrt{\frac{eB(E+q)}{2E}} \zeta^{\frac{m}{2}} L_n^m(\zeta) e^{im\phi} \\ \frac{iseB}{\sqrt{E(E-q)}} \zeta^{m+1} L_{n-1}^{m+1}(\zeta) e^{i(m+1)\phi} \end{pmatrix} e^{iqz} \quad \text{Laguerre polynomial , } (M > 0)$$

$$\chi_{nMs}(\vec{r}) = \frac{1}{2\pi} \sqrt{\frac{n!}{(n+|m|)!}} e^{-\frac{\zeta}{2}} \begin{pmatrix} \sqrt{\frac{eB(E+q)}{2E}} \zeta^{\frac{|m|}{2}} L_n^{|m|}(\zeta) e^{im\phi} \\ -\frac{iseB(n+|m|)}{\sqrt{E(E-q)}} \zeta^{\frac{(|m|-1)}{2}} L_n^{|m|-1}(\zeta) e^{i(m+1)\phi} \end{pmatrix} e^{iqz} \quad (M < 0)$$

式中, $\zeta \equiv \frac{1}{2}eB\rho^2$, $m \equiv M - 1/2$, $n = 0, 1, 2, \dots$ 且 $s = \pm$ 。

Thermodynamic Pressure (UR.)

$$\begin{aligned}
 P = & \frac{T}{\Omega} \sum_{n=0, M>0, q>0} [\ln(1 + e^{-\beta(|q|-M\omega-\mu)}) + \ln(1 + e^{-\beta(|q|+M\omega+\mu)})] \\
 & + \frac{T}{\Omega} \sum_{n=0, M>0, q} [\ln(1 + e^{-\beta(\sqrt{q^2+2neB}-M\omega-\mu)}) + \ln(1 + e^{-\beta(\sqrt{q^2+2neB}+M\omega+\mu)})] \\
 & + \frac{T}{\Omega} \sum_{n\neq 0, M>0, q} [\ln(1 + e^{-\beta(\sqrt{q^2+2(n+M+\frac{1}{2})eB}+M\omega-\mu)}) \\
 & + \ln(1 + e^{-\beta(\sqrt{q^2+2(n+M+\frac{1}{2})eB}-M\omega+\mu)})]
 \end{aligned}$$

With rotation

Strongly degenerate, $\mu \gg T$ (anti-particle's contribution can be neglected)

$$\begin{aligned}
 P = & \frac{T}{\pi R^2} \int_0^\infty \frac{dq}{4\pi} \sum_{M>0} \ln(1 + e^{-\beta(|q|-M\omega-\mu)}) + \frac{T}{\pi R^2} \int_{-\infty}^\infty \frac{dq}{2\pi} \sum_{n>0, M>0} \ln(1 + e^{-\beta(\sqrt{q^2+2neB}-M\omega-\mu)}) \\
 & + \frac{T}{\pi R^2} \int_{-\infty}^\infty \frac{dq}{2\pi} \sum_{n, M>0} \ln(1 + e^{-\beta(\sqrt{q^2+2(n+M+\frac{1}{2})eB}+M\omega-\mu)})
 \end{aligned}$$

$\rightarrow 0$, the 3rd term vanishes in thermodyn. limits

$$\begin{aligned}
 P = & \frac{T}{\pi R^2} \int_0^\infty \frac{dq}{4\pi} \sum_{M>0} \ln(1 + e^{-\beta(|q|-M\omega-\mu)}) + \frac{T}{\pi R^2} \int_{-\infty}^\infty \frac{dq}{2\pi} \sum_{n>0, M>0} \ln(1 + e^{-\beta(\sqrt{q^2+2neB}-M\omega-\mu)}) \\
 \equiv & \frac{1}{\pi R^2} P_M \\
 P_M = & T \int_0^\infty \frac{dq}{4\pi} \ln(1 + e^{-\beta(|q|-\mu_M)}) + T \int_{-\infty}^\infty \frac{dq}{2\pi} \sum_{n>0} \ln(1 + e^{-\beta(\sqrt{q^2+2neB}-\mu_M)}), \quad \mu_M = \mu + M\omega
 \end{aligned}$$

De Hass-van Alphen Effect with Rotation

Shu-Yun Yang, Ren-Da Dong, DF Hou, Hai-Cang Ren, PRD 107, 076020 (2023)

finite T

$$P_{\text{dHvA}} = -\frac{(eB)^{\frac{1}{2}}}{2\pi^2 R^2} \sum_{l=1}^{\infty} \frac{1}{l^{3/2}} \sum_{M>0} \frac{\cos \left[\frac{l\pi}{eB}(\mu + M\omega)^2 - \frac{\pi}{4} \right]}{\sinh \frac{2l\pi^2 T(\mu+M\omega)}{eB}}.$$

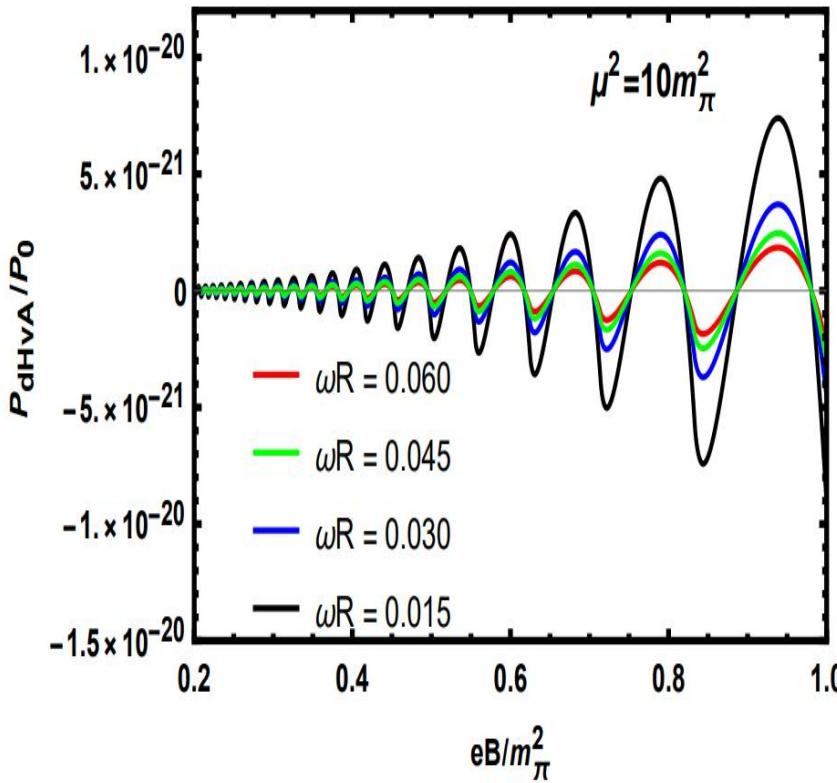
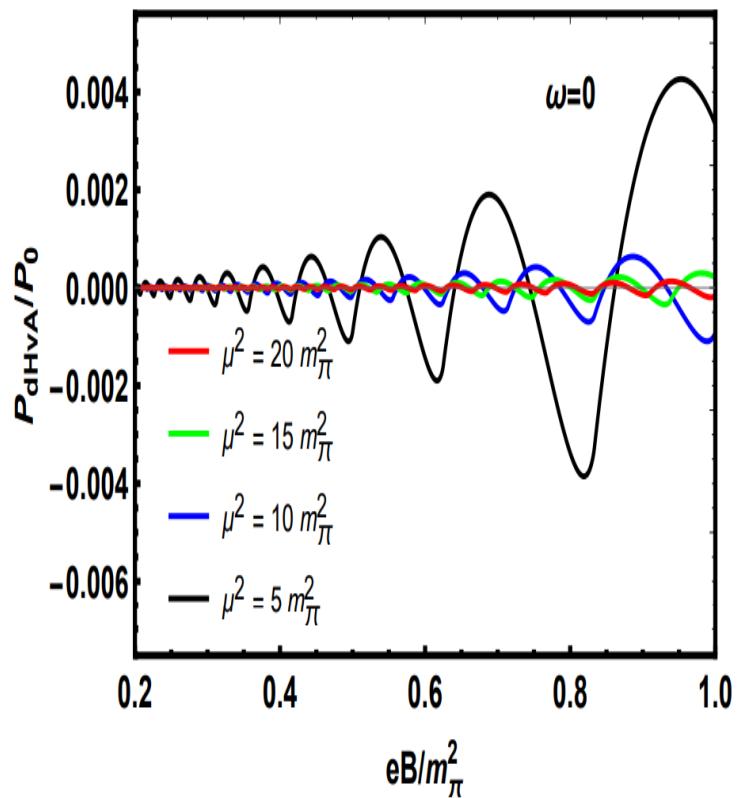
Zero T:

$$P_{\text{dHvA}} = -\frac{(eB)^{\frac{3}{2}}}{4\pi^4 R^2} \sum_{M>0} \frac{1}{\mu + M\omega} \sum_{l=1}^{\infty} \frac{1}{l^{5/2}} \cos \left[\frac{l\pi}{eB}(\mu + M\omega)^2 - \frac{\pi}{4} \right]$$

In rotation, the thermodyn Equl. is established under a AM. The equal distrib. of different AM states within a LL is offset by the nonzero AV with higher AM more favored than lower ones, which amounts to lifting the degeneracy of LL. The dHvA oscillation is thereby expected to be reduced by rotation

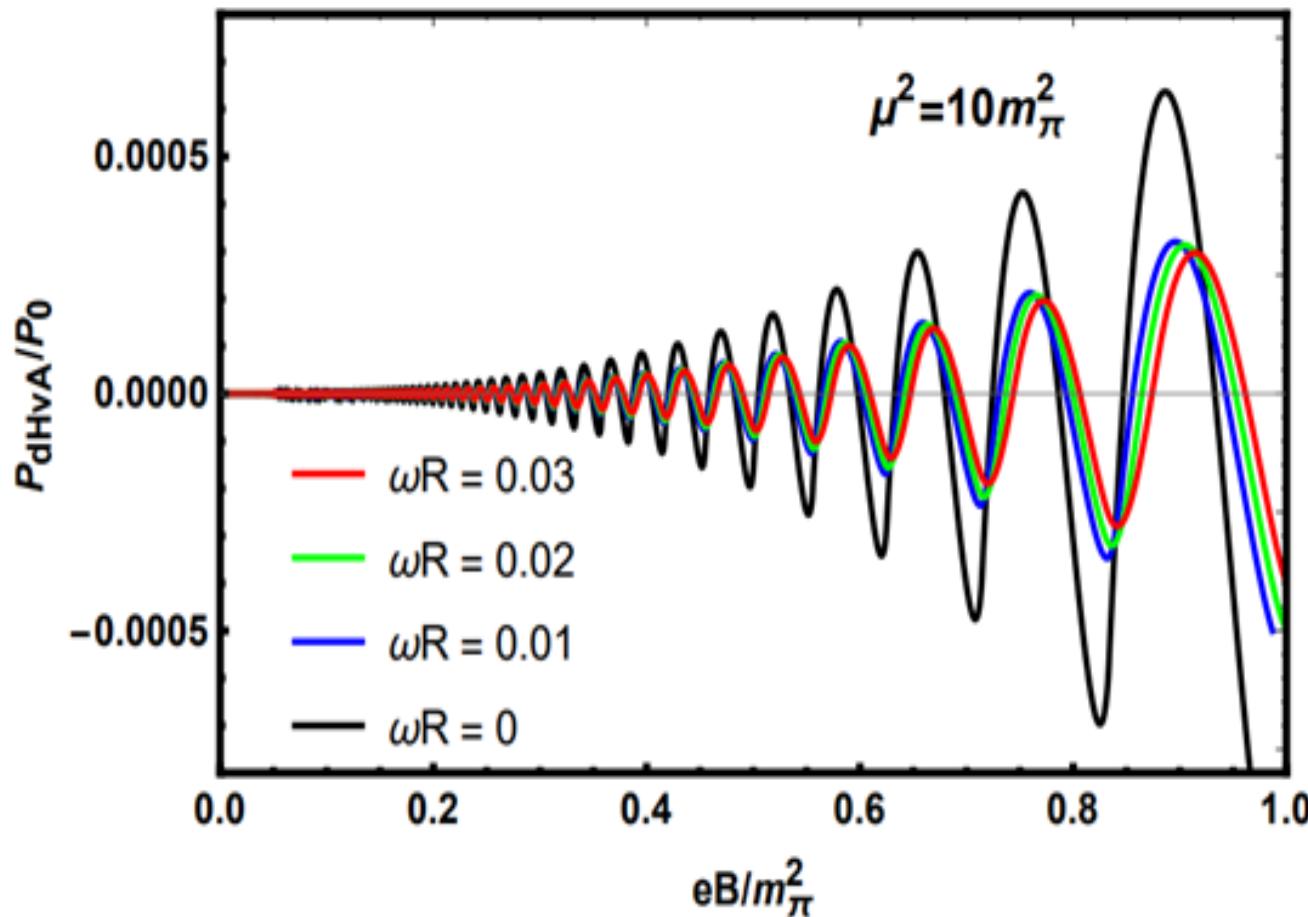
De Hass-van Alphen oscillation

dHvA in NS with rotation ($R=1\text{km}$)

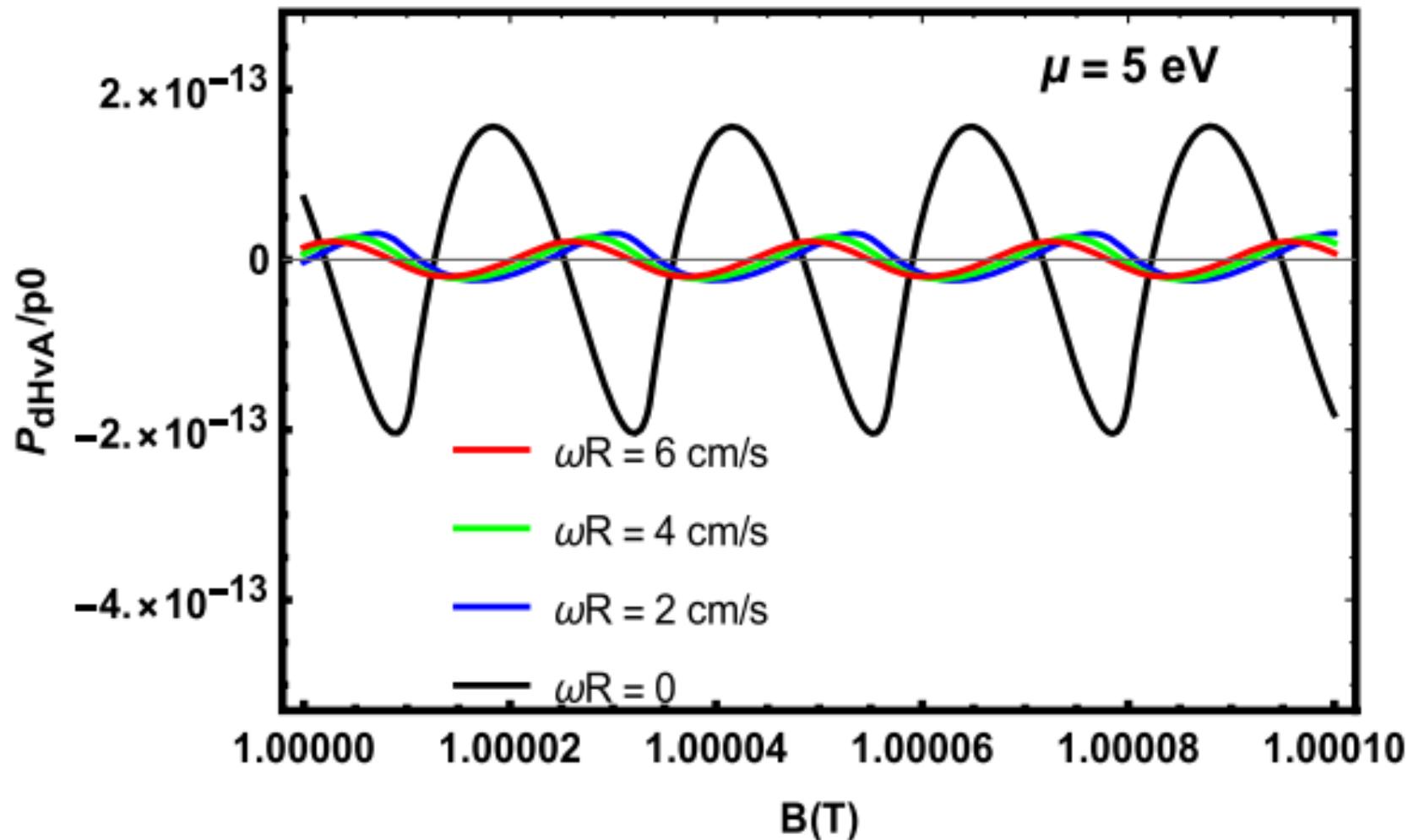


Huge suppression of dHvA oscill. (17 order)
due to large size

dHvA oscillation in cold & dense QGP droplet (R=10fm)



the variation of dHvA oscillation with angular velocity
appears detectable (oscillation remains)



a typical electron gas in a good metal, the variation of dHvA oscillation with AM appears detectable, via magnetization and/or magnetic susceptibility.

Summary

Properties of strongly interacting matter under magnetic field and rotation are interesting

- **We derive the relativistic viscous Hydro with Angular momentum**
- **De Hass-van Alphen Effect with Rotation**

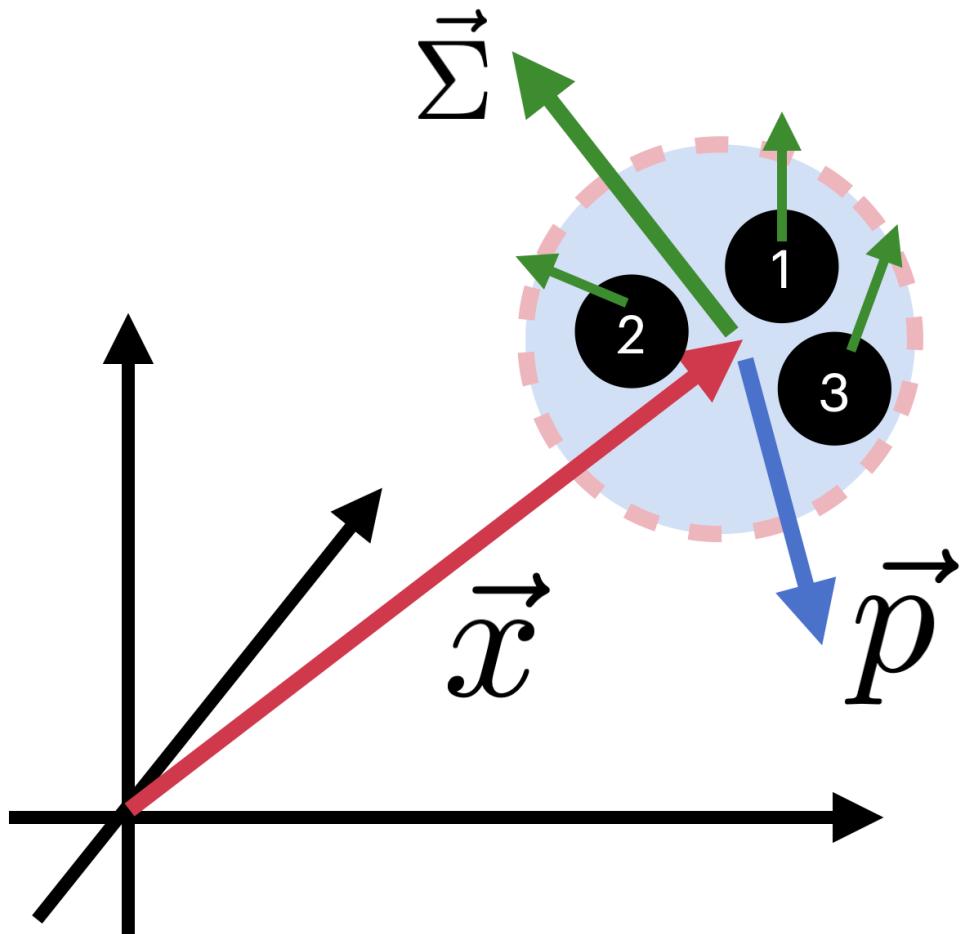
Outlook: How to understand new phase structure

2-nd order viscous hydro. With AM

Compute the new transport coefficents

Thank you very much for your attention!

Transformation of the system of particles and the system of CM



$$\begin{aligned}
 \mathbf{r}_c &= \frac{\sum_{i=1}^n m_i \mathbf{r}_i}{\sum_{i=1}^n m_i} \Rightarrow \mathbf{v}_c = \frac{\sum_{i=1}^n m_i \mathbf{v}_i}{\sum_{i=1}^n m_i}, \quad \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} \\
 M\mathbf{v}_c &= \sum_{i=1}^n m_i \mathbf{v}_i, \quad M = \sum_{i=1}^n m_i \\
 \mathbf{r}_i &= \mathbf{r}_c + \mathbf{r}'_i \Rightarrow \mathbf{v}_i = \mathbf{v}_c + \mathbf{v}'_i \\
 L &= \sum_{i=1}^n [\mathbf{r}_i \times (m_i \mathbf{v}_i) + \mathbf{s}_i] \\
 &= \sum_{i=1}^n [m_i (\mathbf{r}_c + \mathbf{r}'_i) \times (\mathbf{v}_c + \mathbf{v}'_i) + \mathbf{s}_i] \\
 &= \sum_{i=1}^n m_i \mathbf{r}_c \times \mathbf{v}_c + \sum_{i=1}^n m_i \mathbf{r}_c \times \mathbf{v}'_i + \sum_{i=1}^n m_i \mathbf{r}'_i \times \mathbf{v}_c + \sum_{i=1}^n m_i \mathbf{r}'_i \times \mathbf{v}'_i + \sum_{i=1}^n \mathbf{s}_i \\
 &= M \mathbf{r}_c \times \mathbf{v}_c + \sum_{i=1}^n m_i \mathbf{r}'_i \times \mathbf{v}'_i + \sum_{i=1}^n \mathbf{s}_i \\
 \sum_{i=1}^n m_i \mathbf{v}'_i &= \sum_{i=1}^n m_i (\mathbf{v}_i - \mathbf{v}_c) = \sum_{i=1}^n m_i \mathbf{v}_i - \mathbf{v}_c \sum_{i=1}^n m_i = 0 \\
 \sum_{i=1}^n m_i \mathbf{r}'_i &= \sum_{i=1}^n m_i (\mathbf{r}_i - \mathbf{r}_c) = \sum_{i=1}^n m_i \mathbf{r}_i - \mathbf{r}_c \sum_{i=1}^n m_i = 0
 \end{aligned}$$

Equivalence of canonical and phenomenological formulations of spin hydrodynamics

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Canonical framework

		Power counting
$\partial_\mu T^{\mu\nu}$	= 0	$S^{\mu\nu} \sim \mathcal{O}(\partial^0), \omega^{\mu\nu} \sim \mathcal{O}(\partial^1)$
$\partial_\mu J^{\mu\alpha\beta}$	= $\partial_\mu (x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + S^{\mu\alpha\beta})$	
	= $\partial_\mu S^{\mu\alpha\beta} + 2T^{[\alpha\beta]} = 0$	
$T_{\text{can}}^{\mu\nu}$	= $T_{(0)}^{\mu\nu} + T_{\text{can}(1)}^{\mu\nu}, \quad T_{(0)}^{\mu\nu} = \varepsilon u^\mu u^\nu - p \Delta^{\mu\nu}$	
$S_{\text{can}}^{\mu\alpha\beta}$	= $u^\mu S^{\alpha\beta} + u^\beta S^{\mu\alpha} + u^\alpha S^{\beta\mu} + S_{\text{can}(1)}^{\mu\alpha\beta}, \quad S^{\mu\nu} = -S^{\nu\mu}$	

The generalized laws of thermodynamics are

$$\begin{aligned}\varepsilon + p &= Ts + \omega_{\alpha\beta} S^{\alpha\beta} \\ d\varepsilon &= Tds + \omega_{\alpha\beta} dS^{\alpha\beta} \\ dp &= sdT + S^{\alpha\beta} d\omega_{\alpha\beta}\end{aligned}$$

The nonequilibrium entropy current is

$$\begin{aligned}\mathcal{S}_{\text{can}}^\mu &= T_{\text{can}}^{\mu\nu} \beta_\nu + p \beta^\mu - \omega_{\alpha\beta} S^{\alpha\beta} \beta^\mu + \mathcal{O}(\partial^2) \\ &= S_{(0)}^\mu + T_{\text{can}(1)}^{\mu\nu} \beta_\nu + \mathcal{O}(\partial^2)\end{aligned}$$

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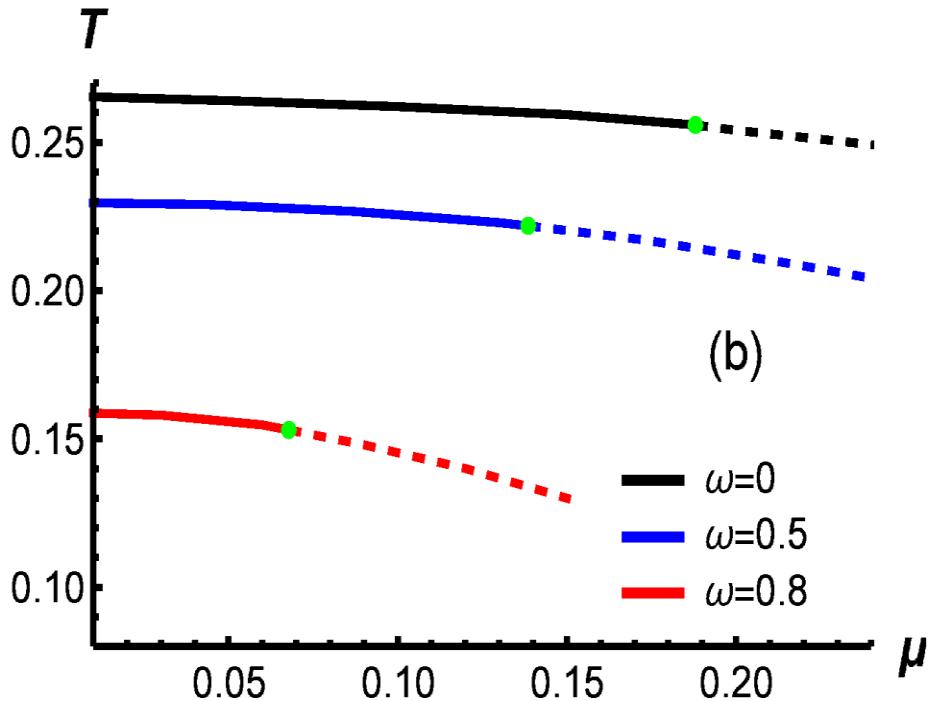
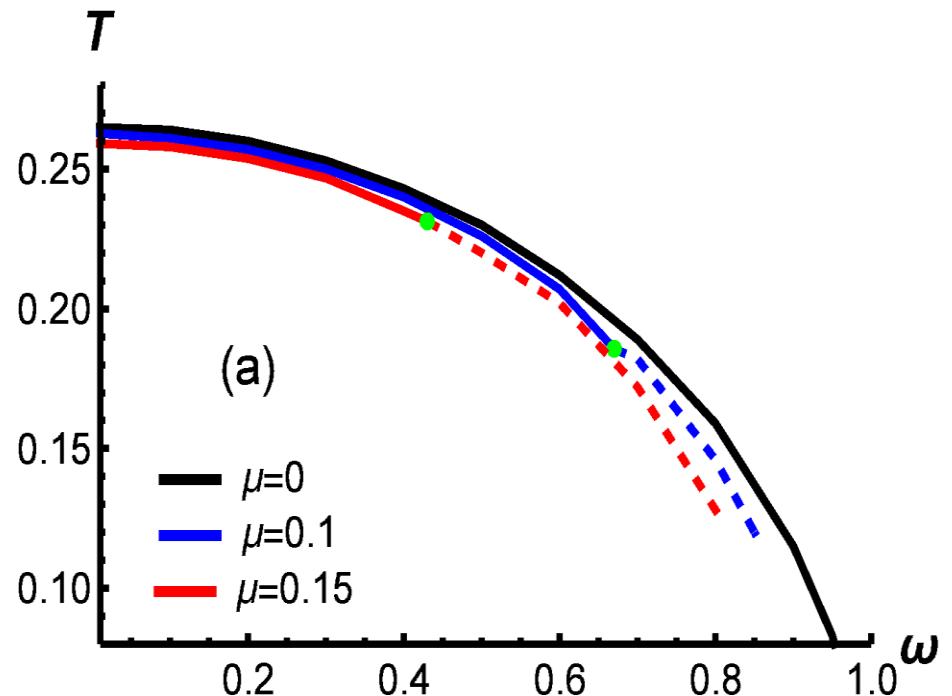
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