

Form Factors and Chiral Kinetic Theory



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Outline

- Introduction
- CKT & Field theory
- In-medium electromagnetic form factors
- Example: Radiative correction
- Summary

Introduction

- Heavy ion collision

↳ ➤ QGP state

➤ Introduction

- Non-central collision

↳ ➤ Strong magnetic field

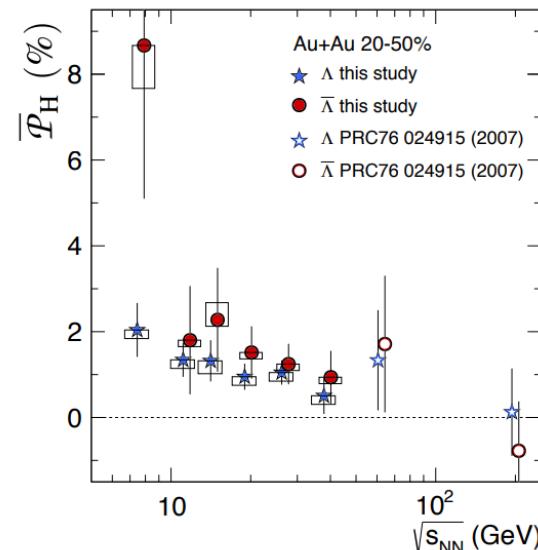
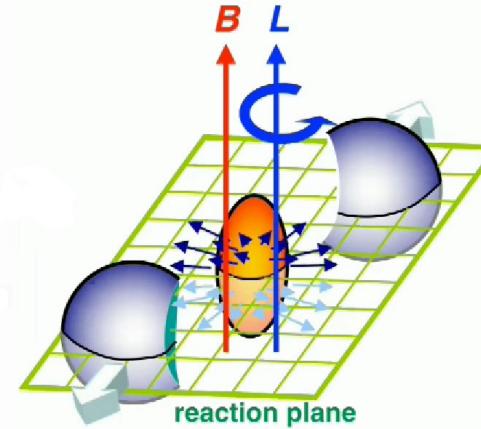
➤ Large angular momentum

➤ CKT & Field theory

- Non-zero background field

↳ ➤ Spin polarization

➤ In-medium
electromagnetic
form factors



STAR collaboration, Nature 2017.

➤ Example: Radiative
correction

➤ Summary

CKT & Field theory

In high temperature region:

- A direct choice: **Chiral Kinetic Theory(CKT)**
 - effective in finite-temperature physics
 - Limitations: weak coupling (may incompatible with phenomenology calculation)
 - New point of view: **Form Factor Formalism** (for the chiral fermions)
 - Describe the **vertex structure**
 - Universal, does **not** require weak coupling
- Describe the spin interaction **non-perturbatively**
- Introduction
 - CKT & Field theory
 - In-medium electromagnetic form factors
 - Example: Radiative correction
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CKT & Field theory

- Wigner function (right-handed component) in **CKT**

Definition: $S^{<\mu}(X, P) = \frac{1}{2}Tr[\sigma^\mu S^<(X, P)]$

$$S_{\alpha\beta}^<(X = \frac{x_1 + x_2}{2}, P) = \int d(x_1 - x_2) e^{iP \cdot (x_1 - x_2)} U(x_2, x_1) \left(-\langle \psi_\beta(x_2) \psi_\alpha(x_1) \rangle \right)$$

➤ Introduction

➤ CKT & Field theory

➤ In-medium electromagnetic form factors

➤ Example: Radiative correction

➤ $\mathcal{O}(\partial_X^0)$: $S_{(0)}^{<i} = -2\pi P^i \delta(P^2) f(p_0)$

➤ $\mathcal{O}(\partial_X)$: $S_{(1)}^{<i} = -2\pi (\underbrace{p_0 B_i}_{\text{spin-magnetic coupling}} - \underbrace{\epsilon^{ijk} P_j E_k}_{\text{spin-Hall effect}}) \delta'(P^2) f(p_0)$

- Wigner function (right-handed component) in **field theory**:

- Fermion-background field coupling
 - Feynman diagrams
- } \implies Spin interaction

➤ Summary

CKT & Field theory

- Field theory calculation

$$S^{<\mu} = -\frac{1}{2} \text{Tr} [\sigma^\mu (S^{ra} - S^{ar}) \tilde{f}]$$

KMS relation

$$S^{ra} = \overbrace{\text{r} \quad \text{a}} + \overbrace{\text{r} \quad \text{a} \underbrace{\text{r}}_{\otimes} \quad \text{a}} + \overbrace{\text{r} \quad \text{a} \underbrace{\text{r}}_{\otimes} \quad \text{a} \underbrace{\text{r}}_{\otimes} \quad \text{a}} + \dots$$

➤ $\mathcal{O}(q^0)$: a “free” propagator carrying **kinetic momentum**.

$$S_{(0)}^{<\mu}(P \rightarrow P - A = K) = -2\pi K^\mu \delta(K^2) \tilde{f}(K_0)$$

$$q \sim \partial_X$$

➤ $\mathcal{O}(q)$: contribute to the **spin interactions**.

$$S_{(1)}^{<0}(K) = -2\pi(\vec{k} \cdot \vec{B}) \delta'(K^2) \tilde{f}(K_0)$$

$$S_{(1)}^{<i}(K) = -2\pi(k_0 B_i - \epsilon^{ijk} K_j E_k) \delta'(K^2) \tilde{f}(K_0)$$

➤ Introduction

➤ CKT & Field theory

➤ In-medium electromagnetic form factors

➤ Example: Radiative correction

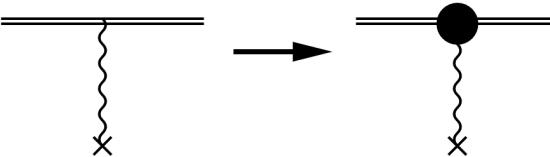
➤ Summary

In-medium electromagnetic form factors (EMFF)

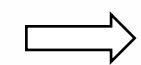
- Vacuum: represent spin interaction as a **QED coupling**. ➤ Introduction
 - Medium:
 - **Lorentz covariance** is broken ➤ CKT & Field theory
 - **coefficients** of spin interaction might be partially changed ➤ In-medium electromagnetic form factors
 - Introduce the EMFF into decomposition of the vertex:
 - Constraint condition: **Ward identity**
 - “no work” condition: $K \cdot E = 0$ ➤ $K \cdot Q = 0 : P_{1,2}$ on shell (up to $\mathcal{O}(Q^2)$) ➤ Example: Radiative correction
 - $Q \cdot u = 0$: electromagnetic field static
- ⇒
$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{k}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu K_\rho Q_\sigma}{2(K \cdot u)^2}$$
 ➤ Summary

In-medium electromagnetic form factors (EMFF)

- Replacement: $\sigma^\mu \rightarrow \Gamma^\mu$:



➤ Introduction



$$S_{(1)}^{<0} = F_2 (\vec{k} \cdot \vec{B}) 2\pi\delta'(K^2) f(K_0),$$

➤ CKT & Field theory

$$S_{(1)}^{<i} = [F_0 \epsilon^{ijk} E_j k_k + F_1 (k_0 B^i - (\vec{B} \cdot \vec{k}) \hat{k}^i) + F_2 (\vec{B} \cdot \vec{k}) \hat{k}^i] 2\pi\delta'(K^2) f(K_0).$$

➤ In-medium
electromagnetic
form factors

where F_0 : spin Hall effect; F_1 : spin-perpendicular magnetic coupling;
 F_2 : spin-parallel magnetic coupling.

- For tree-level diagrams: $F_0 = 1; F_1 = 1; F_2 = 1$ $\xrightarrow{\text{correspond}}$ CKT
- **Nonindependence** of each other:

➤ Example: Radiative
correction

$$u^\dagger(P_1) [\mathbf{A}_1 \sigma^\mu] u(P_2)$$

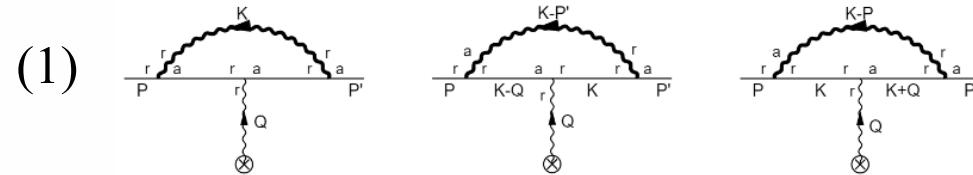
- \mathbf{A}_1 gives $F_{1,2,3}$
- Still in **vacuum** yet

➤ Summary

Example: Radiative correction (in medium)

- An example: **QCD radiative correction** to electromagnetic form factor

➤ Introduction

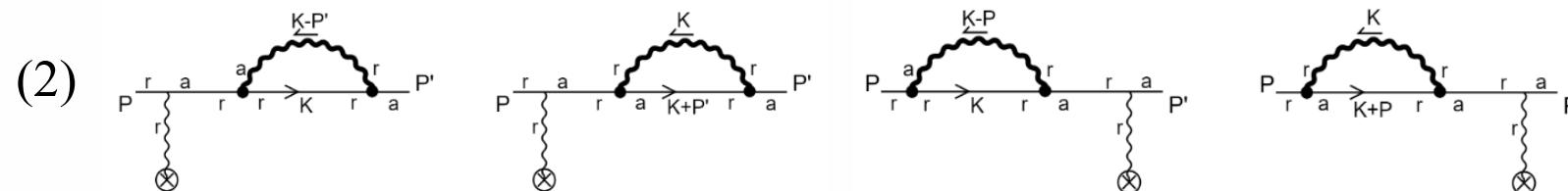


: gluon propagator

➤ CKT & Field theory

$$\Rightarrow \delta\Gamma_{vertex}^\nu A_\nu = 2m_f^2 A_\nu \sigma_\lambda \left[\frac{1}{6p^2} \left(2 \ln \left(\frac{pT}{m_f^2} \right) + \ln \left(\frac{2pT}{m_g^2} \right) - 36 \ln(A) + \ln(16\pi^3) + 3 \right) \left(\hat{l}^\lambda \hat{l}^\nu - \hat{p}^\lambda P^\nu \frac{1}{p} \right) + \hat{p}^\lambda \hat{p}^\nu \frac{1}{p^2} \right]$$

➤ In-medium electromagnetic form factors



➤ Example:
Radiative correction

$$\Rightarrow \delta\Gamma_{self-energy}^\nu A_\nu = 2 \frac{m_f^2}{2p^2} \left(1 - \ln \frac{2p^2}{m_f^2} \right) \sigma^\nu A_\nu$$

where $\hat{l}^i = \frac{1}{pq} \epsilon^{ijk} q_j p_k$, $\hat{p}^\nu = (0, \vec{p})$.

➤ Summary

- Approximation: Hard Thermal Loop, external propagator on-shell

Example: Radiative correction (in medium)

Medium correction to electromagnetic form factors:

$$\left. \begin{aligned} \delta F_0 &= \frac{2m_f^2}{k^2} X + \frac{m_f^2}{k^2} \left(1 - \ln \frac{2k^2}{m_f^2} \right), \\ \delta F_1 &= \frac{2m_f^2}{k^2} (X - 1) + \frac{m_f^2}{k^2} \left(1 - \ln \frac{2k^2}{m_f^2} \right), \\ \delta F_2 &= \frac{2m_f^2}{k^2} X + \frac{m_f^2}{k^2} \left(1 - \ln \frac{2k^2}{m_f^2} \right), \end{aligned} \right\} \quad \boxed{\delta F_0 = \delta F_1 \neq \delta F_2}$$

where

$$\rightarrow X = \frac{1}{6} \left(2 \ln \left(\frac{kT}{m_f^2} \right) + \ln \left(\frac{2kT}{m_g^2} \right) - 36 \ln(A) + \ln(16\pi^3) + 3 \right)$$

$\rightarrow m_f^2 = \frac{1}{8} g^2 T^2 \cdot \frac{4}{3}$: fermion thermal mass;

$\rightarrow m_g^2 = \frac{1}{3} g^2 T^2 (3 + \frac{1}{2} N_f)$: gluon thermal mass.

F_0 : spin Hall effect;

F_1 : spin-perpendicular magnetic coupling;

F_2 : spin-parallel magnetic coupling.

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Summary

- Spin interaction in CKT is reformulated in field theory. ➤ Introduction
- Extending to form factor, CKT propagator correspond to the **tree level diagrams**. ➤ CKT & Field theory
- Medium correction lift the degeneracy of these form factor to a certain extent. ➤ In-medium electromagnetic form factors
- Outlook
 - Go beyond “**no work**” condition ➤ Example: Radiative correction
 - **Fermion-metric perturbation coupling** ➤ Summary

Thanks