Quantum kinetic theory with QED-interaction and its effect on spin polarization

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The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion collisions

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Outline

- Global and local polarization in heavy ion collisions
- Quantum kinetic theory with collisions
- Collisional effects on spin polarization
- Summary

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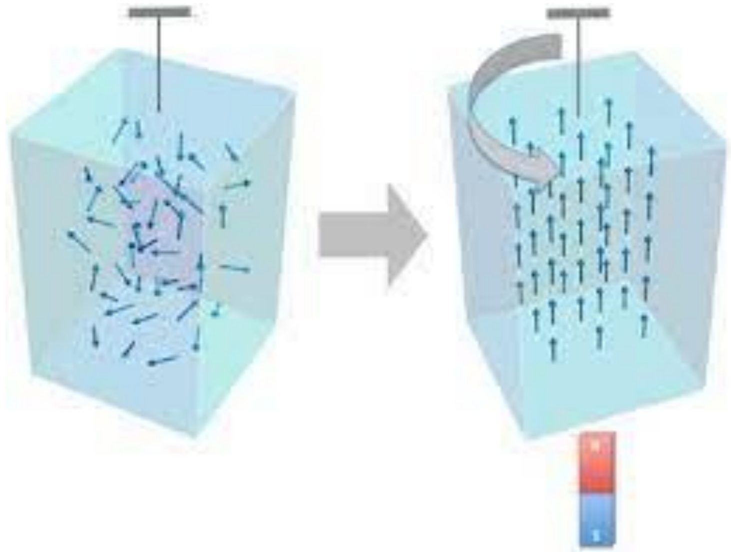
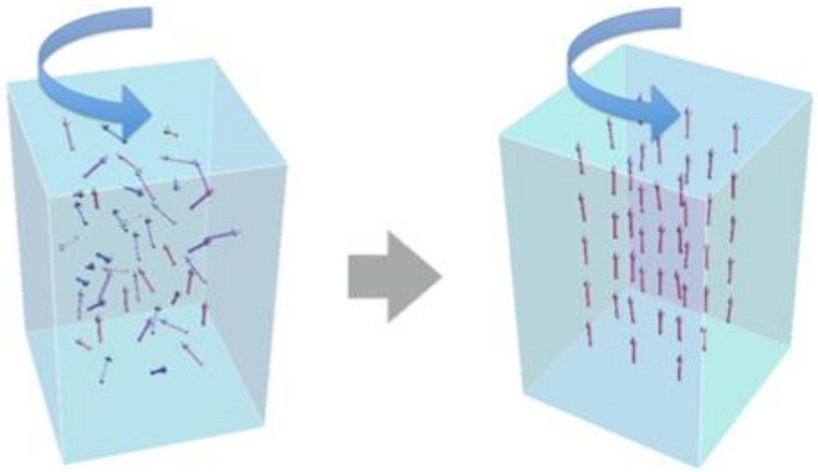
Spin-orbit coupling in many-body system



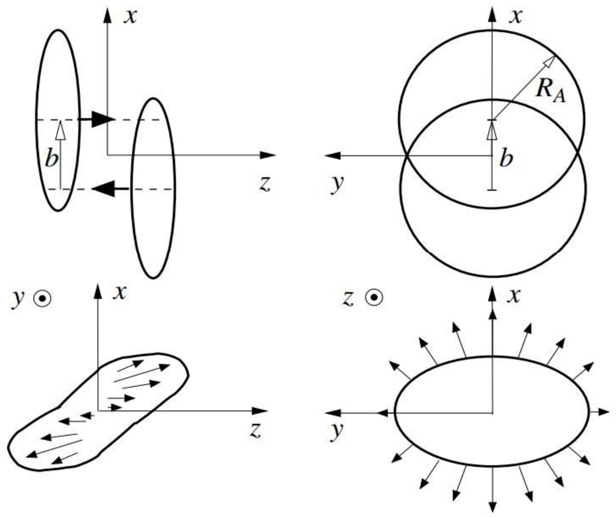
Einstein de-Haas effect
Polarization -> rotation



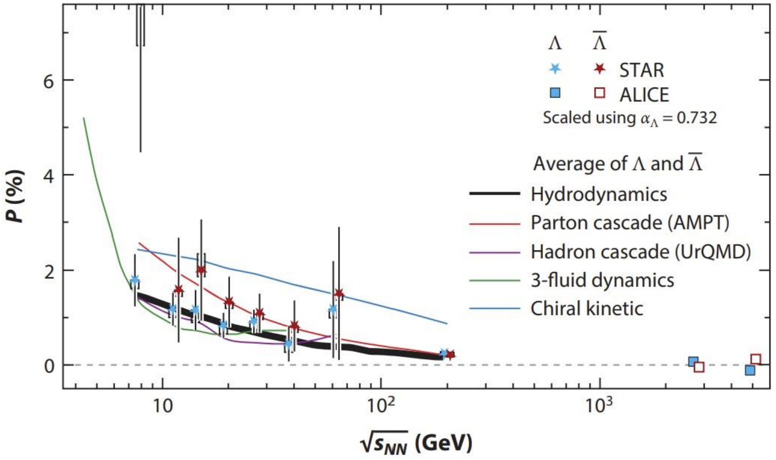
Barnett effect
Rotation -> Polarization

S. J. Barnett, Phys. Rev. 6, 239 (1915)
A. Einstein and W. J. de Haas, Verh. d. Deutsch. Phy. Ges. 17, 152 (1915)

Global polarization



- Large initial orbit angular momentum (OAM) of order of $10^5\hbar$ is produced in peripheral heavy ion collisions.
- The deconfined quarks can be polarized along the direction of the initial OAM via **spin-orbit coupling**, and the hadrons in final state can be further polarized from quark coalescence mechanism.
- QGP is **the most vortical matter in nature**, $\omega \sim 10^{22}s^{-1}$.



Z. T. Liang, X. N. Wang, PRL(2005)
B. Betz, M. Gyulassy, G. Torrieri, PRC (2007)
F. Becattini, F. Piccinini, J. Rizzo PRC (2008)
G.-H. Gao, S.-W. Chen, W.-T. Deng, Z.-T. Liang, Q. Wang PRC (2008)

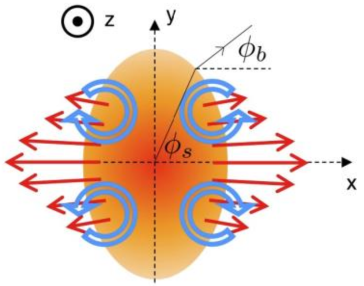
L. Adamczyk et al. [STAR Collaboration], Nature 548, 62(2017)
Francesco Becattini, Michael A. Lisa, ARNPS 70 (2020) 395-423

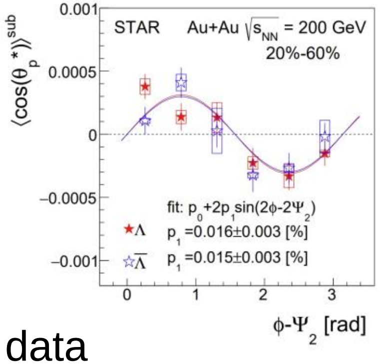
Sign puzzle in local polarization

- **Stronger in-plane expansion** of QGP due to **spacetime anisotropy** can induce **local polarization** along the beam line.

- In the past, the theories in the market **CANNOT** explain the local polarization well, even gives opposite results.

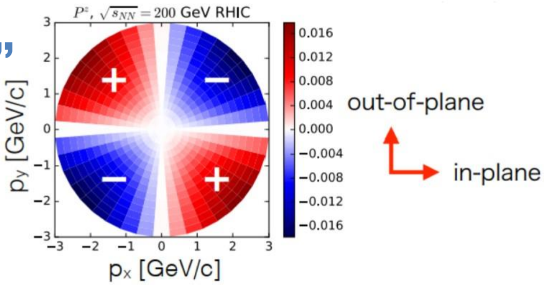
- The **shear effects** can be important to local polarization, and even quantitively explain the experiment by **proper choice of parameter**.





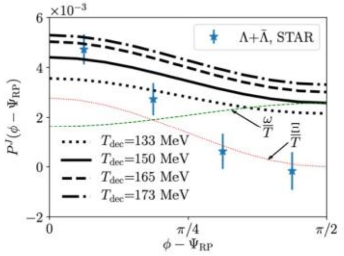
Experimental data
[STAR Collaboration], PRL(2019)

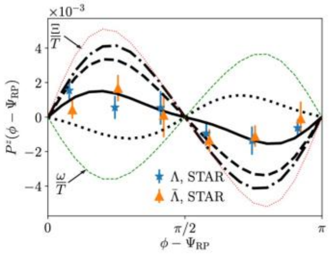
"sign puzzle"

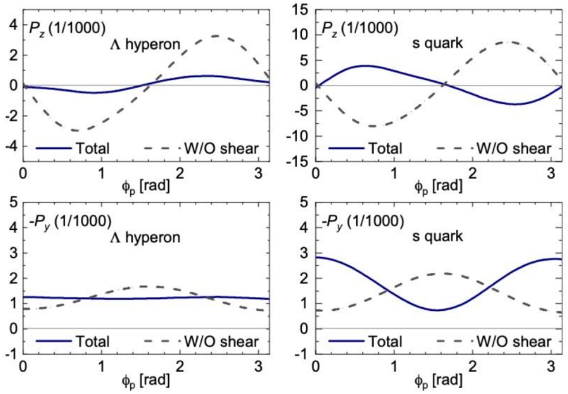


Phenomenological model
Becattini , Karpenko, PRL(2018)

✓ A shear induced polarization:
An "**Intrinsic Hall effect**"







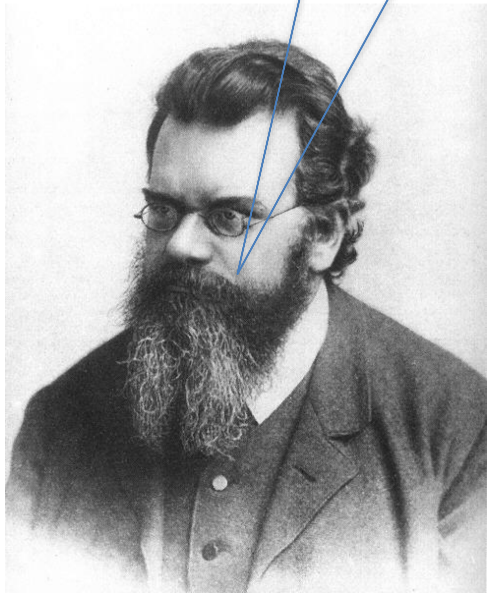
Fu, Liu, Pang et al, PRL. 127 (2021) 14, 142301
Becattini , Buzzegoli, Palermo, PRL. 127 (2021) 27, 272302
Yi , Pu, and Yang, PRC. 104 (2021) 6, 064901

Question: how can we understand spin polarization?

The spin polarization in heavy ion collision is a collective motion, and to understand it we need: a Boltzmann-type equation with spin.

Question: Microscopically, how can we implement the **spin evolution** into the evolution of quarks and gluons?

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \mathbf{F} \cdot \nabla_{\mathbf{p}} f = C[f]$$



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- Global and local polarization in heavy ion collisions
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Present efforts on quantum kinetic theory (I)

Up to now there are plenty of investigations on relativistic quantum kinetic theory and spin transport equations:

- Hamiltonian form and effective field theory:

  D. T. Son, N. Yamamoto, PRL109 (2012) 181602; PRD 87 (2013) 8, 085016; S. Lin, A. Shukla, JHEP 06 (2019) 060

- Path integral formalism:

  M.A. Stephanov, Y. Yin PRL 109 (2012) 162001; J.-Y. Chen, D. T. Son, M. A. Stephanov, H.-U. Yee, Y. Yin, PRL 113 (2014) 18, 182302; J.-W. Chen, J.-Y. Pang, S. Pu, Q. Wang PRD 89 (2014) 9, 094003; J.-Y. Chen, D. T. Son, M. A. Stephanov, PRL 115, 021601 (2015)

- Chiral kinetic theory from quantum field theory:

  J.-W. Chen, S. Pu, Q. Wang, X.-N. Wang, PRL 110 (2013) 26, 262301; Y. Hidaka, S. Pu, D.-L. Yang, PRD 95 (2017) 9, 091901; A. Huang, S. Shi, Y. Jiang, J. Liao, P. Zhuang, PRD 98 (2018) 3, 036010

- Worldline formalism:

  N. Mueller, R. Venugopalan, PRD 97 (2018) 5, 051901;PRD 96 (2017) 1, 016023

- Quantum kinetic theory including quantum corrections to collision terms:

  N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, D. H. Rischke PRD 100 (2019) 5, 056018; PRL 127 (2021) 5, 052301; PRD 104 (2021) 1, 016022; PRD 104 (2021) 1, 016029; K. Hattori, Y. Hidaka, D.-L. Yang PRD 100 (2019) 9, 096011; JHEP 07 (2020) 070; Y.-C. Liu, K. Mameda, X.-G. Huang CPC 45 (2021) 8, 089001; Z. Wang, X. Guo, S. Shi, P. Zhuang PRD 100 (2019) 1, 014015; Z. Wang, X. Guo, P. Zhuang, EPJC 81 (2021) 9, 799; Z. Wang, arXiv:2205.09334; S. Lin, Phys.Rev.D 105 (2022) 7, 076017; S. Lin, Z. Wang, JHEP 12 (2022) 030

Present efforts on quantum kinetic theory (II)

Talks on recent process of **quantum kinetic theory** in the **7th conference on Chirality, 2023**:

- July 15th :

  Jianhua Gao, 16:30-17:00; **X.-L. Sheng**, 17:00-17:30;

- July 17th :

  Qun Wang, 8:30-9:00; **Shi Pu**, 9:30-10:00; **Kazuya Mameda**, 10:00-10:30;
  Shu Lin, 10:50-11:20; **Ziyue Wang**, 11:20-11:50; **Xingyu Guo**, 11:50-12:20;
  David Wagner, 15:00-15:30;

- July 18th :

  Afternoon Session-I: **Jiayuan Tian**, 14:30-14:45; **Peiwei Yu**, 15:15-15:30;

Collisional quantum kinetic theory (I)

- We define the gauge invariant 2-point Wigner function for fermions,

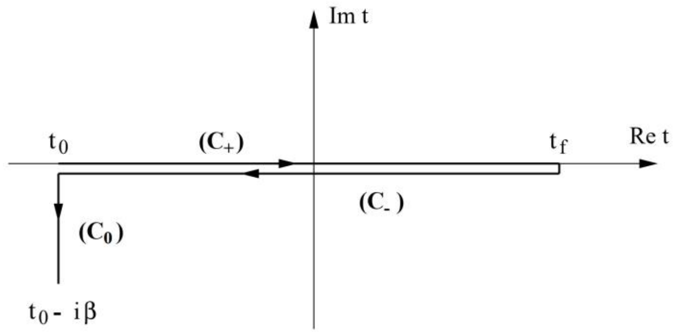
$$S^{<}_{\alpha\beta}(x,p) = \int d^4y e^{-ip\cdot y}\langle :\overline{\psi}_\beta(x+\frac{y}{2})e^{\frac{y}{2}\cdot\overleftarrow{D}(x)}\otimes e^{-\frac{y}{2}\cdot\overrightarrow{D}(x)}\psi_\alpha(x-\frac{y}{2}):\rangle$$

- Along the Schwinger-Keldysh contour, one derives the Kadanoff-Baym equations up to O($\hbar$),

$$p_\mu\gamma^\mu S^< + \frac{i\hbar}{2}\gamma^\mu\partial_\mu S^< = \frac{i\hbar}{2}(\Sigma^< S^> - \Sigma^> S^<) \quad , \quad p_\mu S^<\gamma^\mu - \frac{i\hbar}{2}\partial_\mu S^<\gamma^\mu = -\frac{i\hbar}{2}(S^>\Sigma^< - S^<\Sigma^>)$$

- For massless particle, in 4×4 Clifford basis, we have,

$$S = \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu$$



- We adopt the power counting considering that the axial chemical potential comes from quantum corrections,

$$\mathcal{V}^\mu, \Sigma^\mu_{\rm V}, f_{\rm V} \sim \mathcal{O}(\hbar^0), \qquad \mathcal{A}^\mu, \Sigma^\mu_{\rm A}, f_{\rm A} \sim \mathcal{O}(\hbar^1)$$

J.-P. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528；Y. Hidaka, S. Pu, and D.-L. Yang, Phys. Rev. D 95, 091901 (2017)；D.-L. Yang, K. Hattori, Y. Hidaka, JHEP 07 (2020) 070

Collisional quantum kinetic theory (II)

- The formal solutions to the vector and axial Wigner functions read,

$$\begin{aligned}\mathcal{V}^{\lessgtr,\mu}(p) &= 2\pi\mathrm{sgn}(n\cdot p)p^{\mu}\delta(p^{2})f_{V}^{\lessgtr}(p)\\ \mathcal{A}^{\lessgtr,\mu}(p) &= 2\pi\mathrm{sgn}(n\cdot p)p^{\mu}\delta(p^{2})f_{A}^{\lessgtr}(p)+2\pi\hbar\delta(p^{2})\mathrm{sgn}(n\cdot p)S^{(n),\mu\alpha}\left[\partial_{\alpha}f_{V}^{\lessgtr}-\left(\Sigma_{V,\alpha}^{\lessgtr}f_{V}^{\gtrless}-\Sigma_{V,\alpha}^{\gtrless}(p)f_{V}^{\lessgtr}\right)\right]\end{aligned}$$

where the spin tensor is $S_{(n)}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}p_{\rho}n_{\sigma}/(2n\cdot p)$

- The EoMs of the Wigner functions are,

$$\begin{aligned}\Delta\cdot\mathcal{V}^{<} &= \Sigma_{V}^{<}\cdot\mathcal{V}^{>}-\Sigma_{V}^{>}\cdot\mathcal{V}^{<},\\ \Delta\cdot\mathcal{A}^{<} &= \Sigma_{V}^{<}\cdot\mathcal{A}^{>}-\Sigma_{V}^{>}\cdot\mathcal{A}^{<}-\Sigma_{A}^{<}\cdot\mathcal{V}^{>}+\Sigma_{A}^{>}\cdot\mathcal{V}^{<}.\end{aligned}$$

- For massless fermions, the **equilibrium conditions** are,

$$\begin{aligned}\text{global equilibrium} &: \partial_{(\mu}(\beta u_{\nu)})=0,\omega_{s}^{\mu\nu}=-\Omega^{\mu\nu}=-\partial^{[\mu}(\beta u^{\nu]})\\ \text{local equilibrium} &: \omega_{s}^{\mu\nu}=-\Omega^{\mu\nu}.\end{aligned}$$

Spin chemical potential

Thermal vorticity

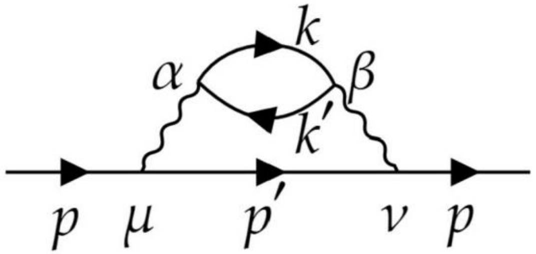
Y. Hidaka, S. Pu, and D.-L. Yang, Phys.Rev.D 97 (2018) 1, 016004

***SF**,S. Pu, D.-L. Yang Phys.Rev.D 106 (2022) 1, 016002*

General discussions and HTL approximations

- We consider the behavior of a hard electron in a thermalized **QED plasma**: a **t-channel** process is firstly considered; **a realistic gauge theory**.

$$e^-(p) + e^-_{eq}(k) \leftrightarrow e^-(p') + e^-_{eq}(k'),$$



- The distribution functions in local equilibrium are taken as

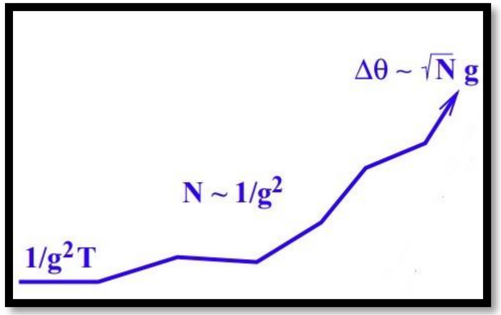
$$\mathcal{N}^{<}_{V,leq}(x,p) = \left[\exp\left(\beta u\cdot p - \beta\mu_V\right) + 1\right]^{-1},$$

$$\mathcal{N}^{<}_{A,leq}(x,p) = -\frac{\hbar}{2}\mathcal{N}^{<}_{V,leq}(x,p)\mathcal{N}^{>}_{V,leq}(x,p)\Omega_{\mu\nu}S^{\mu\nu}_{(u)},$$

so that the axial charge are damped in local equilibrium.

- Then we adopt the momentum hierarchy to consider the **small-angle multi-scattering process** only to get the leading-logarithm result,

$$eT \ll q^\mu \ll T, k^\mu, k'^{,\mu}, p^\mu,$$



We also adopt the q expansion and keep up to q^2: **leading logarithm** results.

Arnold, Moore, Yaffe JHEP 11 (2000) 001; JHEP 05 (2003) 051

Spin Boltzmann equation with collision terms

- A spin Boltzmann equation (SBE) with explicit collision kernel is derived:

$$(p\cdot\partial)f_V^{<}(x,p) = \mathcal{C}_V^{\mathrm{HTL}}[f_V] + \mathcal{O}(\hbar^2),$$

$$(p\cdot\partial)f_A^{<}(x,p) + \hbar\partial_\mu S^{\mu\nu}_{(u)}\partial_\nu f_V^{<}(x,p) = \mathcal{C}_A^{\mathrm{HTL}}[f_V,f_A] + \mathcal{O}(\hbar^2),$$

$$\mathcal{C}_V[f_V] = \frac{e^4\delta(p^2)}{24\beta^2}\ln\frac{T}{m_D}\left[2f_V^{<}(p)f_V^{>}(p) + |\boldsymbol{p}|F(p)\hat{p}_{\perp,\alpha}\partial^{\alpha}_{p_\perp}f_V^{<}(p) - |\boldsymbol{p}|\frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})f_V^{<}(p)\right]$$

$$\begin{aligned}\mathcal{C}_A[f_V,f_A] = -\frac{e^4\delta(p^2)}{8\pi^2|\boldsymbol{p}|}\ln\frac{T}{m_D}\Big\{&\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|F(p)f_A^{<}(p) + \frac{\pi^2}{3\beta^2}|\boldsymbol{p}|^2F(p)[(\hat{p}_\perp\cdot\partial_{p_\perp}) - \frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})]f_A^{<}(p)\\
&-\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|^2f_A^{<}(p)(\hat{p}_\perp\cdot\partial_{p_\perp})f_V^{<}(p) + \hbar F(p)|\boldsymbol{p}|H_{3,\alpha}\partial^{\alpha}_{p_\perp}f_V^{<}(p)\\
&-\hbar\frac{\pi^2}{12\beta^2}F(p)|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\partial_{p_\perp,\rho}\partial_\alpha f_V^{<}(p) + \hbar\frac{\pi^2}{6\beta^3}\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\rho}u_\beta\partial_{p_\perp,\nu}\partial_\alpha f_V^{<}(p)\\
&+\hbar\frac{\pi^2}{6\beta^2}\epsilon^{\mu\xi\lambda\kappa}p_\lambda u_\kappa\partial_\xi f_V^{<}(p)\partial_{p_\perp,\mu}f_V^{<}(p)\\
&-\hbar\frac{\pi^2}{12\beta^3}|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\hat{p}_{\perp,(\gamma}g_{\lambda)\rho}\hat{p}_{\perp,\lambda}\partial^{\lambda}_{p_\perp}\partial^{\gamma}_{p_\perp}\partial_\alpha f_V^{<}(p)\Big\} + \mathcal{O}(\hbar^2).\end{aligned}$$

- It is found that, when the vector and axial **distribution functions of probes are in local equilibrium**, the **collision kernels are zero**: Detailed balance exists for **quantum modes**.

- For certain cases, we find the **spin relaxation rate can be much slower** than particle relaxation rate.

SF, S. Pu, D.-L. Yang Phys.Rev.D 106 (2022) 1, 016002

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Off-equilibrium effects from SBE

- f_A, f_V in **global equilibrium**: the spin polarization pseudovector S^μ receives contributions **only** from **thermal vorticity**;
- f_A, f_V in **local equilibrium**: S^μ also has contributions from shear viscosity, etc. ↔ "**Intrinsic Hall effect**"

$$\mathcal{S}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p\mathcal{J}_5^\mu(p,X)}{2m_\Lambda \int d\Sigma \cdot \mathcal{N}(p,X)},$$

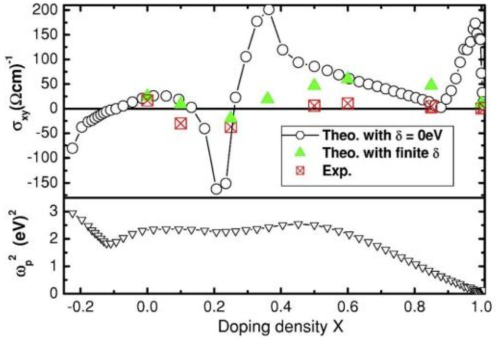
Non-dissipative ↔ Reversible ↔ $C_V = C_A = 0$

- What about the **off-equilibrium effects** from collisions?

Important in condensed matter physics: anomalous Hall effect, spin Hall effect, etc. ↔ "**Extrinsic Hall effect**"

We need to solve the spin Boltzmann equations!

Extrinsic anomalous Hall effects from collisions in condensed matter physics



D. Xiao, M.-C. Chang, Q. Niu RMP 82, 1959
N. A. Sinitsyn 2008, J. Phys.: Condens. Matter 20,023201
Y. Yao, et al. PRB 75, 020401
Hirsch, J. E., 1999, PRL 83, 1834.

- Some common used method: relaxation time approximation, a gradient expansion, method of moments.

SBE with Moller scattering (I)

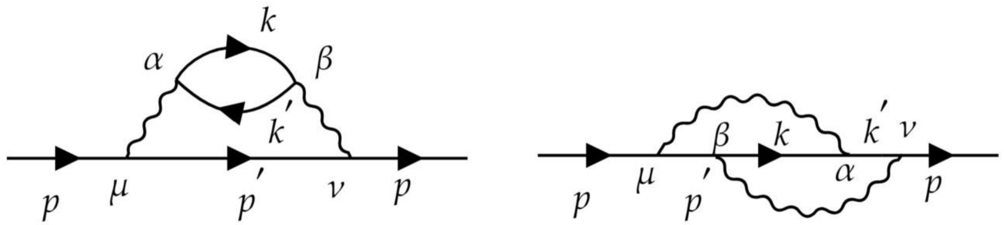
- We discuss the off-equilibrium corrections to the Wigner functions in local equilibrium (the free streaming Wigner functions):

$$\mathcal{A}_{\rm f}^{<,\mu}(X,p) \quad = \quad 2\pi\delta(p^2)p^\mu f^{<}_{\rm A,f} + 2\pi\delta(p^2)\hbar S^{\mu\nu}_{(u)}\partial_\nu f^{<}_{\rm V,f}$$

$$f_{\rm A,f}+\delta f_{\rm A} \qquad\qquad f_{\rm V,f}+\delta f_{\rm V}$$

We did not insert the perturbations to the self-energy terms to avoid double counting.

- Consider a Moller scattering process



- The spin Boltzmann equation array (EoM of f_A, f_V),

$$p\cdot\partial f^{<}_{\rm V} = \frac{1}{4}\int_{p',k,k'} W_{pk\to p'k'}(2\pi)^3\delta(k^2)\delta(p'^{,2})\delta(k'^{,2})$$
$$\times\left(f^{<}_{\rm V}(k')f^{<}_{\rm V}(p')f^{>}_{\rm V}(p)f^{>}_{\rm V}(k) - f^{>}_{\rm V}(k')f^{>}_{\rm V}(p')f^{<}_{\rm V}(p)f^{<}_{\rm V}(k)\right),$$

SF, S. Pu, et al in preparation

$$p^\mu\partial_\mu f^{<}_{\rm A}(p) + \hbar\partial_\mu S^{(u),\mu\alpha}\partial_\alpha f^{<}_{\rm V} = \mathcal{C}^{\rm B}_{\rm A}[f_{\rm V}, f_{\rm A}] + \mathcal{C}^{\partial^2}_{\rm A}[f_{\rm V}] + \mathcal{C}^{\Sigma}_{\rm A}[f_{\rm V}],$$

SBE with Moller scattering (II)

- The spin Boltzmann equation is,

$$\boxed{p^\mu \partial_\mu f^<_A(p) + \hbar \partial_\mu S^{(u),\mu\alpha} \partial_\alpha f^<_V} = \mathcal{C}^{\mathrm{B}}_{\mathrm{A}}[f_{\mathrm{V}}, f_{\mathrm{A}}] + \mathcal{C}^{\partial^2}_{\mathrm{A}}[f_{\mathrm{V}}] + \mathcal{C}^{\Sigma}_{\mathrm{A}}[f_{\mathrm{V}}]$$

∂^2 term, $O(e^4\hbar\partial^2)$

$O(\hbar\partial^2)$

Boltzmann type,
$O(e^4\hbar\partial) + O(e^4\hbar\partial^2)$

Self-energy
coupled, $O(e^8\hbar\partial)$

- If considering the radiative corrections, the formal solution should be written as,

$$\begin{aligned} \delta f_{\mathrm{V}} &= \frac{1}{h(\ln e) e^4 \ln \frac{1}{e}} \times \mathcal{O}(\partial^1) \\ \delta f_{\mathrm{A}} &= \frac{1}{H_1(\ln e) e^4 \ln \frac{1}{e}} \times \left[\mathcal{O}(\hbar\partial^2) + H_2(\ln e) e^4 \ln \frac{1}{e} \times \mathcal{O}(\hbar\partial^1)\right] \end{aligned}$$

h, H1, H2: Possible radiative corrections from thermal photons

Question: Between the leading order terms of gradient and coupling constant, which one is dominant to the spin polarization effect?

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f_V in local equilibrium: Gradient expansion (I)

- To avoid the above problem, we consider f_V in local equilibrium for simplicity: only $C_A^B[f_A, f_V^{leq}]$ survives. This is guaranteed by **$C_A \neq 0$ when f_A is off-equilibrium**.

- Up to **leading logarithm**, the spin Boltzmann equation reduces to,

$$\begin{aligned}
& p^\mu \partial_\mu f_A^<(p) + \hbar \partial_\mu S_{(u)}^{\mu\alpha} \partial_\alpha f_{V,\mathrm{leq}}^<(p) \\
= \; & 8e^4 \int_{p',k,k'} (2\pi)^3 \delta(k'^{,2}) \delta(p'^{,2}) \delta(k^2) \frac{(p\cdot k')^2 + (p\cdot k)^2}{(p-p')^4} \\
& \times f_{V,\mathrm{leq}}^<(k') f_{V,\mathrm{leq}}^<(p') f_{V,\mathrm{leq}}^>(k) f_{V,\mathrm{leq}}^>(p) \left[-\frac{\delta f_A^<(p)}{f_{V,\mathrm{leq}}^<(p) f_{V,\mathrm{leq}}^>(p)} + \frac{\delta f_A^<(p')}{f_{V,\mathrm{leq}}^<(p') f_{V,\mathrm{leq}}^>(p')} \right].
\end{aligned}$$

- Make gradient expansion and match the same order in both side,

$$\begin{aligned}
& \hbar \left(\frac{\hat{\partial}_\mu S_{(u)}^{\mu\nu}}{E_{\mathbf{p}}} \left[-\hat{\partial}_\nu (\beta u \cdot p) + \hat{\partial}_\nu (\beta \mu_V) \right] \right)^{(2)} f_{V,\mathrm{leq}}^<(x,p) f_{V,\mathrm{leq}}^>(x,p) \\
& + L \left(\hat{D} f_A^<(x,p) \right)^{(2)} + L \frac{p^{\langle\mu\rangle}}{E_{\mathbf{p}}} \hat{\nabla}_\mu f_A^{<,(1)}(x,p) = \frac{\lambda^2}{E_{\mathbf{p}}} \mathcal{C}_A^{(2)},
\end{aligned}$$

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SF, S. Pu, et al in preparation

f_V in local equilibrium: Gradient expansion (II)

- By hard calculation, we get,

$$\frac{\lambda^2}{E_{\mathbf{p}}}\mathcal{C}_{\mathrm{A}}^{(2)} = -\frac{\hbar}{2} f^{<}_{V,\mathrm{leq}}(x,p) f^{>}_{V,\mathrm{leq}}(x,p)\left[A_{\mathbf{p}} + B^{\alpha}_{\mathbf{p}} p_{\langle\alpha\rangle} + C^{\alpha\rho}_{\mathbf{p}} p_{\langle\alpha}p_{\rho\rangle} + D^{\mu\alpha\lambda}_{\mathbf{p}} p_{\langle\mu}p_{\alpha}p_{\lambda\rangle}\right]$$

- Then we assume the most general expression for f_A,

$$\begin{aligned} f^{<,(2)}_{\mathrm{A}}(p) = & -\frac{\hbar}{2} f^{<}_{V,\mathrm{leq}}(x,p) f^{>}_{V,\mathrm{leq}}(x,p)\left\{\varphi^{\mathrm{s},1}_{\mathbf{p}}\frac{1}{\beta}\hat{\omega}_{\rho}\hat{\nabla}^{\rho}\beta + \varphi^{\mathrm{s},2}_{\mathbf{p}}\hat{\nabla}_{\alpha}\hat{\omega}^{\alpha} + \varphi^{\mathrm{s},3}_{\mathbf{p}}\hat{\omega}_{\rho}\hat{\nabla}^{\rho}\alpha\right. \\ & + \left[\varphi^{\mathrm{v},1}_{\mathbf{p}}\beta\hat{\theta}\hat{\omega}^{\alpha} + \varphi^{\mathrm{v},2}_{\mathbf{p}}\epsilon^{\mu\nu\alpha\beta}u_{\beta}\hat{\nabla}_{\nu}\alpha\hat{\nabla}_{\mu}\beta + \varphi^{\mathrm{v},3}_{\mathbf{p}}\beta\hat{\sigma}^{\mu\alpha}\hat{\omega}_{\mu}\right]p_{\langle\alpha\rangle} \\ & + \left[\varphi^{\mathrm{ts},1}_{\mathbf{p}}\beta\hat{\omega}^{\langle\alpha}\hat{\nabla}^{\rho\rangle}\beta + \varphi^{\mathrm{ts},2}_{\mathbf{p}}\beta^{2}\hat{\nabla}^{\langle\rho}\hat{\omega}^{\alpha\rangle} + \varphi^{\mathrm{ts},3}_{\mathbf{p}}\beta\epsilon^{\mu\nu\sigma\langle\rho}\hat{\sigma}^{\alpha\rangle}_{\mu}u_{\sigma}\hat{\nabla}_{\nu}\beta + \varphi^{\mathrm{ts},4}_{\mathbf{p}}\beta^{2}\epsilon^{\mu\nu\sigma\langle\rho}\hat{\sigma}^{\alpha\rangle}_{\mu}u_{\sigma}\hat{\nabla}_{\nu}\alpha\right. \\ & \left.\left. + \varphi^{\mathrm{ts},5}_{\mathbf{p}}\beta^{2}\hat{\omega}^{\langle\alpha}\hat{\nabla}^{\rho\rangle}\alpha\right]p_{\langle\alpha}p_{\rho\rangle} + \varphi^{\mathrm{tt}}_{\mathbf{p}}\beta^{3}\hat{\omega}^{\langle\rho}\hat{\sigma}^{\mu\lambda\rangle}p_{\langle\mu}p_{\rho}p_{\lambda\rangle} + a_{0\mathbf{p}}\right\} \end{aligned}$$

- We then impose the matching condition, $n_{\mathrm{a}} = \langle E_{\mathbf{p}}\rangle_{0,\mathrm{a}}$ choose the polynomial basis for the coefficient functions $\varphi_p^{i,j}$ and performing the collision integrals using **HTL approximation**, we can get the off equilibrium corrections.

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f_V in local equilibrium: Gradient expansion (III)

- Finally, we get,

$$
\begin{aligned}
&\delta f_{\mathrm{A}}^{<}(x,p) \\
= \; & -\frac{\hbar}{2} f_{V,\mathrm{leq}}^{<}(x,p) f_{V,\mathrm{leq}}^{>}(x,p) \frac{48\pi\beta^2}{e^4 \ln\frac{T}{m_{\mathrm{D}}}} \left\{ -(-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}) \frac{2\ln 2}{3} (\omega_\rho \nabla^\rho \beta - \beta \nabla_\alpha \omega^\alpha) \right. \\
& + (-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}) \left(\frac{5}{3} - \frac{360\zeta(3)}{7\pi^4} \ln 2 \right) \beta \omega_\rho \nabla^\rho \alpha \\
& + \left[-\frac{45\zeta(3) - 7\pi^2 \ln 2}{14\pi^2 \ln 2} \epsilon^{\mu\nu\alpha\beta} u_\beta \nabla_\nu \alpha \nabla_\mu \beta - \frac{3\pi^2}{10\ln 2} \beta \sigma^{\mu\alpha} \omega_\mu \right] p_{\langle\alpha\rangle} \\
& + \left[-\frac{\beta}{4} \left(\omega^{\langle\alpha} \nabla^{\rho\rangle} \beta + \frac{1}{2} \beta \nabla^{\langle\rho} \omega^{\alpha\rangle} + \epsilon^{\mu\nu\sigma\langle\rho} \sigma_\mu^{\alpha\rangle} u_\sigma \nabla_\nu \beta \right) + \frac{3645\zeta^2(3) + 7\pi^6}{1512\pi^4\zeta(3)} \beta^2 \epsilon^{\mu\nu\sigma\langle\rho} \sigma_\mu^{\alpha\rangle} u_\sigma \nabla_\nu \alpha \right. \\
& \left. \left. + \frac{10935\zeta^2(3) - 14\pi^6}{756\pi^4\zeta(3)} \beta^2 \omega^{\langle\alpha} \nabla^{\rho\rangle} \alpha \right] p_{\langle\alpha} p_{\rho\rangle} + \frac{7\pi^4}{10125\zeta(5)} \beta^3 \omega^{\langle\rho} \sigma^{\mu\lambda\rangle} p_{\langle\mu} p_\rho p_{\lambda\rangle} \right\},
\end{aligned}
$$

- We notice that, the corrections are proportional to combinations of hydrodynamic fluctuations and, therefore, **should not** lead to sizeable contributions to the spin polarization **if f_V is in local equilibrium**.

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Other applications of collisional SBE:
Spin hydrodynamics from method of moments

- Another useful effective way to describe the off-equilibrium effect of SBE is to use the method of moments and investigate only the behavior of several moments.

- The rank-3 spin AMT can be given by QFT,

$$S^{\mu\rho\sigma} = \frac{\hbar}{2}\overline{\psi}\{\gamma^{\mu}, S^{\rho\sigma}\}\psi = \frac{\hbar}{2}\epsilon^{\beta\mu\rho\sigma}\overline{\psi}\gamma_{\beta}\gamma^{5}\psi$$

which in terms of Wigner function is,

$$S^{\mu\rho\sigma}(x) = \epsilon^{\beta\mu\rho\sigma}\frac{\hbar}{2}4\int\frac{d^4p}{(2\pi)^4}\mathcal{A}^{<}_{\beta}(x,p)$$

- Using the solution to Wigner functions, we derive the EoM of the spin density using HTL approximation,

$$\tau_s D s^{\langle\alpha\rangle} + s^{\alpha} = \frac{\hbar^2}{96\beta^2}\omega^{\alpha} + \tau_s\left(s^{\beta}\omega^{\alpha}{}_{\beta} - \frac{3}{5}s^{\beta}\sigma^{\alpha}_{\beta} - s^{\alpha}\theta\right)$$

$$+\mathbf{other\ \ dissipative\ \ correction}$$

where $\tau_s = \frac{108\beta}{e^4\ln\frac{1}{e}\frac{6+5\pi^2}{8\pi^3}}.$

A generalized off-equilibrium spin BMT equation

Also see Prof. Shi Pu, July 17, 9:30,
D.-L. Wang, July 18, 4:00pm Parallel Session A

Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 11 (2021) 150
Denicol, Koide, Rischke, Phys.Rev.Lett. 105 (2010) 162501
***SF**, Fukushima, Pu, Wang, in preparation*

Outline

- Global and local polarization in heavy ion collisions
- Quantum kinetic theory with collisions
- Collisional effects on spin polarization
- **Summary**

Summary

1. We have derived a spin Boltzmann equation with QED collision kernel is derived, the **collision kernel is calculated using HTL approximation**; such SBE can be used for **future simulation**; the **spin relaxes slower** than normal quantity.

2. We investigate the **off-equilibrium correction** to spin polarization pseudo-vector using gradient expansion and RTA separately: **off-equilibrium effects can be important**!

3. An off-equilibrium BMT equation is derived using the method of moments.

Thanks for your attention！

Backup slides

SBE with Moller scattering

- The spin Boltzmann equation is,

$$p^{\mu}\partial_{\mu}f_{A}^{<}(p)+\hbar\partial_{\mu}S^{(u),\mu\alpha}\partial_{\alpha}f_{V}^{<}=\mathcal{C}_{\mathrm{A}}^{\mathrm{B}}[f_{\mathrm{V}},f_{\mathrm{A}}]+\mathcal{C}_{\mathrm{A}}^{\partial^{2}}[f_{\mathrm{V}}]+\mathcal{C}_{\mathrm{A}}^{\Sigma}[f_{\mathrm{V}}]$$

  - The Boltzmann type collision kernel:

$$\begin{aligned}\mathcal{C}_{\mathrm{A}}^{\mathrm{B}}[f_{\mathrm{V}},f_{\mathrm{A}}] &= 8e^{4}\int_{p',k,k'}W_{pk\rightarrow p'k'}(2\pi)^{3}\delta(k'^{,2})\delta(k^{2})\delta(p'^{,2}) \\ &\times\left[f_{V}^{<}(k')f_{V}^{<}(p')f_{V}^{>}(k)f_{A}^{>}(p)-f_{V}^{>}(k')f_{V}^{>}(p')f_{V}^{<}(k)f_{A}^{<}(p)\right]+\ldots\end{aligned}$$

  - The pure ∂^2 term:

$$\begin{aligned}\mathcal{C}_{\mathrm{A}}^{\partial^{2}}[f_{\mathrm{V}}] &= 8e^{4}\int_{p',k,k'}W_{\mu,pk\rightarrow p'k'}^{1}(2\pi)^{3}\delta(k'^{,2})\delta(k^{2})\delta(p'^{,2})\hbar S^{(u),\mu\alpha}(p) \\ &\times\left[f_{V}^{<}(k')f_{V}^{<}(p')f_{V}^{>}(k)\partial_{\alpha}f_{V}^{>}(p)-f_{V}^{>}(k')f_{V}^{>}(p')f_{V}^{<}(k)\partial_{\alpha}f_{V}^{<}(p)\right]+\ldots\end{aligned}$$

  - The self-energy coupled term:

$$\begin{aligned}\mathcal{C}_{\mathrm{A}}^{\Sigma}[f_{\mathrm{V}}] &= 8e^{4}\int_{p',k,k'}W_{\mu,pk\rightarrow p'k'}^{1}(2\pi)^{3}\delta(p'^{,2})\delta(k'^{,2})\delta(k^{2})\hbar S^{(u),\mu\alpha}(p') \\ &\times\left[-f_{V}^{>}(p)f_{V}^{>}(k)f_{V}^{<}(k')\left(\Sigma_{V,\alpha}^{<}(p')f_{V}^{>}(p')-\Sigma_{V,\alpha}^{>}(p')f_{V}^{<}(p')\right)\right. \\ &\left.+f_{V}^{<}(k)f_{V}^{<}(p)f_{V}^{>}(k')\left(\Sigma_{V,\alpha}^{>}(p')f_{V}^{<}(p')-\Sigma_{V,\alpha}^{<}(p')f_{V}^{>}(p')\right)\right]+\ldots\end{aligned}$$

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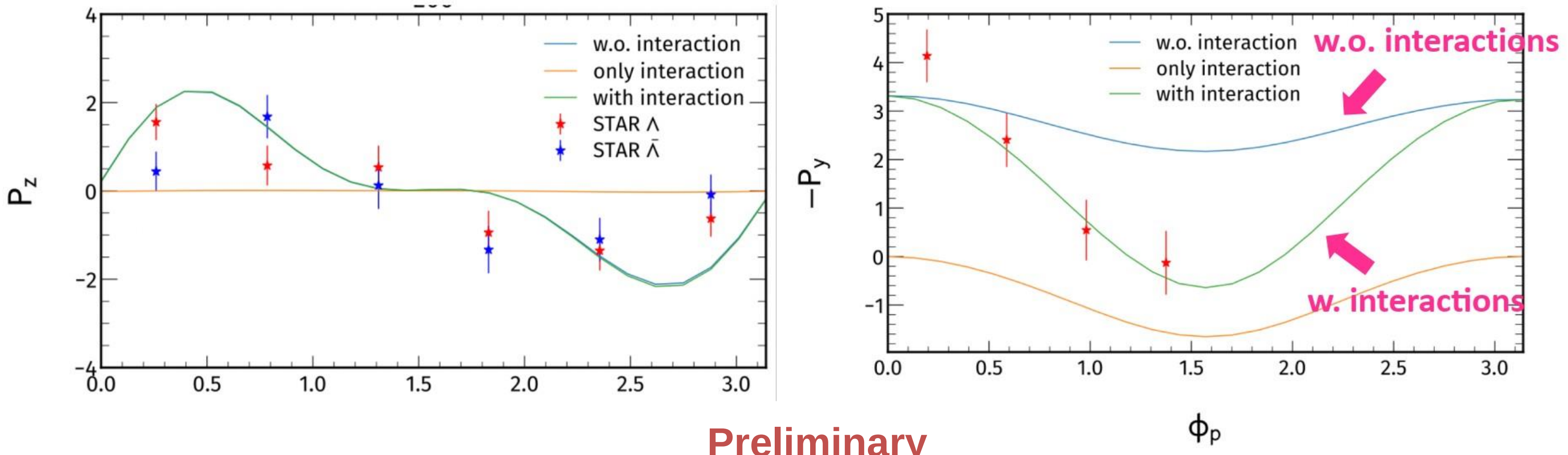
f_V in local equilibrium: another approach

- The off-equilibrium effect can be described using numerical simulation,

$$\mathcal{S}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p, X)}{2m_{\Lambda} \int d\Sigma \cdot \mathcal{N}(p, X)},$$

200GeV, CLVisc hydrodynamics + AMPT initial + EoS: NEOS-BQS



Preliminary

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