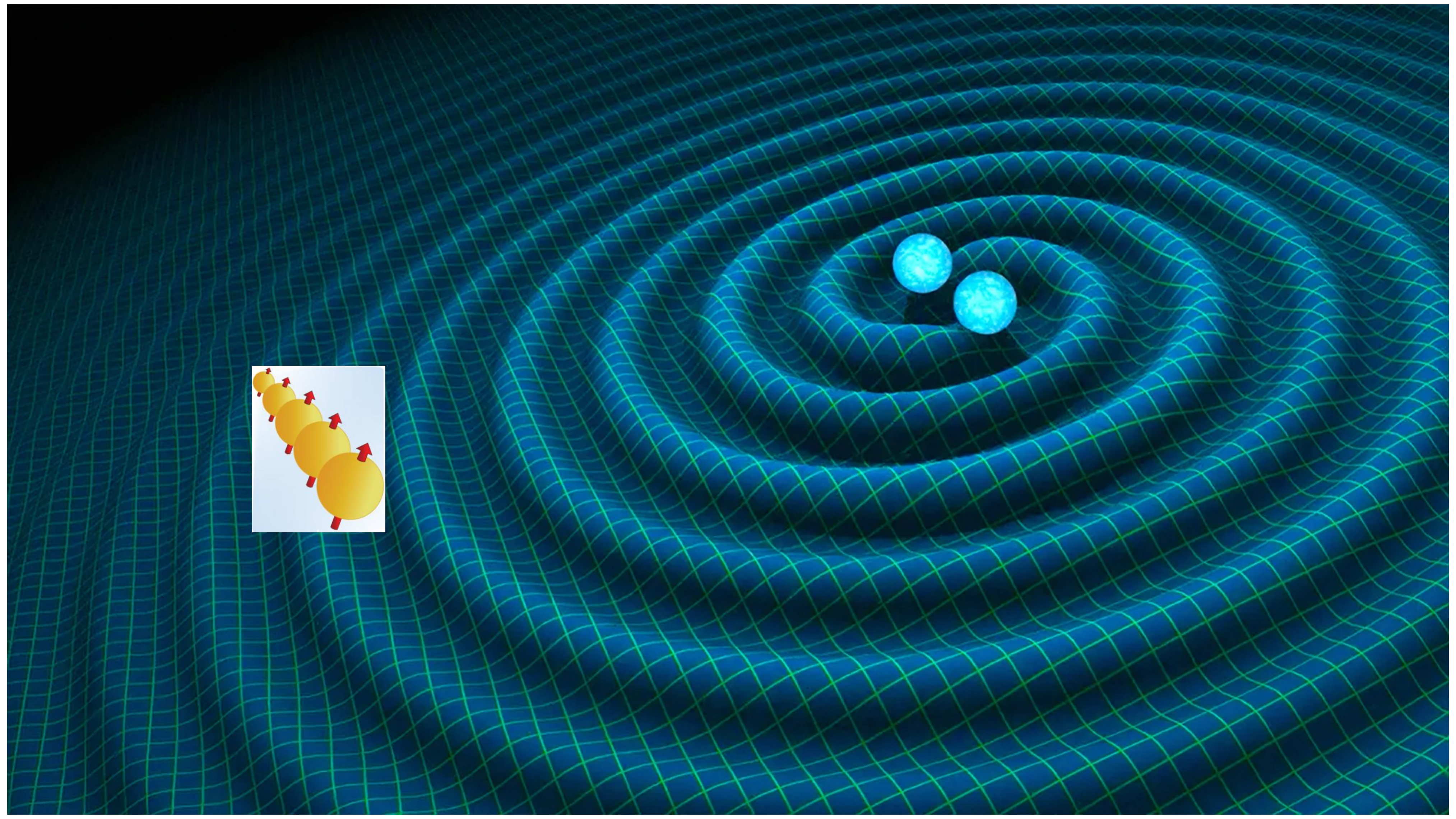
Spin polarization under gravity



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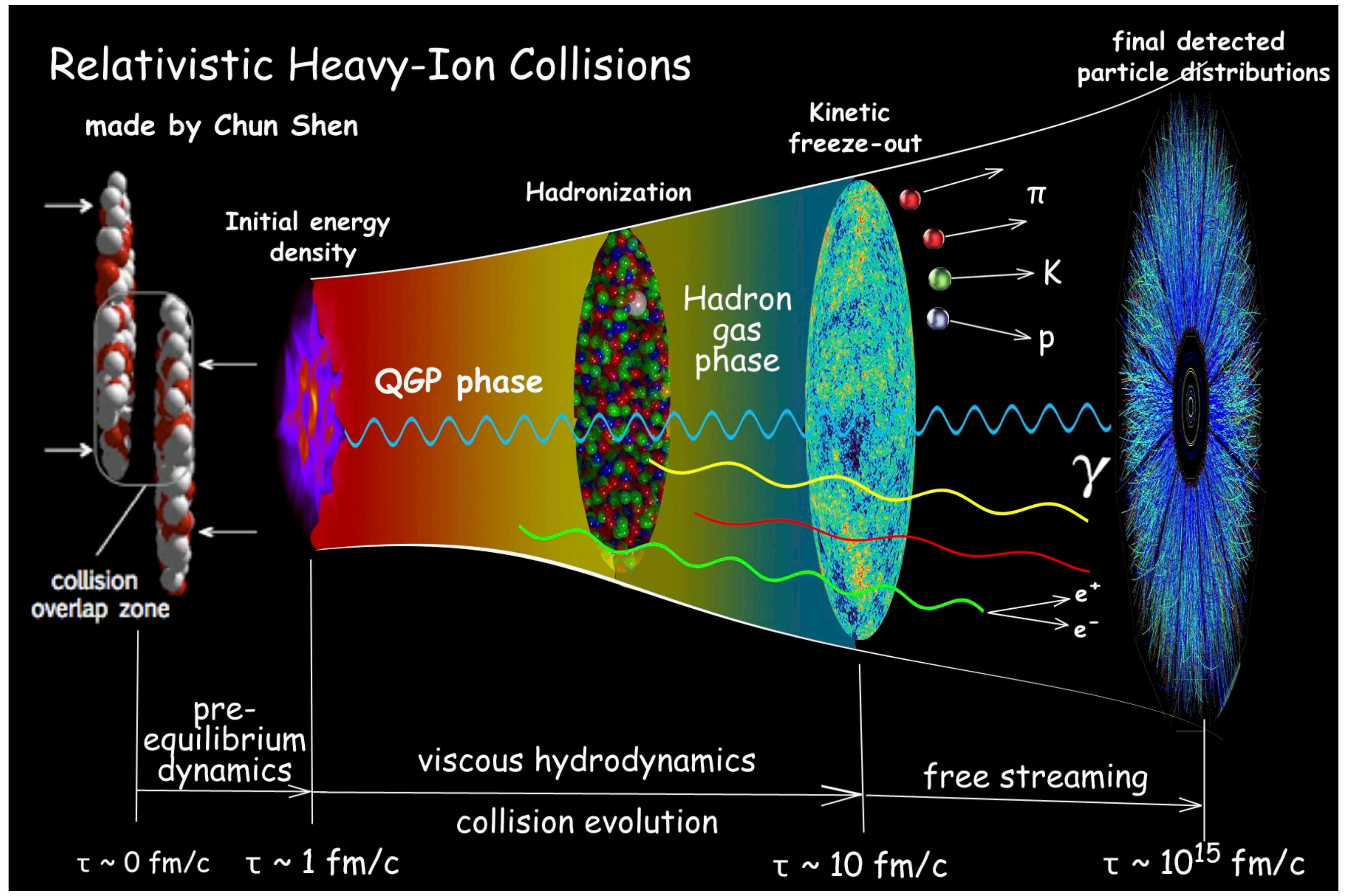
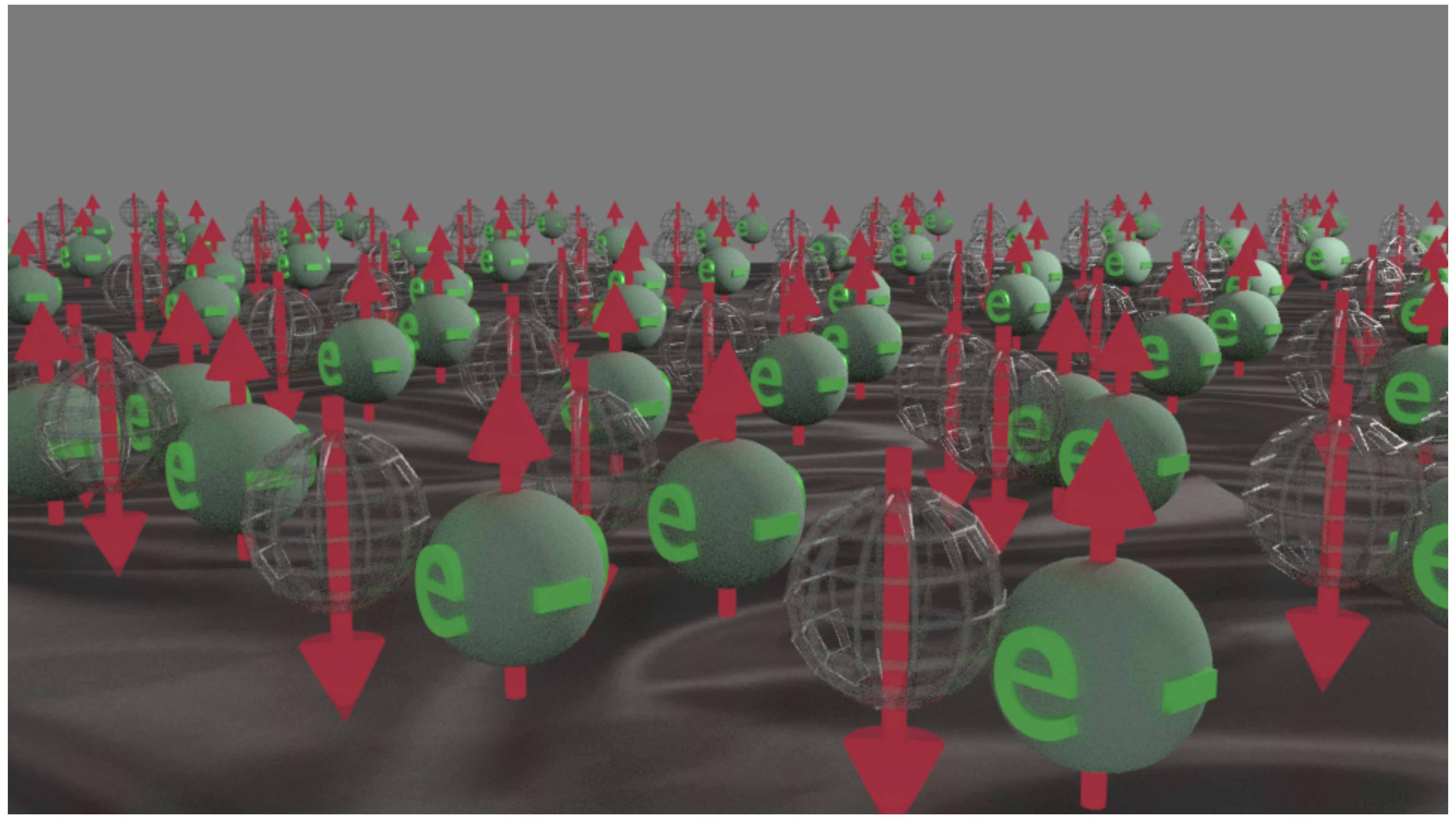
International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Spin polarization in HIC

- Spin observables give an important way to probe the properties of the media
- Heavy-ion collision, condensed matter physics, cosmology





Temperature gradient effect
Vorticity effect
Shear induced polarization (SIP)

$\Rightarrow$ spin polarization

Weak Gravity

- "linear" transport coefficient can be calculated by response to the weak gravitational field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \ |h_{\mu\nu}| \ll 1$$

h_{00} temperature gradient effect
h_{0i} vorticity effect
h_{ij} shear induced polarization

eg. $\sigma^{xy} = \partial_t h_{xy}$

Guy D. Moore, Kiyoumars A. Sohrabi, *Phys.Rev.Lett.* 106 (2011) 122302

- More general questions: How is spin polarization affected by gravity?

  What can it tell us about the above effects?

Gauge for Weak Gravity

- $h_{\mu\nu}$ has 2 physical degrees of freedom (DOF) and 8 gauge DOF
- Coulomb gauge for electromagnetic field $A_0 = 0, \partial^i A_i = 0.$

Nonzero components are perpendicular to the direction of propagation (transverse components)

- Transverse-traceless (TT) gauge, which is generally used in theories and experiments of the gravitational wave, is similar to the Coulomb gauge

$$h_{0\mu} = 0, h^i{}_i = 0, \partial^i h_{ij} = 0 \qquad h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Gauge transformation (diffeomorphism) of weak gravity to achieve TT-gauge

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Response to gravity

- Gauge transformation (diffeomorphism)

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

response in TT-gauge $\longrightarrow$ gauge transformation gives the response to other components

h_{ij} $\downarrow$ shear induced polarization

h_{0i} $\downarrow$ vorticity effect

h_{00} $\downarrow$ temperature gradient effect

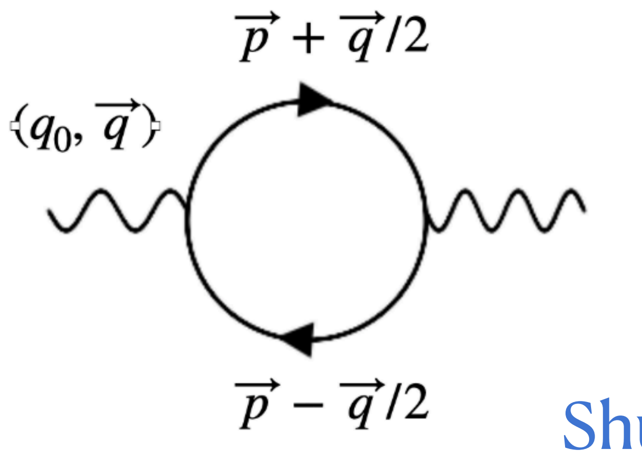
There are connections between T-gradient effect, vorticity effect and SIP

Spin current from field theory

- Finite temperature: Schwinger-Keldysh formalism
- Wigner function

$$W(q,p) = \int_x \int_y e^{-i(q\cdot x + p\cdot y)} \psi(x_+) \underbrace{U(x_+, x_-)}_{\text{gauge link}} \bar{\psi}(x_-) \qquad x_\pm = x \pm \frac{y}{2} \qquad q = (\omega, \mathbf{q})$$

spin dependent current $\quad \mathcal{A}^\mu = \frac{1}{4}\mathrm{Tr}\{\gamma^\mu \gamma^5 W\}$

$G_h^{\mu\nu} =$ 

$+$ Gauge Link $\qquad q_\mu G_h^{\mu\nu} = 0$

Shuai Liu, Yi Yin, *JHEP* 07 (2021) missed the gauge link

$\delta W_h = G_h^{\mu\nu} h_{\mu\nu}$

is gauge covariant

Gravity induced Current

- Spin current in TT-gauge

$$\mathcal{A}^i_h = \frac{1}{4}\mathrm{Tr}\{\gamma^\mu\gamma^5\delta W_h\} = -iE\frac{n'_F}{\omega-\Delta E}[\epsilon^{ilm}q_l v_m(\omega h_{ij}v^i v^j) + v^i(\omega v_j\epsilon^{jlm}q_l h_{mk}v^k)] + n'_F\epsilon^{ilm}v_l(\omega h_{mj}v^j)$$

$$= v^i\delta f_1 + \mu^{ij}q_j\delta f_0 + iEn'_F[\mathbf{v}\times(\omega\mathbf{h}\cdot\mathbf{v})]^i$$

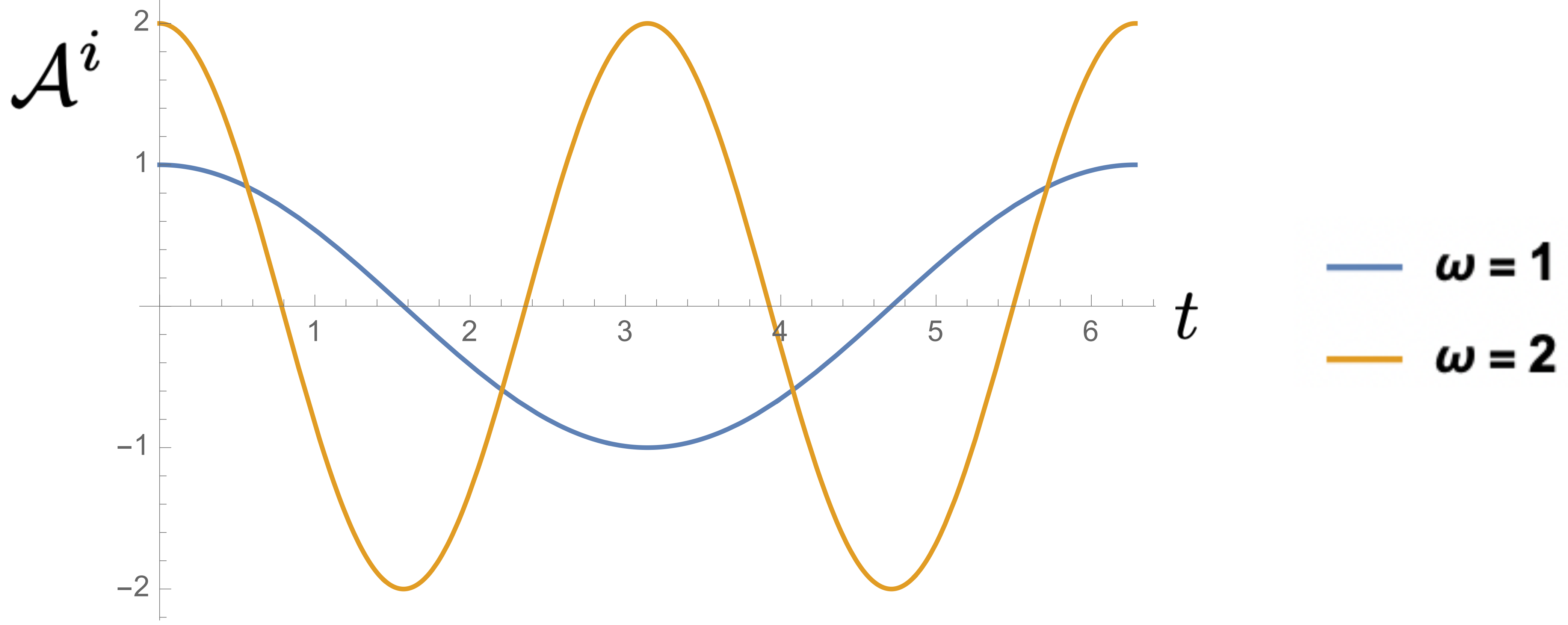
$f = n_F + \delta f_0 + \delta f_1$ is the solution of the Boltzmann equation

δf_0 and δf_1 are the zeroth and the first order in gradient expansion

- The last term is spin Hall effect (SHE) triggered by force $-\partial_t h_{ij}v^j$ $\quad \mathbf{s}\propto\mathbf{v}\times\mathbf{F}$

eg. $h_{ij} = C_{ij}\cos(\omega t - \mathbf{q}\cdot\mathbf{x})$ for a fixed $\mathbf{x}=\mathbf{x_0}$

When $\omega \gg |\mathbf{q}|$ it gives the shear response



Gravity induced Current

- Gauge transformation gives the response to other components

eg. $\xi^\mu = (0, \xi^i)$ $\quad \mathcal{A}^i = G_h^{i,jk} h_{jk} \rightarrow G_h^{i,jk}(h_{jk} + \partial_j \xi_k + \partial_k \xi_j) + 2G_h^{i,0j} \partial_t \xi_j$

$\mathcal{A}^i$ is covariant $\Rightarrow G_h^{i,0j} = \omega^{-1} q_k G_h^{i,jk}$

response to h_{0j}

$$\mathcal{A}_h^i = -iE\frac{n_p'}{\omega - \Delta E}[\epsilon^{ilm} q_l v_m (\Delta E h_{0j} v^j) + v^i (\Delta E v_j \epsilon^{jlm} q_l h_{0m})] + iE n_F' \Delta E \epsilon^{ilm} v_l h_{0m}$$

But one can not obtain the h_{ij} response from h_{0j} response, inversely

The same story in the field theory language is the Ward identity

Kinetic theory as an EFT

Kinetic theory is an effective description of many-body systems

Distribution $f(t, \mathbf{x}, \mathbf{p})$ as an effective degree of freedom

Jingyuan Chen, Dam T. Son, Annals Phys. 377 (2017)

$$J^\mu = \boxed{v^\mu f + \cdots} + \qquad$$

the expression of the observable in terms of the distribution function

eg. Hall conductivity

$\downarrow$

Boltzmann equation

$\downarrow$

The functional relation $J^\mu[f]$ is general

We compute the current under electromagnetic field and the weak gravity from field theory. The first part, functional relations, are the same; while the second part depends on which kind of field it is.

Summary and outlook

- We compute the spin current under weak gravity, using field theory
  - Different components of $h_{\mu\nu}$ can be connected by gauge transformation.
  - Response to TT-gauge components can give the response to other components by gauge transformation
  - Gauge tran. gives connections between T-gradient effect, vorticity effect and SIP
- The field theory results correspond to the kinetic theory
- Kinetic theory should be regarded as an effective theory, and the observables containing 2 parts
- In the early universe with QGP and gravitational wave

Thank you for your attention!