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Quantum transport theory with vector interaction

Presenter: *Peiwei Yu (余沛伟)*

Collaborator: *Xingyu Guo (郭星雨)*

South China Normal University - Institute of Quantum Matter

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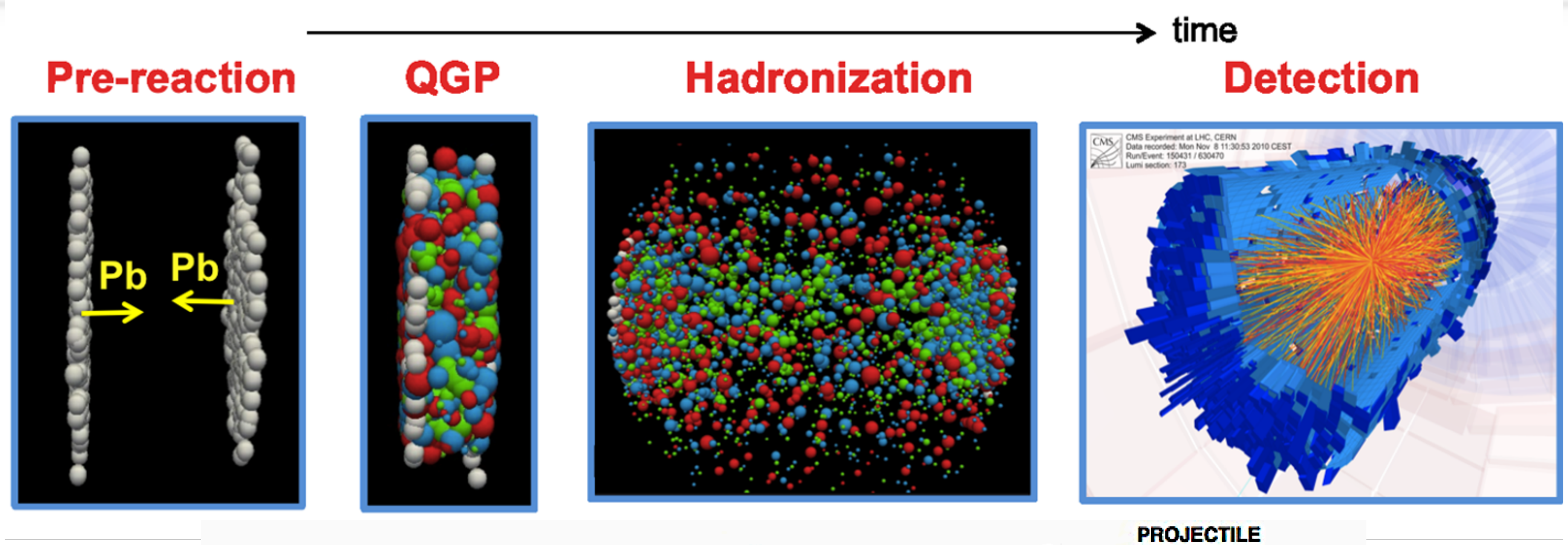
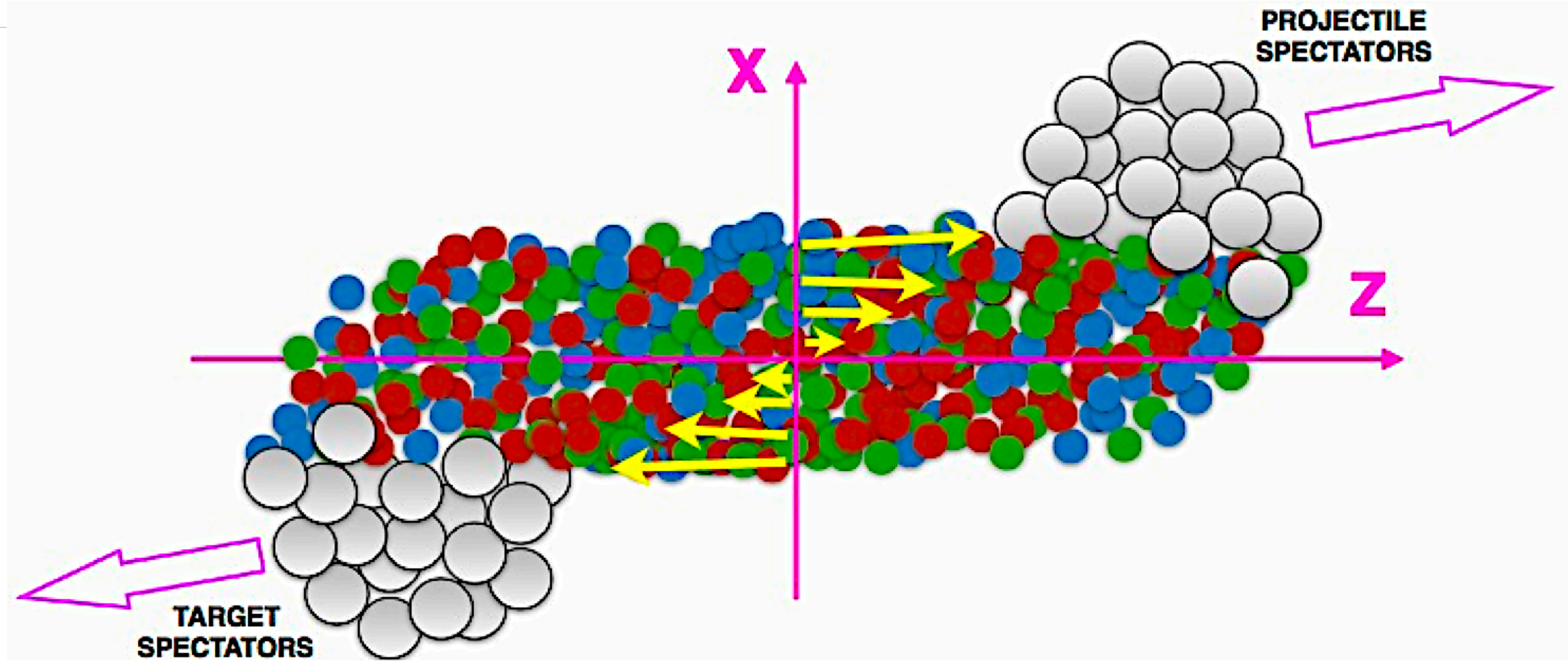
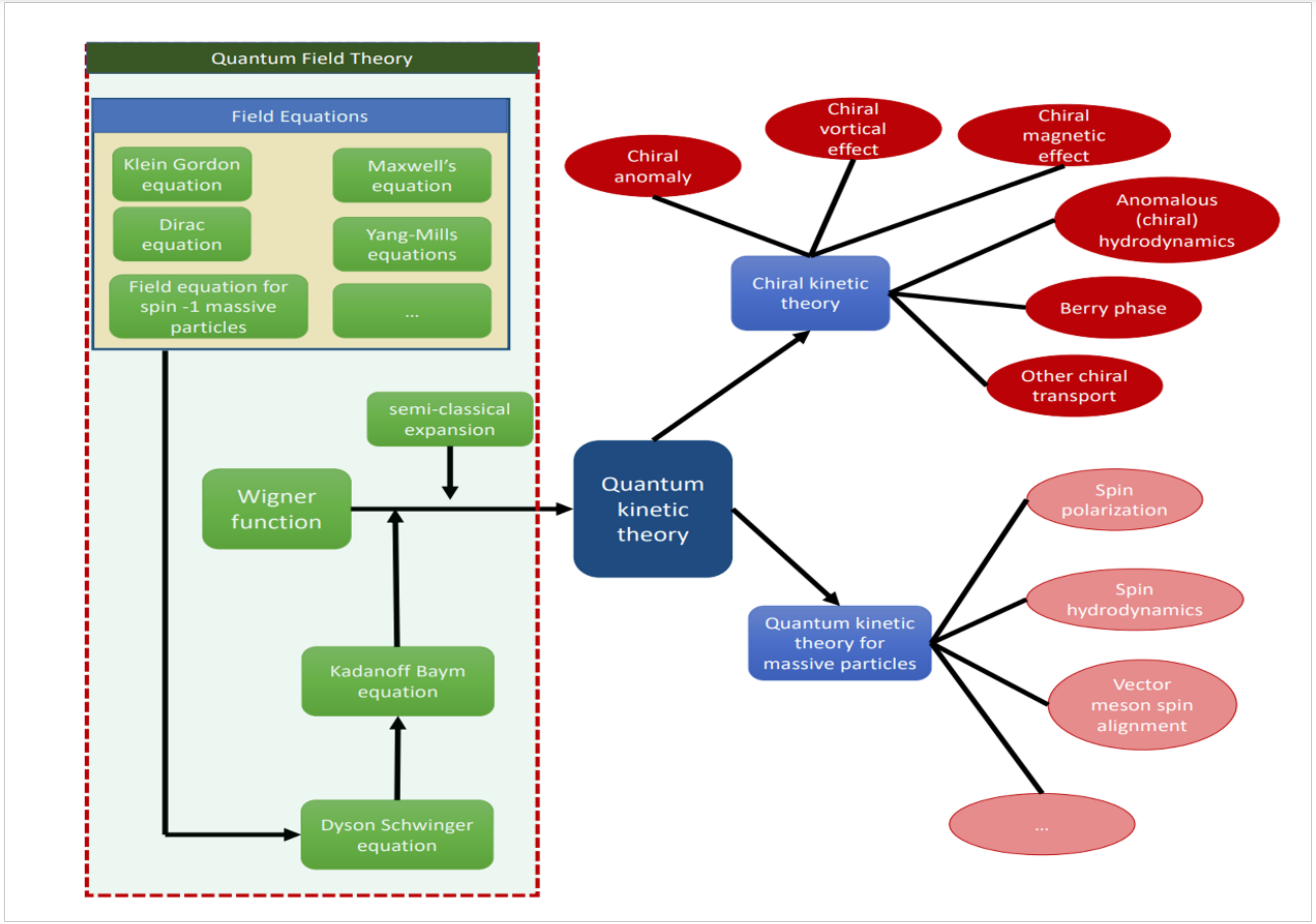
Contants

Introduction





Introduction



Introduction

Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla_x + \vec{F} \cdot \nabla_p\right) f(\vec{x},\ \vec{p},\ t) = C[f]$$

Boltzmann equation is an equation to describe phase space distribution, which is affected by particle flow， force and collision term.

Wigner function

$$W(x,p) = \int d^4y e^{ipy} \left\langle \psi\left(x + \frac{y}{2}\right) \overline{\psi}\left(x - \frac{y}{2}\right) \right\rangle$$

Equation of motion for the Wigner function

The Lagrange function for vector interaction and axial interaction:

$$\mathcal{L} = i\gamma^\mu \overline{\psi}\partial_\mu \psi + G\left[\left(\overline{\psi}\gamma^\mu\psi\right)^2 + \left(\overline{\psi}i\gamma_5\gamma^\mu\psi\right)^2\right]$$

The mean-field approximation:

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu\partial_\mu + \gamma^\mu J_{V_\mu} + \gamma_5\gamma^\mu J_{A_\mu}\right)\psi,$$

The covariant kinetic equation:

$$\left(\gamma^\mu K_\mu - \gamma_5\gamma^\mu K_{5\mu}\right)W = 0$$

$$\Pi_{5\mu} = cos\left(\frac{\hbar}{2}\nabla\right)J_{A_\mu}$$

$$D_{5\mu} = -sin\left(\frac{\hbar}{2}\nabla\right)J_{A_\mu}$$

$$\nabla = \partial_x \cdot \partial_p$$

$$K_\mu = \Pi_\mu + iD_\mu$$

$$K_{5\mu} = \Pi_{5\mu} + iD_{5\mu}$$

$$\Pi_\mu = p_\mu + cos\left(\frac{\hbar}{2}\nabla\right)J_{V_\mu}$$

$$D_\mu = \frac{\hbar}{2}\partial_\mu - sin\left(\frac{\hbar}{2}\nabla\right)J_{V_\mu}$$

Equation of motion for the Wigner function

Wigner function is a 4×4 matrix and satisfies the relationship $\gamma_0 W^{+}\gamma_0 = W$. It can be decomposed in term of 16 independent generations of Clifford algeba.

$$W(x,p) = \frac{1}{4}\left(F + i\gamma_5 P + \gamma_\mu V^\mu + \gamma_5\gamma_\mu A^\mu + \frac{1}{2}\sigma^{\mu\nu}S_{\mu\nu}\right)$$

So we can get 10 equations

$$\Pi_\mu V^\mu + \Pi_{5\mu} A^\mu = 0$$
$$D_\mu A^\mu + D_{5\mu} V^\mu = 0$$
$$\Pi_\mu F + D^\nu S_{\mu\nu} - D_{5\mu} P + \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}\Pi_5^\nu S^{\sigma\rho} = 0$$
$$D_\mu P - \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}\Pi^\nu S^{\sigma\rho} - \Pi_{5\mu} F - D_5^\nu S_{\mu\nu} = 0$$
$$D_\mu V_\nu - D_\nu V_\mu - \varepsilon_{\mu\nu\sigma\rho}\Pi^\sigma A^\rho - \varepsilon_{\mu\nu\sigma\rho}\Pi_5^\sigma V^\rho + D_{5\mu} A_\nu - D_{5\nu} A_\mu = 0$$
$$D_\mu V^\mu + D_{5\mu} A^\mu = 0$$
$$\Pi_\mu A^\mu + \Pi_{5\mu} V^\mu = 0$$
$$D_\mu F - \Pi^\nu S_{\mu\nu} + \Pi_{5\mu} P + \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}D_5^\nu S^{\sigma\rho} = 0$$
$$\Pi_\mu P + \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}D^\nu S^{\sigma\rho} + D_{5\mu} F - \Pi_5^\nu S_{\mu\nu} = 0$$
$$\Pi_\mu V_\nu - \Pi_\nu V_\mu + \varepsilon_{\mu\nu\sigma\rho}D^\sigma A^\rho + \varepsilon_{\mu\nu\sigma\rho}D_5^\sigma V^\rho + \Pi_{5\mu} A_\nu - \Pi_{5\nu} A_\mu = 0$$

Constraint equation

$$J_V^\mu(x) = G\int d^4p V^\mu(x,p)$$
$$J_A^\mu(x) = G\int d^4p A^\mu(x,p)$$

Semi-classical expansion：

expand by $\hbar$ order.

$$W = W^{(0)} + W^{(1)}\hbar + \dots$$

Transport equation

The 0-th order

$$\left(p_\mu + J^{(0)}_{V_\mu}\right) V^{(0)\mu} + J^{(0)}_{A_\mu} A^{(0)\mu} = 0$$

$$\left(\partial_\mu - \nabla J_{V_\mu}\right) A^{(0)\mu} - \nabla J^{(0)}_{A_\mu} V^{(0)\mu} = 0$$

$$\left(p_\mu + J^{(0)}_{V_\mu}\right) F^{(0)} + \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho} J^{(0)\nu}_{A} S^{(0)\sigma\rho} = 0$$

$$-\frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}\left(p^\nu + J^{(0)\nu}_{V}\right) S^{(0)\sigma\rho} - J^{(0)}_{A_\mu} F^{(0)} = 0$$

$$-\varepsilon_{\mu\nu\sigma\rho}\left(p^\sigma + J^{(0)\sigma}_{V}\right) A^{(0)\rho} - \varepsilon_{\mu\nu\sigma\rho} J^{(0)\sigma}_{A} V^{(0)\rho} = 0$$

$$\left(\partial_\mu - \nabla J^{(0)}_{V_\mu}\right) V^{(0)\mu} - \nabla J^{(0)}_{A_\mu} A^{(0)\mu} = 0$$

$$\left(p_\mu + J^{(0)}_{V_\mu}\right) A^{(0)\mu} + J^{(0)}_{A_\mu} V^{(0)\mu} = 0$$

$$-\left(p^\nu + J^{(0)\nu}_{V}\right) S^{(0)}_{\mu\nu} + J^{(0)}_{A_\mu} P^{(0)} = 0$$

$$\left(p_\mu + J^{(0)}_{V_\mu}\right) P^{(0)} - J^{(0)\nu}_{A} S^{(0)}_{\mu\nu} = 0$$

$$\left(p_\mu + J^{(0)}_{V_\mu}\right) V^{(0)}_\nu - \left(p_\nu + J^{(0)}_{V_\nu}\right) V^{(0)}_\mu + J^{(0)}_{A_\mu} A^{(0)}_\nu - J^{(0)}_{A_\nu} A^{(0)}_\mu = 0$$

Transport equation

On-shell condition

$$\left(\left(p+J_V^{(0)}\right)^2+J_A^{(0)2}\right)F^{(0)}=0$$

$$\left(\left(p+J_V^{(0)}\right)^2+J_A^{(0)2}\right)P^{(0)}=0$$

$$\left(\left(p+J_V^{(0)}\right)^2+J_A^{(0)2}\right)S^{(0)\mu\nu}=0$$

$$\left(p+J_V^{(0)}+\chi J_A^{(0)}\right)^2 V_{\chi\nu}^{(0)}=0$$

$$V_{+\nu}^{(0)}=V_\nu^{(0)}+A_\nu^{(0)}$$
$$V_{-\nu}^{(0)}=V_\nu^{(0)}-A_\nu^{(0)}$$

Constraint equation

$$J_{\chi\mu}=G\int d^4pV_{\chi\mu}$$

The tensor components can be expressed by the scalar and pseudoscalar components

$$S^{(0)\sigma\rho}=\frac{1}{J_A^{(0)2}}\left(\varepsilon^{\mu\nu\sigma\rho}\left(p_\mu+J_{V_\mu}^{(0)}\right)J_{A_\nu}^{(0)}F^{(0)}-\left(p^\rho+J_V^{(0)\rho}\right)J_A^{(0)\sigma}P^{(0)}+\left(p^\sigma+J_V^{(0)\sigma}\right)J_A^{(0)\rho}P^{(0)}\right)$$

Transport equation

The transport equation for chiral components

$$\begin{aligned}\widetilde{p}_{\chi\mu}V_{\chi}^{(0)\mu} &= 0\\ \widetilde{p}_{\chi\mu}V_{\chi\nu}^{(0)} - \widetilde{p}_{\chi\nu}V_{\chi\mu}^{(0)} &= 0\\ \nabla_{\chi\mu}V_{\chi}^{(0)\mu} &= 0\end{aligned}$$

Where $\tilde{p}_{x\mu} = p_\mu + xJ^{(0)}_{x\mu}$ and $J_{x\mu} = J_{V_\mu} + xJ_{A_\mu}$

The chiral fermion distribution function is

$$V_{\chi\mu}^{(0)} = \widetilde{p}_{\chi\mu}f_{\chi}^{(0)}\delta\left(\widetilde{p}_{\chi}^{2}\right) \quad \longrightarrow \quad f_{\chi\ eq}^{(0)} = \frac{1}{e^{\frac{|p\cdot u_\chi|}{T}}+1},$$

Where $u_{\chi\mu} = J^{(0)}_{\chi\mu}/n_\chi$ is chiral component and n_χ is the particle number density.

Transport equation

The equilibrium condition at zeroth order

$$(p^\mu p^\nu + J_\chi^{(0)\mu} J_\chi^{(0)\nu}) \left[\partial_\mu(\frac{u_{\chi\nu}}{T}) + \partial_\nu(\frac{u_{\chi\mu}}{T})\right] + (2J_\chi^{(0)} - p) \cdot \partial(\frac{n_\chi}{T}) = 0.$$

Where the shear tensor $\partial_\mu(u_\upsilon/T) + \partial_\nu(u_\mu/T)$ depend on particle density.

When the particle number density is a constant, the equilibrium condition can be reduced as

$$\boldsymbol{\partial_\mu}\left(\frac{\boldsymbol{u_\upsilon}}{\boldsymbol{T}}\right) + \boldsymbol{\partial_\nu}\left(\frac{\boldsymbol{u_\mu}}{\boldsymbol{T}}\right)=0$$

(the killing condition)

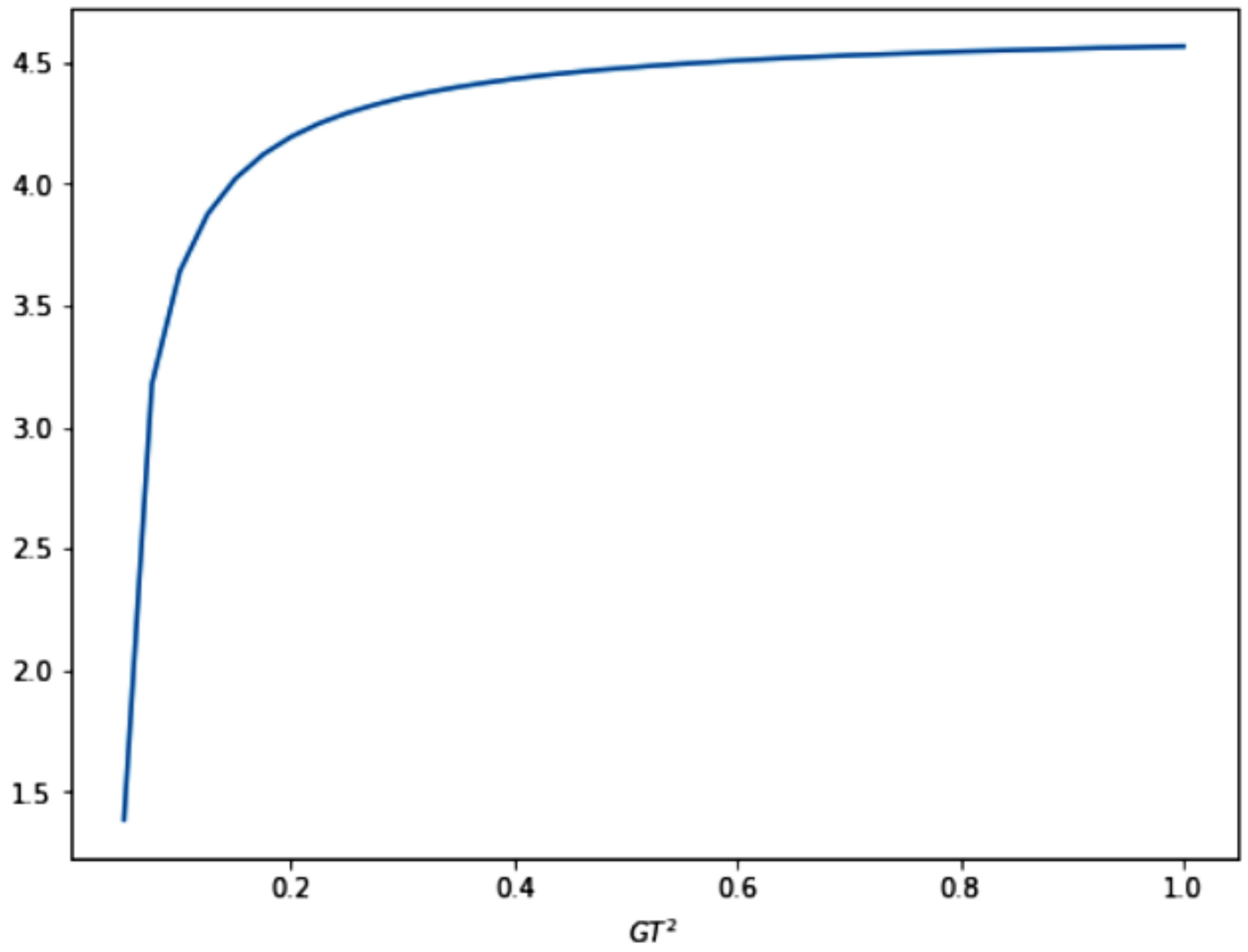
Transport equation

Then we consider the constraint equation.

$$J_{\chi\mu}^{(0)} = G \int d^4 p \widetilde{p}_{\chi\mu} f_\chi^{(0)} \delta\left(\widetilde{p}_\chi^2\right)$$

we can get the constrain for chiral current

$$\frac{n}{T} = \frac{2}{3}\pi^3 \frac{n}{T} GT^2 + \left[4\ln 2 \cdot \pi \frac{n^2}{T^2} GT^2 + 6\pi\zeta(3) GT^2 + 4\pi GT^2 Li_3\left(-e^{\frac{n}{T}}\right) + 4\pi GT^2 Li_3\left(-e^{-\frac{n}{T}}\right)\right] \mathrm{sgn}(n),$$



By Taylor expansion, we get

$$\frac{n}{T} \approx GT^2\left(\frac{2}{3}\pi^3 \frac{n}{T} - \frac{\pi}{12} \frac{n^4}{T^4} \mathrm{sgn}(n)\right).$$

Therefore when $GT^2 < 0.05$, the constraint equation has only trivial solution $n = 0$. When $GT^2 \geq 0.05$, there are two additional opposite non-zero solutions

Transport equation

The transport equation of $\hbar$ order

$$\widetilde{p}^{(0)}_{\chi\mu} V^{(1)\mu}_{\chi} + J^{(1)}_{\chi\mu} V^{(1)\mu}_{\chi} = 0,$$

$$\nabla_{\chi\mu} V^{(1)\mu}_{\chi} - \nabla J^{(1)}_{\chi\mu} V^{(0)\mu}_{\chi} = 0,$$

$$\varepsilon_{\mu\nu\sigma\rho} \nabla^{\sigma}_{\chi} V^{(0)\rho}_{\chi} = -2\chi\left(\widetilde{p}^{(0)}_{\chi\mu} V^{(1)}_{\chi\nu} - \widetilde{p}^{(0)}_{\chi\nu} V^{(1)}_{\chi\mu} + J^{(1)}_{\chi\mu} V^{(0)}_{\chi\nu} - J^{(1)}_{\chi\nu} V^{(0)}_{\chi\mu}\right).$$

The solution of $V_{\chi\mu}$ to $\hbar$ order

$$V_{\chi\mu} = \widetilde{p}_{\chi\mu} f_{\chi} \delta\left(\widetilde{p}^{2}_{\chi}\right) - \frac{\chi}{2\left(\widetilde{p}_{\chi} \cdot u'\right)} \varepsilon_{\mu\nu\alpha\sigma} u'^{\nu} \widetilde{p}^{\alpha}_{\chi} \left(\nabla^{\sigma}_{\chi} f^{(0)}_{\chi}\right) \delta\left(\widetilde{p}^{2}_{\chi}\right)$$
$$+ \frac{\chi}{\widetilde{p}^{2}_{\chi}} \varepsilon_{\mu\nu\sigma\rho} \widetilde{p}^{\nu}_{\chi} \Omega^{\sigma\rho}_{\chi} f^{(0)}_{\chi} \delta\left(\widetilde{p}^{2}_{\chi}\right) - \frac{2\widetilde{p}_{\chi\mu} \widetilde{p}_{\chi\nu} J^{(1)\nu}_{\chi}}{\widetilde{p}^{2}_{\chi}} f^{(0)}_{\chi} \delta\left(\widetilde{p}^{2}_{\chi}\right) + J^{(1)}_{\chi\mu} f^{(0)}_{\chi} \delta\left(\widetilde{p}^{2}_{\chi}\right),$$

$$\Omega_{\chi\mu\nu} = \frac{1}{2}\left(\partial_{\mu} J^{(0)}_{\chi\nu} - \partial_{\nu} J^{(0)}_{\chi\mu}\right) = \frac{n_{\chi}}{2}(\partial_{\mu} u_{\chi\nu} - \partial_{\nu} u_{\chi\mu}) + \frac{1}{2}(u_{\chi\nu}\partial_{\mu} - u_{\chi\mu}\partial_{\nu}) n_{\chi}$$

Transport equation

The transport equation for f_x :

$$\delta\left(\tilde{p}_\chi^2+\frac{2\chi\hbar}{\tilde{p}_\chi\cdot J_\chi^{(0)}}\tilde{p}_\chi^\alpha\tilde{\Omega}_{\chi\alpha\nu}J_\chi^{(0)\nu}+2\hbar\tilde{p}_{\chi\nu}J_\chi^{(1)\nu}\right)\{\tilde{p}_\chi\cdot\nabla_\chi$$
$$+\frac{\chi\hbar}{2\left(\tilde{p}_\chi\cdot J_\chi^{(0)}\right)^2}\left[\left(\partial^\mu J_\chi^{(0)\beta}\right)\tilde{p}_{\chi\beta}-\Omega_\chi^{\beta\mu}J_{\chi\beta}^{(0)}\right]\varepsilon_{\mu\nu\alpha\sigma}J_\chi^{(0)\nu}\tilde{p}_\chi^\alpha\nabla_\chi^\sigma$$
$$-\frac{\chi\hbar}{\tilde{p}_\chi\cdot J_\chi^{(0)}}\tilde{\Omega}_{\chi\alpha\sigma}\tilde{p}_\chi^\alpha\nabla_\chi^\sigma+\frac{\chi\hbar}{2\tilde{p}_\chi\cdot J_\chi^{(0)}}J_{\chi\nu}^{(0)}\tilde{p}_{\chi\alpha}\left(\partial^\beta\tilde{\Omega}_\chi^{\nu\alpha}\right)\partial_\beta^p+\hbar J_{\chi\mu}^{(1)}\nabla_\chi^\mu$$
$$-\frac{\hbar}{\tilde{p}_\chi^2}\tilde{p}_{\chi\delta}\Omega_\chi^{\mu\delta}J_{\chi\mu}^{(1)}-\hbar\partial^\beta J_\chi^{(1)\mu}\tilde{p}_{\chi\mu}\partial_\beta^p\}f_\chi=0.$$

$$\Rightarrow$$

$$\delta\left(\overline{p}_\chi^2\right)\left(\overline{p}_\chi^\mu\partial_\mu\right)f_\chi=0$$

We do not consider vorticity and shear tensor

Transport equation

Then we consider $J^{(0)}_{A_\mu}$ to be zore, so $V_{\chi\mu}$ become

$$V_{\chi\mu} = \tilde{p}_\mu f_\chi \delta\left(\tilde{p}^2\right) - \frac{\chi\hbar}{2\tilde{p}\cdot u}\varepsilon_{\mu\nu\alpha\sigma}u^\nu\tilde{p}^\alpha\left(\nabla^\sigma f^{(0)}\right)\delta\left(\tilde{p}^2\right) + \frac{\chi\hbar}{\tilde{p}^2}\varepsilon_{\mu\nu\sigma\rho}\tilde{p}^\nu\Omega^{\sigma\rho}f^{(0)}\delta\left(\tilde{p}^2\right)$$

$$V_\mu = \frac{1}{2}\left(V_+ + V_-\right) = \tilde{p}_\mu f^{(0)}\delta\left(\tilde{p}^2\right)$$

$$A_\mu = \frac{1}{2}\left(V_+ - V_-\right) = \frac{1}{2}\tilde{p}_\mu\left(f_+^{(1)} - f_-^{(1)}\right)\delta\left(\tilde{p}^2\right) - \frac{\hbar}{2\tilde{p}\cdot u}\varepsilon_{\mu\nu\alpha\sigma}u^\nu\tilde{p}^\alpha\left(\nabla^\sigma f^{(0)}\right)\delta\left(\tilde{p}^2\right) + \frac{\hbar}{\tilde{p}^2}\varepsilon_{\mu\nu\sigma\rho}\tilde{p}^\nu\Omega^{\sigma\rho}f^{(0)}\delta\left(\tilde{p}^2\right)$$

When we do not consider vorticity, A_μ become

$$A_\mu = \frac{1}{2}\tilde{p}_\mu\left(f_+^{(1)} - f_-^{(1)}\right)\delta\left(\tilde{p}^2\right) - \frac{\hbar}{2\tilde{p}\cdot u}\varepsilon_{\mu\nu\alpha\sigma}u^\nu\tilde{p}^\alpha\left(\nabla^\sigma f^{(0)}\right)\delta\left(\tilde{p}^2\right) + \frac{\hbar}{\tilde{p}^2}\varepsilon_{\mu\nu\sigma\rho}\tilde{p}^\nu u^\rho\left(\partial^\sigma n\right)f^{(0)}\delta\left(\tilde{p}^2\right)$$

Summary

1. The right-hand and left-hand component has different on-shell condition in zero order, which can be viewed as a shift of masses or as the modification of mechanical momenta.
2. By the constraint equation, we know the relationship between particle number density and temperature, which mean the non-zero solution of particle number density will serve as local fluctuations with zero chemical potential.
3. The shear tensor play an important role in our theory. The shear tensor depends on whether the particle number density changes. The spin polarization will be produced by the non-zero shear tensor.

Thanks