The 7th International Conference on
Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

Effects of First-order Chiral Phase Transition in Heavy-Ion Collisions

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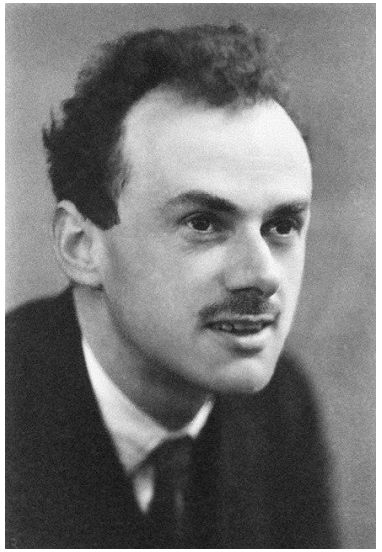
Chiral Symmetry



P. Dirac, 1928

$$(i\partial\!\!\!/ - m)\psi(x) = 0$$

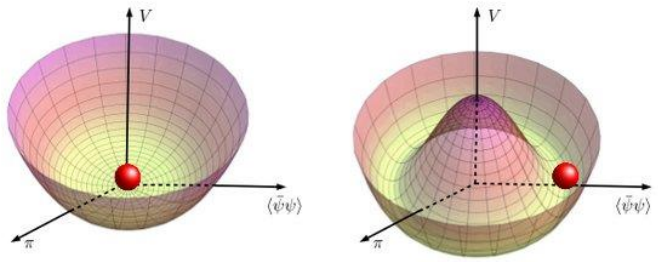
Y. Nambu, 1961

$$\langle \bar{q}^a_{\mathsf{R}}\, q^b_{\mathsf{L}} \rangle = v\,\delta^{ab}$$

$$M \propto -< \bar{q}q >$$

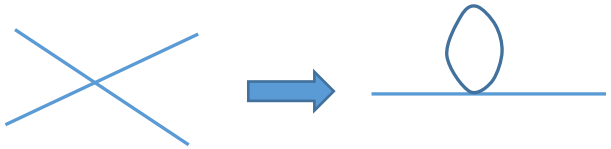
In analogy with BCS theory for superconductivity

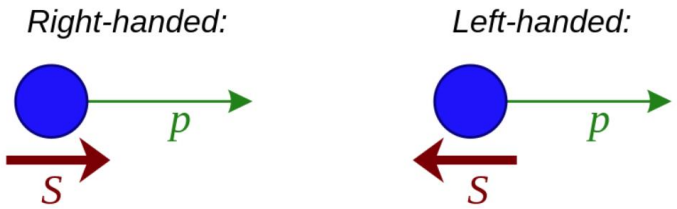
Spontaneous symmetry breaking:



$$U(1)_V \times SU(N_f)_L \times SU(N_f)_R$$
$$\rightarrow U(1)_V \times SU(N_f)_V$$

Mass generation:





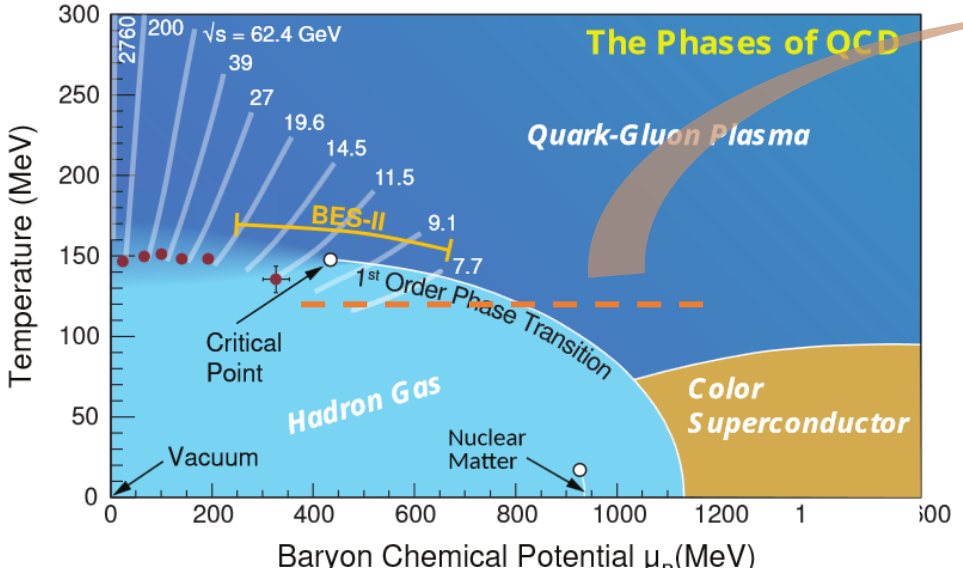
$$\partial^\mu J_{\mu 5} = 2m_f i\bar{\psi}_f\gamma_5\psi_f - \frac{N_f g^2}{16\pi^2} G^{\mu\nu}_\alpha \tilde{G}_{\alpha\mu\nu}$$

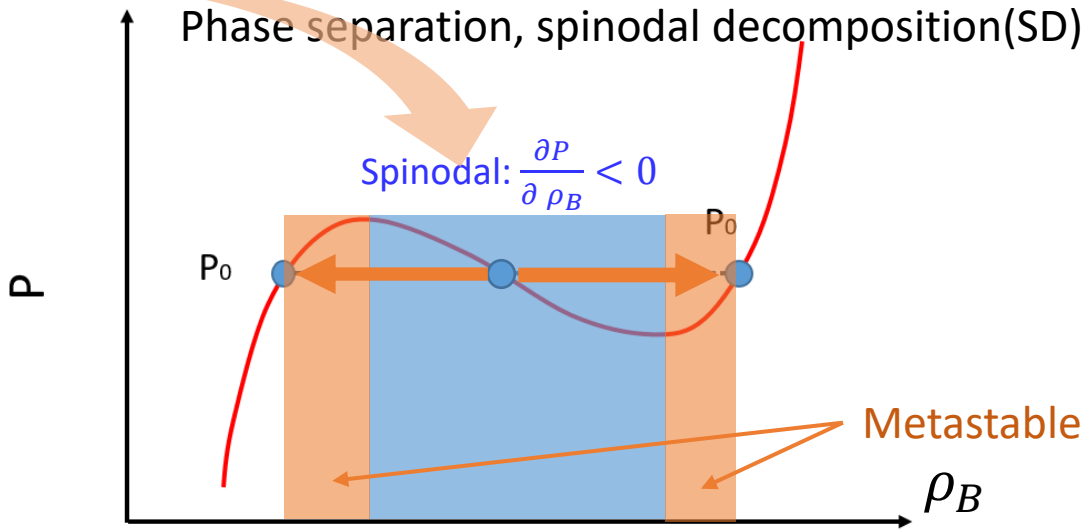
Chiral anomaly
(when coupled to gauge fields)

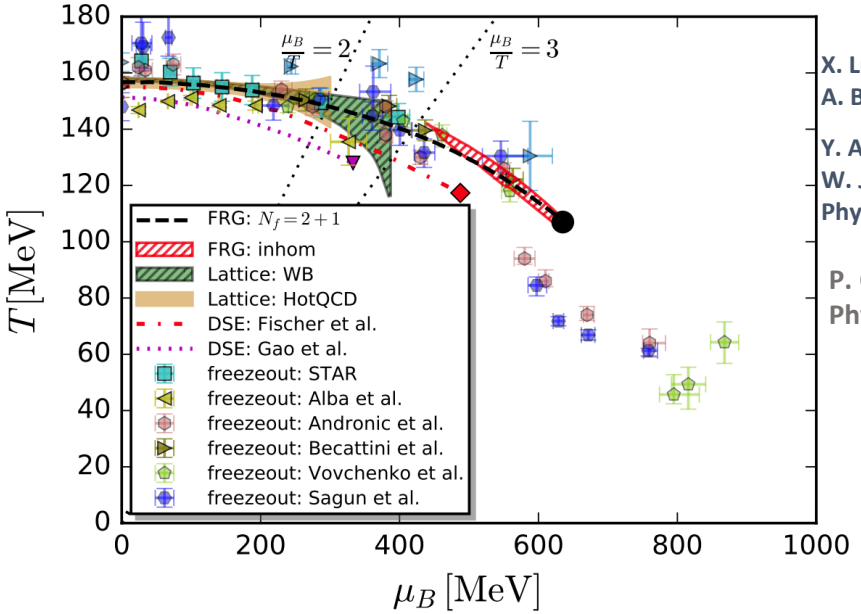
$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 G^{\mu\nu}_\alpha \tilde{G}_{\alpha\mu\nu}$$

Strong CP violation

QCD phase diagram and first-order PT (2)



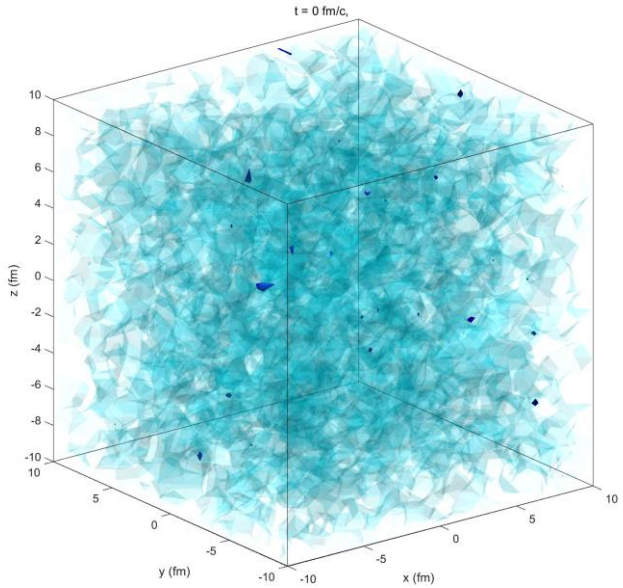




X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)
A. Bzdak et al., Phys. Rept. 853, 1 (2020)

Y. Aoki et al., Nature 443, 675 (2006)
W. J. Fu, J. M. Pawlowski, F. Renneke, Phys. Rev. D101, 054032 (2020)

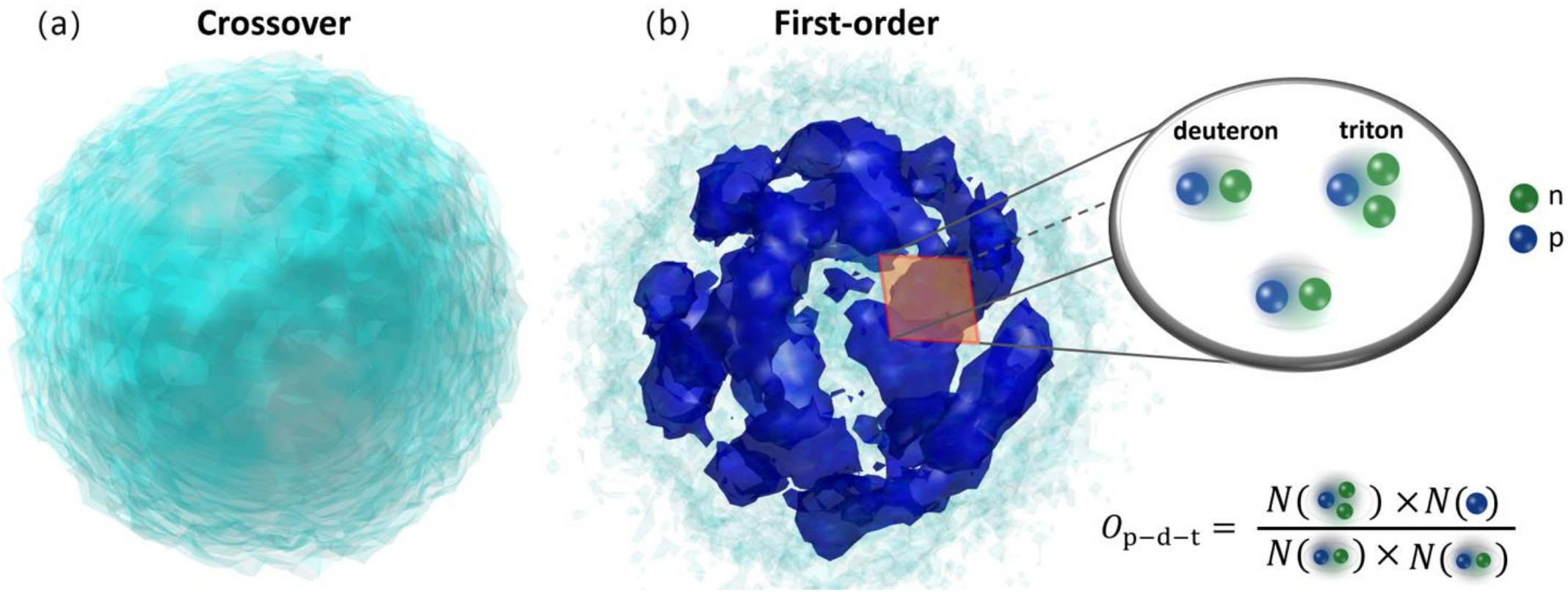
P. Chomaz, M. Clonna, and J. Randrup, Phys. Rep. 389, 263 (2004)



Probing QCD phase transition with light nuclei (3)

C. M. Ko, NST 34, 80 (2023).

Talk by Shanjin Wu (Jul. 19th)



Large density fluctuations could lead to enhancements of $N_t N_p / N_d^2$

K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu, Phys. Lett. B 774, 103 (2017); K. J. Sun, C. M. Ko, and F. Li, PLB 816, 136258 (2021);
E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019); S. Wu et al., PRC 106,034905(2022).

Why $N_t N_p / N_d^2$?

Density matrix formulation

$$N_d \propto Tr[\hat{\rho}_s \hat{\rho}_d]$$

Phys. Lett. B 774, 103 (2017);781, 499 (2018);816, 136258 (2021)

Encodes many-body density fluctuation/correlation

Phase-space representation:

$$N_d = \frac{3}{4}\int d\Gamma\, f_{pn}(\vec{p}_1, \vec{r}_1, \vec{p}_2, \vec{r}_2) \times W_d(\vec{r}, \vec{p})$$

$$W_d(\vec{r}, \vec{p}) = \frac{1}{\pi\hbar}\int d\vec{r}'\psi_d^*(\vec{r}+\vec{r}')\psi_d(\vec{r}-\vec{r}')e^{2i\vec{p}\cdot\vec{r}'}$$

Wigner function(Gaussian):

$$W_d(r,k) = 8\exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 p^2\right) \qquad \sigma_d \approx 2.26 \text{ fm}$$

with density fluctuation and correlation:

$$f_{np}(x_1, p_1; x_2, p_2) = \rho_{np}(x_1, x_2)(2\pi mT)^{-3} e^{-\frac{p_1^2+p_2^2}{2mT}}$$

$$\rho_{np}(x_1, x_2) = \rho_n(x_1)\rho_p(x_2) + C_2(x_1, x_2)$$

$$\rho_n(x) = <\rho_n> + \delta\rho_n(x) \qquad \rho_p(x) = <\rho_p> + \delta\rho_p(x)$$

$\delta\rho(x)$ denotes density fluctuation over space or inhomogeneity,

$$\boxed{\begin{aligned} C_{\rm np} &= \langle\delta\rho_n(x)\delta\rho_p(x)\rangle/(\langle\rho_n\rangle\langle\rho_p\rangle) \\ \Delta\rho_{\rm n} &= \langle\delta\rho_n(x)^2\rangle/\langle\rho_n\rangle^2 \end{aligned}} \qquad <\cdots> \equiv \frac{1}{V}\int dx$$

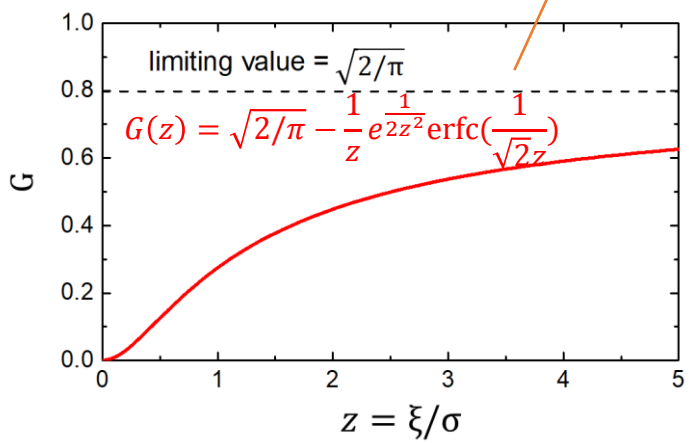
$$C_2(x_1 - x_2) \approx \lambda\langle\rho_n\rangle\langle\rho_p\rangle\frac{e^{-|x_1-x_2|/\xi}}{|x_1-x_2|^{1+\eta}} \quad (singular\ part\ only)$$

with ξ being the density − density correlation length

$$0 < \langle\delta N^2\rangle \sim \int dx C_2(x) \sim \lambda\xi^2 \rightarrow \lambda > 0$$

$$N_d \approx \frac{3}{\sqrt{2}}\left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p\langle\rho_n\rangle\left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

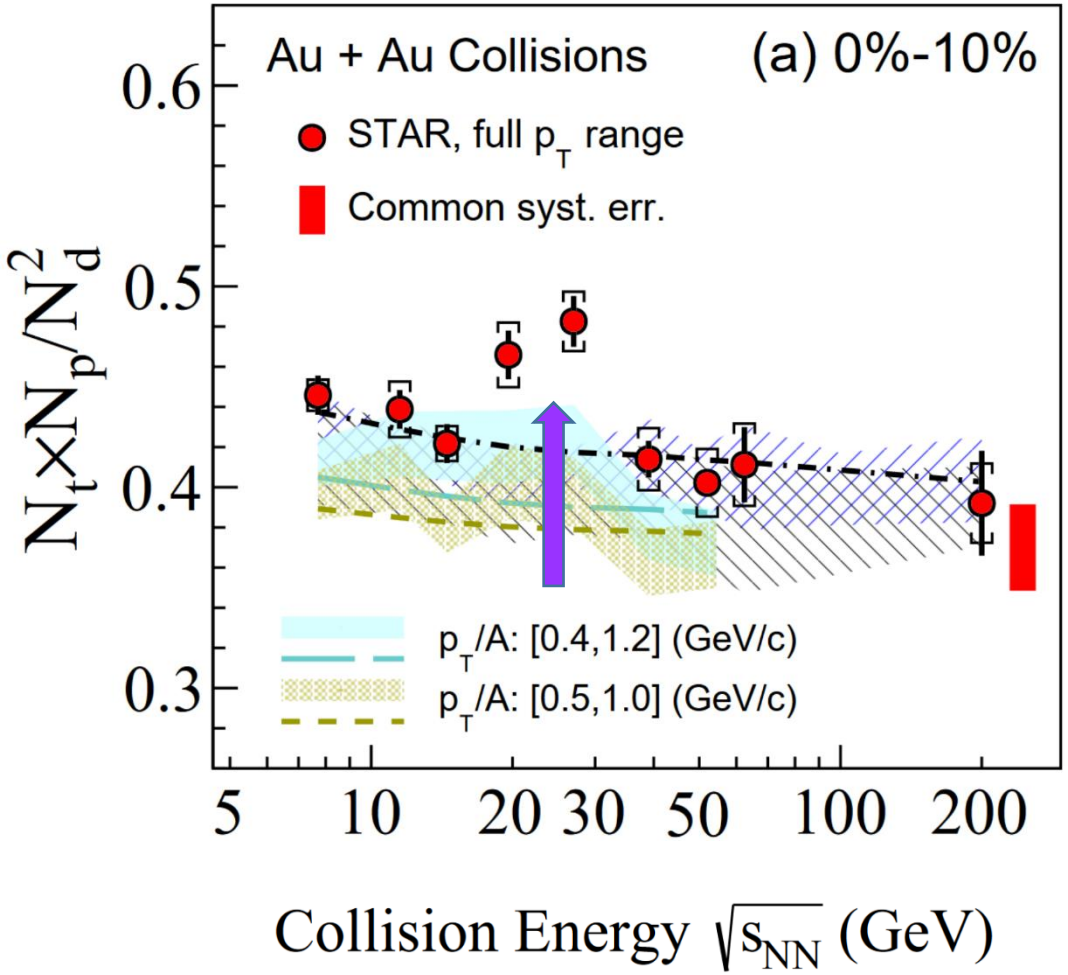
$$N_t = \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2\left[1 + 2C_{np} + \Delta\rho_n + \frac{3\lambda}{\sigma_t} G\left(\frac{\xi}{\sigma_t}\right) + O(G^2)\right]$$

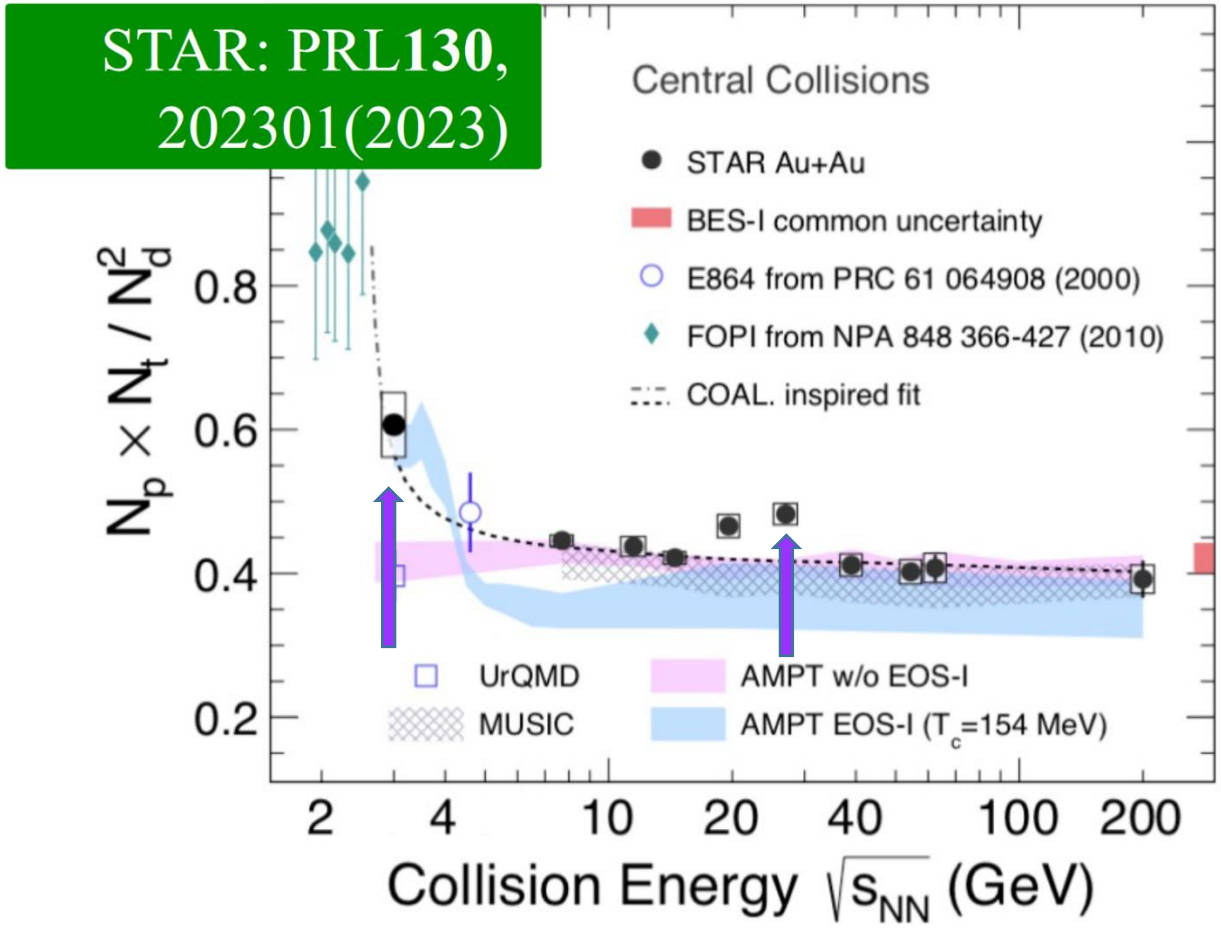


Ratio: $$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

Recent STAR Measurements (5)

STAR: PRL130, 202302 (2023)





In the following, I will discuss how the first-order chiral phase transition modify light nuclei production

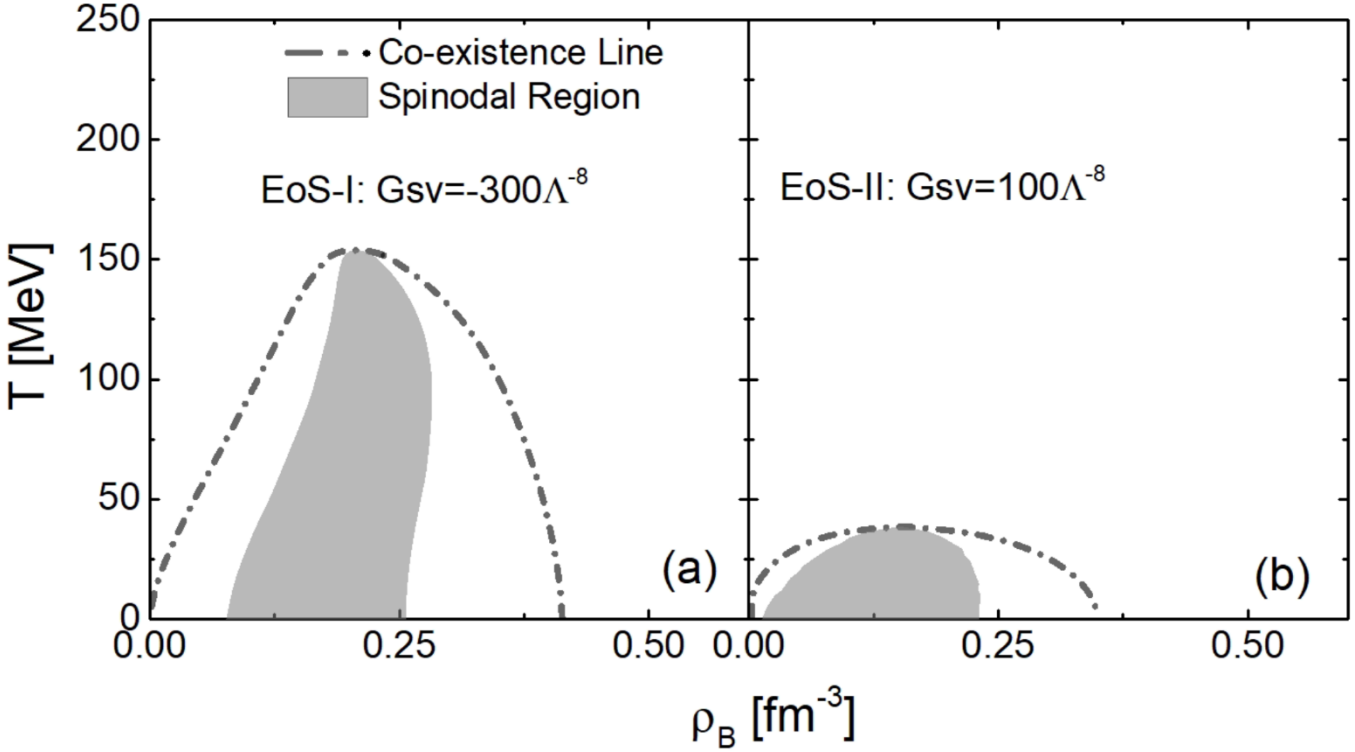
Equation of State (extended NJL model) (6)

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for an extended Nambu-Jona-Lasinio (eNJL) model

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ &- K\{[\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\} \\ &+ \boxed{G_{SV}\left\{\sum_{a=1}^{3}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]\right\} \times \left\{\sum_{a=1}^{3}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2]\right\}},\end{aligned}$$

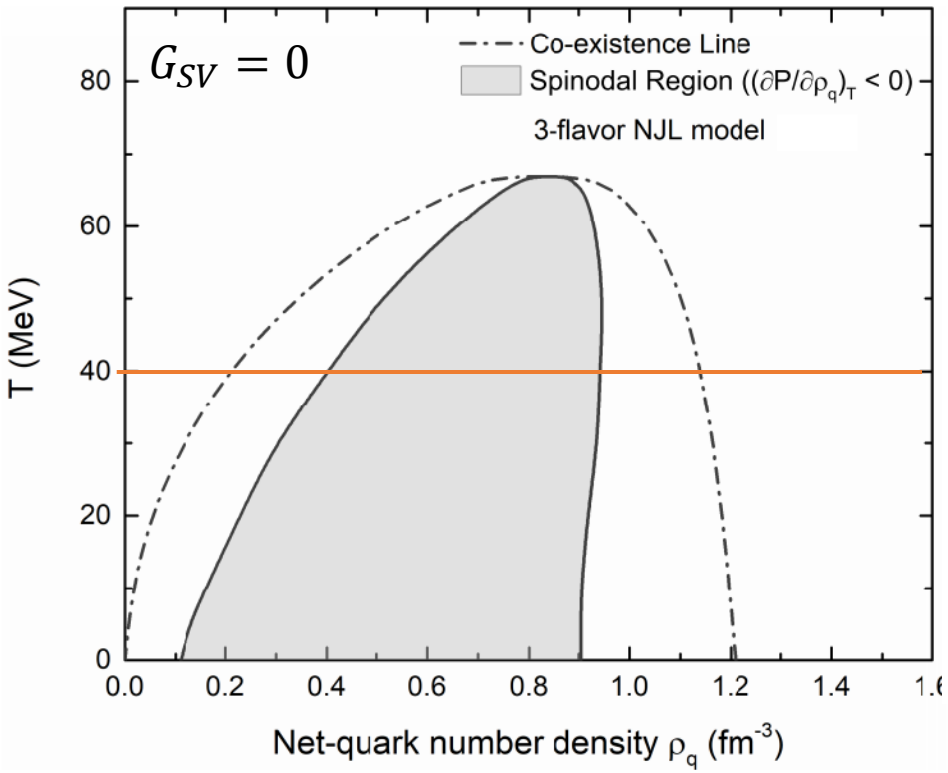
Λ [MeV]	602.3	$M_{u,d}$ [MeV]	367.7
$G\Lambda^2$	1.835	M_s [MeV]	549.5
$K\Lambda^5$	12.36	$(\langle\bar{u}u\rangle)^{1/3}$ [MeV]	-241.9
$m_{u,d}$ [MeV]	5.5	$(\langle\bar{s}s\rangle)^{1/3}$ [MeV]	-257.7
m_s [MeV]	140.7		

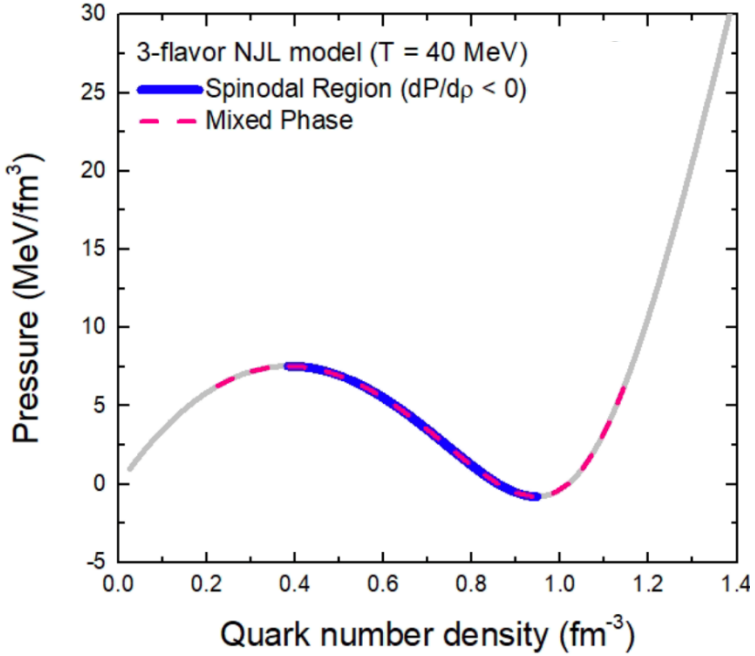


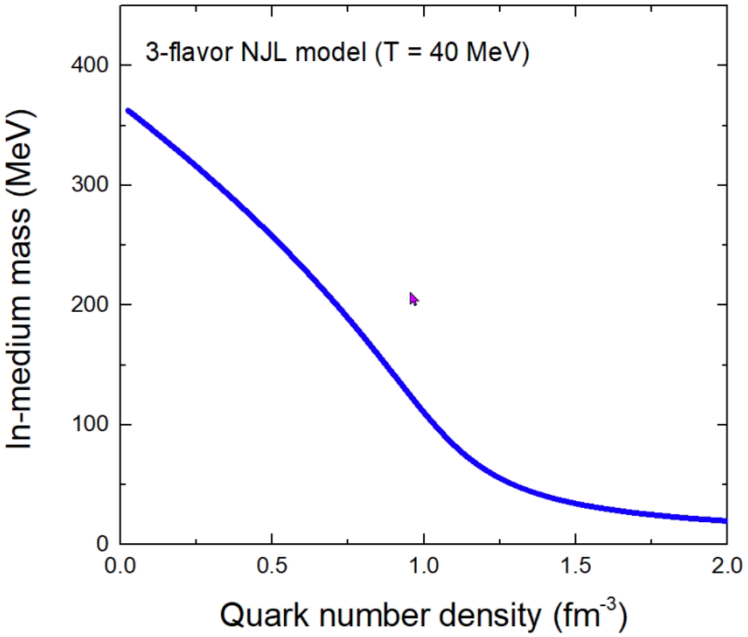
M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

In-medium quark mass (7)







K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

Box Simulation (8)

Effective mass:

$$M_u = m_u - 4G_S\phi_u + 2K\phi_d\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_d = m_d - 4G_S\phi_d + 2K\phi_u\phi_s - 2G_{SV}(\rho_u+\rho_d)^2(\phi_u+\phi_d),$$
$$M_s = m_s - 4G_S\phi_s + 2K\phi_u\phi_d$$

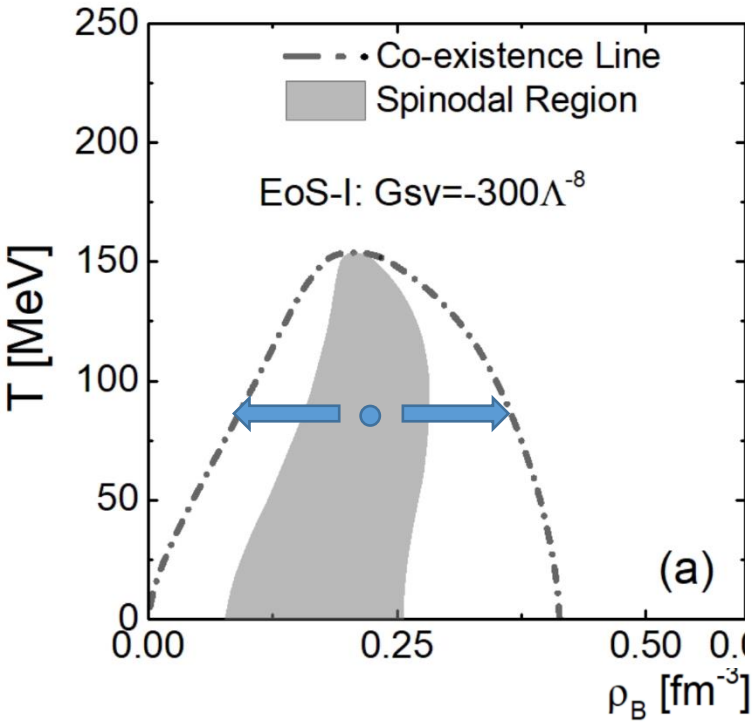
$$\phi_i = -2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}\frac{M_i}{E_i}(1-f_i-\bar{f}_i)$$

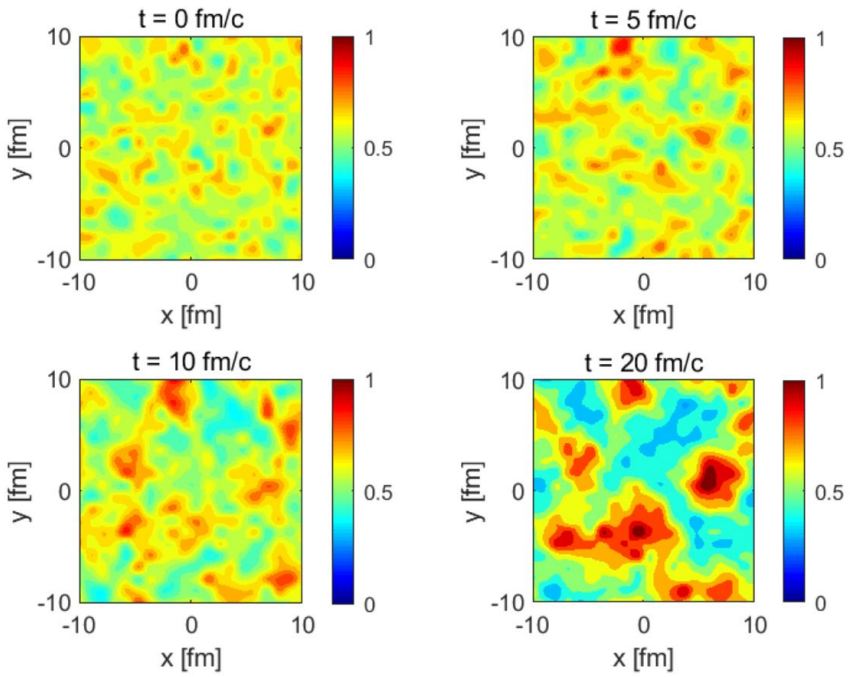
$$\rho_i = 2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}(f_i-\bar{f}_i)$$

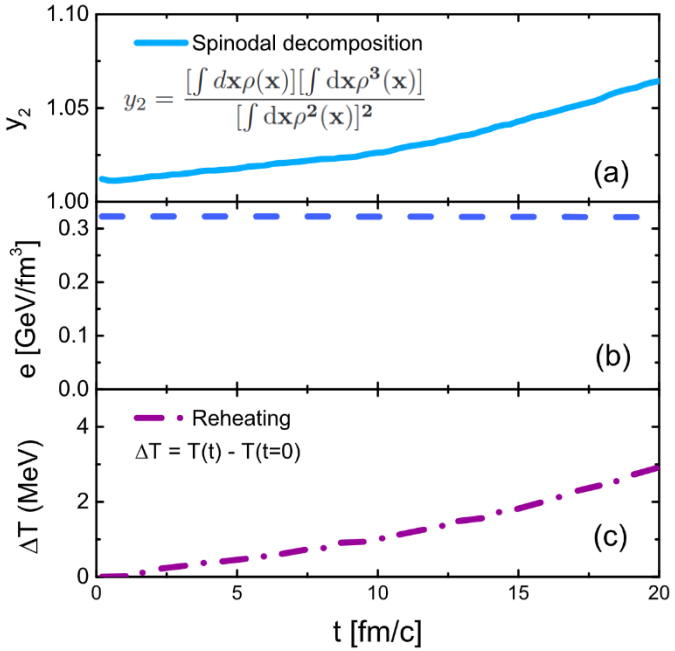
Test –particle method: J. Xu, arXiv:1904.00131 (2019)

$$\frac{d\mathbf{r}}{dt} = \mathbf{v},$$
$$\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*}\nabla_r M \pm \mathbf{E} \pm \mathbf{v}\times\mathbf{B}$$

Strong EM fields







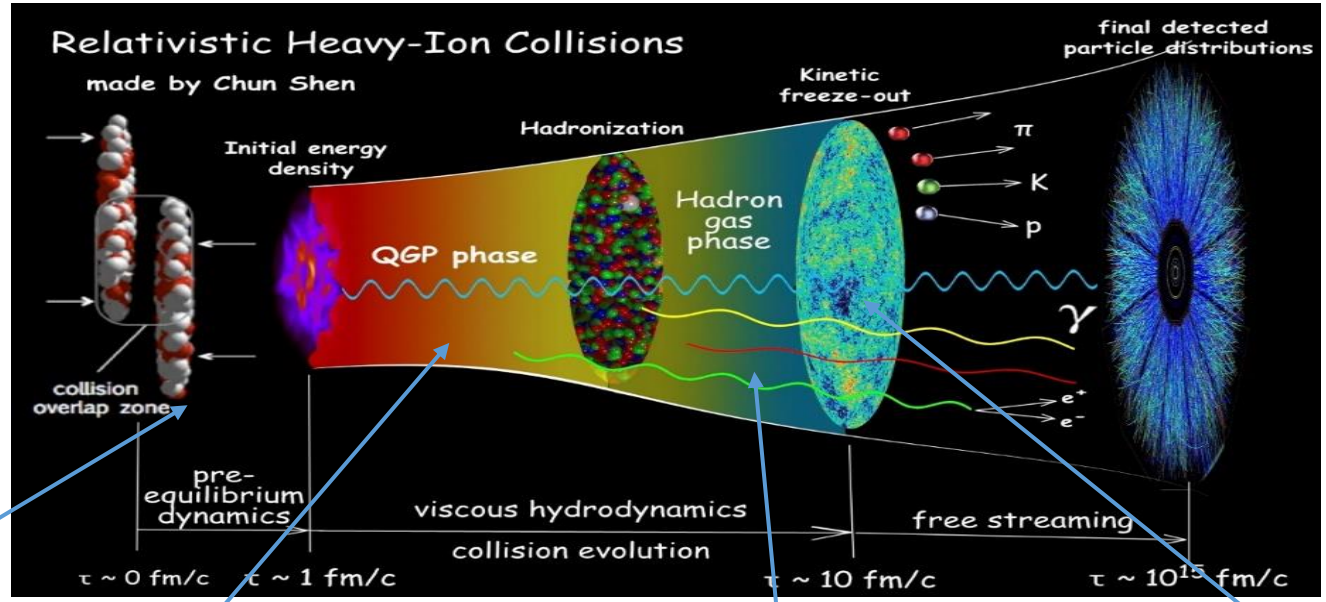
Small irregularities will grow exponentially and soon the evolution becomes 'chaotic'.

For small k, $\Gamma_k = -i\,vk$,

$v^2 = \frac{n}{\varepsilon+p}\left(\frac{\partial p}{\partial n}\right)_S$ or $v^2 = \frac{n}{\varepsilon+p}\left(\frac{\partial p}{\partial n}\right)_T < 0$

M. Buballa, Phys. Rept. 407, 205 (2005)
K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

Relativistic Heavy-Ion Collisions (9)



Hadronization:
Quark coalescence

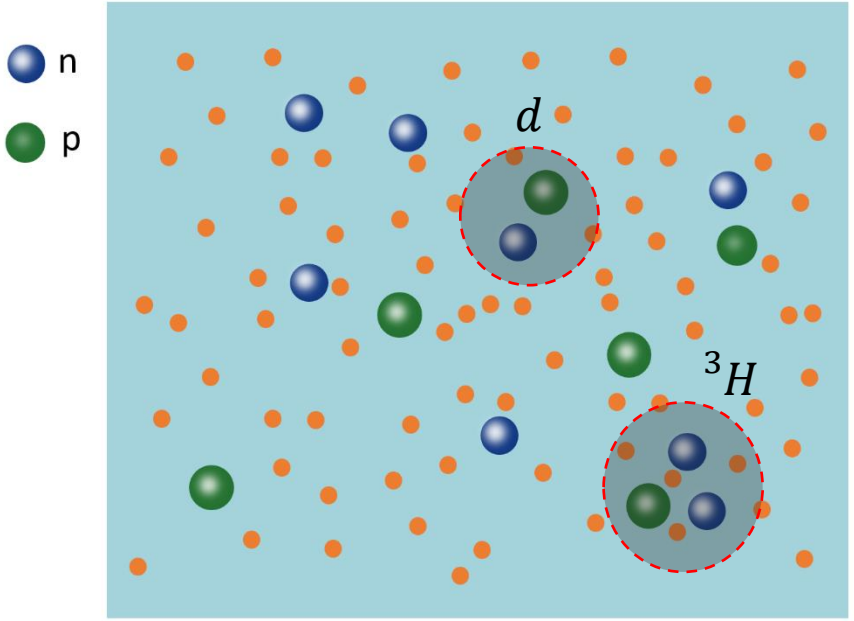
Initialization → Partonic evolution → Relativistic hadronic transport → Light nuclei

Mean field + scattering

Exhibits dynamical chiral phase transition

Nucleon coalescence

Light Nuclei Production



$$N_d \propto W_d\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}\right)$$

$$N_t \propto W_t\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}, \frac{x_1 + x_2 - 2x_3}{\sqrt{6}}, \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)$$

Wigner function: $W_d(r, k) = 8\exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 k^2\right)$ $\sigma_d \approx 2.26$ fm

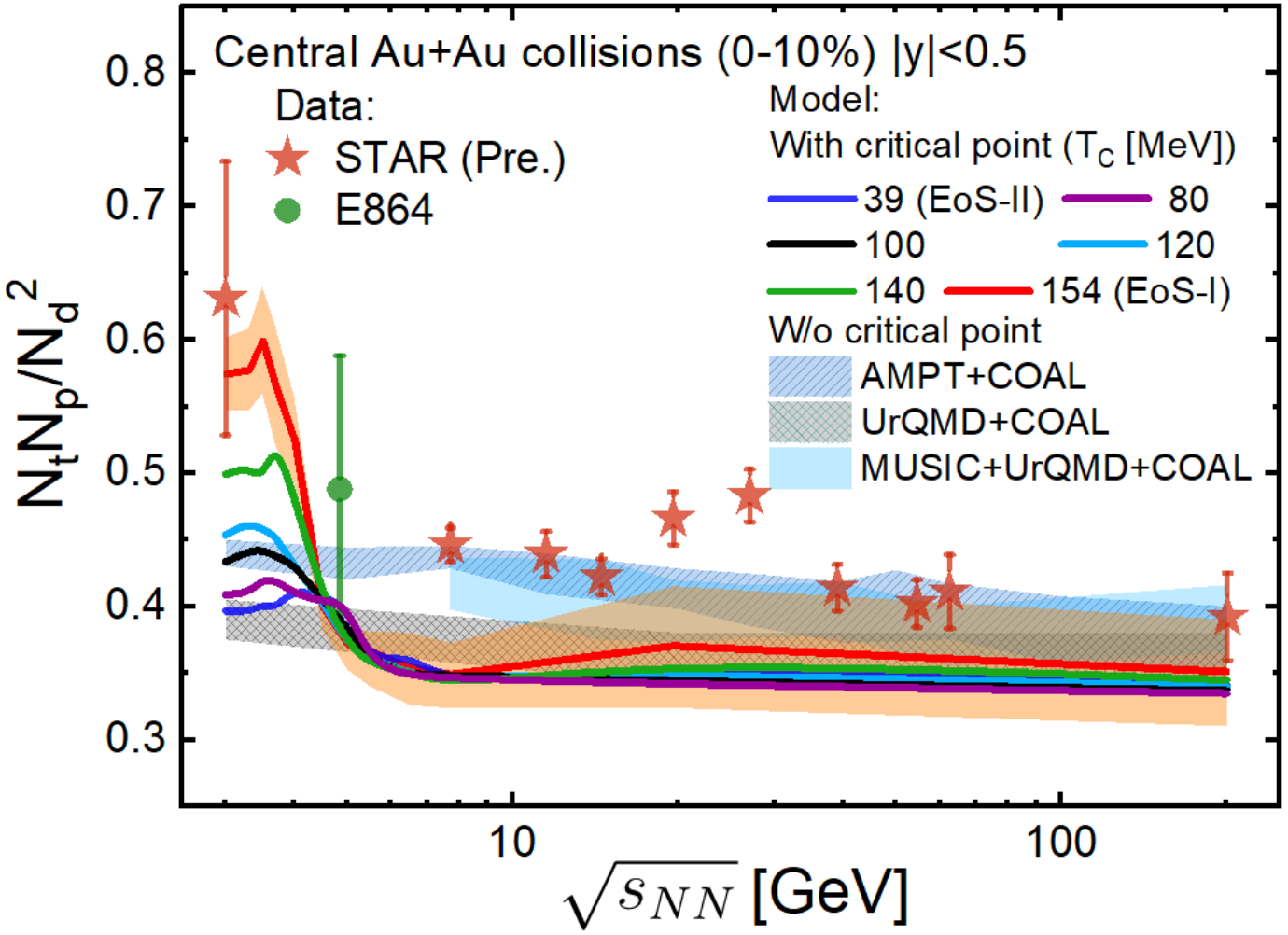
$$W_t(\rho, \lambda, k_\rho, k_\lambda) = 8^2 \exp\left(-\frac{\rho^2}{\sigma_t^2} - \frac{\lambda^2}{\sigma_t^2} - \sigma_t^2 k_\rho^2 - \sigma_t^2 k_\lambda^2\right)$$ $\sigma_t \approx 1.59$ fm

See K. J. Sun et al., arXiv:2207.12532(2022) for the more advanced treatment

R. Scheibl and U. W. Heinz, Phys. Rev. C59. 1585(1999)

Collision Energy Dependence

K. J. Sun, et al., arxiv:2205.11010 (2022)

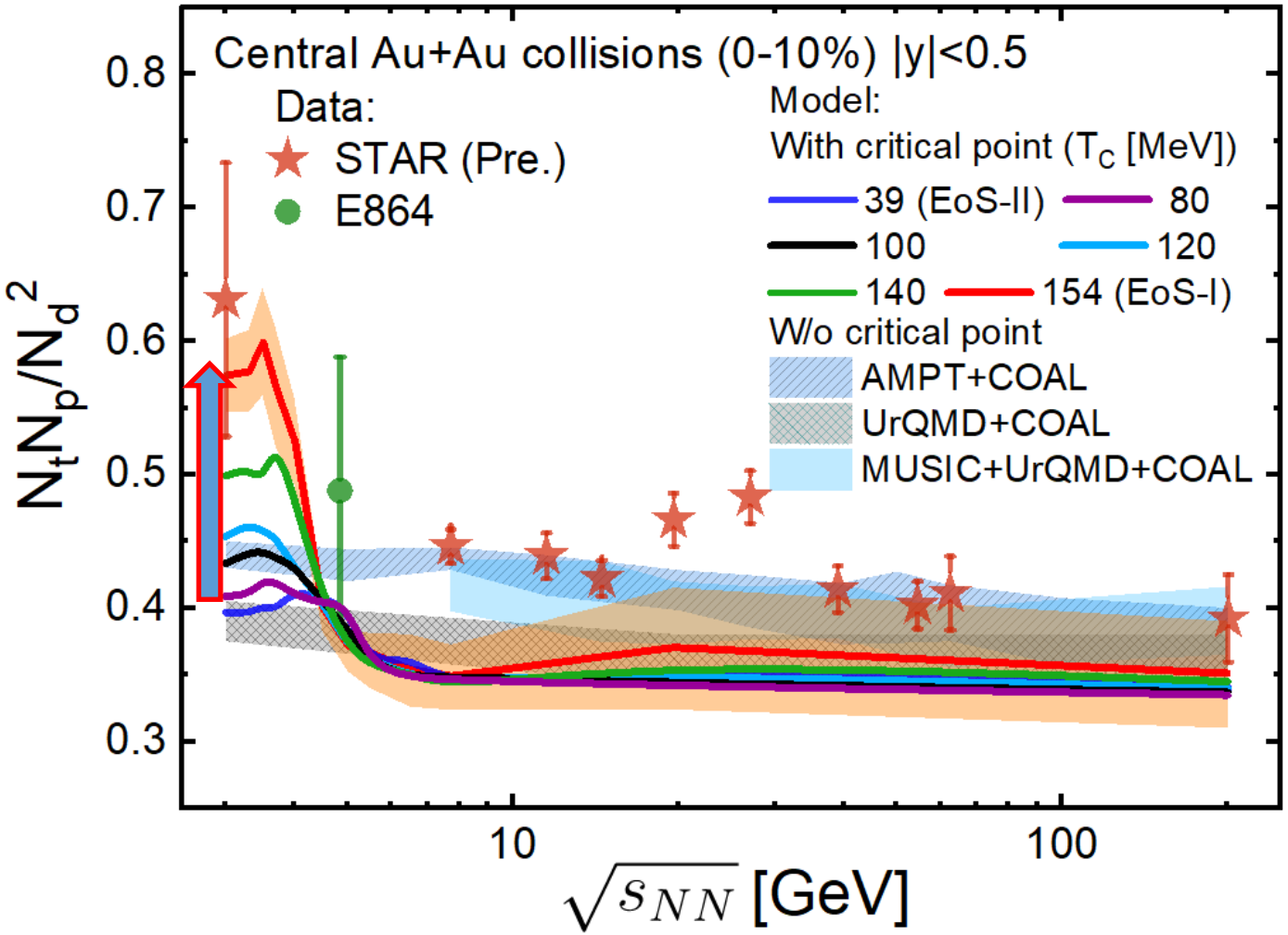


1. Without a critical point:
   The energy dependence of tp/d^2 is almost flat.

2. With a first-order phase transition:
   The spinodal-instability-induced enhancement of tp/d^2 increases as increasing the critical temperature.

STAR, arXiv:2209.08058(2022)
Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

Collision Energy Dependence

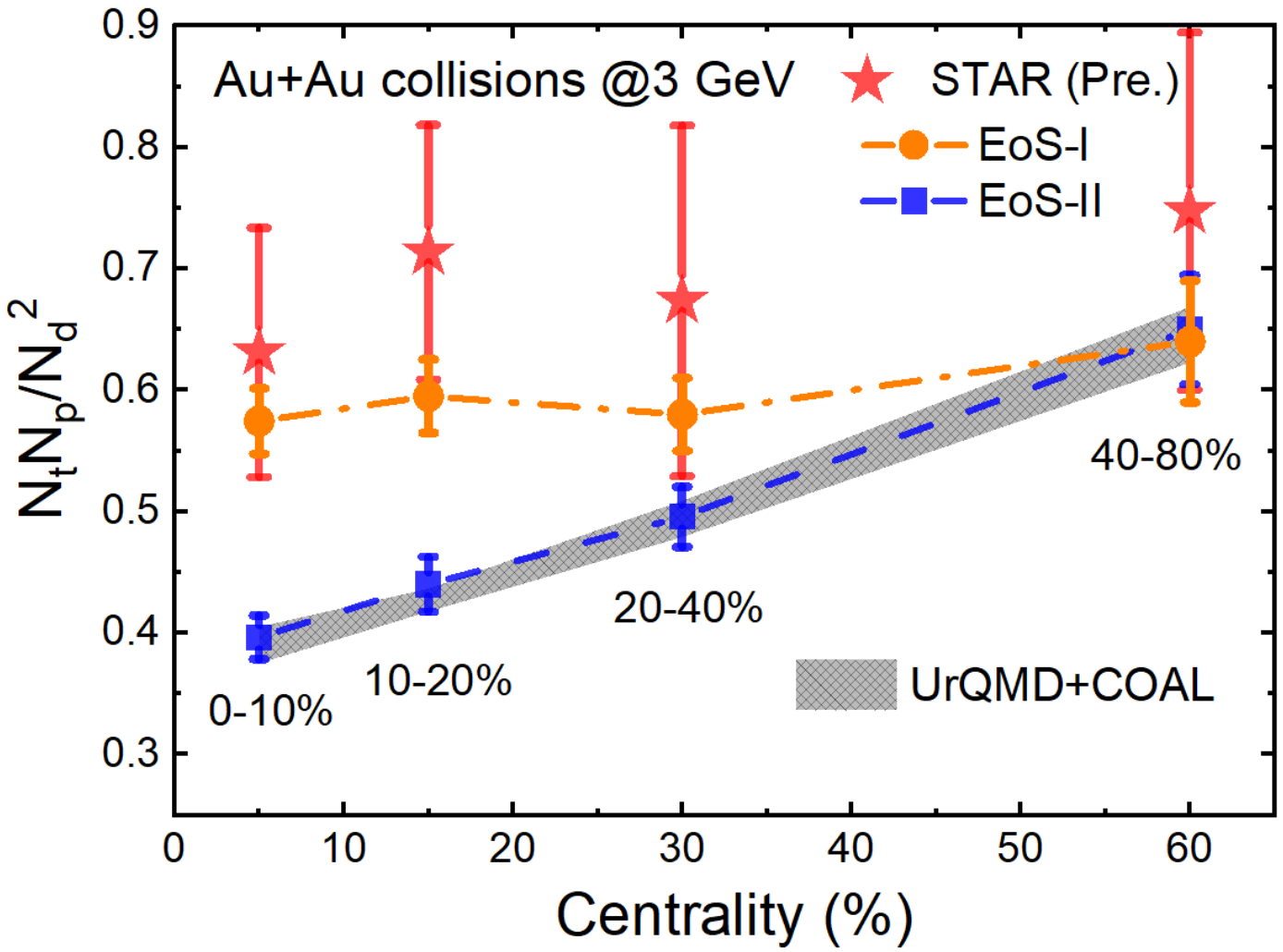


1. Without a critical point:
   The energy dependence of tp/d^2 is almost flat.

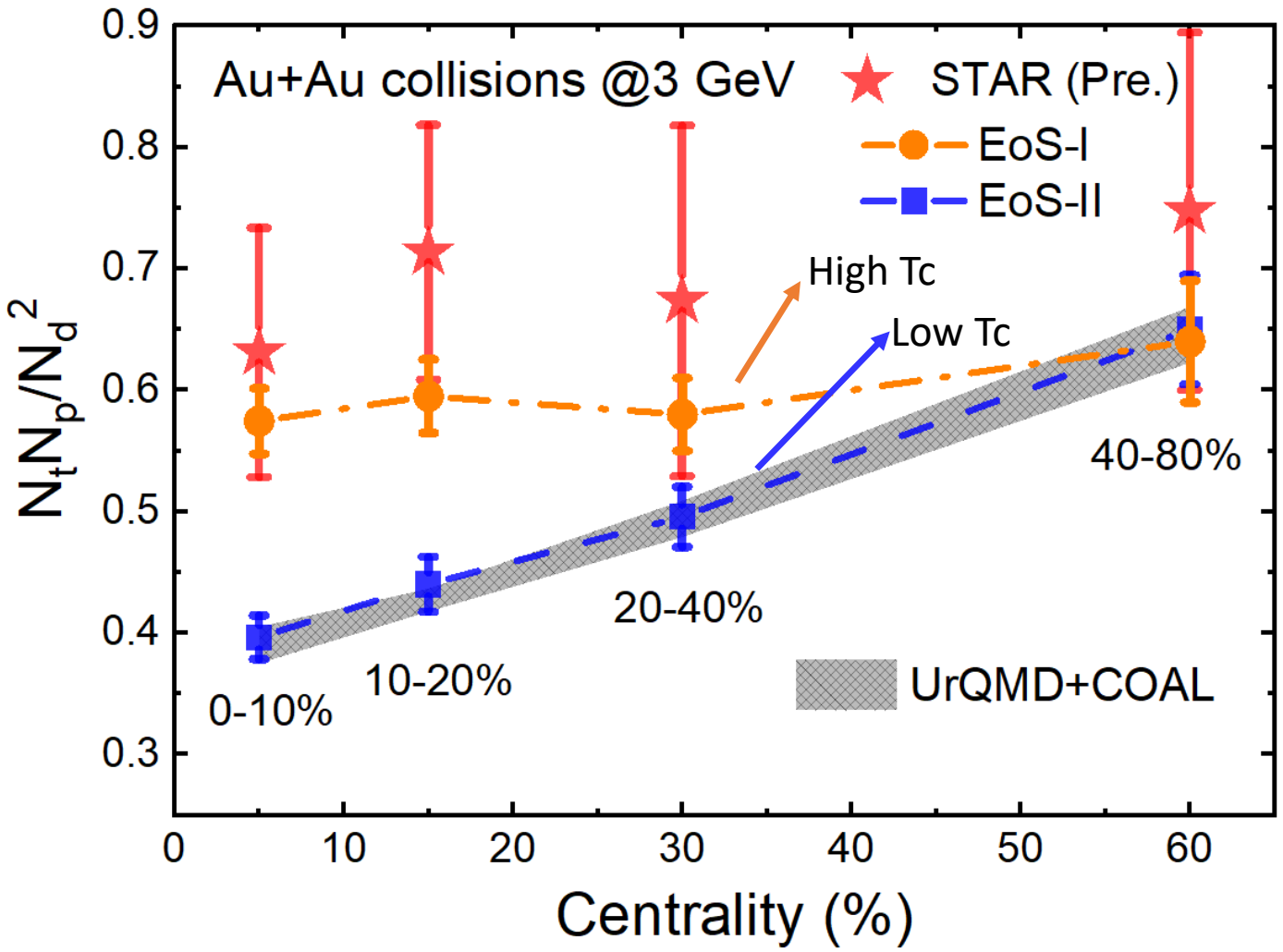
2. With a first-order phase transition:
   The spinodal instability induced enhancement of tp/d^2 during the first-order phase transition increases as increasing the critical temperature.

STAR, arXiv:2209.08058(2022)
Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

Centrality Dependence



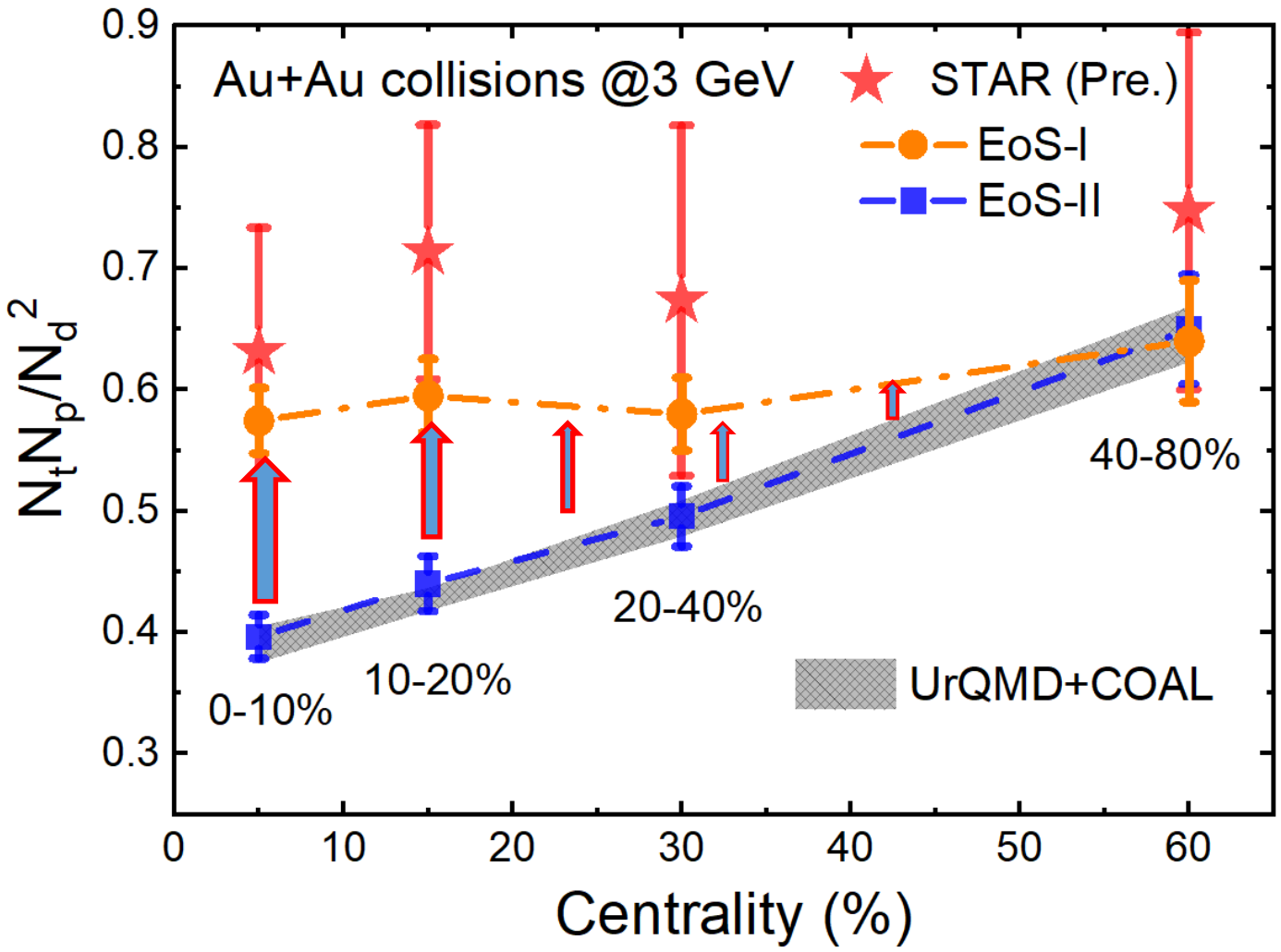
Centrality Dependence



The spinodal enhancement of tp/d^2 subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

Centrality Dependence

K. J. Sun, et al., arxiv:2205.11010 (2022)



The spinodal enhancement of tp/d^2 subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller

Summary and Outlook

1. **Light nuclei production provides a promising probe to non-smooth QCD phase transition.**

2. **Spinodal instability during the first-order chiral phase transition could induce an enhancement of tp/d^2 in central Au+Au collisions at $\sqrt{s_{NN}}$ =3 – 5 GeV ($T_c \geq 80$ MeV).**

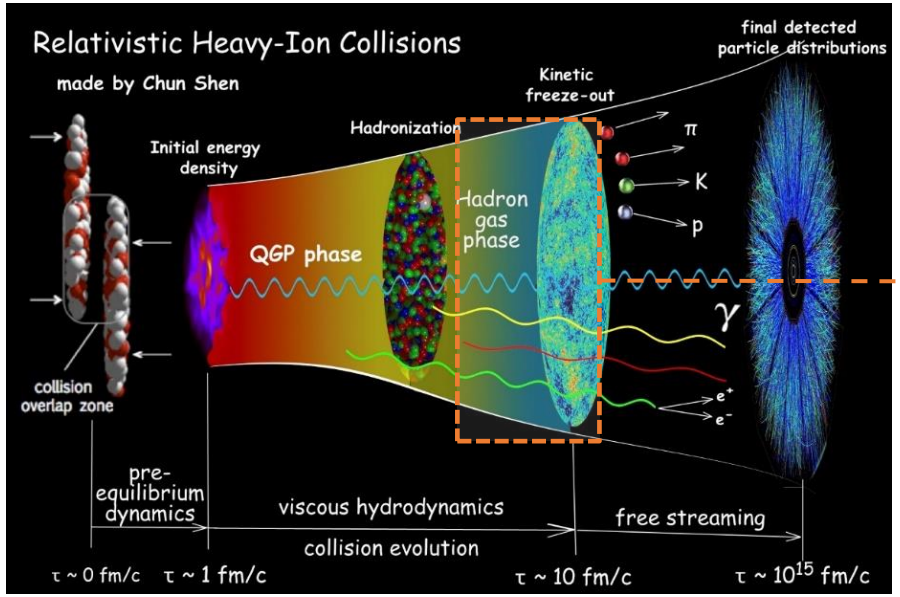
Effects on other observables e.g. flow, HBT, di-lepton production?
More efforts are needed to understand the enhancement of tp/d^2 shown in the STAR data.

Backup

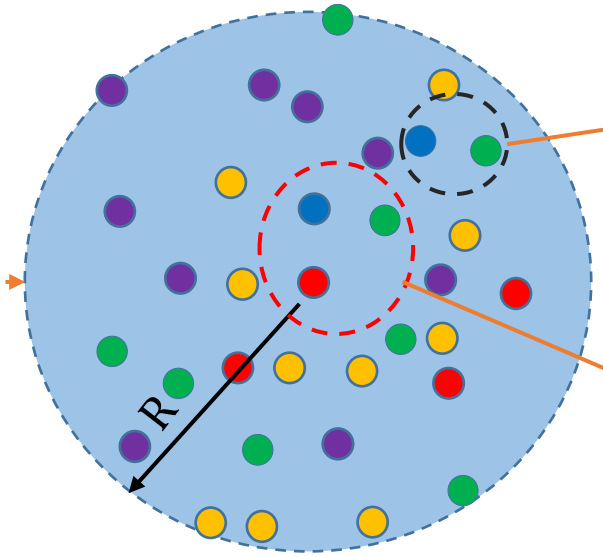
Final-state coalescence

R. Scheibl and U. W. Heinz, PRC59. 1585(1999);

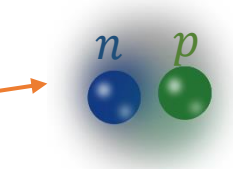
R. Scheibl and U. W. Heinz, PRC59. 1585(1999)



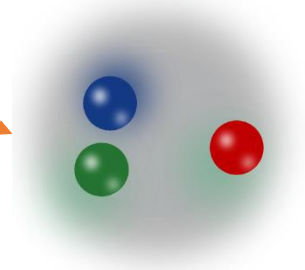
Coalescence Model



Deuteron



$^{3}_{\Lambda}\mathrm{H}$



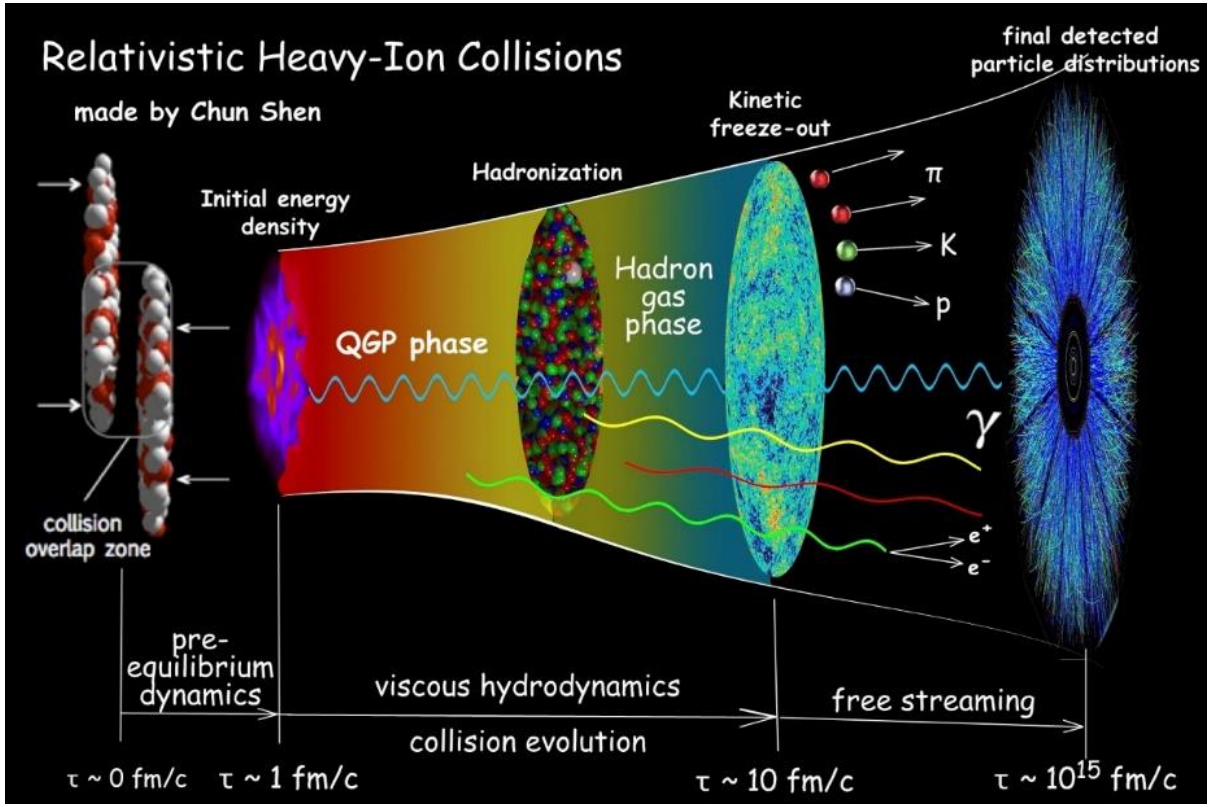
Density Matrix Formulation
(sudden approximation)

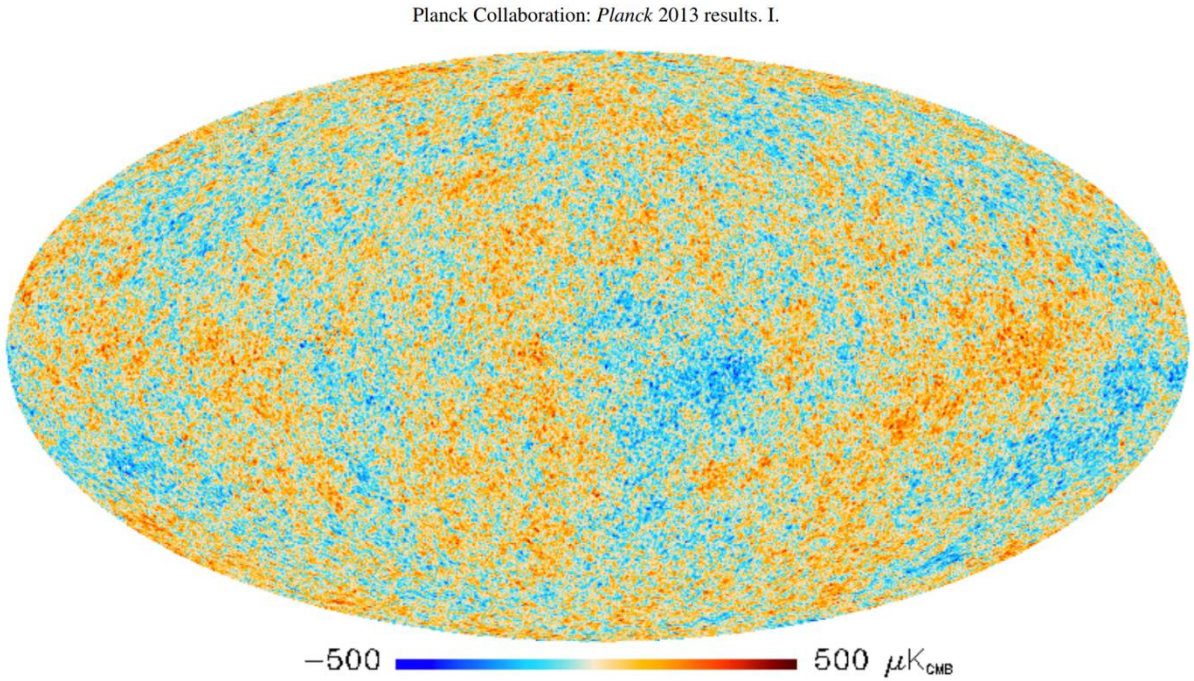
Wigner function of light cluster

$$N_A = Tr(\hat{\rho}_s \hat{\rho}_A)$$
$$= g_c \int d\Gamma\, \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Overlap between source distribution function and Wigner function of light nuclei

Remnant density fluctuation in expanding systems

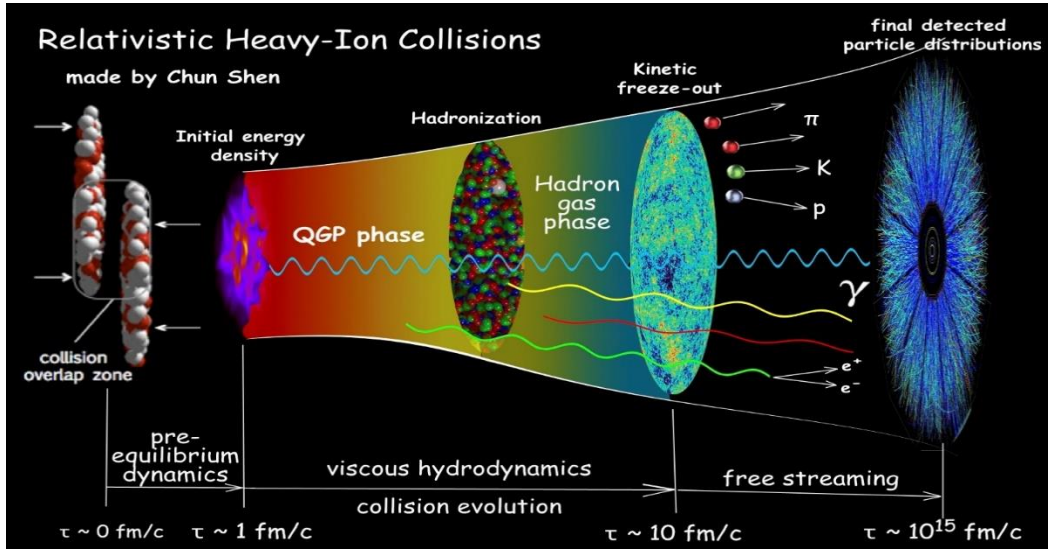




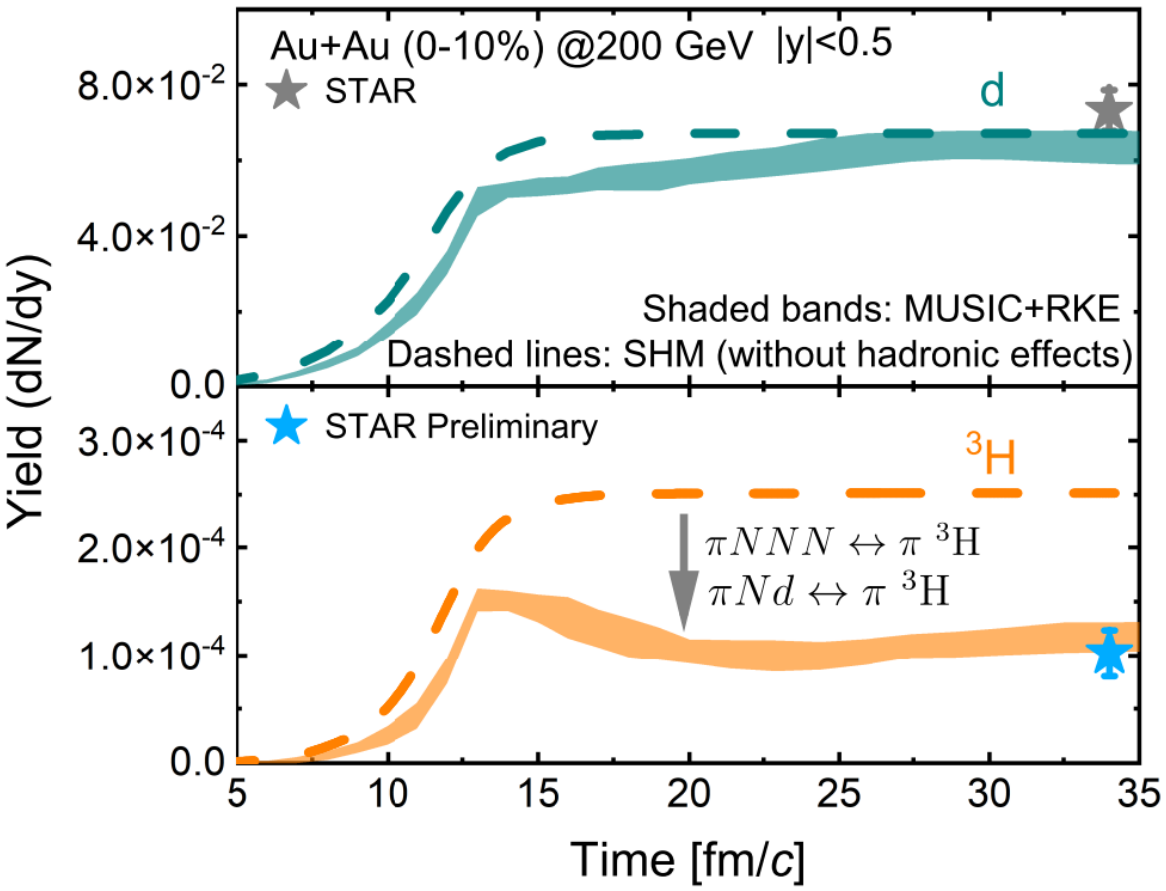
Temperature anisotropy in the cosmic microwave background, which is now considered as the remnant of quantum fluctuations in the primordial Universe during the rapid inflation epoch.

Plank Collaboration, Astron. Astrophys. 571, A1 (2014)

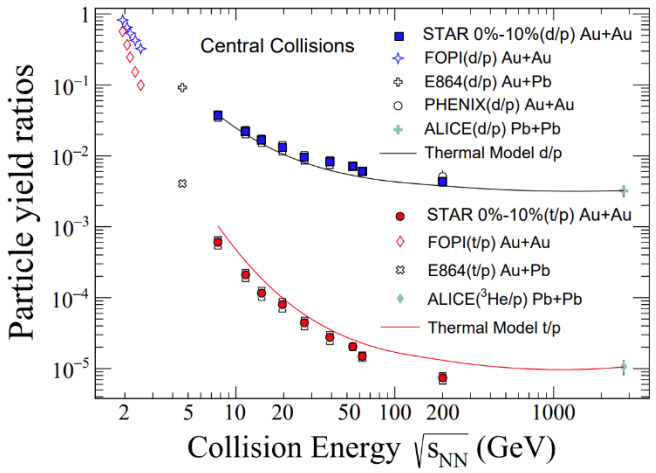
Effects of post-hadronization dynamics

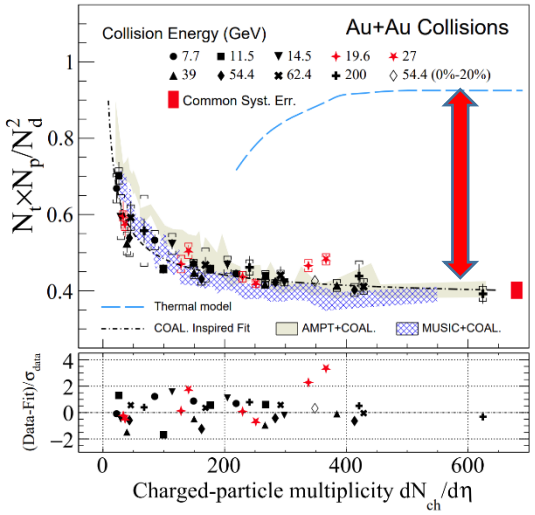


K. J. Sun et al., arXiv:2207.12532(2022)



STAR: arXiv:2209.08058(2022)



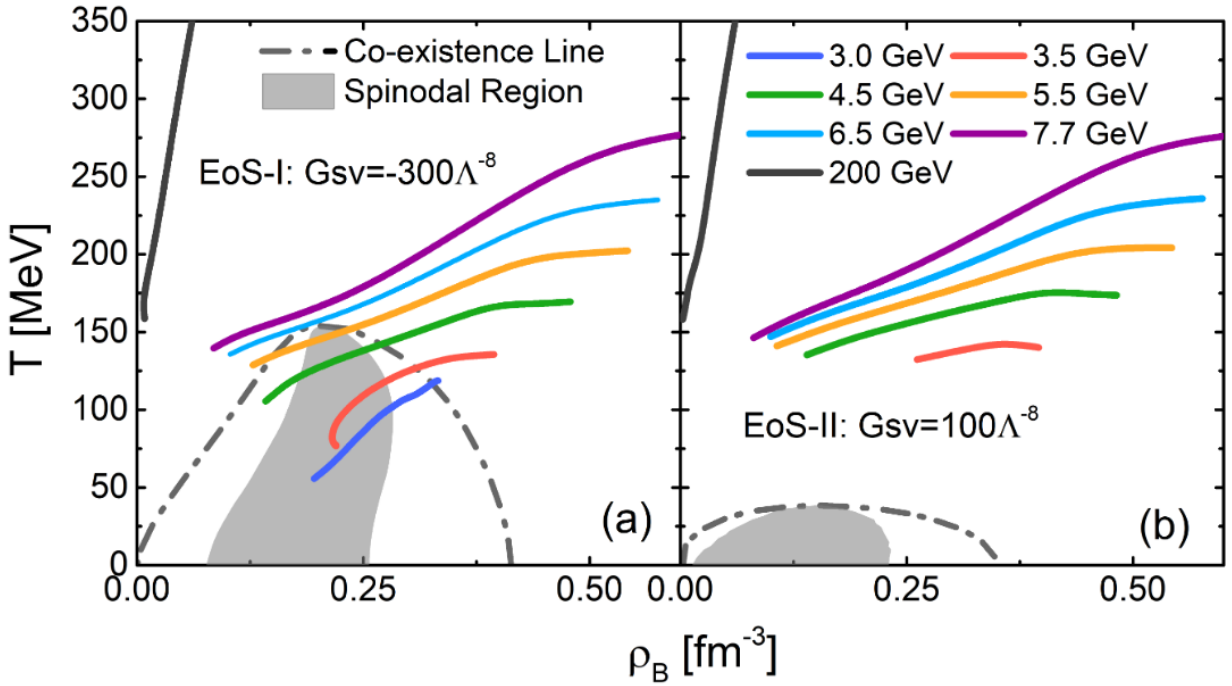


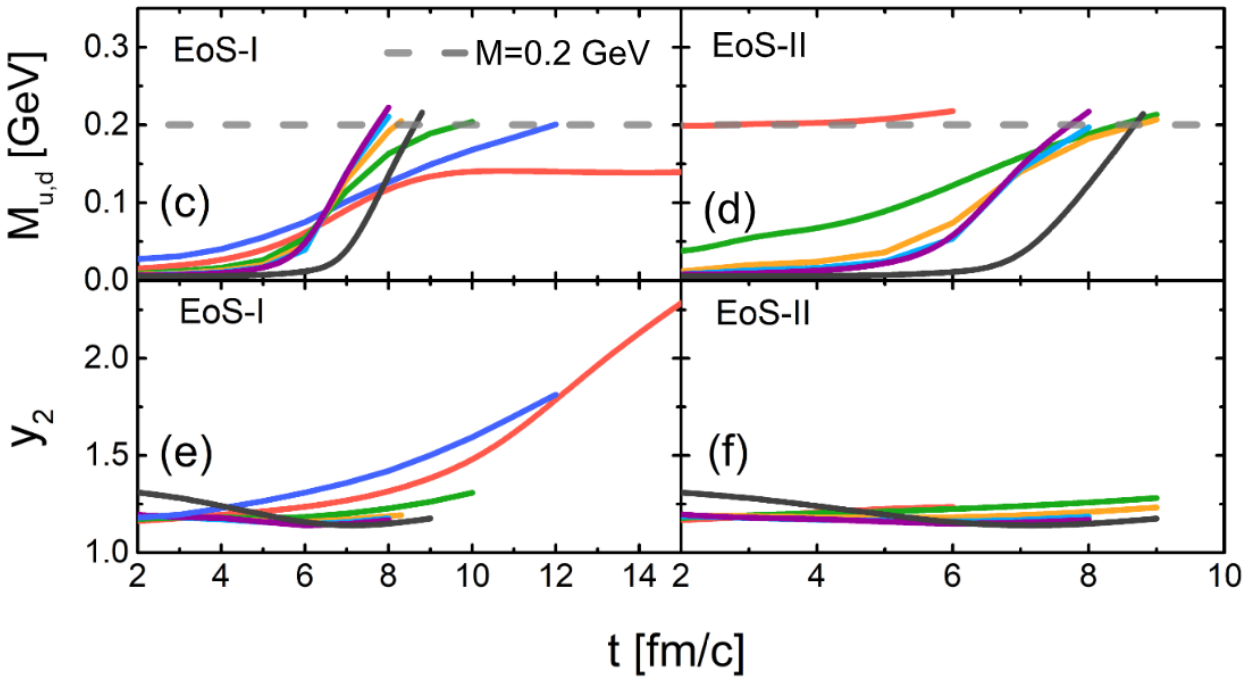
Triton yields are reduced by about a factor of 2 due to hadronic re-scatterings.
Post-hadronization dynamics have visible effects!

Trajectories in the phase diagram

Phase trajectories of central cells in the phase diagram

$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})} \qquad y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int d\mathbf{x}\rho^3(\mathbf{x})]}{[\int d\mathbf{x}\rho^2(\mathbf{x})]^2}$$





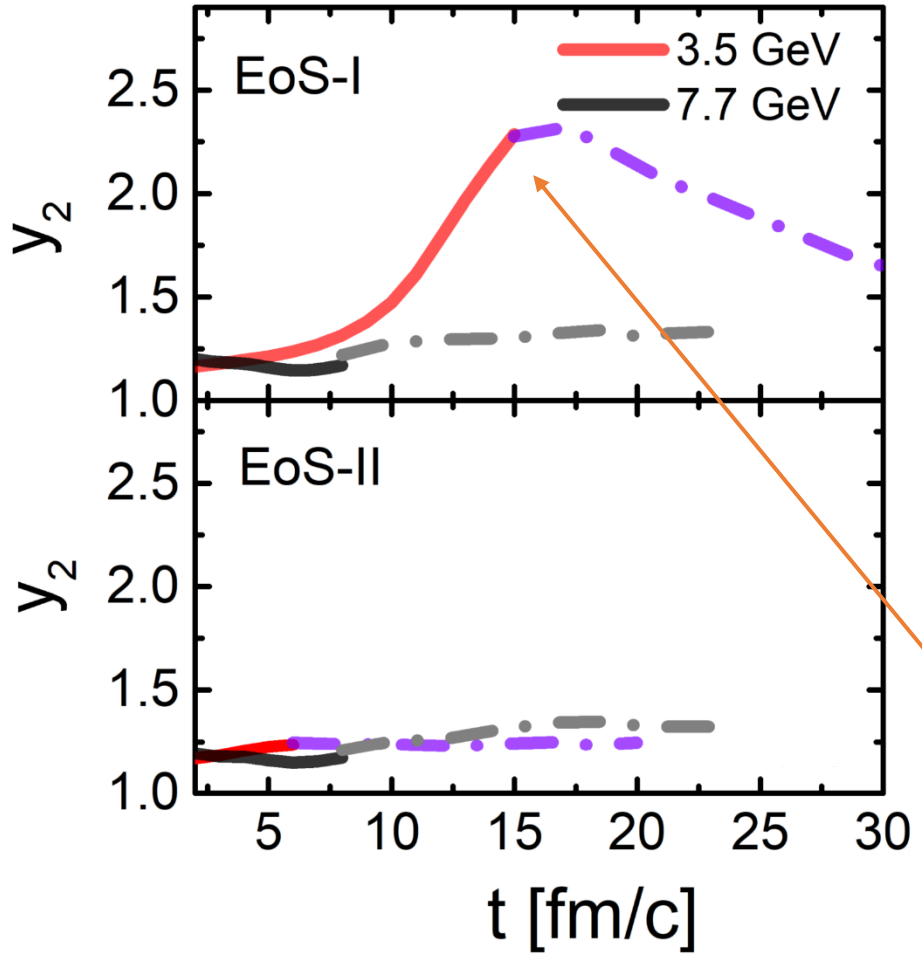
K. J. Sun, et al., arxiv:2205.11010 (2022)

Memory Effects

Survival of density fluctuation in an expanding fireball

Off-equilibrium effects

K. J. Sun et al., Eur.Phys.J.A 57 (2021) 11, 313
K. J. Sun, et al., arxiv:2205.11010 (2022)



Density moment:

$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int \mathrm{d}\mathbf{x}\rho^{\mathbf{3}}(\mathbf{x})]}{[\int \mathrm{d}\mathbf{x}\rho^{\mathbf{2}}(\mathbf{x})]^{\mathbf{2}}}$$

$$\approx 1 + \frac{\int \mathrm{d}\mathbf{x}(\delta\rho(\mathbf{x}))^{\mathbf{2}}}{\int \mathrm{d}\mathbf{x}\rho_{\mathbf{0}}^{\mathbf{2}}} \equiv 1 + \Delta\rho.$$

If the expansion is self-similar or scale invariant

$$\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$$

then $y_2(t) = y_2(t_h)$, i.e., remains a constant

'Memory effects' in the talk by Lijia Jiang
"Dynamical effects on the phase transition signal"

'Memory effects': Large density inhomogeneity survives to kinetic freezeout