

# Chiral phase transition, thermalization and prethermalization in the soft-wall AdS/QCD model



Danning Li (李丹凝)  
Department of Physics, Jinan University

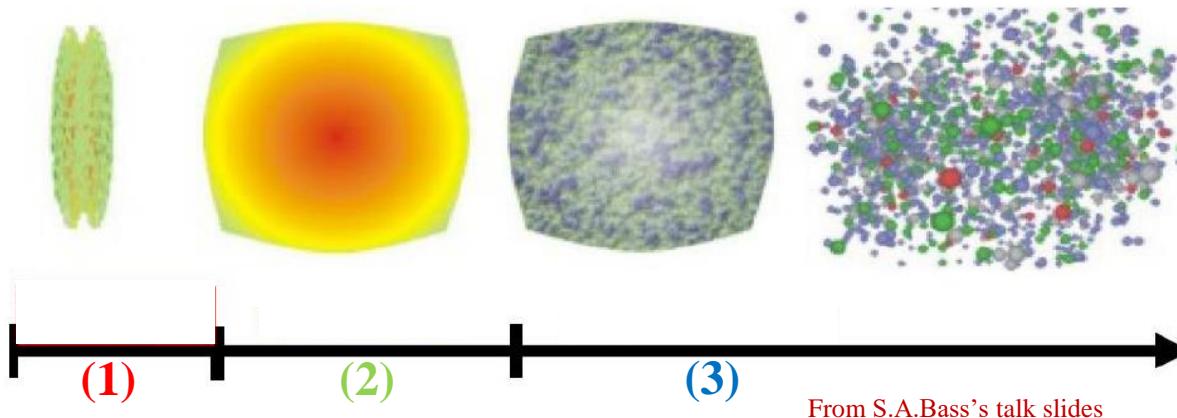
Based on: Phys.Rev.D107(2023)8,086001

Collaborators: Xuanmin Cao, Jingyi Chao, Hui Liu

# Outlines

- Introduction
- Soft-wall model and chiral phase transition
- Thermalization and prethermalization
- Summary

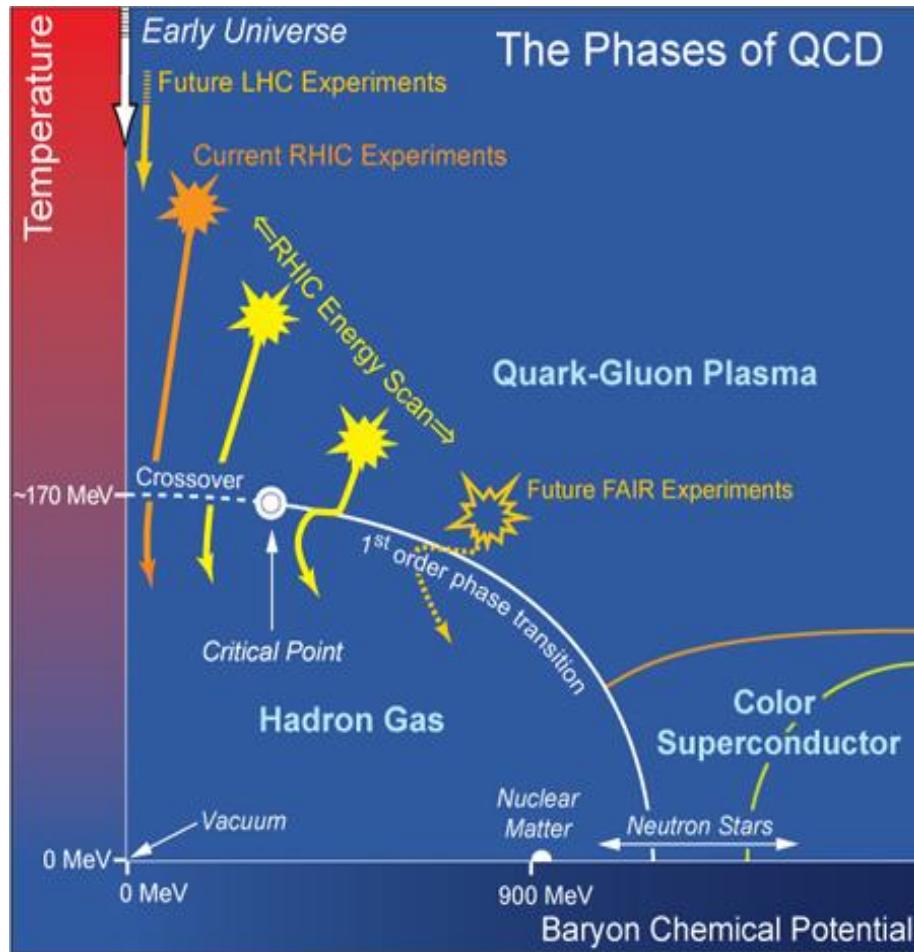
# Nonequilibrium stage in HIC



- fluid dynamics: initial conditions, EOS, transport coefficients  
local equilibrium:  $\tau \sim 0.1\text{-}1\text{fm}/c$ , fast thermalization?
- transport models: hadronic cascade model, UrQMD models
- nonequilibrium stage:
  - Kinetic theory:  $f(x, p)$ , relativistic Boltzman equation
  - Color glasma: gluon saturation
  - weakly coupled picture

( but:  $\eta/s \sim 0.2$ , which indicating a strongly coupled QGP in hydrodynamics.  
At least, at the end of the non-equilibrium period. )

# Real time dynamics of phase transition



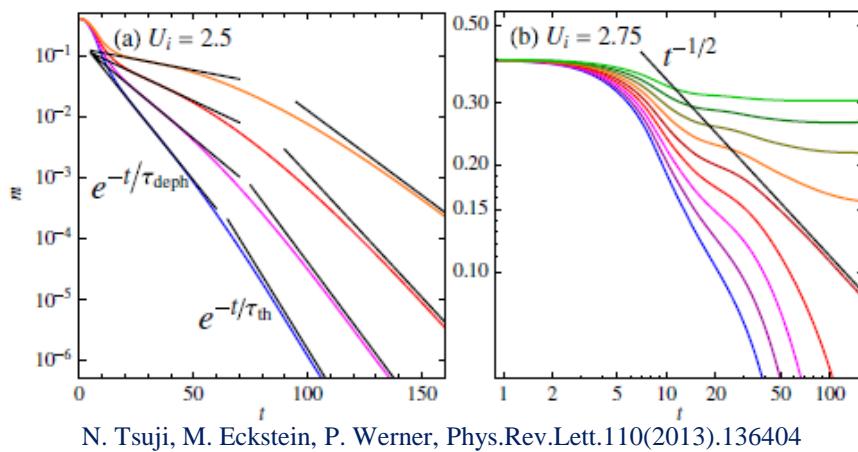
1<sup>st</sup> order region:  
inhomogeneous effect on possible  
signals

Near phase transition: non-  
perturbative!

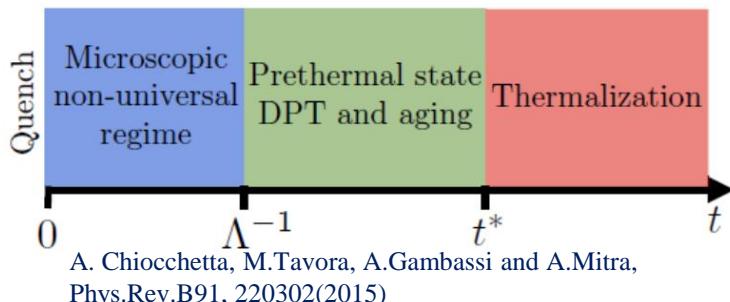
# Prethermalization: short time

Originally, the term “prethermalization” was introduced by Berges, Borsányi, and Wetterich for matter under extreme conditions in a quasisteady state far from equilibrium.

J. Berges, Sz. Borsanyi, and C.Wetterich, Phys.Rev.Lett. 93 (2004) 142002



N. Tsuji, M. Eckstein, P. Werner, Phys.Rev.Lett.110(2013).136404

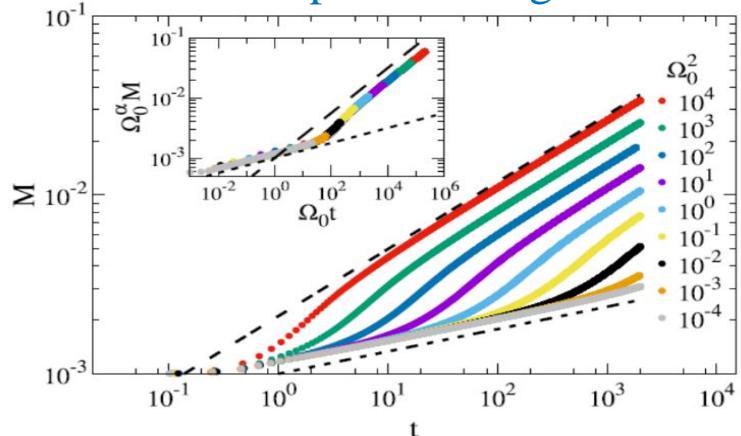


Now widely used in many fields:  
cold atom(theoretical + experiments)  
condensed matter physics,  
statistical physics

Integrable system: generalized Gibbs ensembles

RG point of view: non-thermal fixed point

Near critical point: strong correlation



A. Chiocchetta, A. Gambassi, S.Diehl, and J.Marino, Phys.Rev.Lett. 118 (2017) 13, 135701

# Holographic Method

Closed strings in AdS background

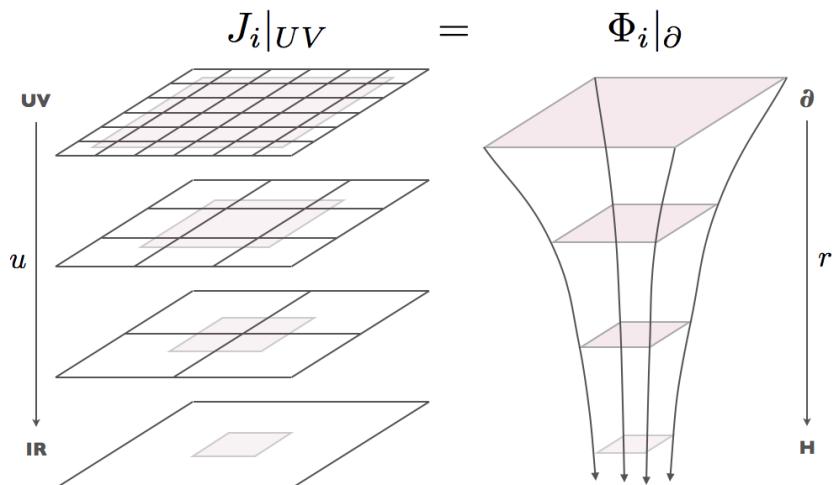
$$\frac{R^4}{l_s^4} = 4\pi g_s N_c \gg 1$$

Holography: 5th dimension( $r$ ) plays as energy scale

J.Maldacena,Adv.Theor.Math.Phys.2:231-252,1998

SYM in Minkovski Space Time

$$\lambda = g_{YM}^2 N = 4\pi g_s N \gg 1$$



application to QCD:  
try to break the conformal symmetry

Correspondence:

$$\Phi(t, x, r) = J(t, x)r^{\Delta_1} + \dots + O(t, x)r^{\Delta_2} + \dots$$

$$Z_{Gauge} \sim Z_{Gravity} \sim e^{-S_{Gravity}} \sim e^{-\int a(z) JO}$$

$$\left. \begin{aligned} & \left[ \partial_z^2 - \frac{(d-1-2J)}{z} \partial_z - \frac{(\mu R)^2}{z^2} - U_J(z) + \mathcal{M}^2 \right] \Phi_J(z) = 0 \\ & k \partial_k \{ k^{-\delta} k \partial_k \Gamma_k^{(n)} \} = k \partial_k (k^{-\delta} \beta_\Gamma^{(n)}) \end{aligned} \right\} k = 1/z$$

Mapped the flow equation of FRG to the wave equation of HQCD:

F.Gao, M.Yamada, Phys.Rev.D 106.126003(2022)

# Holographic thermalization

Hard to deal with in traditional field tools, but easy(relatively) in HQCD:

$T_{ab} \leftarrow \rightarrow g_{ab}$  (appear in its expanding coefficients)  
Time evolution: Einstein equations  
 $g_{ab}(t) \rightarrow T_{ab}(t)$

➤ Early studies: linearize Einstein equations

G.T.Horowitz et al., Phys.Rev.D62,024027(2000)

K.Murata et al. JHEP 0808,027(2008)

R.A.Janik et al., Phys.Rev.D73,045013(2006)

➤ Direct mimicking: full dynamics in SYM( $R+\Lambda$ )

P.M.Chesler et al., Phys.Rev.Lett.106,021601(2011)

M.P.Heller et al., Phys.Rev.Lett.108,201602(2012)

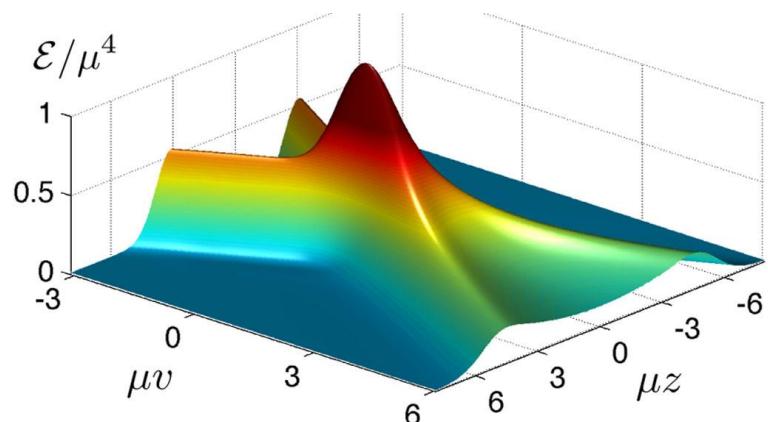
M.P.Heller et al., Phys.Rev.D85,126002(2012)

➤ Describing the experimental data:

W.V.Schee et al., Phys.Rev.Lett.111,222302(2013)

W.V.Schee et al., Phys.Rev.C92,064907(2015)

K.Rajagopal et al., Phys.Rev.Lett.116,211603(2016)



P.M.Chesler et al., Phys.Rev.Lett.106,021601(2011)

# Soft-Wall Model

A.Karch, E.Katz, D.T.Son, M.A.Stephonov, Phys.Rev.D74:015005,2006

**Promote 4D global chiral symmetry to 5D:**

$$I = \int d^5x e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\} \quad SU_L(N_f) \times SU_R(N_f)$$

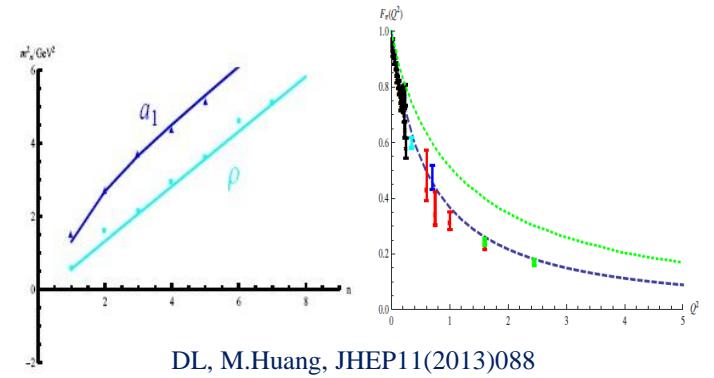
**Field and Operator Correspondence:**

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	$p$	$\Delta$	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A_{L\mu}^a$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A_{R\mu}^a$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$\begin{aligned} A_{L,\mu}^a(z) &= j_{L,\mu}^a(x) + J_{L,\mu}^a(x)z^2 + \dots \\ A_{R,\mu}^a(z) &= j_{R,\mu}^a(x) + J_{R,\mu}^a(x)z^2 + \dots \\ X(z) &= m_q z + \langle \bar{q}q \rangle z^3 + \dots \\ Z_{QCD} \sim Z_{Gravity} &\sim e^{-S_{Gravity}} \sim e^{-\int c(z) j_i J^i + d(z) m \bar{q} q} \end{aligned}$$

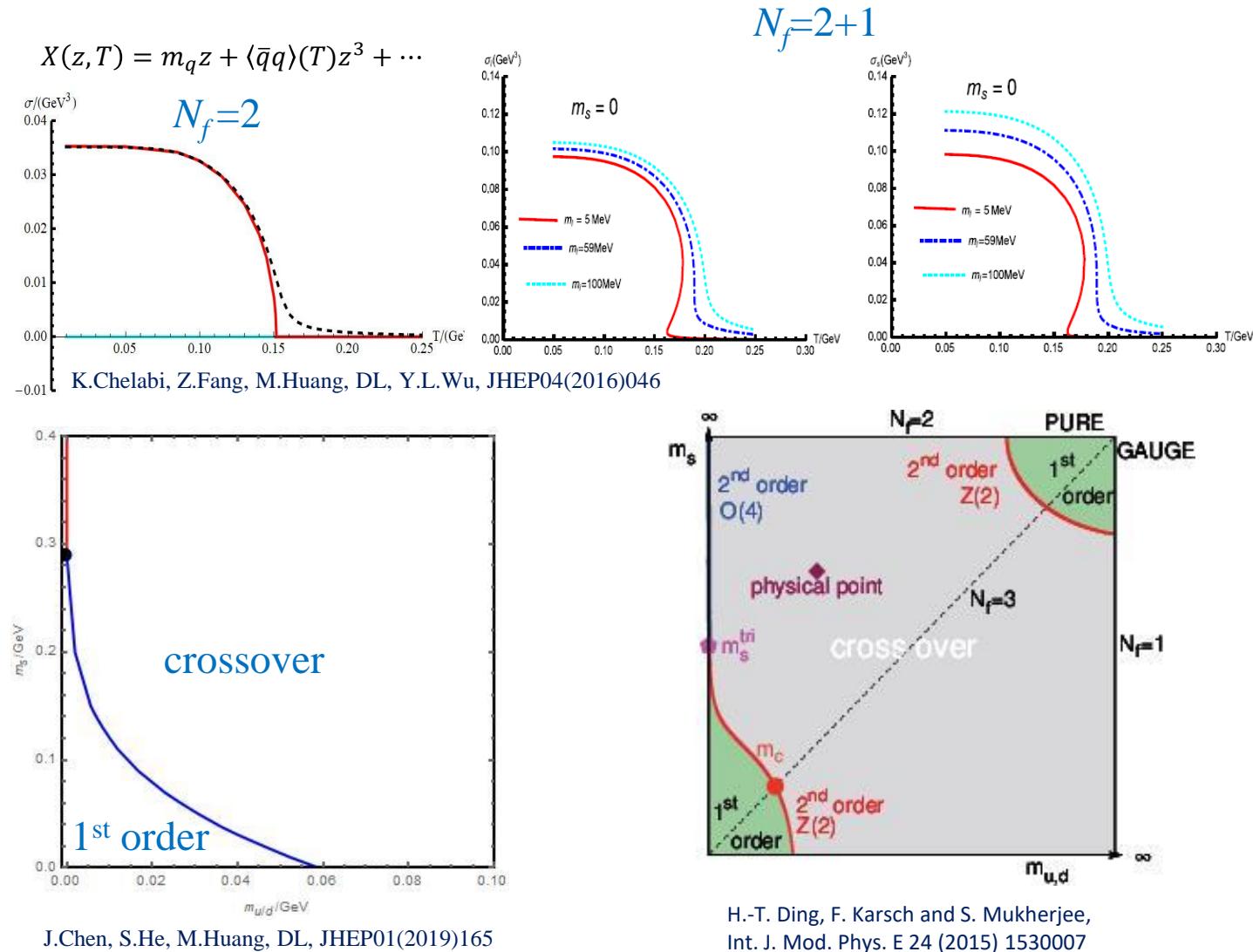
**Extensions:**

- T. Gherghetta, J. I. Kapusta and T. M. Kelley, Phys. Rev. D **79** (2009) 076003
- T. M. Kelley, S. P. Bartz and J. I. Kapusta, Phys. Rev. D **83** (2011) 016002
- Y. -Q. Sui, Y. -L. Wu, Z. -F. Xie and Y. -B. Yang, Phys. Rev. D **81** (2010) 014024
- Y.Q.Sui, Y.L.Wu, Y.B.Yang, Phys.Rev. D83 (2011), 065030
- DL, M.Huang, Q.S.Yan, Eur.Phys.J. C73 (2013) 2615
- S.He, S.Y.Wu, Y.Yang and P.H.Yuan, JHEP 1304 (2013) 093
- Y.Chen, M. Huang, Phys.Rev. D105 (2022), 026021
- .....



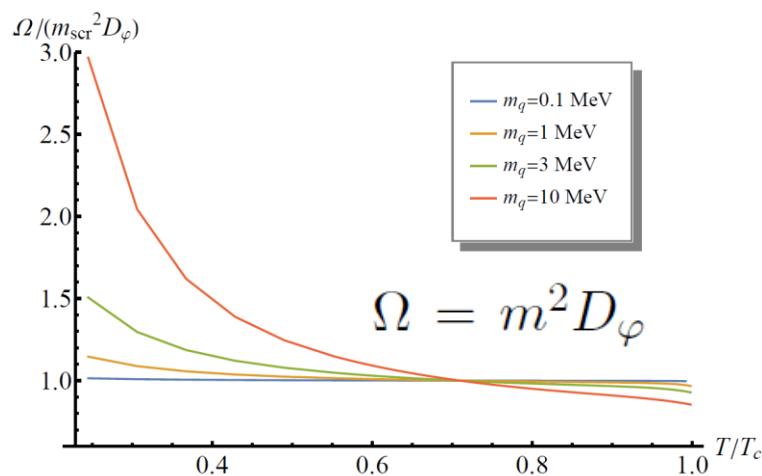
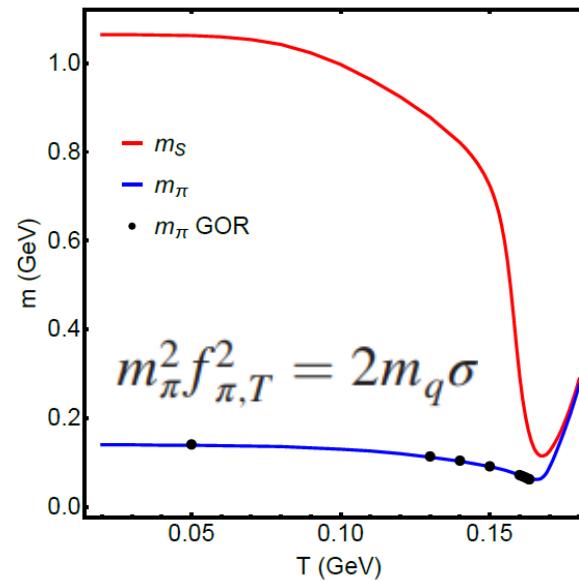
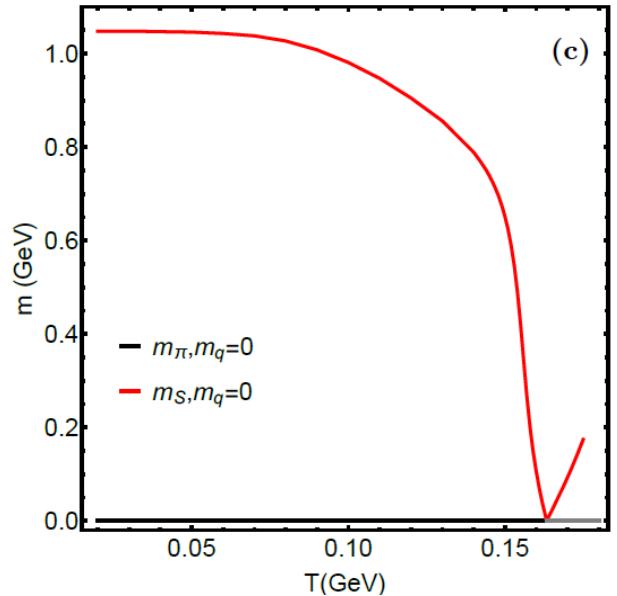
DL, M.Huang, JHEP11(2013)088

# Chiral phase transition in SW model



Mean field exponents: large N suppression?

# Thermal spectra and phase transition



Scaling law near the critical point:

$$\langle \bar{\psi} \psi \rangle \sim t^\beta \quad f \sim t^{\nu/2} \quad \beta \approx 0.38 \quad \nu \approx 0.73$$

$$m_p^2 \equiv u^2 m^2 = - \frac{m_q \langle \bar{\psi} \psi \rangle}{\chi_{15}} \sim m_q t^\beta$$

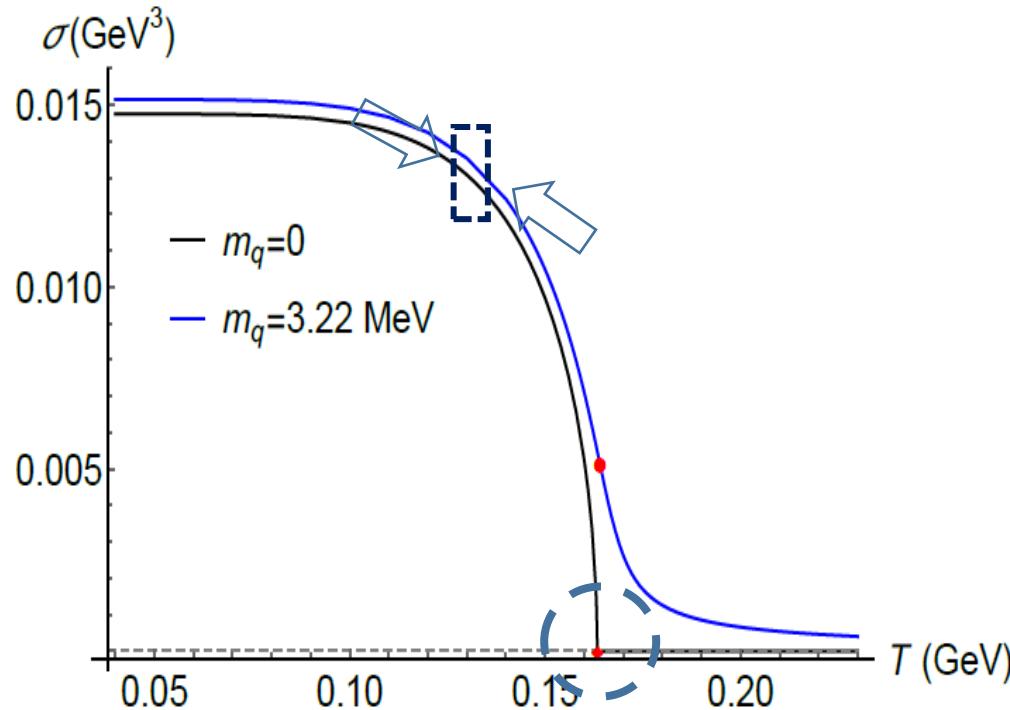
D.T.Son and M.A.Stephanov, Phys.Rev.Lett. 88 (2002) 202302

# Real-time dynamics in soft-wall model

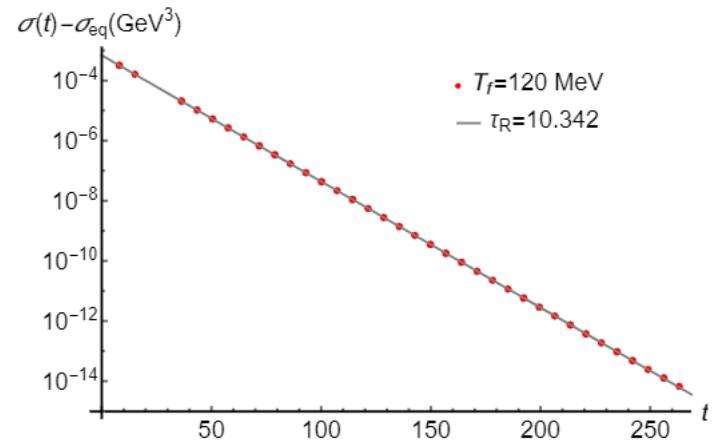
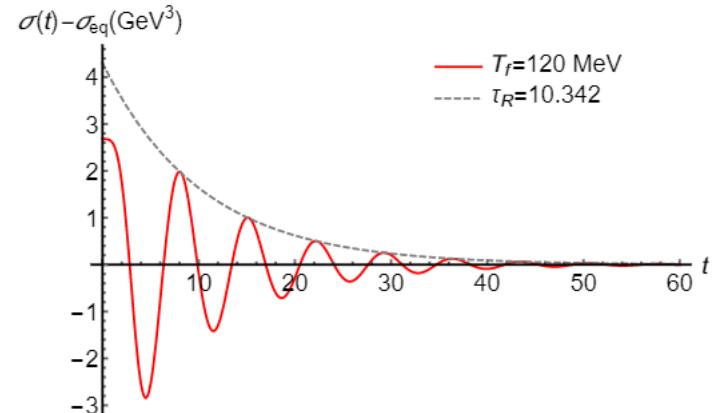
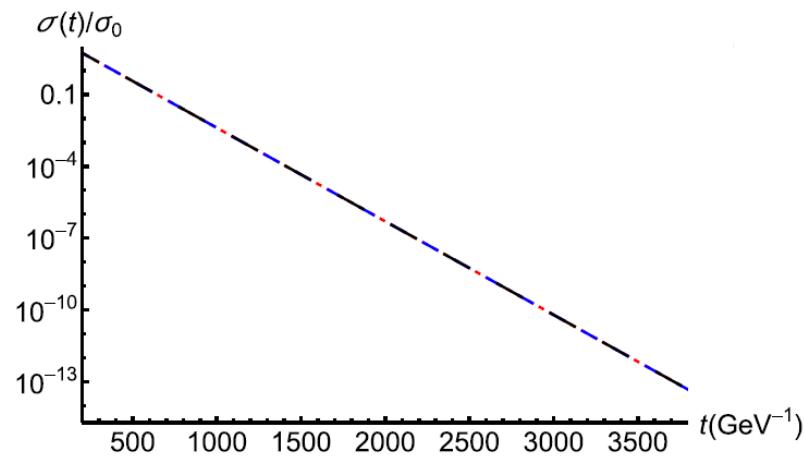
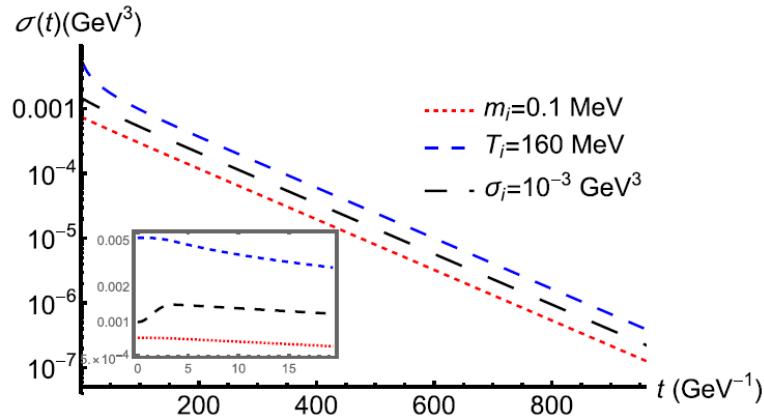
Extension to nonequilibrium case:  $\chi(z) \Rightarrow \chi(t, z)$

Time dependent chiral condensate:  $\chi(z \rightarrow 0) = m_q \gamma z + \frac{\sigma(t)}{\gamma} z^3$

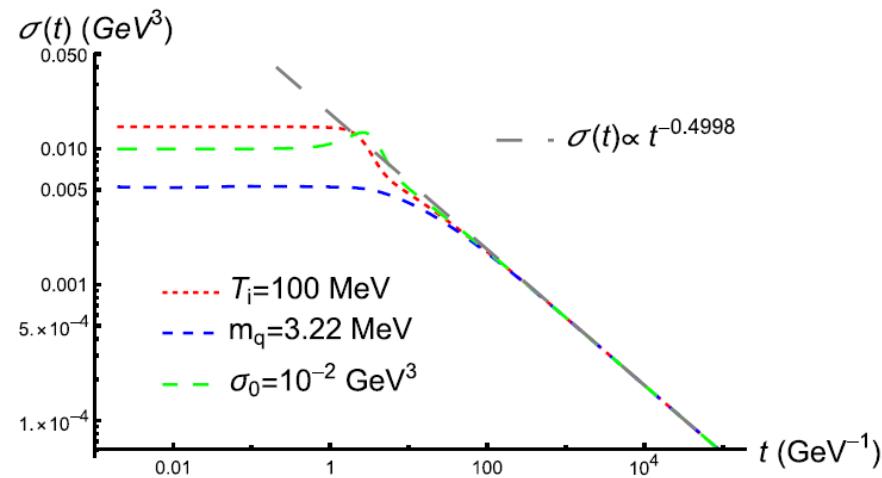
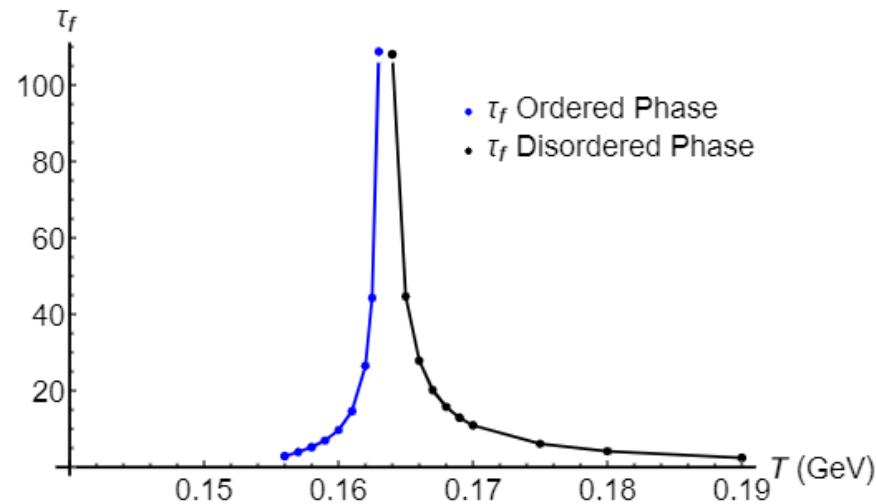
Evolve from nonequilibrium state to equilibrium state:



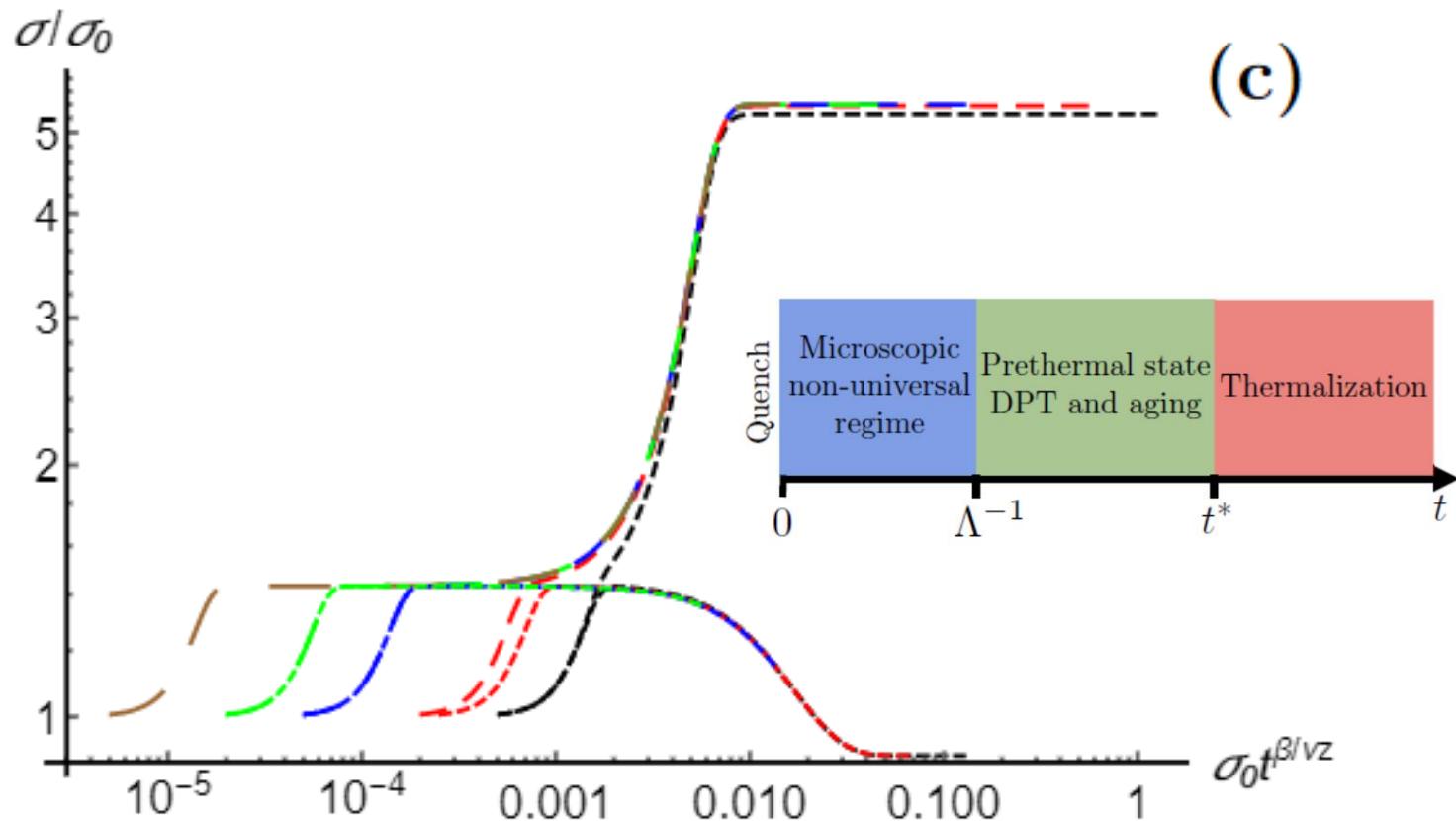
# Thermalization



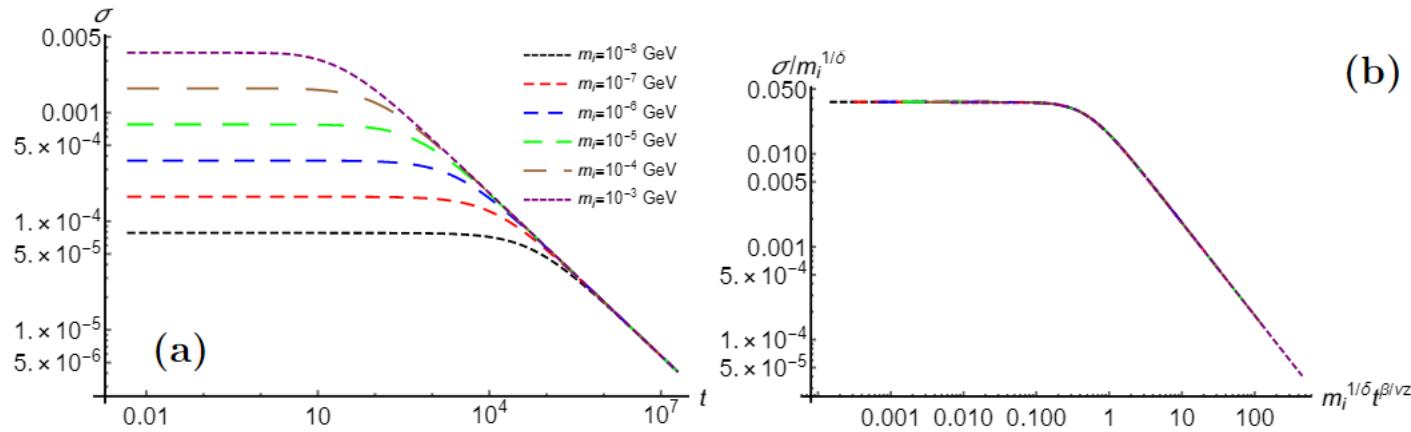
# Relaxation time and critical slowing down



# Prethermalization



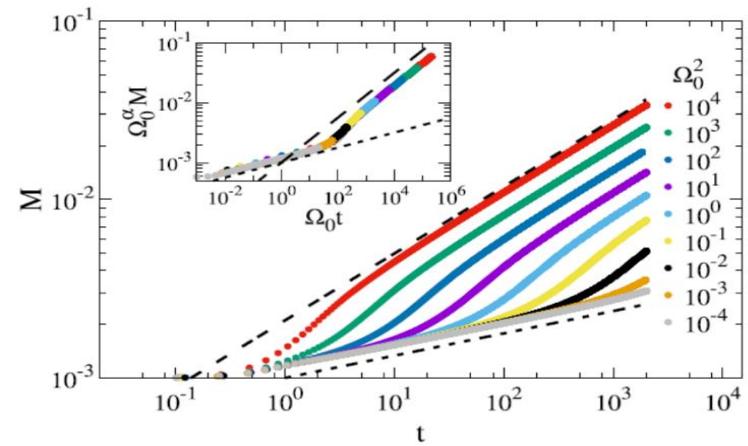
# Dynamic critical scaling



However, the initial-slip exponent  $\theta=0$ , again mean field!

$$\sigma(\sigma_i, t) \propto \sigma_i t^{(x-1)\beta/\nu z}$$

$$\theta \equiv (x-1)\beta/\nu z$$



But qualitatively they have similar patterns

# Summary

- Soft-wall AdS/QCD model provides a good start point to consider chiral phase transition in holographic framework. The phase diagram is in agreement with the Columbia plot. It is like a 5D finite temperature CPT.
- The real-time dynamics of chiral phase transition show non-trivial behavior in the intermediate time. This might be related with the so called “prethermalization” phenomena.
- In the future: beyond the mean field, beyond the probe limit, close to the CEP in  $T\text{-}\mu$  plane, .....

Thanks for your attentions!