

Spin alignment of vector mesons from quark dynamics in a rotating medium

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Based on:
arxiv:2303.01897
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Collaborators: Mei Huang, Yin Jiang

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- ② QCD Phase diagram
- ③ Mass Spectra and Spin Alignment
- ④ Summary and Outlook
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① Introduction

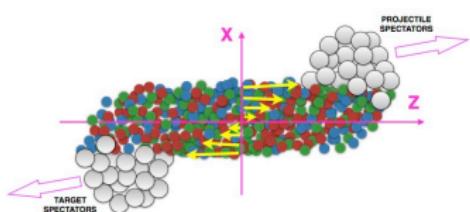
② QCD Phase diagram

③ Mass Spectra and Spin Alignment

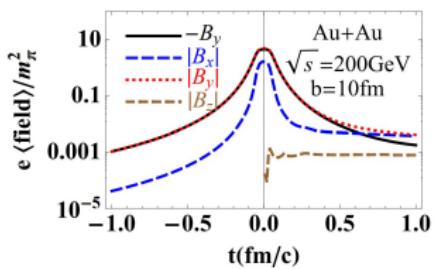
④ Summary and Outlook

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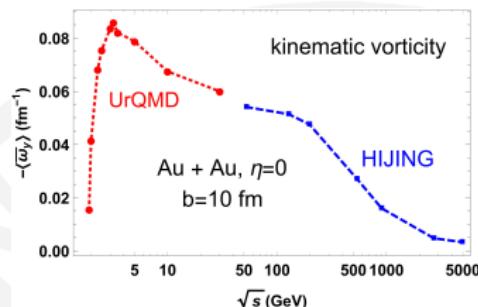
Magnetic Field and Vorticity in Noncentral Collisions



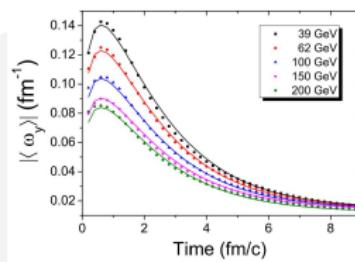
Becattini F, Karpenko I, Lisa M, et al. PRC2017



W.-T. Deng, X.-G. Huang, PRC 85, 044907 (2012)



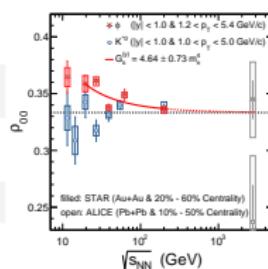
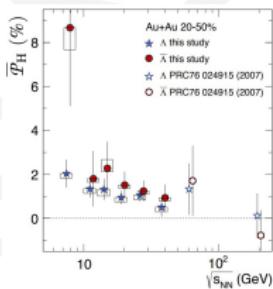
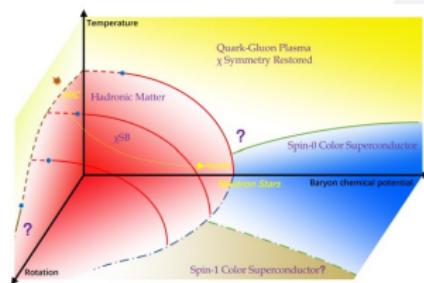
Deng W T, Huang X G, PRC2016;
Deng X G, Huang X G, Ma Y G, PRC2020



Yin Jiang, Jinfeng Liao Ziwei Lin PRC2016

Phenomena with Vorticities

- QCD Phase diagram under rotation[arxiv:2108.00586]
- Spin Polarization[arxiv:1701.06657],
- Spin hydrodynamics[arxiv:1705.00587; 2003.03640,etc];
- Spin alignment[arxiv:1910.14408;2204.02302]



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Methods for QCD Phase diagram under rotation

- NJL model with **spinor connection**[arxiv:1606.03808]

$$\mathcal{L} = \bar{\psi} [i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - m] \psi + G_S \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right] \quad (1)$$

- 3-flavor NJL model with **spinor connection**

$$\begin{aligned} \mathcal{L}_{3\text{NJL}} = & \bar{\psi} [i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - m_f] \psi \\ & + G_S \sum_{a=0}^8 \left[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2 \right] \\ & - G_V \sum_{a=0}^8 \left[(\bar{\psi}\gamma_\mu\lambda^a\psi)^2 + (\bar{\psi}i\gamma_\mu\gamma_5\lambda^a\psi)^2 \right] \\ & - K \left[\det \bar{\psi} (1 + \gamma_5) \psi + \det \bar{\psi} (1 - \gamma_5) \psi \right], \end{aligned} \quad (2)$$

Geometry in rotating frame

- Metric in a co-moving frame

$$g_{\mu\nu} = \eta_{\mu\nu} + \eta_{\mu j} \delta_\nu^0 v_j + \eta_{i\nu} \delta_\mu^0 v_i + \eta_{ij} \delta_\mu^0 \delta_\nu^0 v_i v_j \quad (3)$$

- Dirac equation in a co-moving frame

$$[i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - M_f] \psi = 0. \quad (4)$$

- Spinor connection is $\Gamma_\mu = \frac{1}{4} \times \frac{1}{2} [\gamma^a, \gamma^b] \Gamma_{ab\mu}$, and nonzero term of Spin connection is

$$\begin{aligned} \Gamma_{ij0} &= \frac{1}{2} (\partial_i v_j - \partial_j v_i), \textcolor{blue}{rotation} \\ \Gamma_{i0j} &= \frac{1}{2} (\partial_i v_j + \partial_j v_i), \textcolor{red}{expansion} \\ \Gamma_{0i0} &= -\frac{1}{2} (v_j \partial_i v_j + v_j \partial_j v_i). \textcolor{green}{shear} \end{aligned} \quad (5)$$

- In a uniform rotating frame, i.e. $\vec{v} = \vec{\Omega} \times \vec{x}$, Spinor connection is:

$$\Gamma_{ij0} = \Omega^k \epsilon_{ijk} \quad \Gamma_0 = \frac{1}{8} [\gamma^i, \gamma^j] \Omega^k \epsilon_{ijk} \quad (6)$$

QCD Phase diagram

- Grand potential

$$\Omega_f(r) = \frac{N_c}{8\pi^2} T \sum_n \int dk_t^2 \int dk_z \left[J_n(k_t r)^2 + J_{n+1}(k_t r)^2 \right] \\ \times \left[E_k/T + \ln \left(1 + e^{-(E_k - (n+\frac{1}{2})\Omega)/T} \right) \right. \\ \left. + \ln \left(1 + e^{-(E_k + (n+\frac{1}{2})\Omega)/T} \right) \right]. \quad (7)$$

$$\Omega_{\text{tot}}(r) = \sum_{f=u,d,s} (2G_S\sigma_f^2 - \Omega_f) + 4K\sigma_u\sigma_d\sigma_s. \quad (8)$$

- Gap equations and dynamical quark masses

$$\frac{\partial \Omega_{\text{tot}}}{\partial \sigma_f} = 0, \quad \frac{\partial^2 \Omega_{\text{tot}}}{\partial \sigma_f^2} > 0. \quad (9)$$

$$M_f \equiv m_f - 4G_S\sigma_f + 2K \prod_{f' \neq f} \sigma_{f'}. \quad (10)$$

Chiral condensates and dynamical quark masses

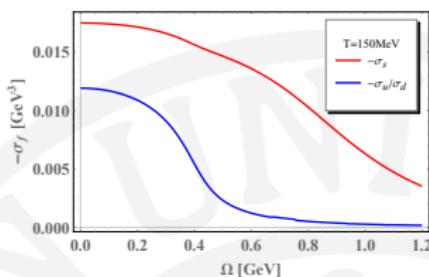
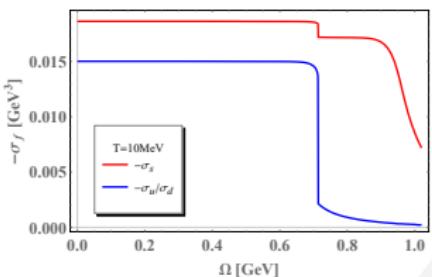


Figure 1: Chiral condensates as functions of angular velocity.

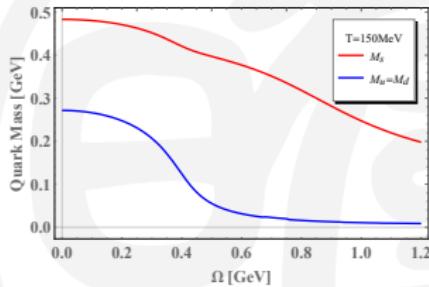
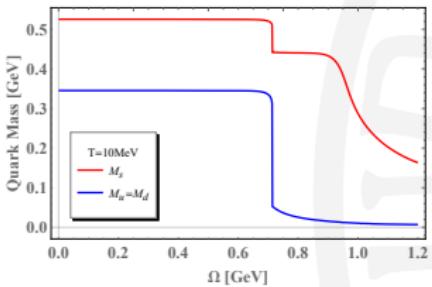


Figure 2: Dynamical quark masses as functions of angular velocity.

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Random Phase Approximation

- RPA

$$\langle \text{---} \rangle \simeq \times + \langle \times \times \rangle + \langle \times \times \times \rangle + \dots = \frac{\times}{1 - \langle \times \rangle}$$

- Pole mass

$$D_\sigma(q^2) = \frac{2G_S}{1 - 2G_S\Pi_s(q^2)}, \quad (11)$$

- Polarization function

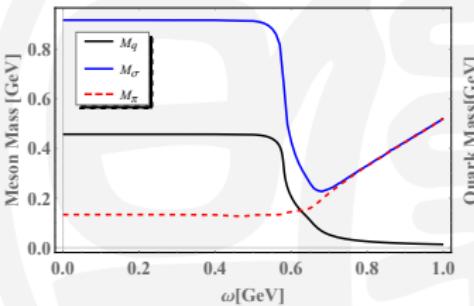
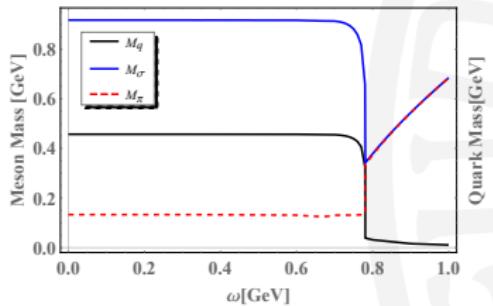
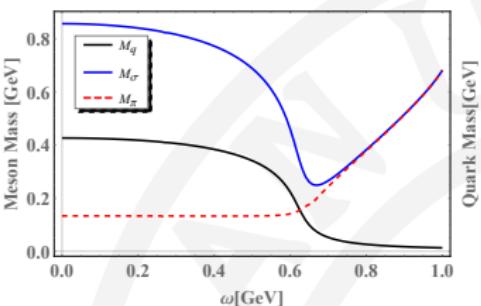
$$\Pi_s(q) = -i \int d^4\tilde{r} Tr_{sfC}[iS(0; \tilde{r})iS(\tilde{r}; 0)]e^{iq \cdot \tilde{r}}, \quad (12)$$

- Quark propagator

$$\begin{aligned} S(\tilde{r}; \tilde{r}') &= \frac{1}{(2\pi)^2} \sum_n \int \frac{dk_0}{2\pi} \int k_t dk_t \int dk_z \frac{e^{in(\theta - \theta')}}{[k_0 + (n + \frac{1}{2})\Omega]^2 - k_t^2 - k_z^2 - M_f^2} e^{-ik_0(t - t') + ik_z(z - z')} \\ &\times \left\{ \left[[k_0 + (n + \frac{1}{2})\Omega + \mu]\gamma^0 - k_z\gamma^3 + M_f \right] \right. \\ &\times \left[J_n(k_t r)J_n(k_t r')\mathcal{P}_+ + e^{i(\theta - \theta')} J_{n+1}(k_t r)J_{n+1}(k_t r')\mathcal{P}_- \right] \\ &- i \gamma^1 k_t e^{i\theta} J_{n+1}(k_t r)J_n(k_t r')\mathcal{P}_+ - \gamma^2 k_t e^{-i\theta'} J_n(k_t r)J_{n+1}(k_t r')\mathcal{P}_- \left. \right\}, \end{aligned} \quad (13)$$

Scalar Meson Mass

- Scalar Meson Mass at $T=150$ MeV, $\mu=100$ MeV, and $\mu=200$ MeV



Vector Meson Mass

- Polarization function

$$\Pi^{\mu\nu,ab}(q) = -i \int d^4\tilde{r} Tr_{sfc}[i\gamma^\mu \tau^a S(0; \tilde{r}) i\gamma^\nu \tau^b S(\tilde{r}; 0)] e^{iq \cdot \tilde{r}}. \quad (14)$$

- Propagator can be decomposed into three spin states

$$\Pi_\rho^{\mu\nu} = A_1^2 P_1^{\mu\nu} + A_2^2 P_2^{\mu\nu} + A_3^2 L^{\mu\nu} + A_4^2 u^\mu u^\nu, \quad (15)$$

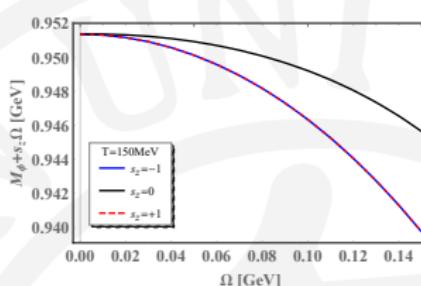
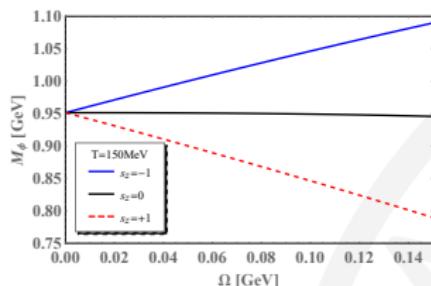
$$D_\rho^{\mu\nu}(q^2) = D_1(q^2) P_1^{\mu\nu} + D_2(q^2) P_2^{\mu\nu} + D_3(q^2) L^{\mu\nu} + D_4(q^2) u^\mu u^\nu, \quad (16)$$

- Pole mass

$$D_i(q^2) = \frac{4G_V}{1 + 4G_V A_i^2}, \quad 1 + 4G_V A_i^2 = 0 \quad (17)$$

Vector Meson Mass

- Mass splitting of ϕ meson at $T = 150$ MeV



- Deviation from the linear relation

$$\begin{aligned}M_\phi(\Omega, s_z = +1) &= 0.95 - 1.00\Omega - 0.54\Omega^2, \\M_\phi(\Omega, s_z = 0) &= 0.95 + 0.01\Omega - 0.31\Omega^2, \\M_\phi(\Omega, s_z = -1) &= 0.95 + 1.00\Omega - 0.54\Omega^2.\end{aligned}\quad (18)$$

Spin Alignment with Thermal Equilibrium

- Meson spectral function[arxiv:2209.01872]

$$\xi_\lambda(k) \equiv \frac{1}{\pi} \operatorname{Im} D_\lambda(k) = \frac{(4G_V)^2 \operatorname{Im} A_\lambda(k)}{\pi \left\{ [1 + 4G_V \operatorname{Re} A_\lambda(k)]^2 + [4G_V \operatorname{Im} A_\lambda(k)]^2 \right\}} \quad (19)$$

- Assuming thermal equilibrium, particle number density for

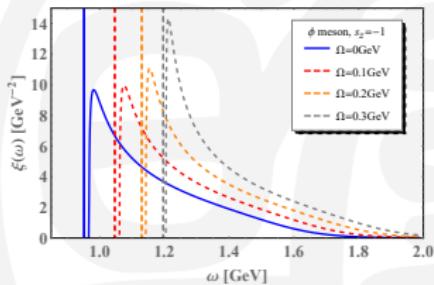
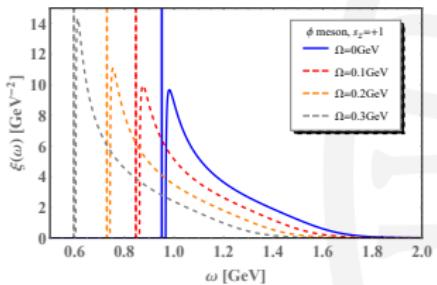
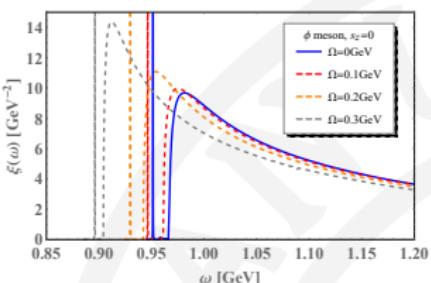
$$f_\lambda = \frac{1}{\exp(M_\lambda/T) - 1} + \int d\omega \frac{2\omega \xi_\lambda^*(\omega)}{\exp(\omega/T) - 1} \quad (20)$$

- Spin alignment

$$\rho_{00} \equiv \frac{f_0}{\sum_{\lambda=0,\pm 1} f_\lambda} \quad (21)$$

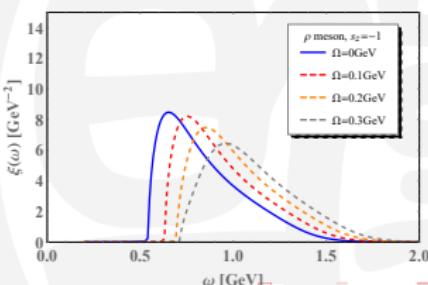
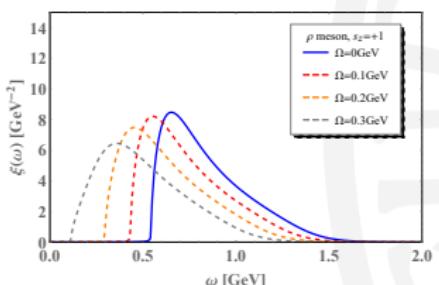
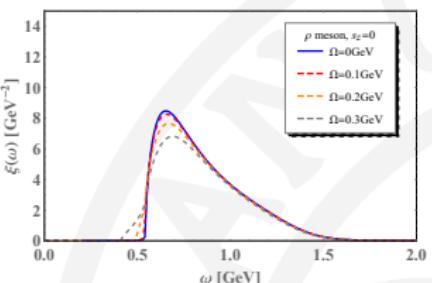
Spectral functions for Vector Meson ϕ

- The spectral function is shifted to the left/right side.



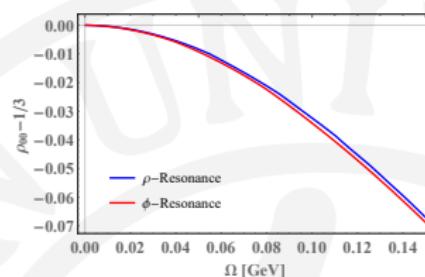
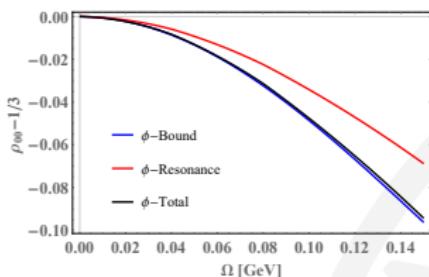
Spectral functions for Vector Meson ρ

- For ρ mesons at the temperature $T = 150$ MeV, spectral functions only have continuum parts and appear as single peaks.



Spin alignment for Vector Meson ϕ

- Spin alignment for Vector Meson ϕ and ρ in 150 MeV



- Comparison with leading order of quark coalescence model[arxiv:1711.06008]

$$\rho_{00}^{\phi,coal} = \frac{1}{3} - \frac{1}{9}(\beta\Omega)^2 \quad (22)$$

$$\rho_{00}^\phi(\Omega) = \frac{1}{3} + 0.0048\Omega - 5.58\Omega^2 + 6.40\Omega^3 + 16.87\Omega^4, \quad (23)$$

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Summary and Outlook

- The rotating angular velocity induces mass splitting of spin components for vector ϕ, ρ mesons, i.e.
$$M_{\phi,\rho}(\Omega) \simeq M_{\phi,\rho}(\Omega = 0) - s_z \Omega;$$
- $\rho_{00} - 1/3$ is negative in the rotating medium. Compared with the quark coalescence model, results from NJL model don't have remarkable improvement currently.
- Spin alignment is a multifactorial phenomenon and an open question.

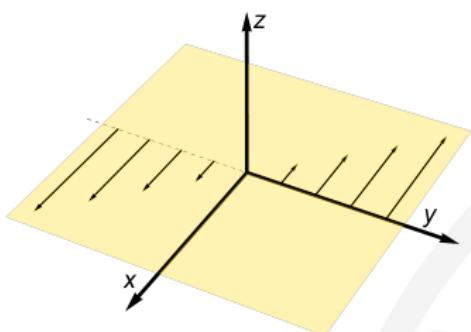
$$\rho_{00} \approx \frac{1}{2} + c_{\text{hydro}} + c_{\text{EM}} + c_F + c_A + c_h + c_{\text{strong}} \quad (24)$$

- Other hydrodynamic gradients(vorticity, expansion, shear tensor) should take into account(see Shuai Liu's talk).

Thanks for your attention!

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An optional profile



$$\begin{aligned} v_1 &= v_x = -\Omega y \\ v_2 &= v_y = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} \Gamma_{120} &= \frac{1}{2} (\partial_x v_y - \partial_y v_x) = \frac{1}{2} \Omega \\ \Gamma_{102} &= -\frac{1}{2} \Omega \quad \Gamma_{201} = -\frac{1}{2} \Omega \\ \Gamma_{010} &= \frac{1}{2} (v_j \partial_x v_j + v_j \partial_j v_x) = \frac{1}{2} v_y \cdot \Omega = 0 \\ \Gamma_{020} &= \frac{1}{2} (v_j \partial_y v_j + v_j \partial_j v_y) = \frac{1}{2} v_x \Omega = -\frac{1}{2} \Omega^2 \end{aligned} \quad (26)$$

Components for $\text{Im}\Pi^{ab}$

$$\begin{aligned} \text{Im}\Pi^{00}(\omega, \vec{q}) = & \frac{1}{8\pi} N_f N_c \sum_{\eta=\pm 1} \int_{p_-}^{p_+} p dp \left\{ \frac{4\omega E_p - 4E_p^2 - M^2}{2qE_p} \right. \\ & \times \left. \left[f(E_p - \mu - \frac{\eta\Omega}{2}) + f(E_p + \mu - \frac{\eta\Omega}{2}) - 1 \right] \right\}, \end{aligned} \quad (27)$$

$$p_{\pm} = \pm \frac{|\vec{q}|}{2} + \frac{\omega}{2} \sqrt{1 - \frac{4M_f^2}{\omega^2 - \vec{q}^2}} \quad (28)$$

Components for $\text{Im}\Pi^{ab}$

$$\begin{aligned}
 & \text{Im}[\Pi^{11}(\omega, \vec{q}) + \Pi^{22}(\omega, \vec{q})] \\
 &= -\frac{1}{2}\pi N_f N_c \sum_{\eta=\pm 1} \int_{p_-^\Omega}^{p_+^\Omega} \frac{pd\mu}{(2\pi)^2} \cdot \frac{1}{|\vec{q}|E_p} \\
 &\quad \times \left\{ 2E_p(\omega - E_p + \eta\Omega) + [(3\frac{q_z^2}{q^2} - 1)(p \cos \theta_1)^2 \right. \\
 &\quad \left. + p^2(1 - \frac{q_z^2}{q^2}) + 2\frac{q_z^2}{q}p \cos \theta_1] + 2M_f^2 \right\} \\
 &\quad \times [1 - f(E_p - \mu - \frac{\eta\Omega}{2}) - f(E_p + \mu - \frac{\eta\Omega}{2})]. \tag{29}
 \end{aligned}$$

$$p_\pm^\Omega = \pm \frac{|\vec{q}|}{2} + \frac{\omega + \eta\Omega}{2} \sqrt{1 - \frac{4M_f^2}{(\omega + \eta\Omega)^2 - \vec{q}^2}} \tag{30}$$

Components for $\text{Im}\Pi^{ab}$

$$\text{Im}\Pi^{33}(\omega, \vec{q})$$

$$\begin{aligned}
&= -\frac{1}{2}\pi N_f N_c \sum_{\eta=\pm 1} \int_{p_-}^{p_+} \frac{p dp}{(2\pi)^2} \cdot \frac{1}{|\vec{q}| E_p} \\
&\times \left\{ E_p (\omega - E_p) + [(1 - 3\frac{q_z^2}{q^2})(p \cos \theta_0)^2 + \frac{p^2 q_z^2}{q^2} + \frac{q_x^2 + q_y^2 - q_z^2}{q} p \cos \theta_0] \right. \\
&\left. + M_f^2 \right\} \times [1 - f(E_p - \mu - \frac{\eta \Omega}{2}) - f(E_p + \mu - \frac{\eta \Omega}{2})]. \tag{31}
\end{aligned}$$

$$p_{\pm} = \pm \frac{|\vec{q}|}{2} + \frac{\omega}{2} \sqrt{1 - \frac{4M_f^2}{\omega^2 - q^2}}, \quad \cos \theta_0 = \frac{\omega^2 - 2\omega \sqrt{p^2 + M_f^2} - q^2}{2|\vec{p}||\vec{q}|}. \tag{32}$$

Other Topics in QCD Phase Diagram with Vorticity and EM Field

- Finite size effect and boundary conditions[arxiv:1606.03808]

$$\int d\theta \bar{\psi} \gamma^r \psi \Big|_{r=R} = 0 \quad (33)$$

- Rotating fermions with background magnetic field[arXiv:1512.08974v2];

$$[i\gamma^\mu (D_\mu + \Gamma_\mu) - m] \psi = 0 \quad (34)$$

- Deconfinement phase transition.[arxiv:2012.04924]

T- μ - Ω Phase diagram

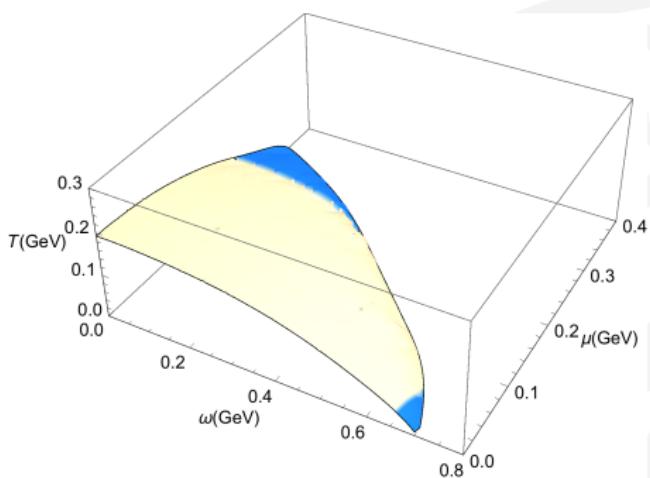


Figure 3: The 3D phase structure for chiral transition on (T, μ, Ω) frame with $G_V = -0.5G_S$.

Expansion

$$\frac{1/(\text{Exp}[(M)/T] - 1)}{1/(\text{Exp}[(M + \Omega)/T] - 1) + 1/(\text{Exp}[(M)/T] - 1) + 1/(\text{Exp}[(M - \Omega)/T] - 1)} \quad (35)$$

The result is:

$$\rho_{00}^\phi = \frac{1}{3} - \frac{\left(e^{M/T} (1 + e^{M/T})\right) \Omega^2}{9 \left((-1 + e^{M/T})^2 T^2\right)} + \frac{e^{M/T} \left(-1 - 7e^{M/T} - 3e^{\frac{2M}{T}} + 3e^{\frac{3M}{T}}\right)}{108 \left(-1 + e^{M/T}\right)^4 T^4} \quad (36)$$