The butterfly effect in a holographic chiral system

Navid Abbasi
Lanzhou University

Motivation

- Quantum chaos is associated with **energy** dynamics (holographic system)

- In chiral systems, **energy** is transported through the CME

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→ Any connection between "Quantum chaos" and "CME" ?

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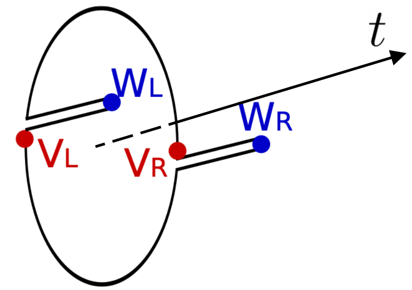
"Butterfly effect"

- This chaotic behavior is referred to as Scrambling.

Quantifying the butterfly effect

Exponential decrease of "Out-of-time-ordered correlators" (OTOC)

OTOC = $F(t) = \langle \Psi | V_L W_R(t) V_R W_L(t) | \Psi \rangle$



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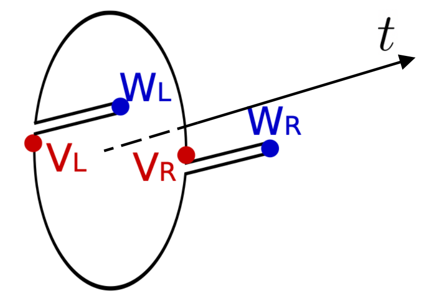

- For large N holographic CFTs

$$F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$$

[Maldacena, Shenker, Stanford 1503.01406]

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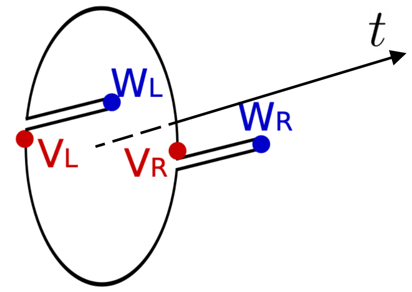

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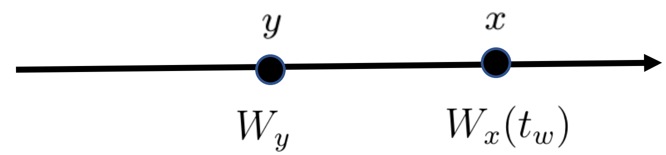
Lyapunov exponent

Butterfly velocity

- OTOC of spatial operators:



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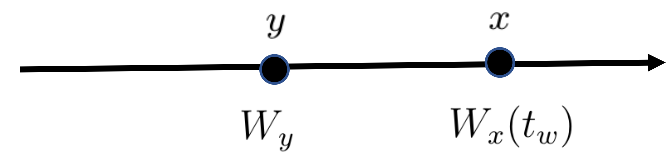

$$\mathrm{Re}\,\langle TFD|\, W_y W_x\,(t_w) W_y W_x(t_w)\,|TFD\rangle \;\sim\; 1 \,-\, \frac{1}{N^2}\, e^{\lambda\left(t_w - \frac{|x-y|}{v_B}\right)}$$

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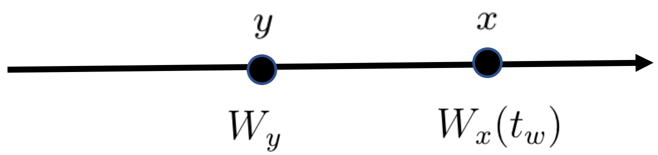

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Butterfly speed

[Roberts, Shenker, Stanford, 1409.8180]

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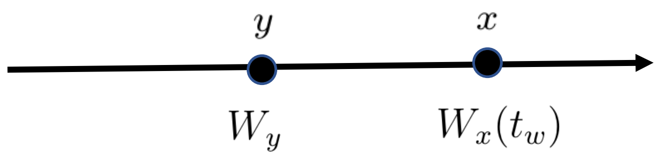

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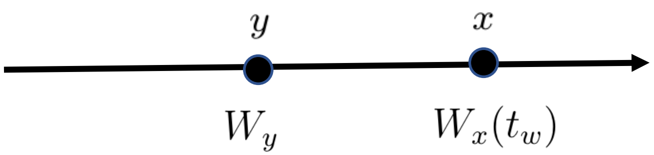

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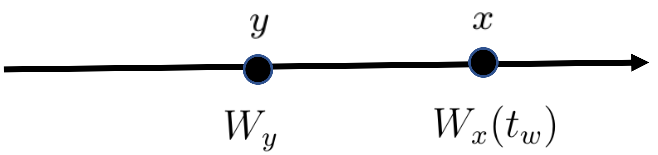

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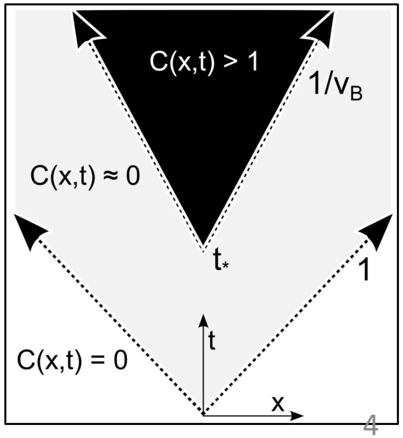
- Butterfly-cone

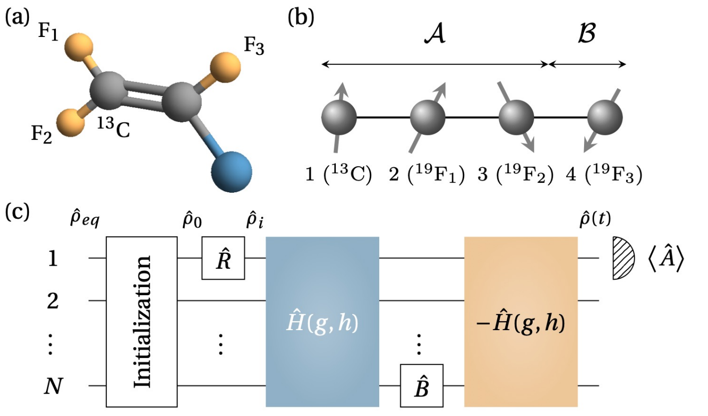


$$C(t_w, |x-y|) = \mathrm{tr}\left\{\rho(\beta)[W_x(t_w), W_y]^\dagger [W_x(t_w), W_y]\right\}$$
$$= 2 - 2\,\mathrm{Re}\,\langle TFD|\, W_y W_x\,(t_w) W_y W_x(t_w)\,|TFD\rangle$$

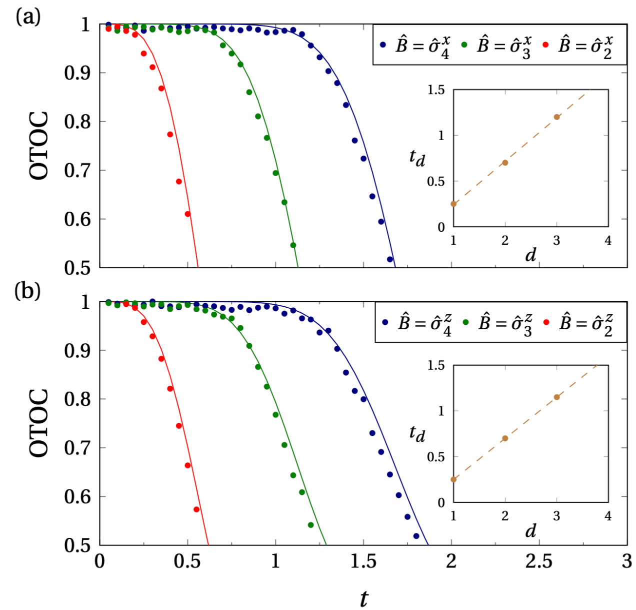
OTOC and $v_{\rm B}$ from experiment

Ising spin chain on a nuclear magnetic resonance (NMR) quantum simulator

[Li, Fan Wang, Ye, Zeng, Zhai, Peng, Du 1609.01246]



$$\hat{H}_{\rm NMR} = -\sum_{i=1}^{4} \frac{\omega_{0i}}{2} \hat{\sigma}_i^z + \sum_{i<j,=1}^{4} \frac{\pi J_{ij}}{2} \hat{\sigma}_i^z \hat{\sigma}_j^z$$



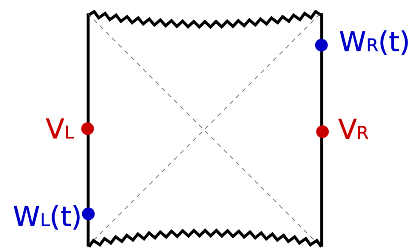
Several other experiments:

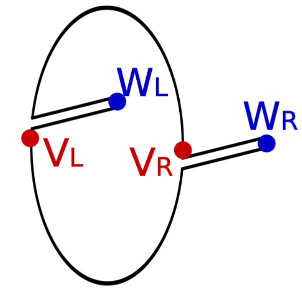
[Garttner, Bohnet, Safavi, Wall, Bollinger, Rey 1608.08938]
[Cao, Zhu, Del Campo 2111.12475]
[Swingle, Bentsen, Schleier-Smith, Hayden 1602.06271]
[...]

OTOC from eternal black hole

[Shenker, Stanford 1412.6987]

$$\langle V_L W_R(t) V_R W_L(t) \rangle$$





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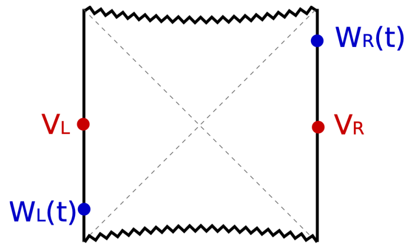

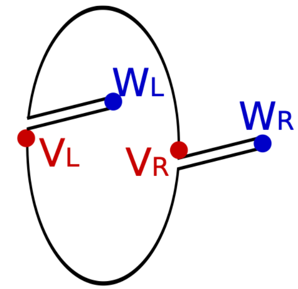

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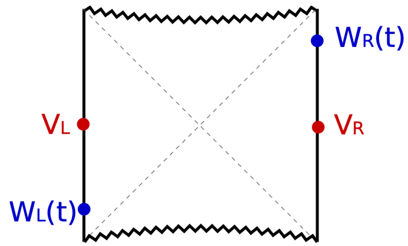

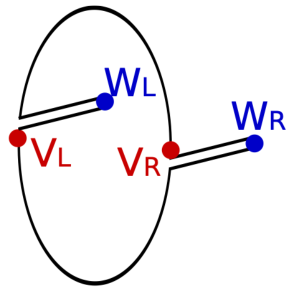

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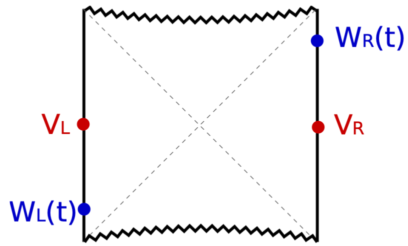

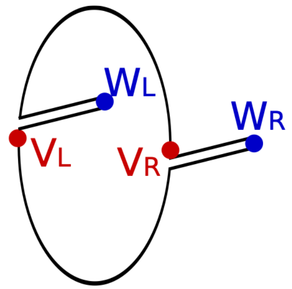

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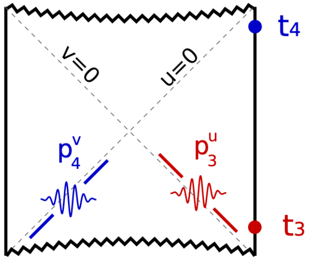
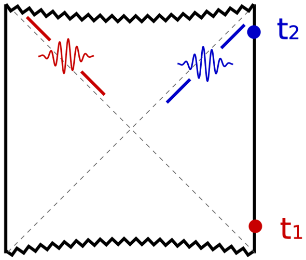


V(t3)W(t4)|TFD⟩



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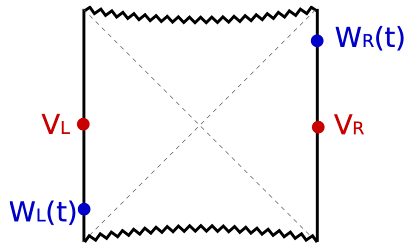

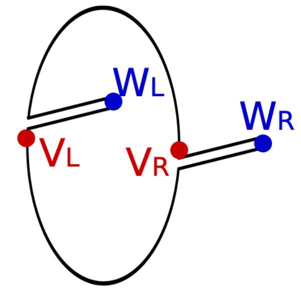

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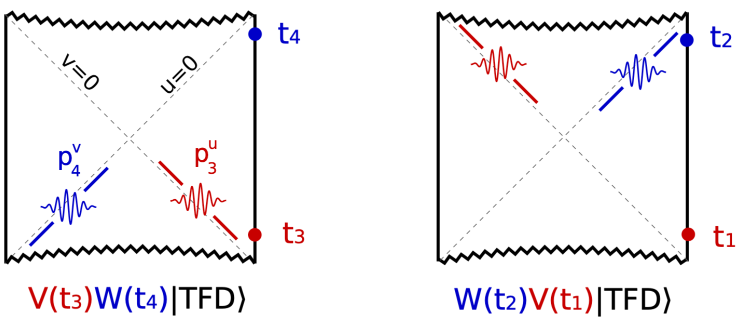

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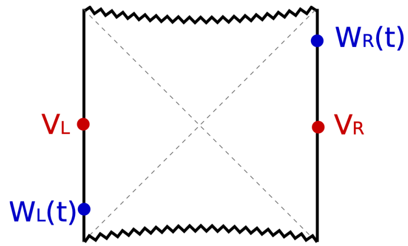

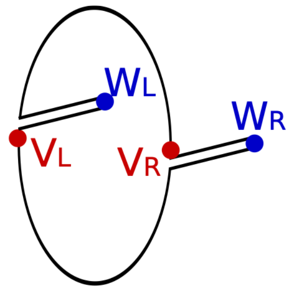

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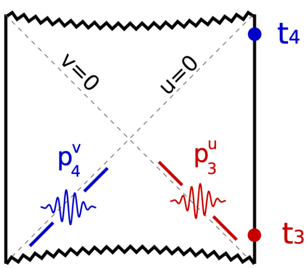

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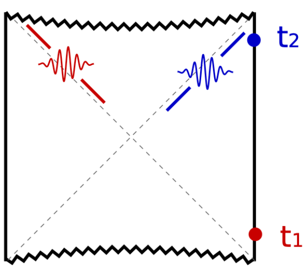

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$$|p_1^u, x_1; p_2^v, x_2\rangle_{out} \approx e^{i\delta(s,b)} |p_1^u, x_1; p_2^v, x_2\rangle_{in} + |\chi\rangle$$

Shock wave geometry in AdS

[Shenker, Stanford 1306.0622]
[Sfetsos 9408169]
[Aichelburg, Sexl 1971]
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Calculating δ is equivalent to see

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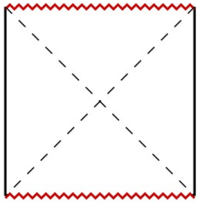

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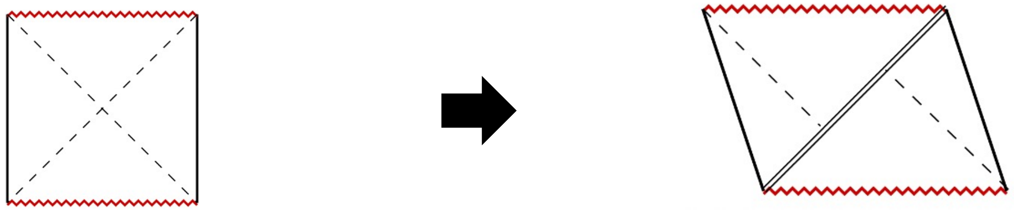

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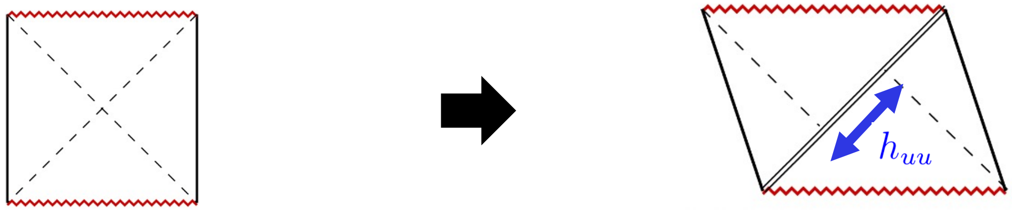

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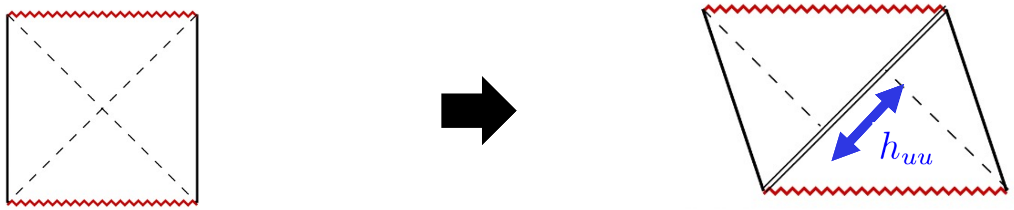

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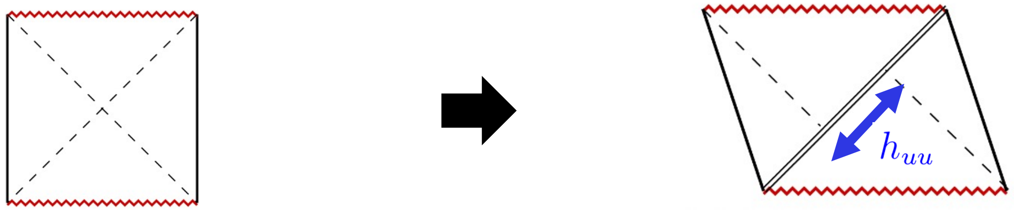

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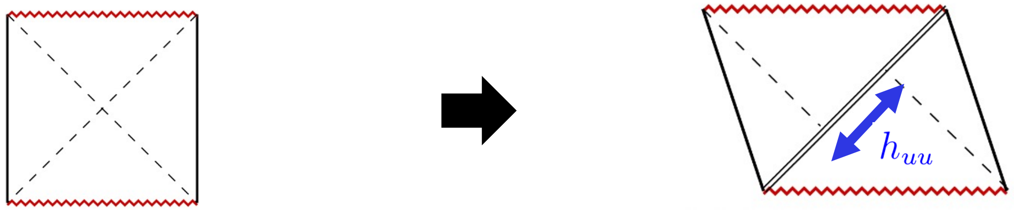

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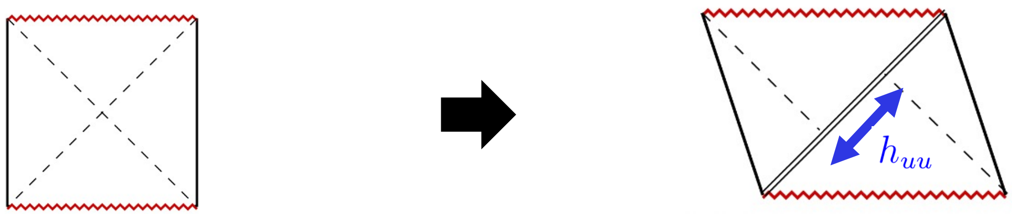

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$$\boxed{\lambda = 2\pi T}$$

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A comment

The butterfly speed obtained from Einstein gravity is isotropic:

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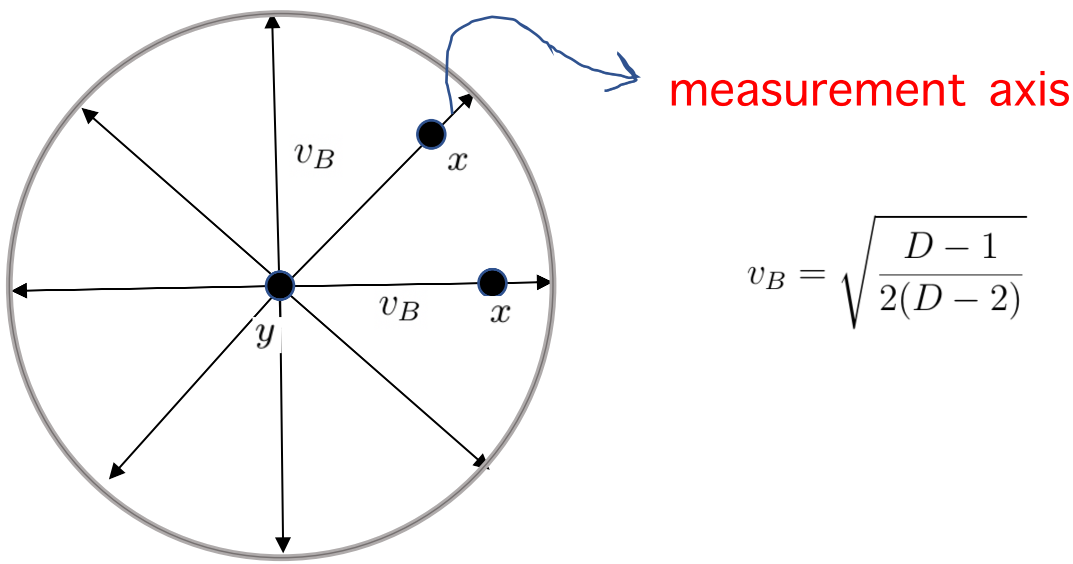

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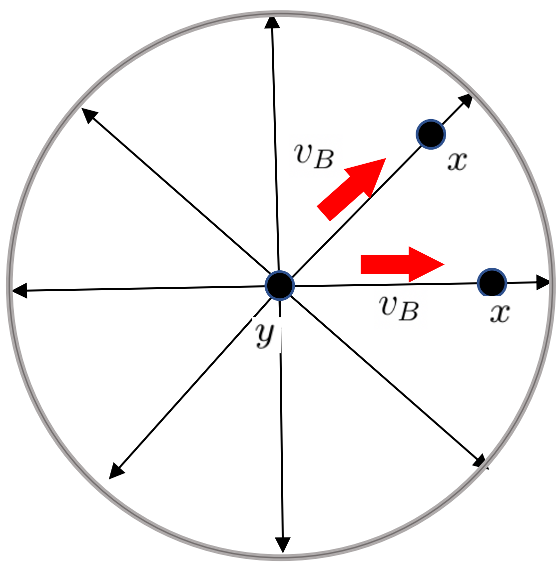

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[Erdmenger, Haack, Kaminski, Yarom 0809.2488]
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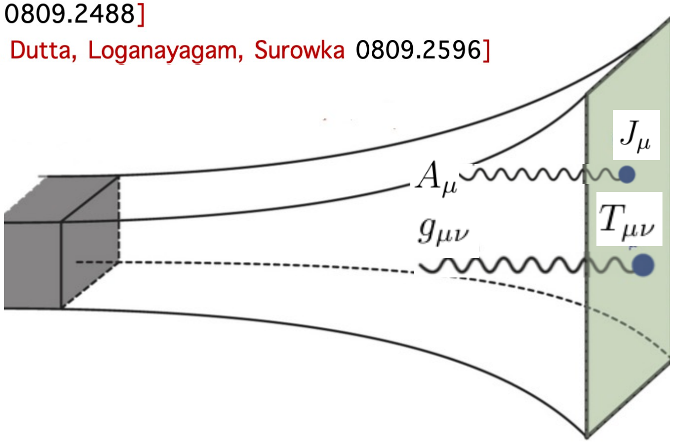

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Our first goal

We want to calculate vB

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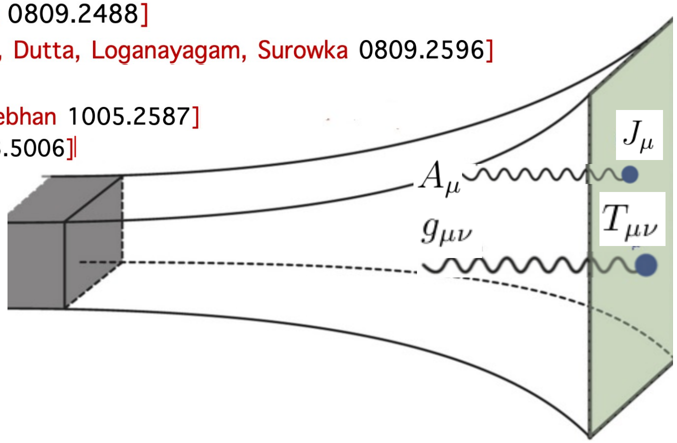

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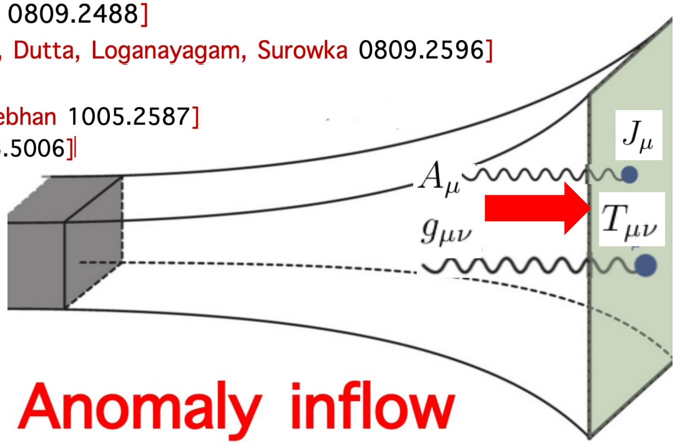

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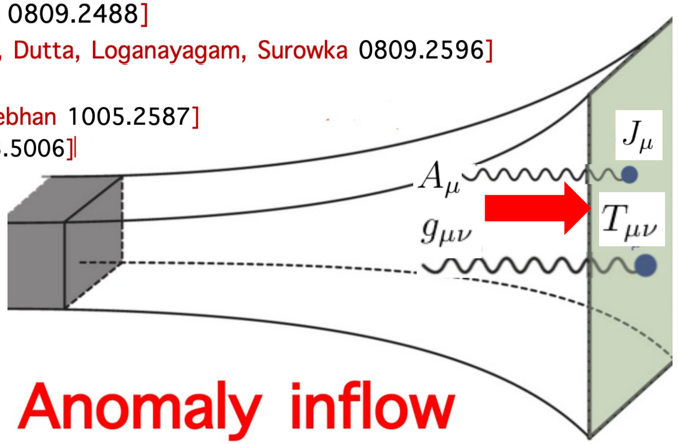

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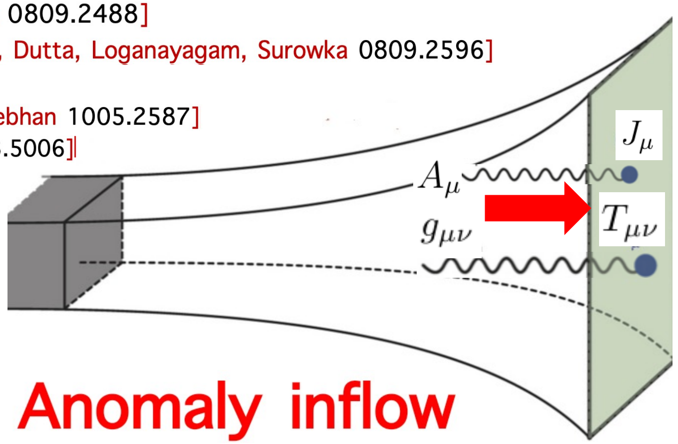

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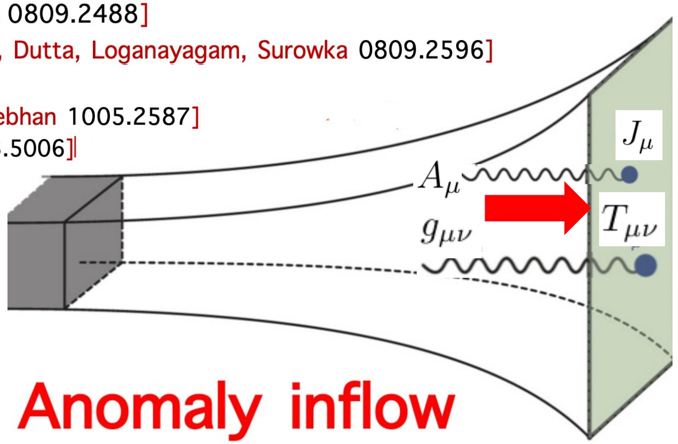

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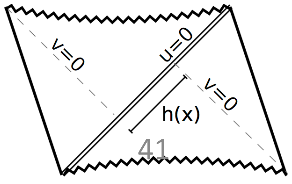
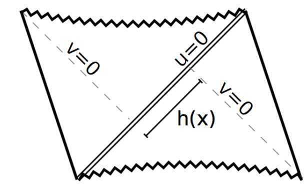
- What we need to find: the function h(x) on this background



A bit technicality

Applying $V \to V + h(x)\delta(U)$



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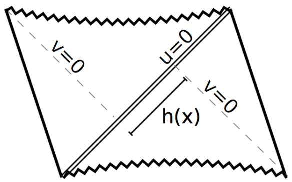

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[NA, Tabatabaei 1910.13696]

$$\mathfrak{p} = (\pi T)\,\kappa\,(\log(4) - 1)\nu^2$$

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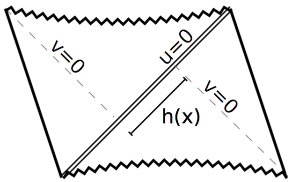

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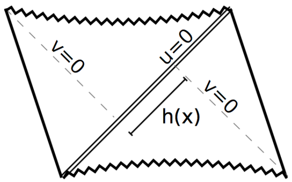

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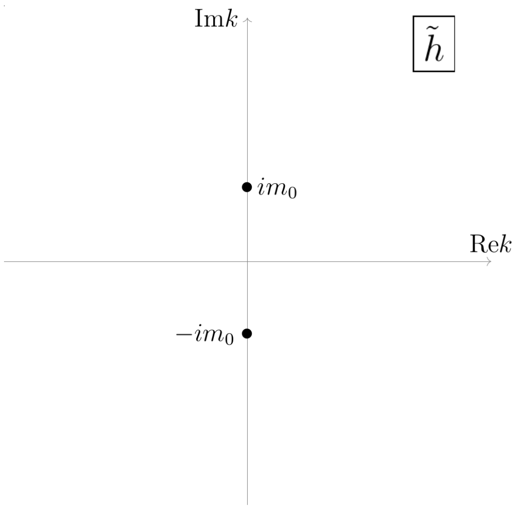

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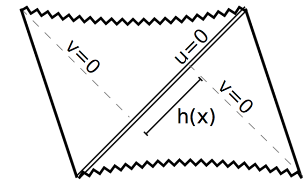

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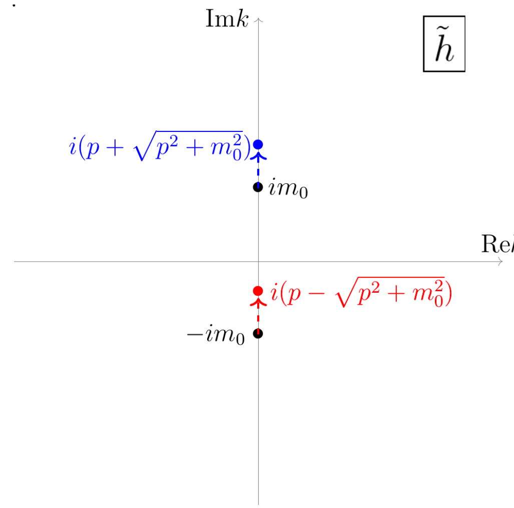

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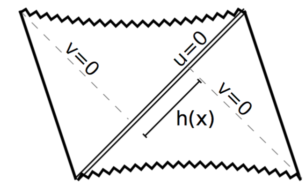

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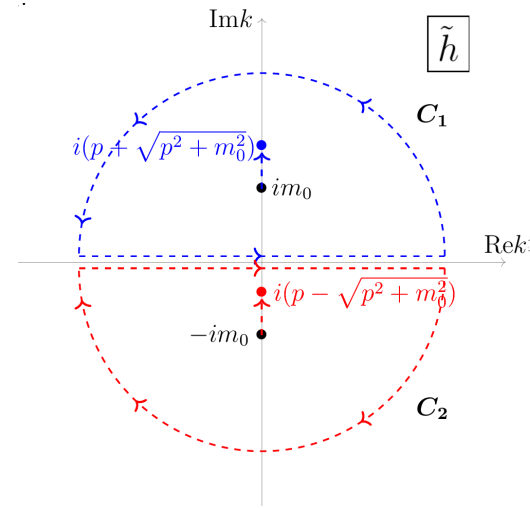
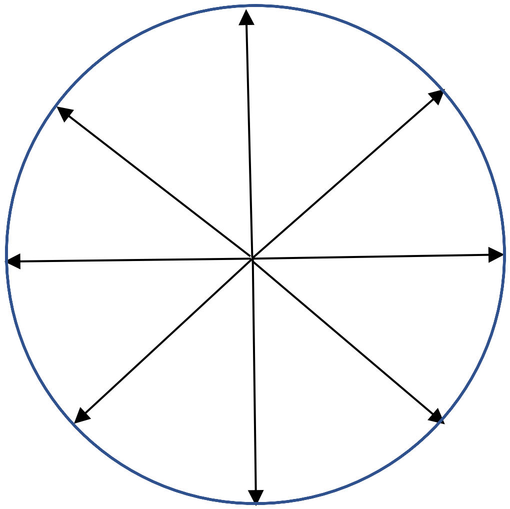


Magnetic field – No anomaly

Isotropic butterfly



$$\vec{B} = \mathbf{0}$$

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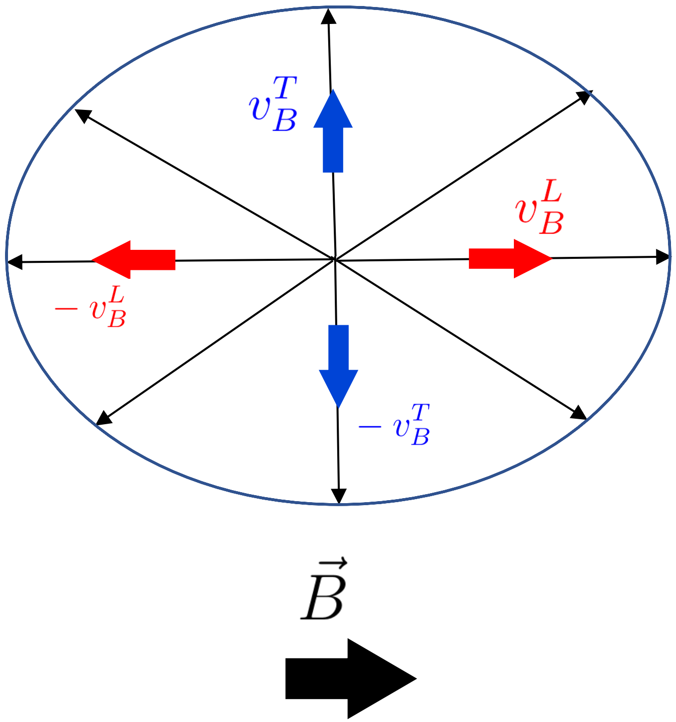

[Blake, Davison, Sachdev 1705.07896]
[Li, Lin, Mei, 1905.07684]

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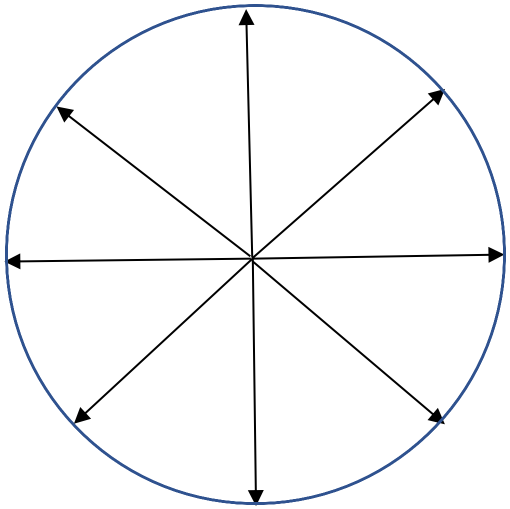

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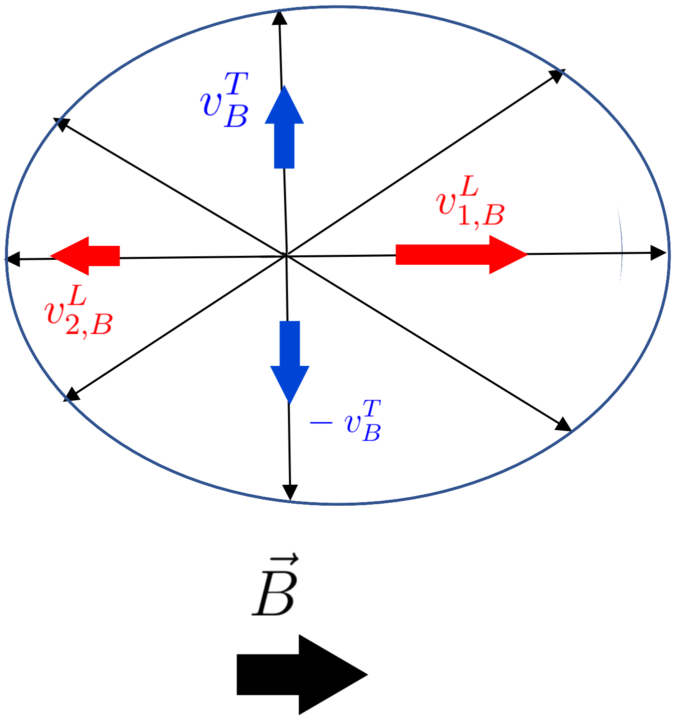

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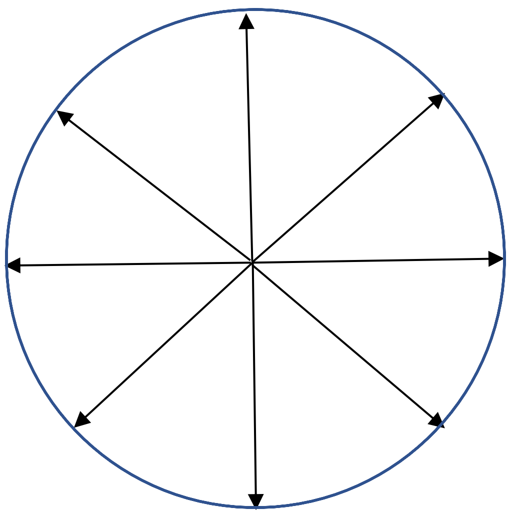

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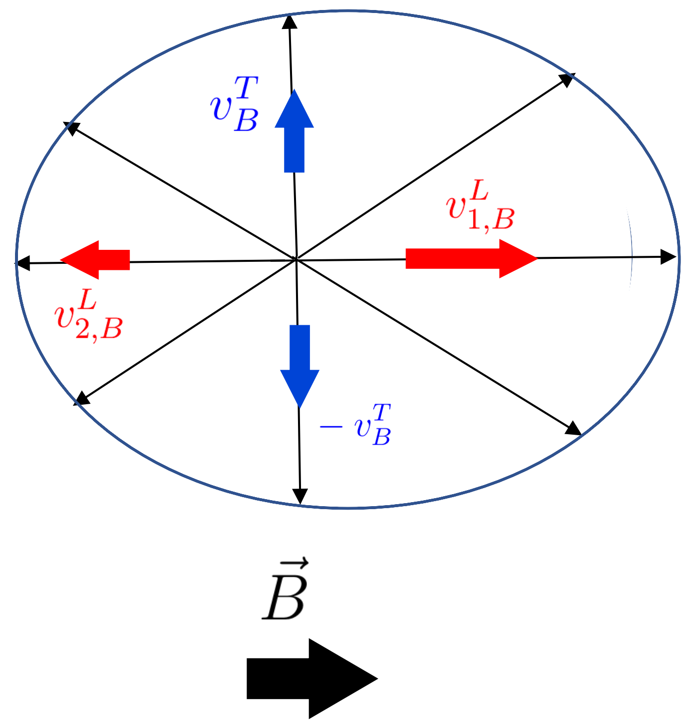

$$\Delta v_B^L \equiv v_{1,B}^L - |v_{2,B}^L| = a\ \kappa \left(\frac{\mu}{T}\right)^2 \left(\frac{B}{T^2}\right)$$

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Generalization to $U(1)_A \times U(1)_V$

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Let's move on to the small T limit

We'll perform high precision **numerical** calculations in the bulk of AdS

Numerical approach

Due to the scale invariance, we choose to work with two new parameters:

$$\hat{B} = \frac{b}{\rho^{2/3}}\,, \quad \hat{T} = \frac{T}{(b^3+\rho^2)^{1/6}}$$

[D'Hoker, Kraus 1003.302]

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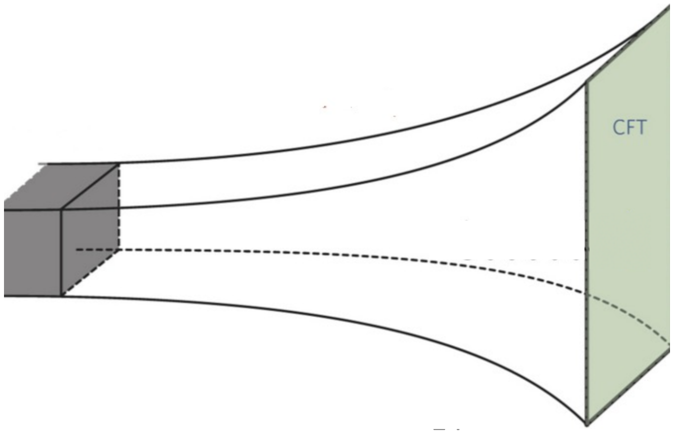
The idea is to analytically find the butterfly velocity in terms of horizon data

and then

translate it into the boundary data



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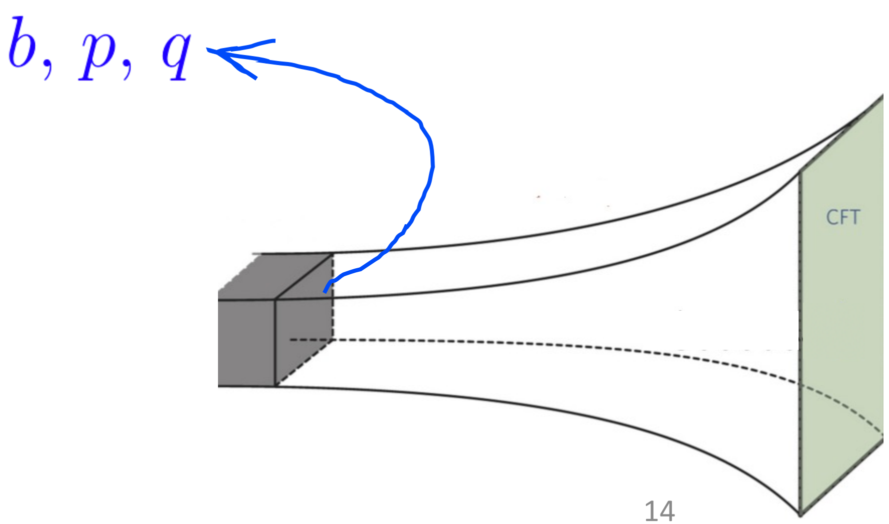

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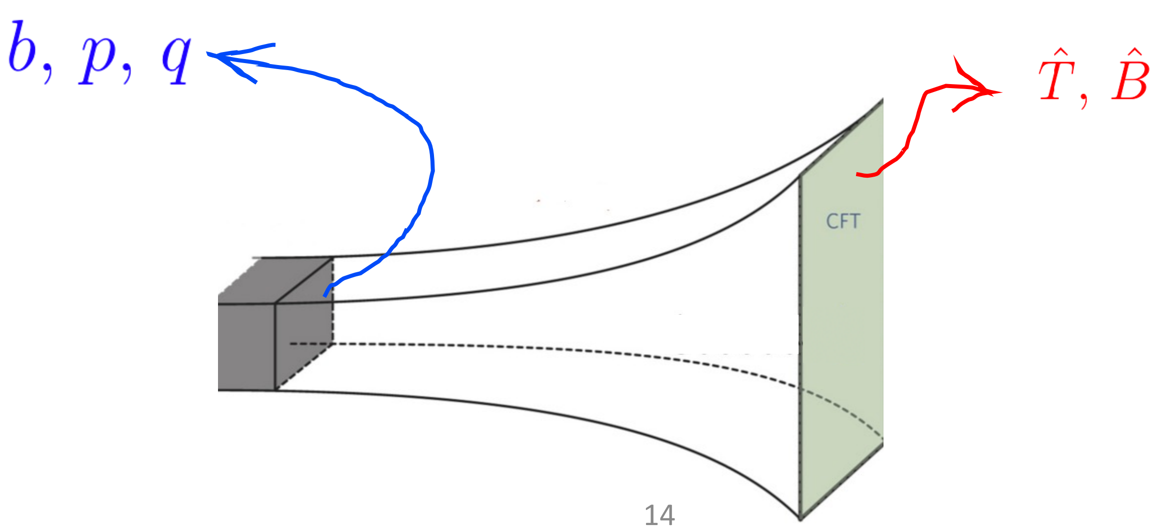
The idea is to analytically find the butterfly velocity in terms of horizon data

and then

translate it into the boundary data



Numerical approach

Due to the scale invariance, we choose to work with two new parameters:

$$\hat{B} = \frac{b}{\rho^{2/3}}\,, \qquad \hat{T} = \frac{T}{(b^3+\rho^2)^{1/6}}$$

[D'Hoker, Kraus 1003.302]

The idea is to analytically find the butterfly velocity in terms of horizon data

and then

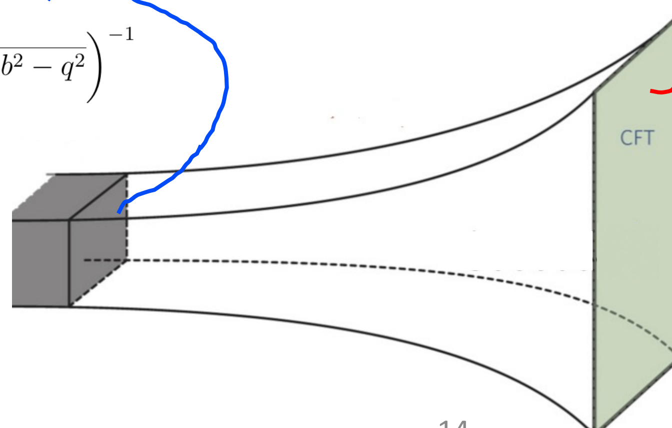
translate it into the boundary data

$b,\ p,\ q$

$$v_B^{(H)} = \frac{\omega_p}{k_p} = \left(\frac{p}{q} + 2\kappa b \pm 2\sqrt{6-b^2-q^2}\right)^{-1}$$

$\hat{T},\ \hat{B}$

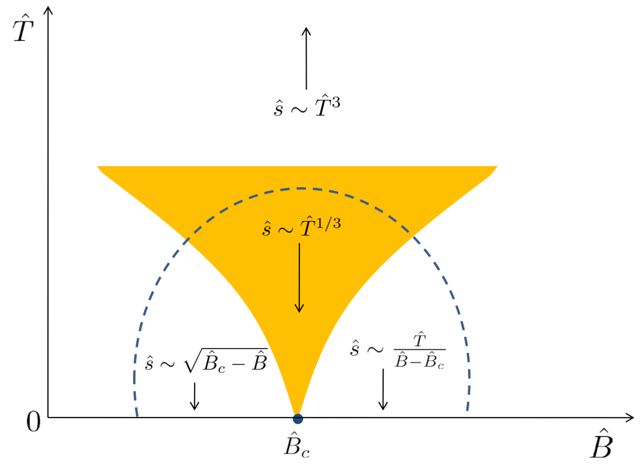
$$v_B^L = \frac{\sqrt{w_L}}{\frac{p}{q} + 2\kappa b \pm 2\sqrt{6-b^2-q^2}}$$



Numerical "known" results

D'Hoker and Kraus show that this has a phase transition at T=0

[D'Hoker, Kraus 1003.302]



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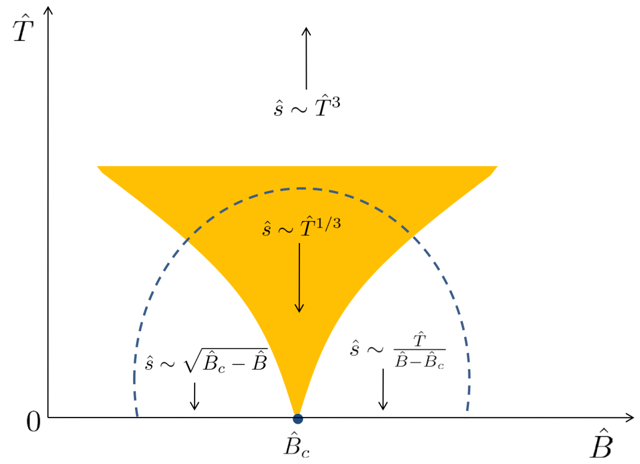

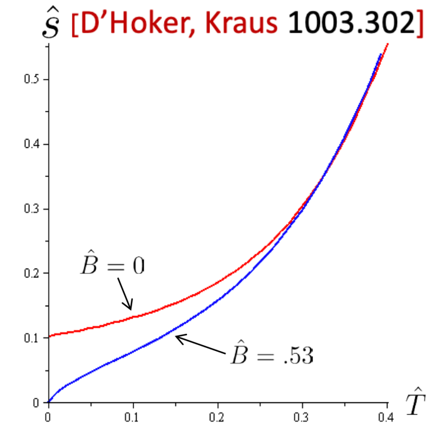

- There is no change in symmetry!
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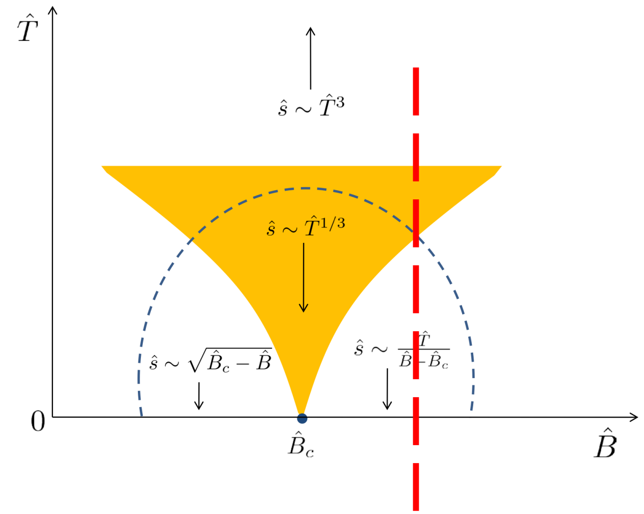
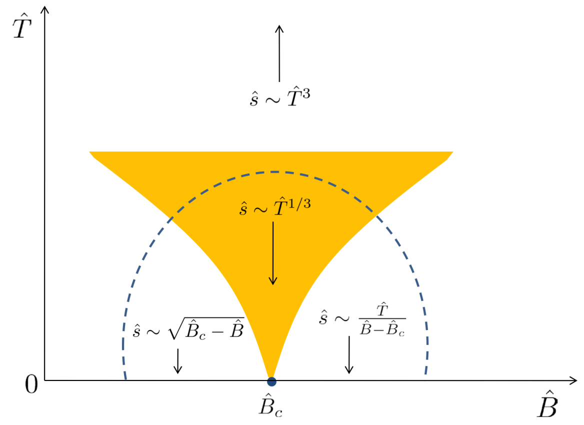


[D’Hoker, Kraus 1003.302]

- There is no change in symmetry!
- The non-analytic behavior of quantities change
- At the critical point: $B_c \approx 0.499$
- What we want to do is to move on vertical cuts towards $T = 0$, and read off v_B^L

Our numerical results

We find



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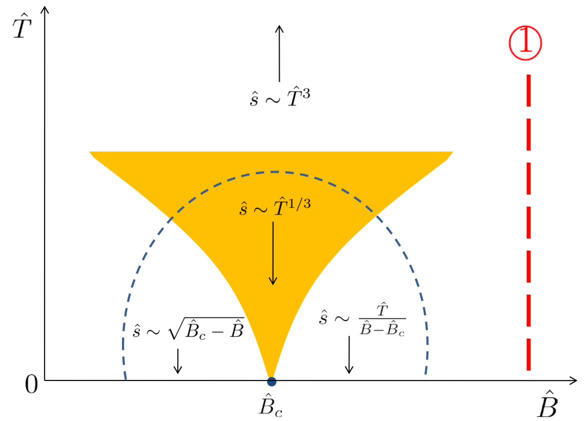

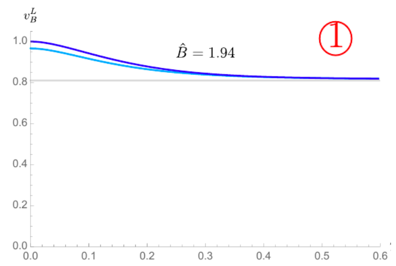

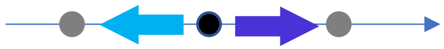

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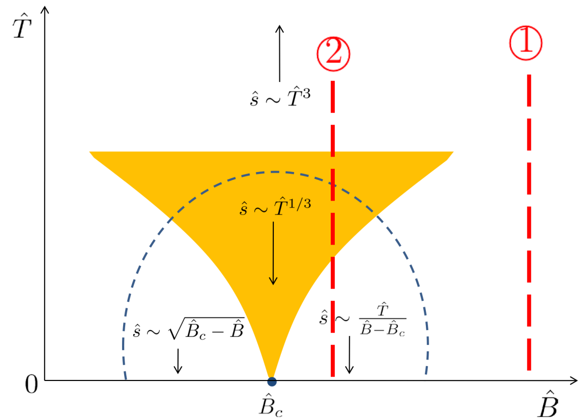

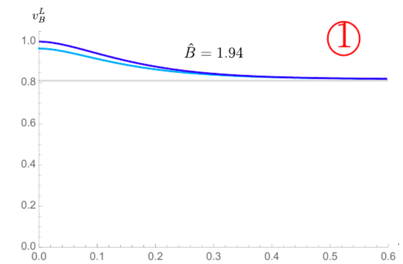

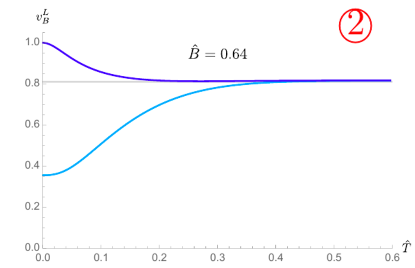

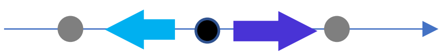

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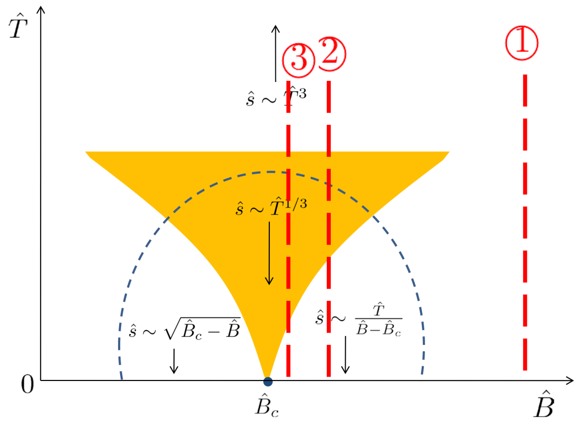

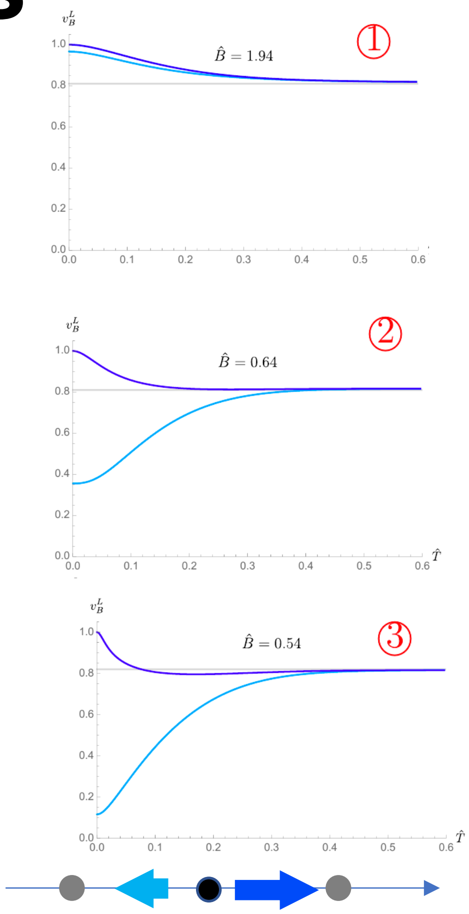

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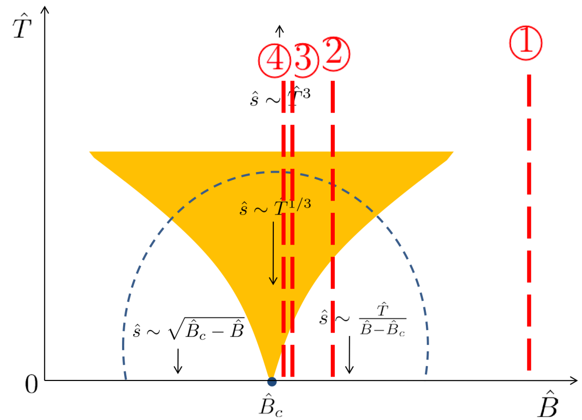

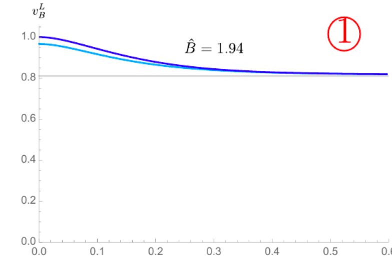

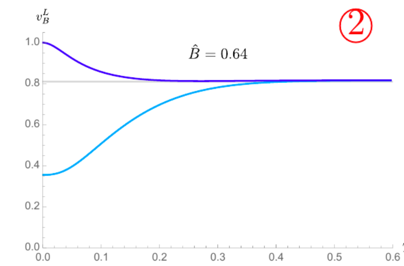

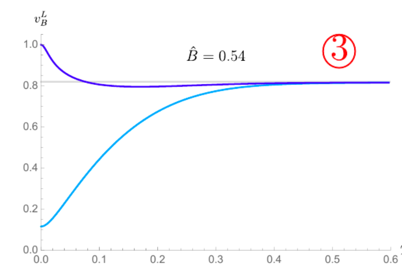

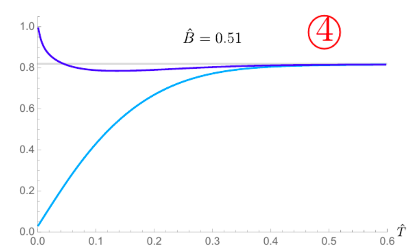

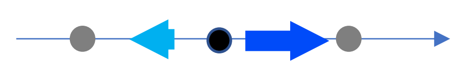

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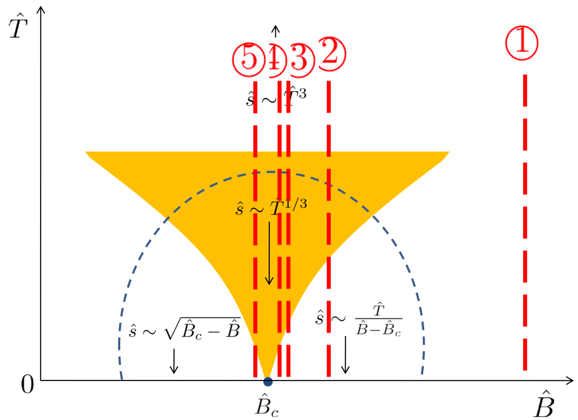

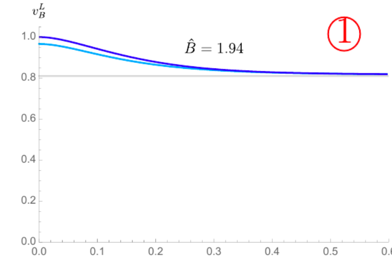

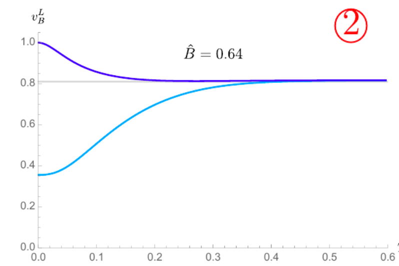

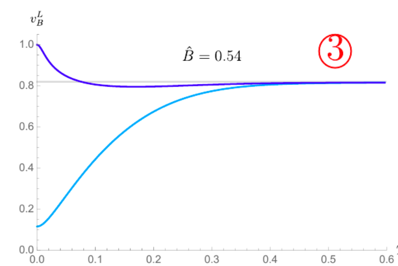

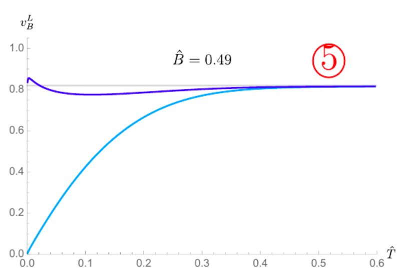

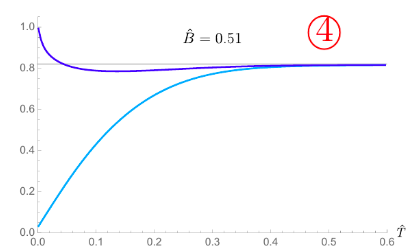

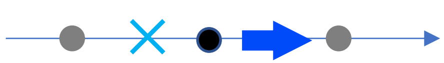

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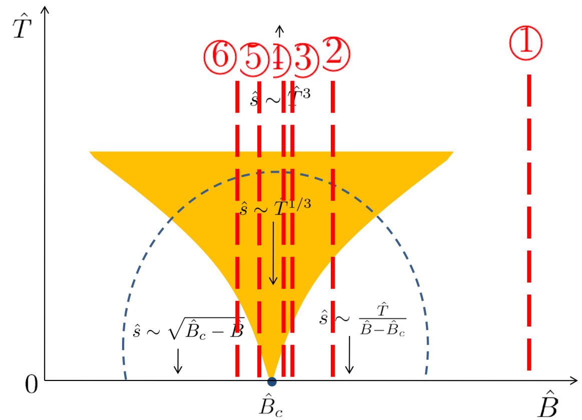

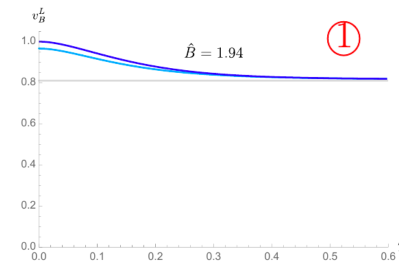

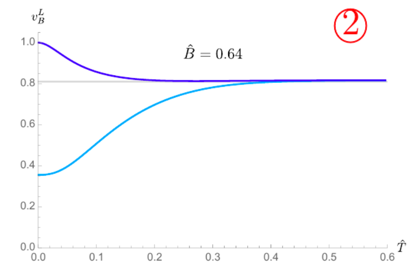

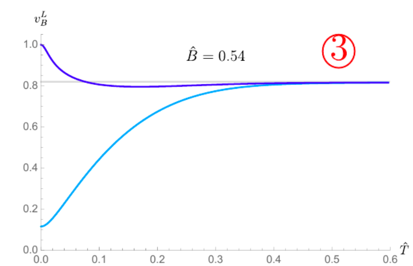

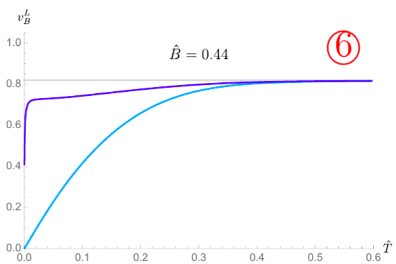

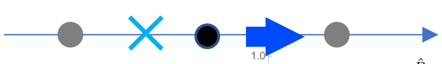

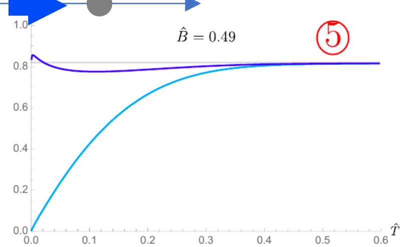

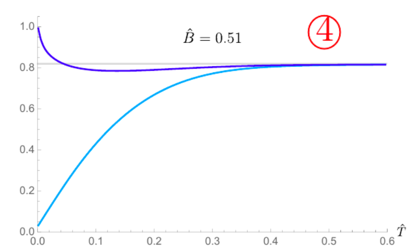

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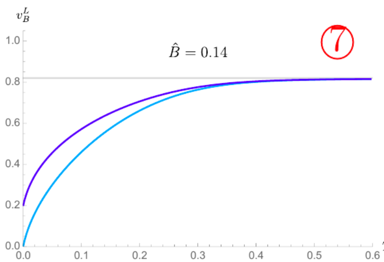

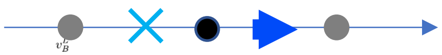

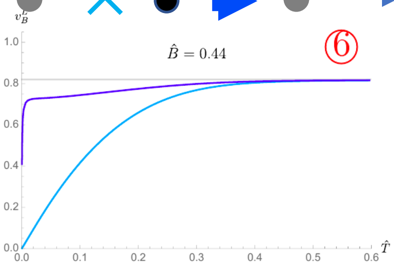

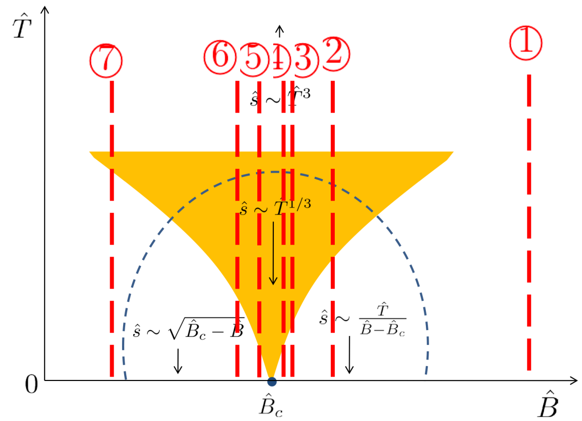

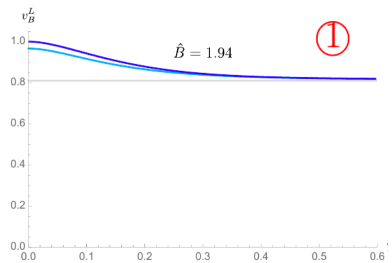

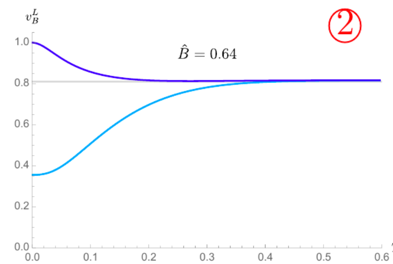

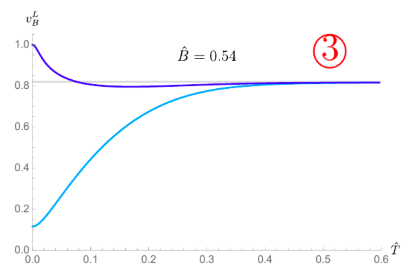

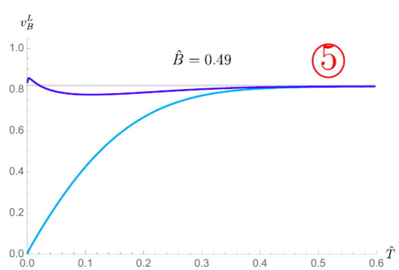

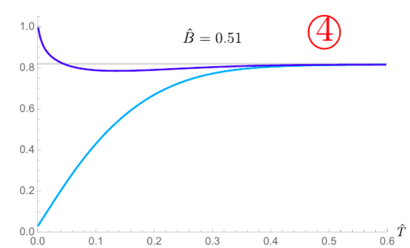

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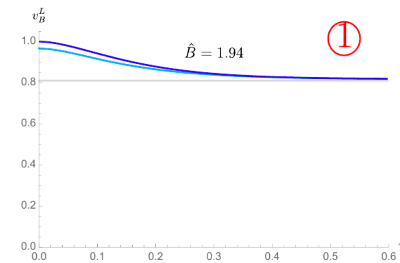

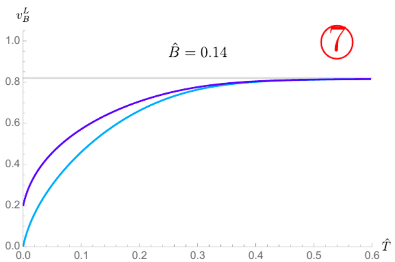

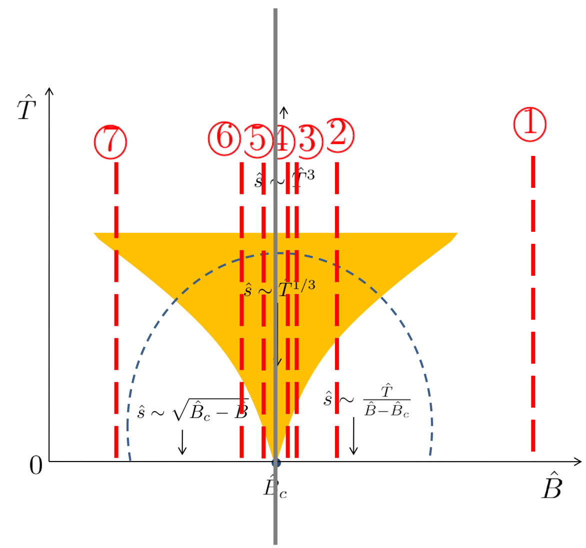

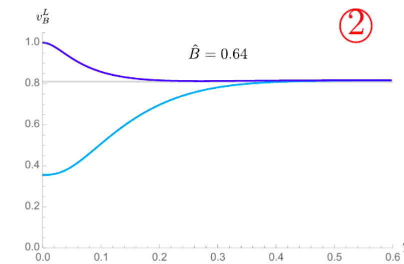

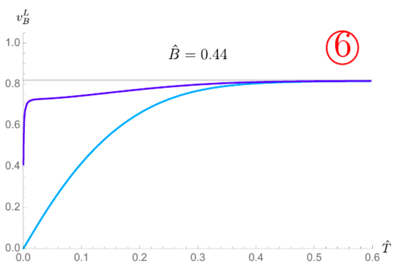

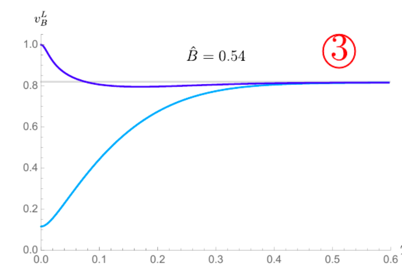

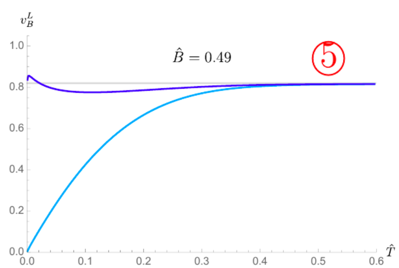

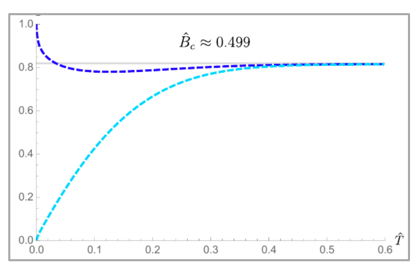

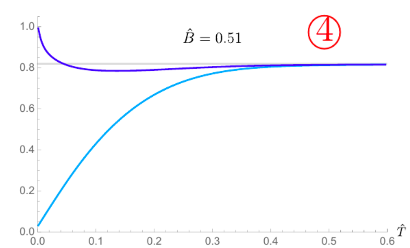

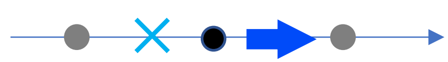

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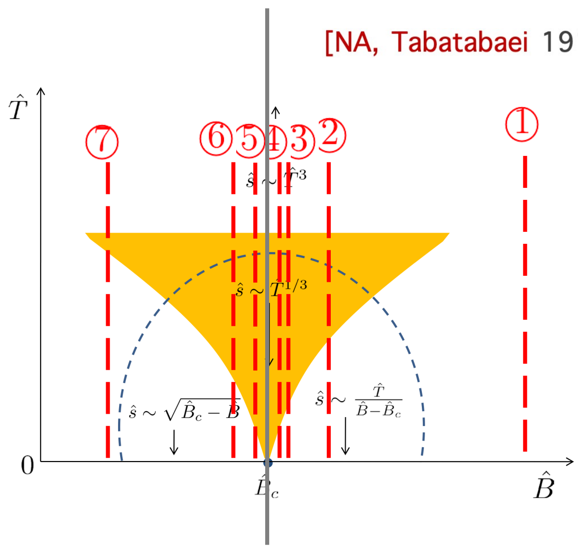
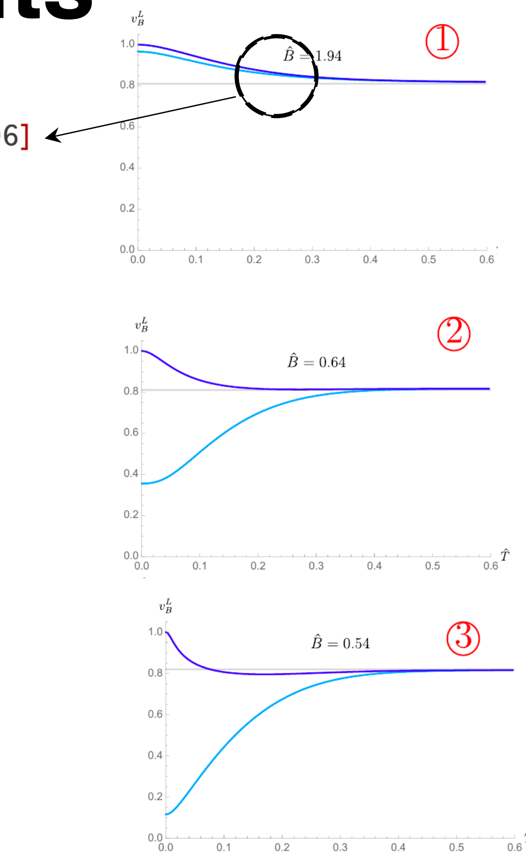
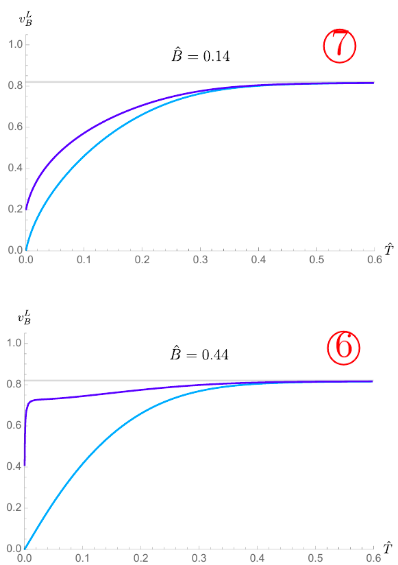
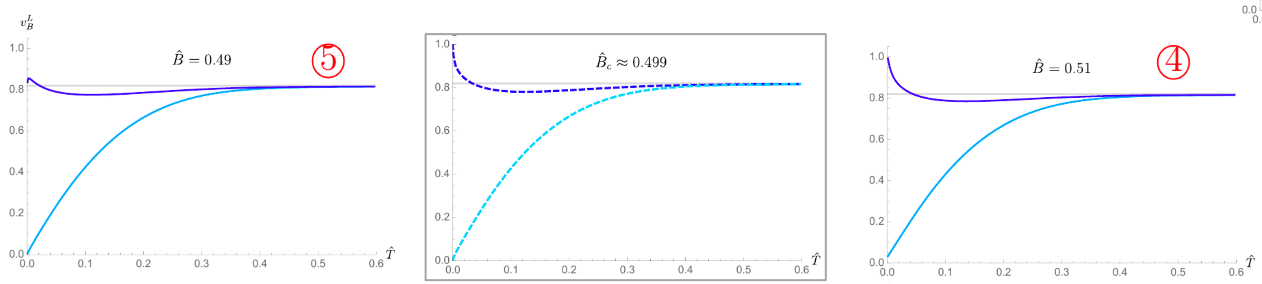




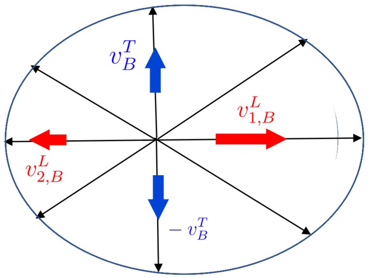




[in preparation]

Identification of QCP by butterfly

At finite T:



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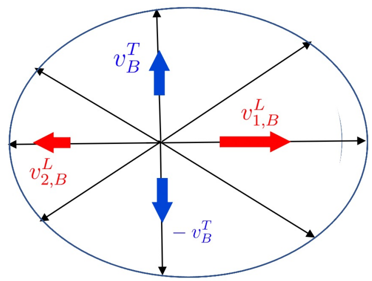

At T=0:

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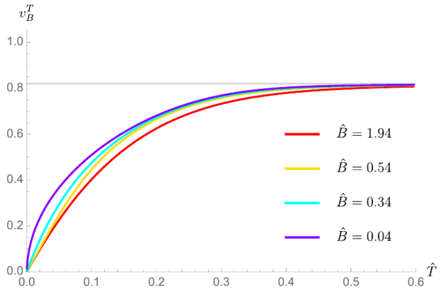

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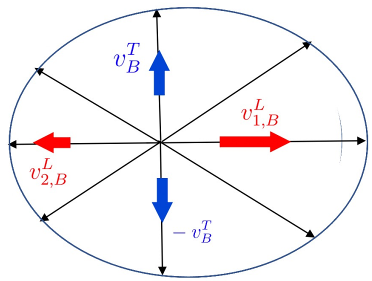

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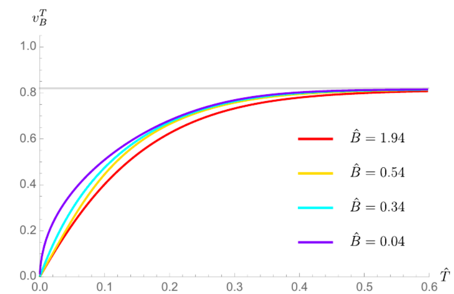
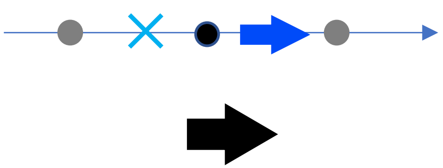


- We may define the QCP by the two following specific butterfly speeds:

$v_{2,B}^L = 0 \quad v_{1,B}^L = 1$



$$\hat{T} = 0, \quad \hat{B} = \hat{B}_c$$

Towards Pole-skipping at T=0

So far we perturbed the theory by a tensor operator with the scaling dimension $\Delta = 4$.

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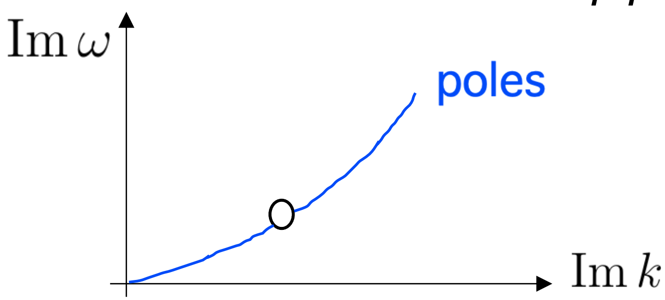
[Grozdanov, Schalm, Scopelliti 1710.00921]



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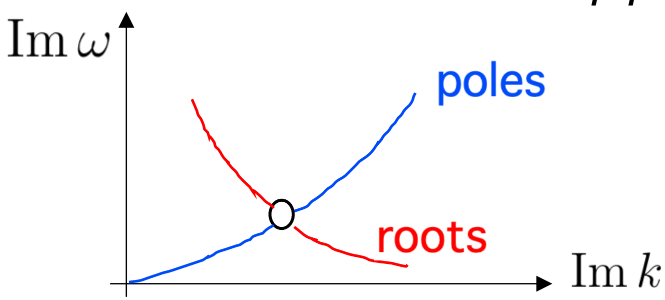

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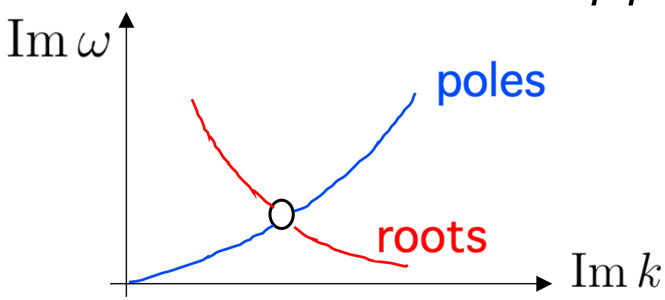
Towards Pole-skipping at T=0

So far we perturbed the theory by a tensor operator with the scaling dimension $\Delta = 4$.

- Now (for technical issues) we proceed with a scalar operator $\psi(t, x)$, with the same dimension.

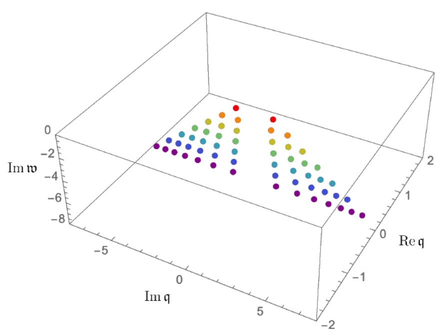
- We calculate the pole-skipping point of $G^R_{\psi\psi}(\omega, k)$:

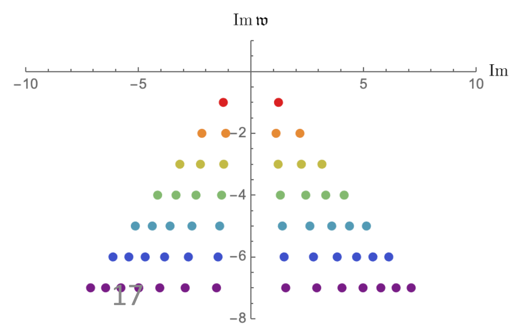


[Grozdanov, Schalm, Scopelliti 1710.00921]

- Previous studies (far away any QCP)

[Blake, Lee, Liu 1801.00010]
[Blake, Davison, Grozdanov, Liu 1809.01169]
[Yuan, Ge, Kim, Ji, Ahn, 2303.04801]
[Wang, Wang, 2208.01047]



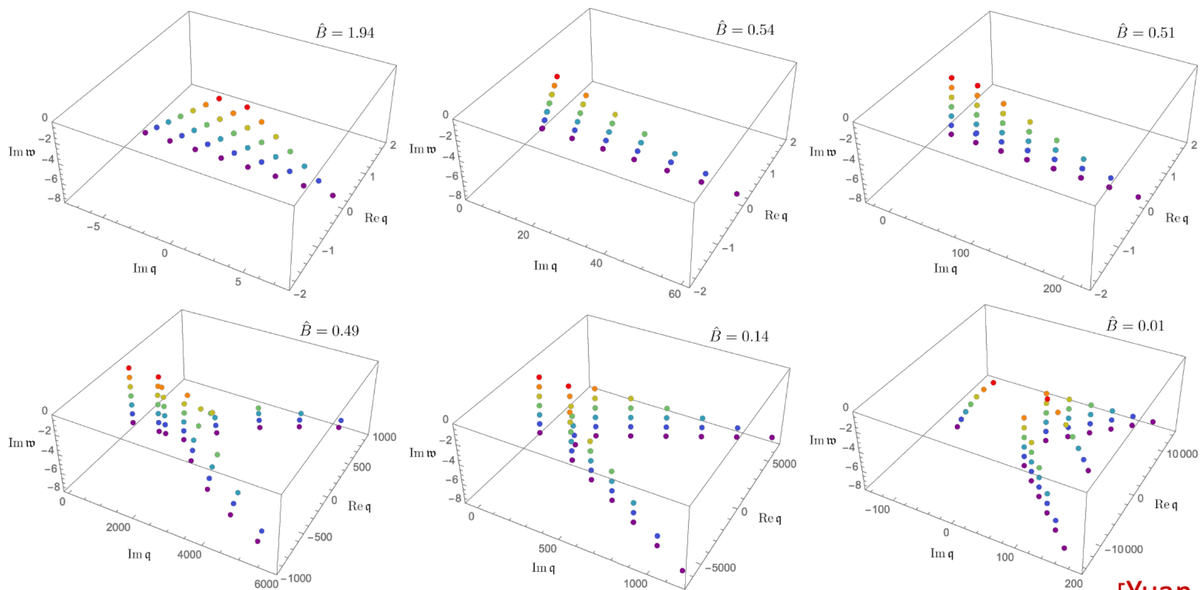


$$\mathfrak{w} = \frac{\omega}{2\pi T}$$

What we find:

Near a quantum critical point:

[in preparation]



[Yuan, Ge, 2012.15396]

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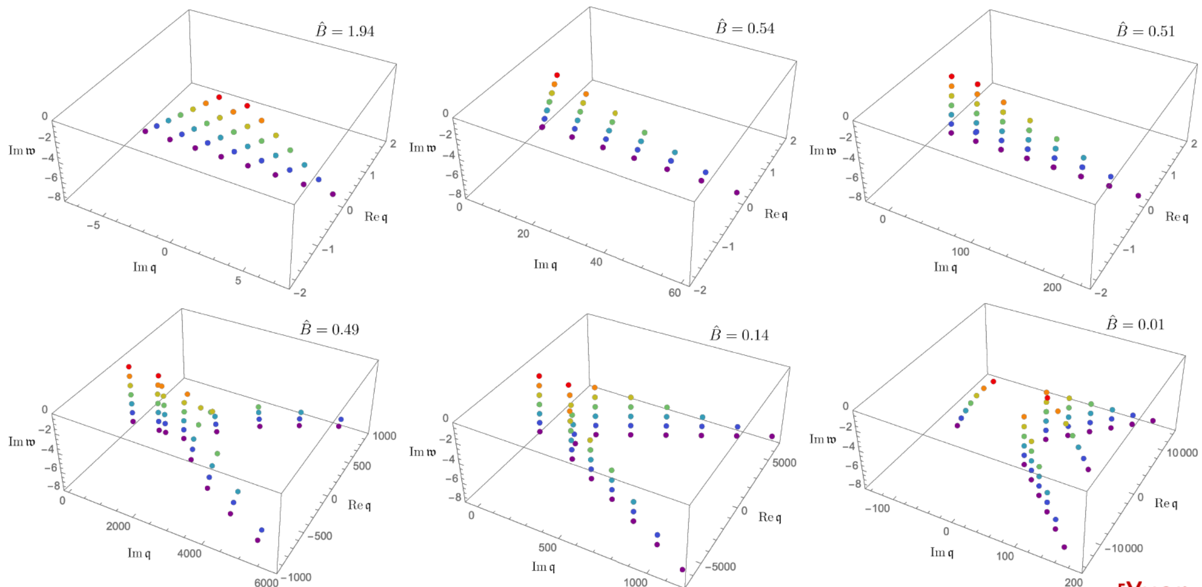

[Yuan, Ge, 2012.15396]

This suggests to take the following quantity as the order parameter:

$$\boxed{\text{order parameter} = \sum_{n,j} |\mathrm{Re}\,\mathfrak{q}^*_{n,j}| \qquad j = 1, \cdots, n}$$

[in preparation]

Discussion

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[Rost, Perry, Mercure, Mackenzie Science Vol. 325. no. 5946 2009]

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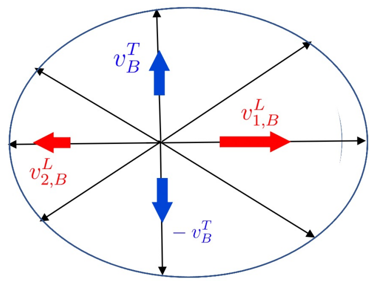
[Rost, Perry, Mercure, Mackenzie Science Vol. 325. no. 5946 2009]

➢ Measuring butterfly speed in this (or in a similar) system

Thank you for your attention

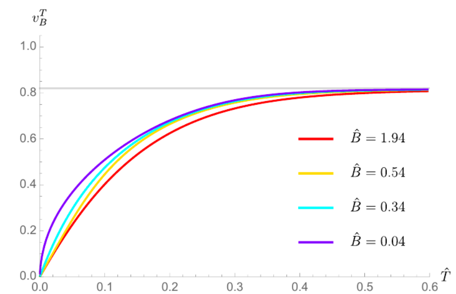
Identification of QCP by butterfly

At finite T:



At T=0:

- There is no transverse propagation



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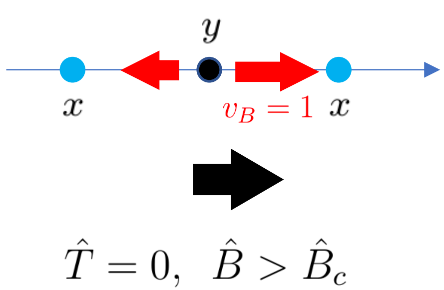

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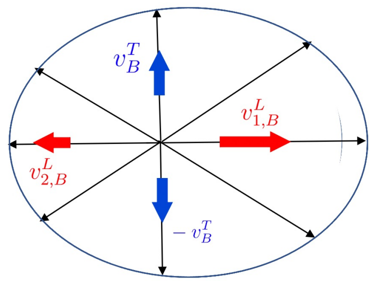

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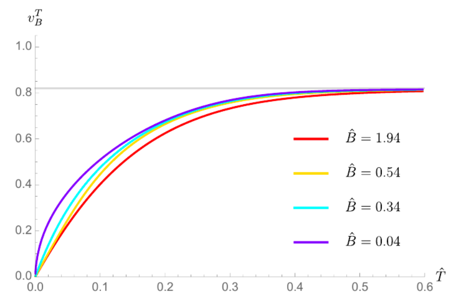

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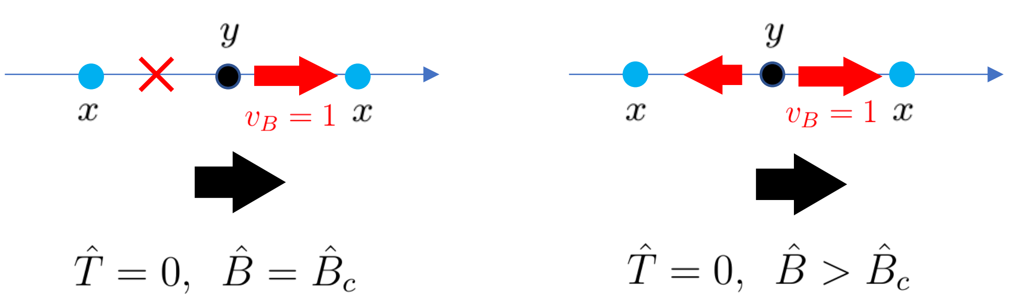

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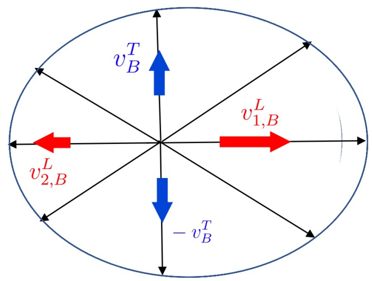

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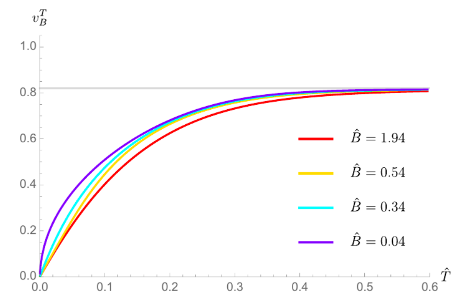

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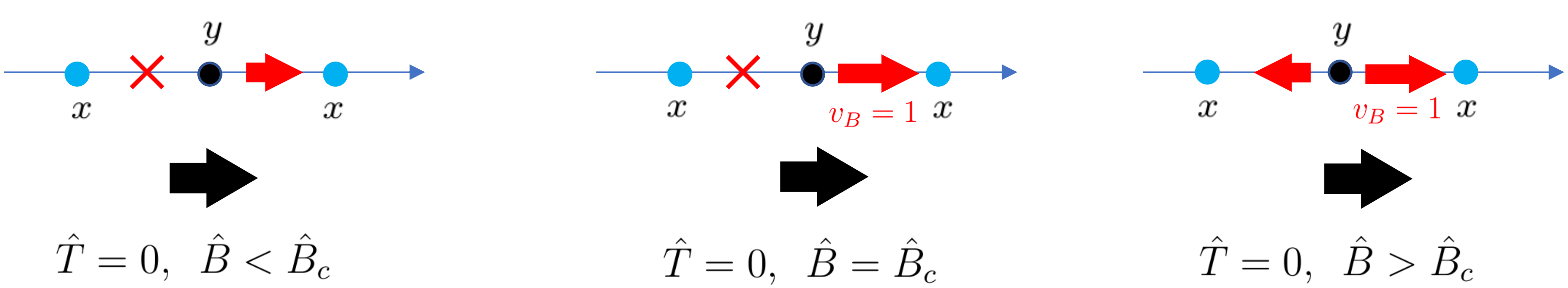
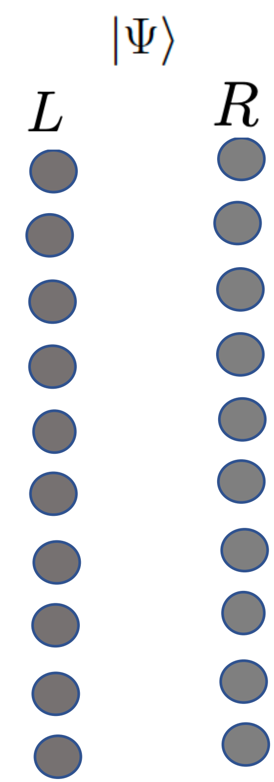


Scrambling measures

In a qubit system with $H_L = \sum_{i=1}^{10} \left\{ \sigma_z^{(i)} \sigma_z^{(i+1)} - 1.05\ \sigma_x^{(i)} + 0.5\ \sigma_z^{(i)} \right\}$

Prepare the system in the thermofield state $|\Psi\rangle = \frac{1}{Z^{1/2}} \sum_n e^{-\beta E_n/2}\ |n\rangle_L |n\rangle_R$

[Shenker, Stanford 1306.0622]



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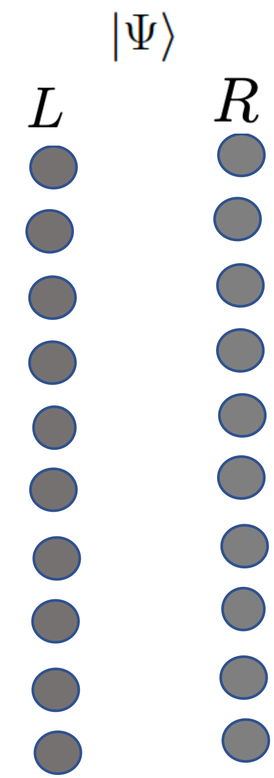

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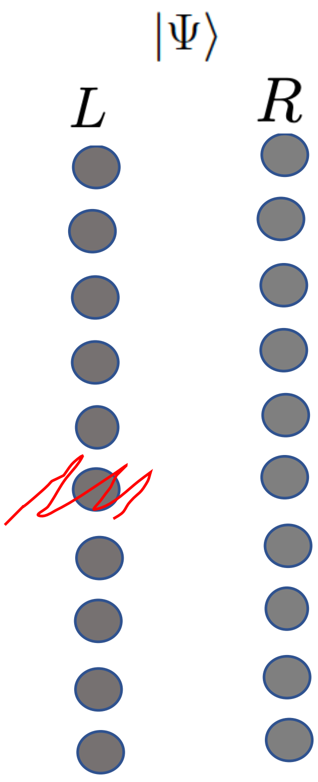

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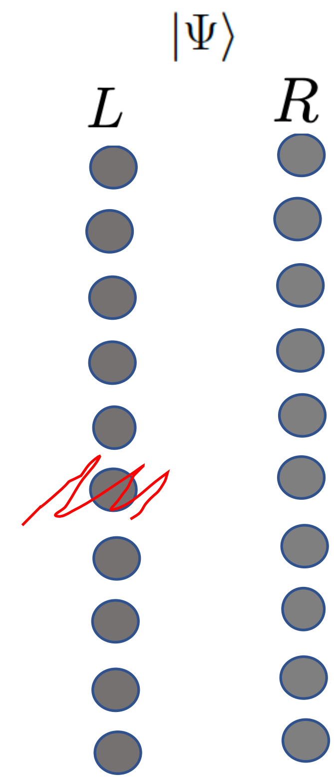

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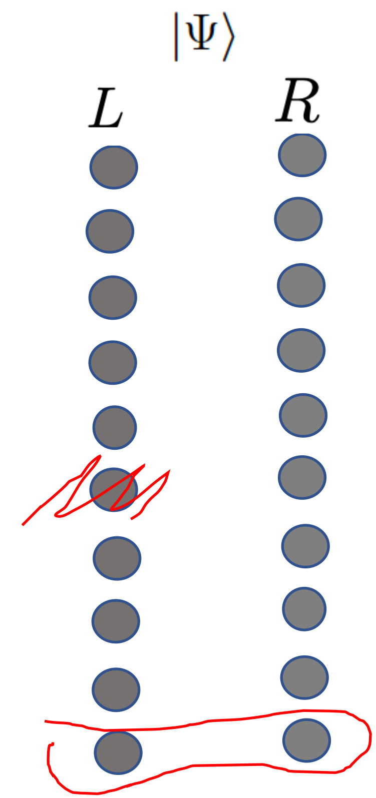

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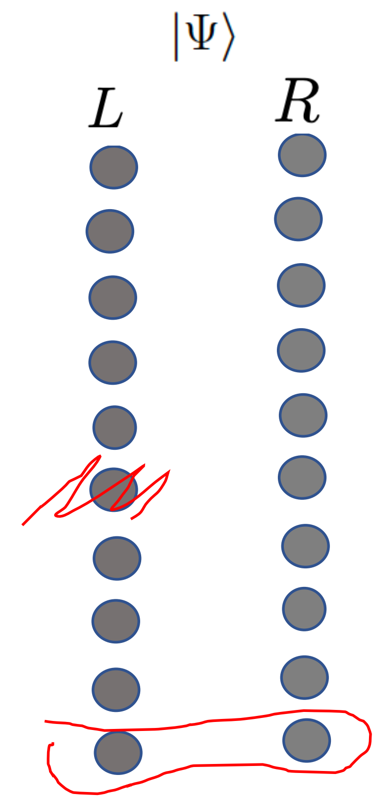

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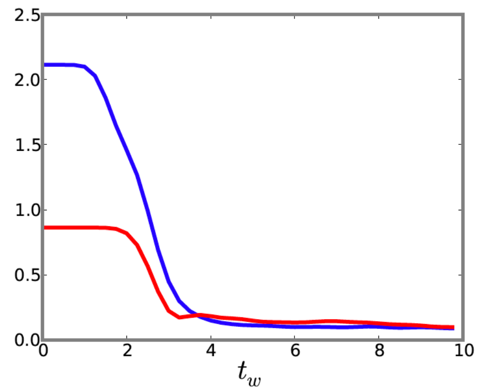

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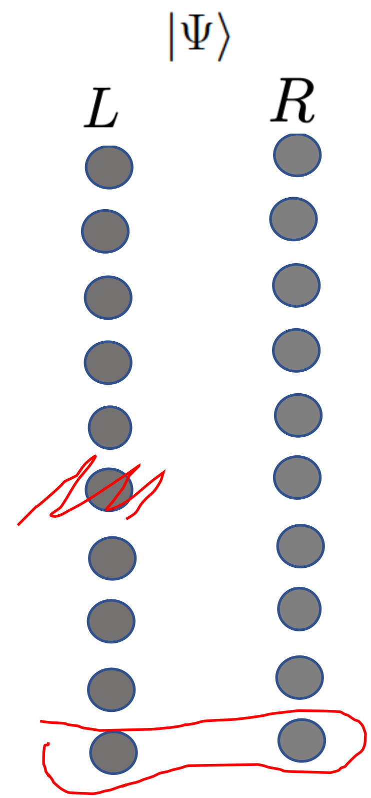

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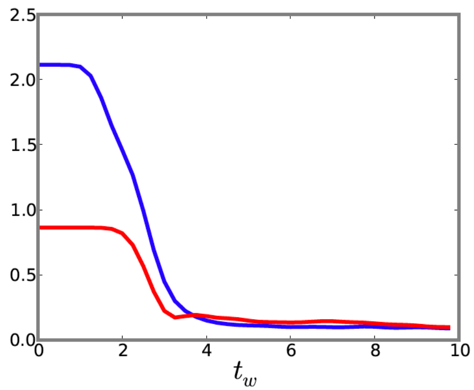

- Scrambling destroys spin correlation

Out-of-time-order correlator: OTOC

The correlator $\langle \Psi' | \sigma_z^{(1,L)} \sigma_z^{(1,R)} | \Psi' \rangle$ is in fact: $|\Psi'\rangle = e^{-iH_L t_w} \sigma_z^{(5,L)} e^{iH_L t_w} |\Psi\rangle$

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- Or more generally: $\langle \Psi | W_L(t) V_L V_R W_L(t) | \Psi \rangle$

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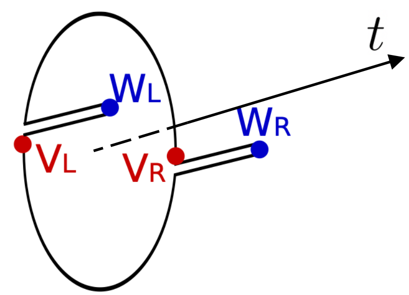

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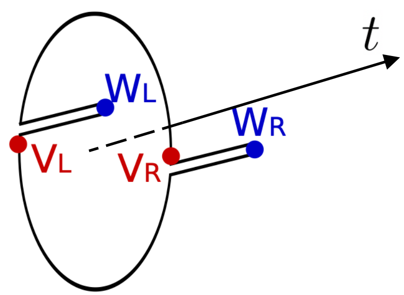

- For large N holographic CFTs

$$F(t) = f_0 - \frac{f_1}{N^2}\exp\frac{2\pi}{\beta}t + \mathcal{O}(N^{-4})$$

[Maldacena, Shenker, Stanford 1503.01406]

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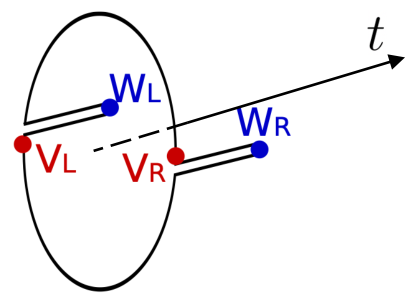

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Lyapunov exponent

Size of precursor growth

- The squared commutator

$$C(t_w, |x-y|) = \mathrm{tr}\left\{\rho(\beta)[W_x(t_w), W_y]^\dagger [W_x(t_w), W_y]\right\}$$

$$= \; 2 - 2\,\mathrm{Re}\,\langle TFD|\, W_y W_x\,(t_w) W_y W_x(t_w)\,|TFD\rangle$$

- Size of precursor $s[W_x(t_w)]$ is the volume of region in y such that $C \geq 1$

= a ball centered at x of the radius

$$r[W_x(t_w)] \approx v_B(t_w - t_*)$$

$$C(t) = 2 - 2\langle W(t,\vec{x})V(0)W(t,\vec{x})V(0)\rangle_\beta \sim \frac{1}{N} e^{\lambda\left(t - \frac{x}{v_B}\right)}$$

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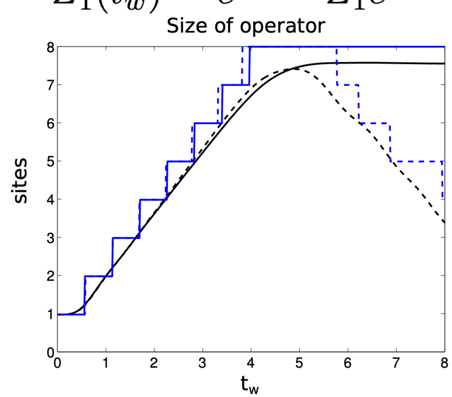
- Linear growth in time is checked numerically

in the spin chain $H = -\sum_i Z_i Z_{i+1} + g X_i + h Z_i$

$$Z_1(t_w) = e^{-iHt_w} Z_1 e^{iHt_w}$$



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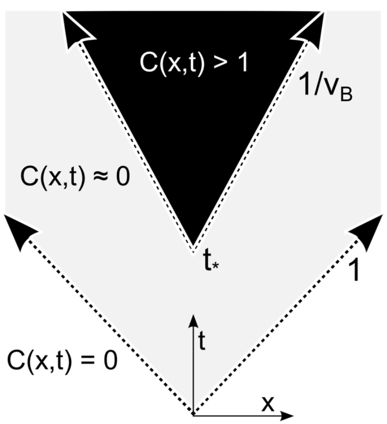
Butterfly speed

[Roberts, Shenker, Stanford, 1409.8180]



$y = 0$

Sound vs butterfly velocity

- We see that $\Delta v_{sound} = \dfrac{8}{9\sqrt{3}}\nu^2 b$ and $\Delta v_B^L = v_B^{L_1} - |v_B^{L_2}| = \dfrac{8}{3\sqrt{3}}(\log 4 - 1)\,\dfrac{\mu^2\, B}{(\pi\, T)^4}$

$$\boxed{\frac{\Delta v_B^L}{\Delta v_{\text{sound}}} = \frac{8\sqrt{3}/9\,(\log 4 - 1)}{8\sqrt{3}/9} = \log 4 - 1}$$

- This suggests:

1. Splitting of butterfly velocities might be originated just from chiral magnetic effects.

$$\begin{aligned} v_{sound} \;&=\; \pm c_s + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(-1 + \frac{(n\alpha_2 - w\beta_2)\partial_T - (n\alpha_1 - w\beta_1)\partial_\mu}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma^{\mathcal{B}} \qquad (2.41)\\ &+\frac{B}{w}\left(1 - \frac{[\gamma,\beta]}{[\alpha,\beta]}\right)\sigma_\epsilon^{\mathcal{B}} + \frac{B}{2w}\frac{[\gamma,\alpha]}{[\beta,\alpha]}\left(\frac{(n\alpha_1 - w\beta_1)\partial_\mu - (n\alpha_2 - w\beta_2)\partial_T}{n[\gamma,\alpha] - w[\gamma,\beta]}\right)\sigma_\epsilon^{\mathcal{B}} \end{aligned}$$

2. There might be a relation between hydrodynamics and quantum chaos in anomalous systems. Note that Hydro works well at scales $\omega \ll T$, while chaos is sensitive to scales $\omega \sim T$.

Confirmed by Pole-skipping $\quad (\omega, k) = (i\lambda\,,\, i\dfrac{\lambda}{v_B})$

[Grozdanov, Schalm, Scopelliti 1710.00921]
[Blake, Lee, Liu 1801.00010]
[Blake, Davison, Grozdanov, Liu 1809.01169]
[NA, Tabatabaei 1910.13696]