



Mass splitting and spin alignment for ϕ mesons in a magnetic field in NJL model

Shu-Yun Yang

Xin-Li Sheng, SY Y, Yao-Lin Zou, De-Fu Hou, arxiv: 2209.01872

Outline

- Motivation
- Nambu-Jona-Lasinio model
- Propagator and spin alignment of vector mesons
- Numerical Results
- Summary

Motivation

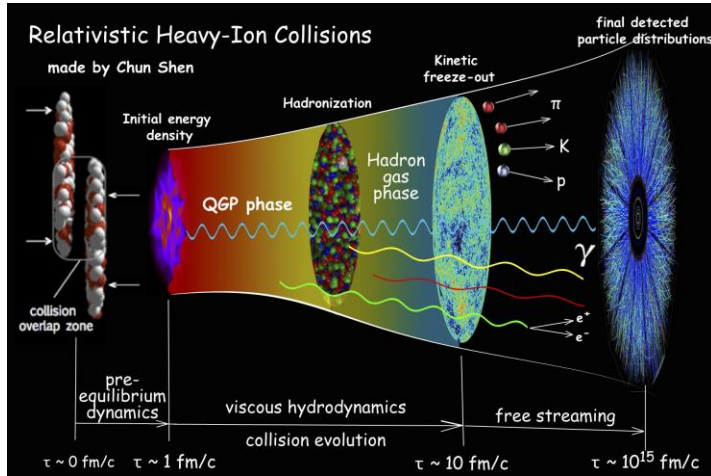
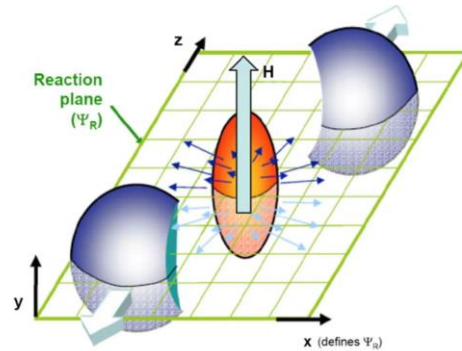


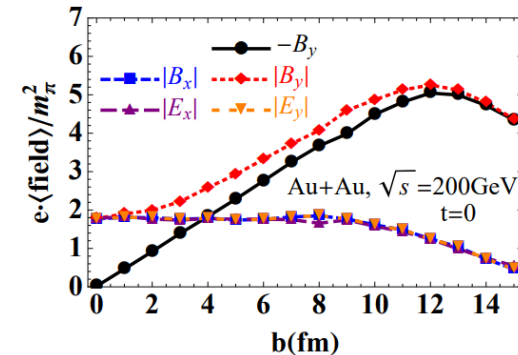
Diagram of relativistic heavy ion collision evolution

- Strong magnetic field in non-central relativistic heavy-ion collisions.
- Sizeable contributions to chiral magnetic field, Λ 's polarization, charge odd directed flow, magnetic catalysis, inverse magnetic catalysis, and phase structure of QGP.



Non-central collisions

Dmitri E. Kharzeev Prog. Part. Nucl. Phys. 75 (2014)



Collision energy dependence of the electromagnetic fields

Weitian Deng, Xuguang Huang, Phys. Rev. C 85 (2012) 044907

Motivation

Z.-T. Liang and X.-N. Wang, Phys. Lett. B 629, 20 (2005), nucl-th/0411101.
A. H. Tang, B. Tu, and C. S. Zhou, Phys. Rev. C 98, 044907 (2018), 1803.05777.

Spin alignment refers to 00-element of normalized spin density matrix for a vector meson with spin-1.

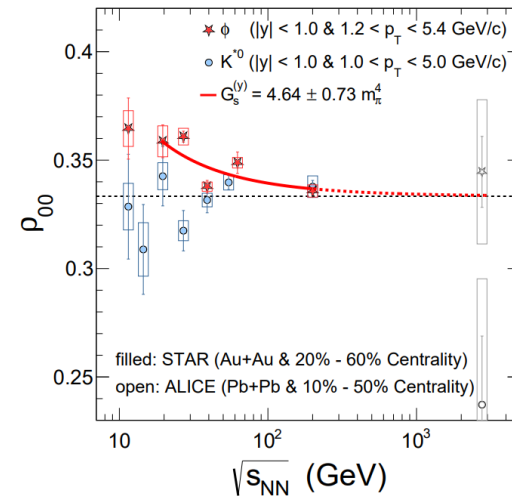
$$\rho_{00}(\mathbf{k}) \equiv \frac{\bar{\rho}_{00}(\mathbf{k})}{\sum_{\lambda=0,\pm 1} \bar{\rho}_{\lambda\lambda}(\mathbf{k})}.$$

$$\bar{\rho}_{\lambda\lambda'}(\mathbf{0}) = \begin{pmatrix} \bar{\rho}_{11} & 0 & 0 \\ 0 & \bar{\rho}_{00} & 0 \\ 0 & 0 & \bar{\rho}_{-1,-1} \end{pmatrix}$$

$\rho_{00} = \frac{1}{3}$	No spin alignment
$\rho_{00} > \frac{1}{3}$	Longitudinally polarized
$\rho_{00} < \frac{1}{3}$	Transversely polarized

M. Abdallah et al. (STAR) Nature 614 (2023), 2204.02302.

ϕ and K^{*0} meson's spin alignment along the out-of-plane direction



The STAR collaboration has measured the ϕ and K^{*0} meson's spin alignment along the out-of-plane direction and observes a significant positive deviation from $1/3$ for the ϕ meson.

Relevant talks: Xin-Li Sheng, Yan-Qing Zhao, Minghua Wei

3 flavor Nambu--Jona-Lasinio model

In the NJL model, gluons are integrated out and quarks interact via local four-fermion interactions, which have the form that keeps the chiral symmetry.

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961).
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124, 246(1961).
S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).

➤ 3 flavor Nambu--Jona-Lasinio model:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_q + G_S \sum_{a=0}^8 [(\bar{\psi} \lambda_a \psi)^2 + (\bar{\psi} i \gamma_5 \lambda_a \psi)^2] & \longleftarrow & \text{scalar channels} \\ & - G_V \sum_{a=0}^8 [(\bar{\psi} \gamma_\mu \lambda_a \psi)^2 + (\bar{\psi} i \gamma_\mu \gamma_5 \lambda_a \psi)^2] & \longleftarrow & \text{vector channels} \\ & - K \{ \det_f [\bar{\psi} (1 + \gamma_5) \psi] + \det_f [\bar{\psi} (1 - \gamma_5) \psi] \} & \swarrow & \text{Six - quark Kobayashi - Maskawa - } \tau' \text{ Hooft interaction} \end{aligned}$$

➤ Lagrangian for quarks:

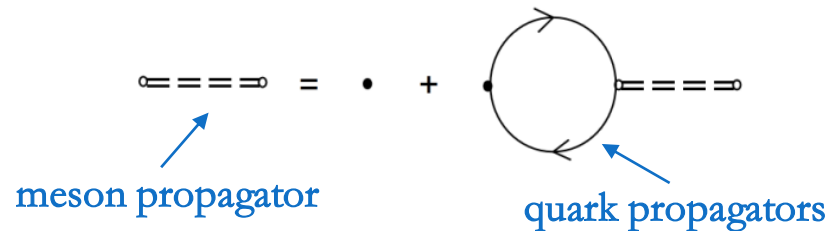
$$\mathcal{L}_q = \sum_{f=u,d,s} \bar{\psi}_f \left(i \gamma_\mu D_f^\mu - m_f - \frac{1}{2} q_f \kappa_f F_{\mu\nu} \sigma^{\mu\nu} \right) \psi_f,$$

current quark masses
AMMs(anomalous magnetic moments)

Discussions of spin alignment in NJL model may help us better understand meson's spin structure in strong interacting system.

Propagator of vector mesons

- Dyson-Schwinger equation under Random phase approximation



- Vector meson's propagator :

$$D^{\mu\nu}(k) = 4G_V \Delta^{\mu\nu}(k) + 4G_V \Delta^{\mu\alpha}(k) \Sigma_{\alpha\beta}(k) D^{\beta\nu}(k)$$

Projection operator Self-energy

- Covariant form of spin polarization vectors :

$$\epsilon^\mu(\lambda, k) = \left(\frac{\mathbf{k} \cdot \boldsymbol{\epsilon}_\lambda}{m_V}, \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{k} \cdot \boldsymbol{\epsilon}_\lambda}{m_V(\omega + m_V)} \mathbf{k} \right)$$

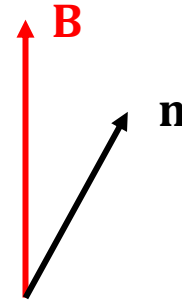
- Projecting Dyson-Schwinger equation :

$$D_{\lambda\lambda'}(k) = -4G_V \delta_{\lambda\lambda'} - 4G_V \sum_{\lambda_1=0,\pm 1} \Sigma_{\lambda\lambda_1}(k) D_{\lambda_1\lambda'}(k) \quad D_{\lambda\lambda'}(k) = - \left[\frac{4G_V}{1 + 4G_V \Sigma(k)} \right]_{\lambda\lambda'}$$

Spin alignment of vector mesons

- Spin alignment in direction of magnetic field :

$$\rho_{00}^B(\mathbf{0}) \equiv \frac{\rho_{00}(\mathbf{0})}{\rho_{00}(\mathbf{0}) + 2\rho_{11}(\mathbf{0})}$$



- Spin alignment in an arbitrary direction $\mathbf{n}$ (denoted by Euler angles (α, β, γ))

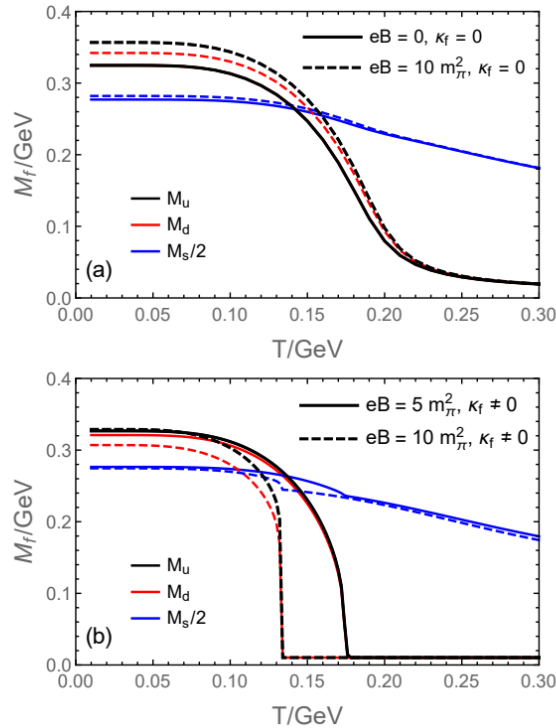
$$\begin{aligned}\rho_{00}(\mathbf{0}; \alpha, \beta, \gamma) &= \frac{\rho_{00}(\mathbf{0}) \cos^2 \beta + \rho_{11}(\mathbf{0}) \sin^2 \beta}{\rho_{00}(\mathbf{0}) + 2\rho_{11}(\mathbf{0})} \\ &= \frac{1}{2} \{1 - \rho_{00}^B(\mathbf{k}) + [3\rho_{00}^B(\mathbf{k}) - 1] \cos^2 \beta\}\end{aligned}$$

Spin alignment is independent to Euler angles α and γ

For a fluctuating magnetic field, one have to take an average over the β -angle and the field strength.

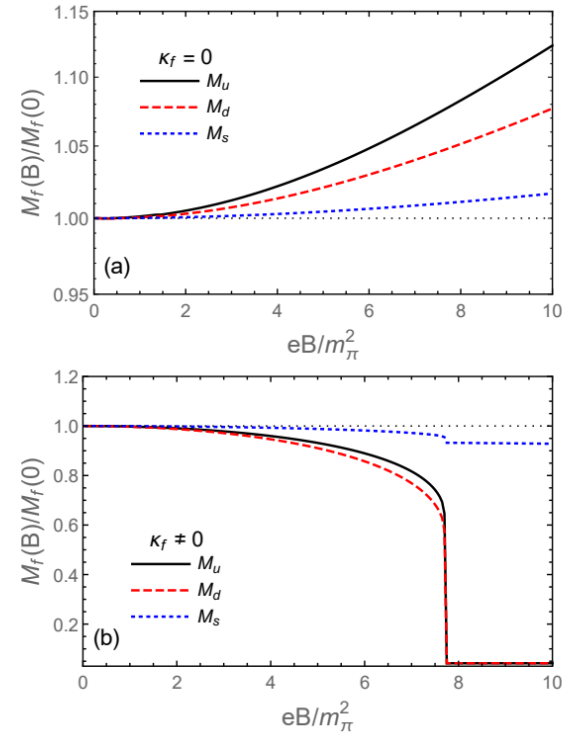
Dynamic masses for quarks

Dynamical masses as functions of the temperature for quarks.



- Behavior of magnetic catalysis.
- With AMMs, for light quarks transition is first order, for the s quark, the transition is still second order.

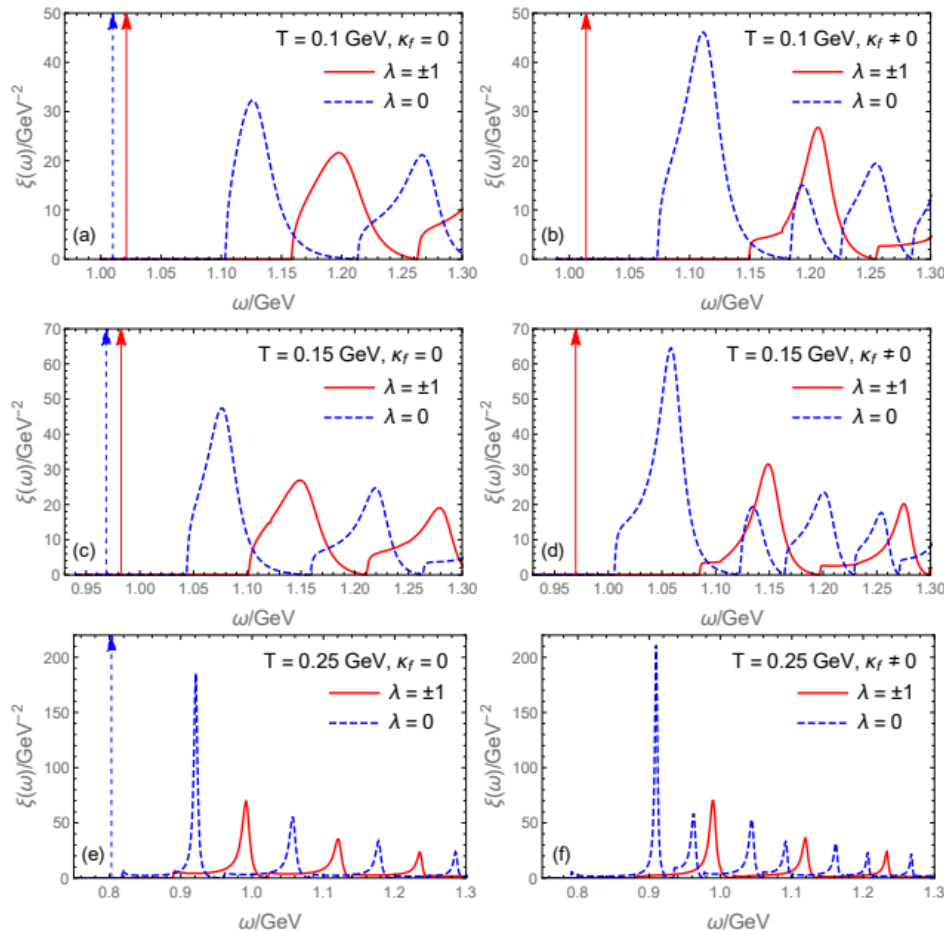
Dynamical masses as functions of the magnetic field strength at $T = 150$ MeV.



- Quark mass $M_u > M_d$, because $|q_u| > |q_d|$.
- Nonvanishing AMMs, inverse magnetic catalysis.

Mass spectra for ϕ meson

Mass spectra for ϕ mesons in a constant magnetic field with $eB = 5 m_\pi^2$.



Spectral function:

$$\xi_\lambda(\omega, \mathbf{0}) = \delta(\omega^2 - M_{\phi, \lambda}^2) + \xi_\lambda^*(\omega)$$

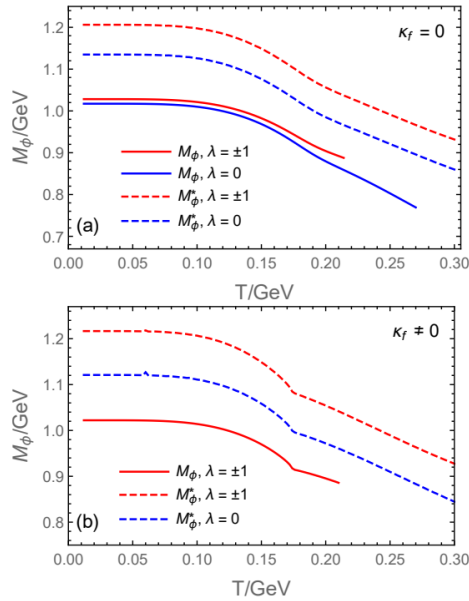
stable bound state

unstable resonance excitations

- In the presence of a magnetic field, the dispersion relation of s quark is quantized as Landau levels. We observe multi-peak structures.
- Bound states will dissociate when T is large enough, indicating a Mott transition.

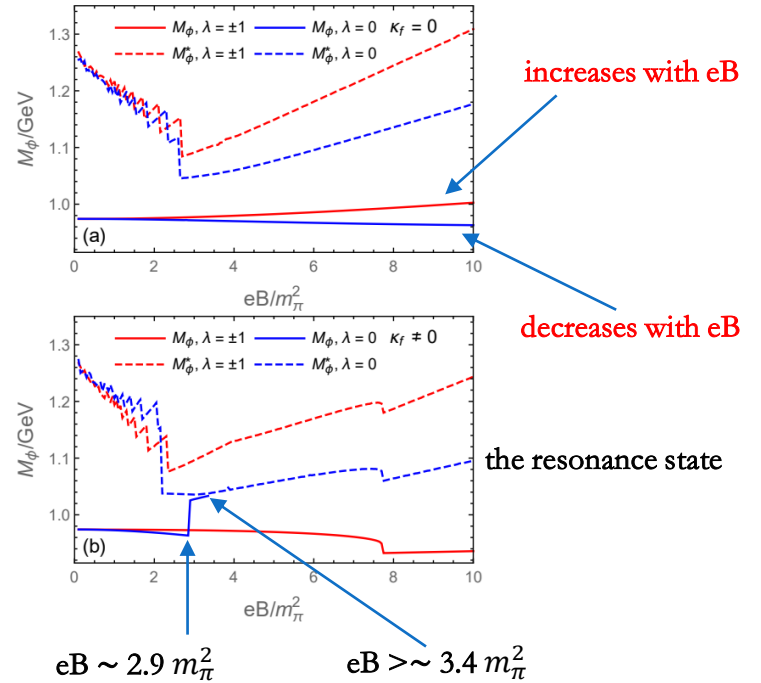
Dynamical masses for ϕ meson

Dynamical masses as functions of the temperature for ϕ mesons at $eB = 5 m_\pi^2$.



- Observed **mass splitting** between states with $\lambda = 0$ and $\lambda = \pm 1$.
 $M_{\phi, \lambda = 0} < M_{\phi, \lambda = \pm 1}$.
- Nonzero AMMs, $M_{\phi, \lambda = 0} = 0$ even at lower temperatures.

Dynamical masses as functions of eB for ϕ mesons at $T = 150$ MeV.

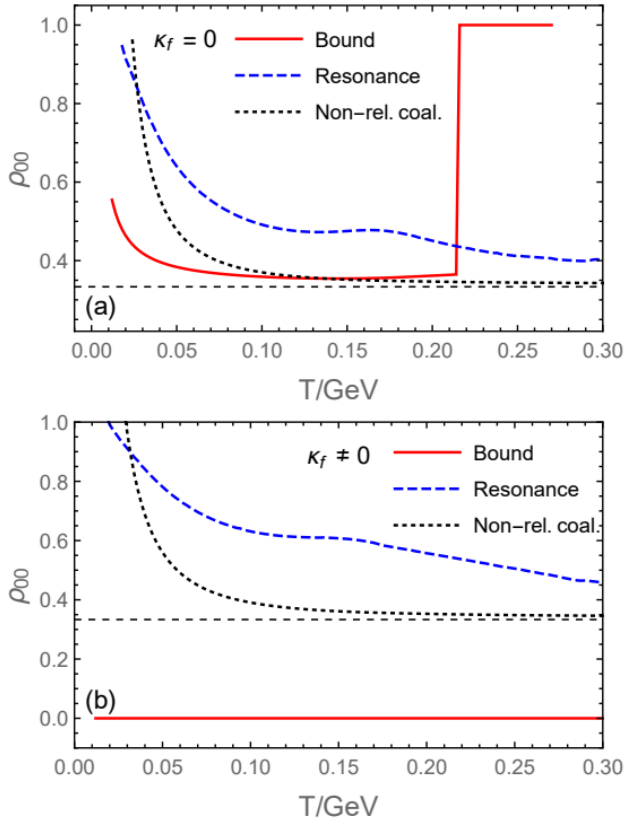


- Corresponding to the Mott transition.

Spin alignment for ϕ meson

Spin alignments as functions of the temperature. The magnetic field strength is taken as $eB = 5 m_\pi^2$.

Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang,
Phys. Rev. C 97, 034917 (2018), 1711.06008.



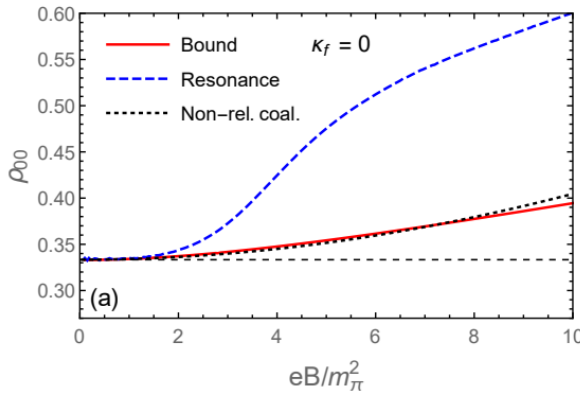
The coalescence model:

$$\rho_{00}^{\text{Non-rel. coal.}} = \frac{1}{3} + \frac{4}{9T^2} \mu_s^2 B^2$$

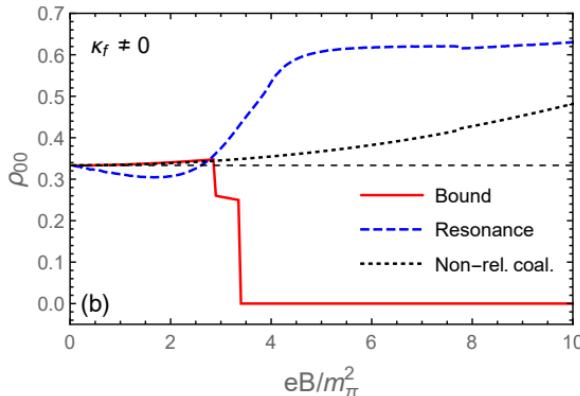
- ρ_{00}^{Bound} decreases towards $1/3$ with an increasing T .
- $T = 150 \text{ MeV}$, $\rho_{00}^{\text{Bound}} \approx \rho_{00}^{\text{Non-rel.coal}}$, $T > 150 \text{ MeV}$, $\rho_{00}^{\text{Bound}} > \rho_{00}^{\text{Non-rel.coal}}$
- Especially, bound states with $\lambda = \pm 1$ vanish when $T > 215 \text{ MeV}$, leading to the result of $\rho_{00} = 1$.
- Nonvanishing AMMs, $\rho_{00}^{\text{Bound}} = 0$ because all bound states have $\lambda = \pm 1$.

Spin alignment for ϕ meson

Spin alignments as functions of the magnetic field strength at temperature $T = 150$ MeV.



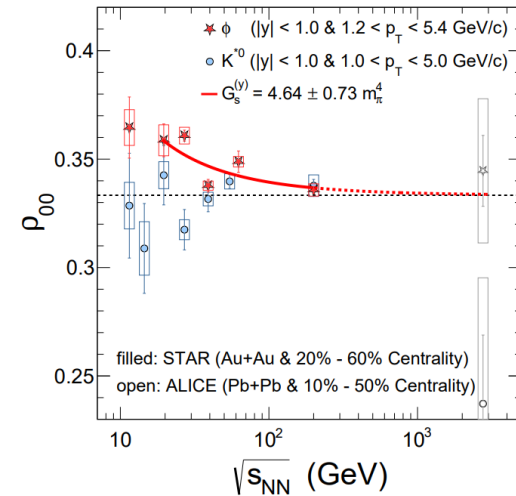
When $eB \rightarrow 0$, $\rho_{00} \approx 1/3$.
 $\rho_{00}^{\text{Bound}} \approx \rho_{00}^{\text{Non-rel.coal.}}$
 $\rho_{00}^{\text{Resonance}} > \rho_{00}^{\text{Bound}}$,
 $\rho_{00}^{\text{Resonance}} > \rho_{00}^{\text{Non-rel.coal.}}$



- When $eB < 2.5 m_\pi^2$, $\rho_{00}^{\text{Bound}} \approx \rho_{00}^{\text{Non-rel.coal.}}$.
- When $eB \sim 2.9 m_\pi^2$, $\rho_{00}^{\text{Bound}} < \frac{1}{3}$, which is a straightforward result of the Mott transition for states with $\lambda = 0$.

M. Abdallah et al. (STAR) Nature 614 (2023), 2204.02302.

ϕ and K^{*0} meson' s spin alignment along the out-of-plane direction



- Positive deviation from $1/3$ observed in experiments indicates that the spin of ϕ meson is preferred to align in the reaction plane.

Summary

- Calculated spectral functions for ϕ mesons in a constant magnetic field.
- Observed mass splitting between longitudinally-polarized and transversely-polarized mesons.
- Spin alignment for ϕ meson bound states is larger than $1/3$, qualitatively agree with recent STAR data. Spin alignment for resonance states gives a larger result.
- AMMs significantly modify the mass spectra and the spin alignment for ϕ meson.

Thank you for your attention!

