

Two-point functions from chiral kinetic theory in magnetized plasma

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The 7_{th} Chirality, Vorticity and Magnetic Field in HIC
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Based on LXY, PRD 105, 074039, 2022
S. Lin, LXY, JHEP 06, 054, 2021

Outline

- Motivation: chiral transports in magnetized plasma
- Chiral kinetic theory from Landau level basis
- Response functions as functional derivatives
- Summary and outlook

Anomalous chiral transports in QGP

Talks by Kharzeev, H. Huang, Q. Shou, J. Gao, F. Wang, X. Huang, S. Shi, Mameda, Y. Lin, X. Zhao, W. Wu ...

CME/CSE

$$\mathbf{J} = C\mu_5 e \mathbf{B}$$

$$\mathbf{J}_5 = C\mu e \mathbf{B}$$

$$J_5^\mu = -\epsilon^{\mu\nu} J_\nu$$

CMW: expected in CKT

Vector/Axial CVE

$$\mathbf{J} = C\mu\mu_5 \boldsymbol{\omega}$$

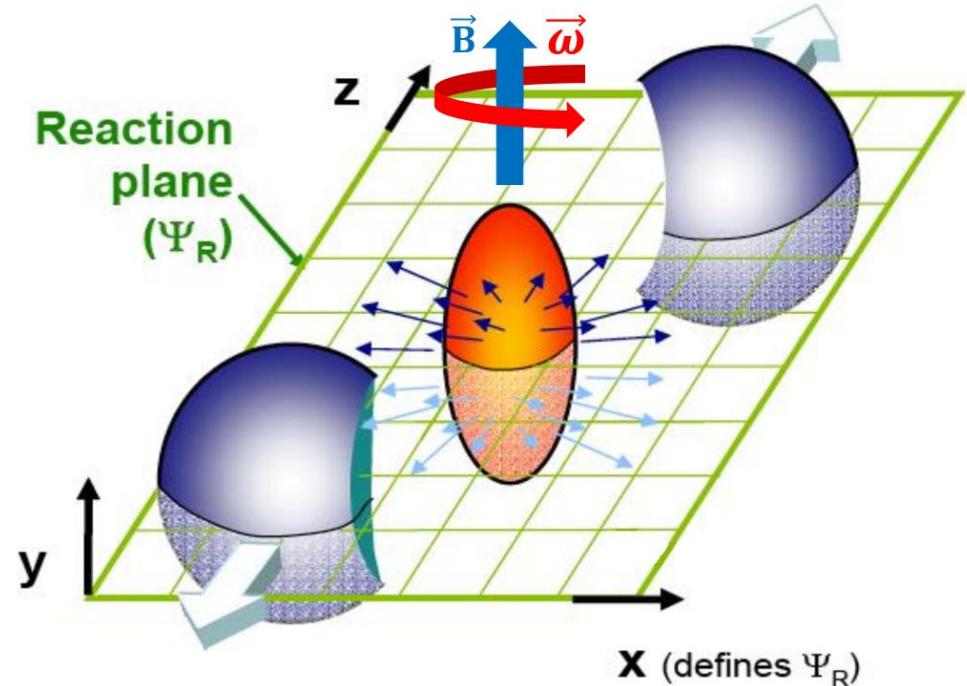
$$\mathbf{J}_5 = C \left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3} \right) \boldsymbol{\omega}$$

CESE

$$\mathbf{J}_5 \propto \frac{\mu\mu_5}{T^2} \sigma e \mathbf{E}$$

Ohm/Hall currents (non-anomalous)

$$\mathbf{J} = \sigma e \mathbf{E} + \sigma_H e^2 \mathbf{E} \times \mathbf{B}$$



CKT with Landau level basis

S. Lin, LXY, PRD 2020

CKE for chiral fermion in collisionless limit

chirality
right-handed

$$p_\mu j^\mu = 0$$

$$\Delta_\mu j^\mu = 0$$

$$p^\mu j^\nu - p^\nu j^\mu = -\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \Delta_\rho j_\sigma$$

Wigner functions in LL basis $\rightarrow 2W(x, p) = j^\mu \bar{\sigma}_\mu$

Valid to all order of background B & first order $O(a)$ $O(\partial a)$

$$B \sim p^2 \sim O(1) \quad a_\mu \sim O(a) \quad \mathcal{B} \& E \sim O(\partial a)$$

Background LLL

$$j_{(0)}^\mu = (u + b)^\mu \delta(\bar{p}_0 - \bar{p}_3) f(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

1+1D, fluid velocity: u^μ
direction of B^μ : b^μ

Current & stress energy tensor

$$J^\mu = \int d^4 p j^\mu \quad T^{\mu\nu} = \int d^4 p \frac{1}{2} (p^\mu j^\nu + p^\nu j^\mu)$$

H. Gao, S. Lin, Z. Mo, PRD 2020

$$A_\mu \rightarrow A_\mu + a_\mu$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho B^\sigma$$

$$f_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho \mathcal{B}^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu} (F_{\mu\nu} + f_{\mu\nu})$$

Perturbation: vector/axial gauge field

CKE in collisionless limit

chirality
 $s=\pm 1$

$$\begin{aligned} p_\mu j_s^\mu &= 0 \\ \Delta_\mu j_s^\mu &= 0 \\ p^\mu j_s^\nu - p^\nu j_s^\mu &= -\frac{s}{2} \epsilon^{\mu\nu\rho\sigma} \Delta_\rho^s j_\sigma^s \end{aligned}$$

$$\begin{aligned} A_\mu &\rightarrow A_\mu + a_\mu^s \\ F_{\mu\nu} &= \epsilon_{\mu\nu\rho\sigma} u^\rho B^\sigma \\ f_{\mu\nu}^s &= \epsilon_{\mu\nu\rho\sigma} u^\rho \mathcal{B}_s^\sigma + E_\mu^s u_\nu - E_\nu^s u_\mu \\ \Delta_\mu^s &= \partial_\mu - \frac{\partial}{\partial p_\nu} (F_{\mu\nu} + f_{\mu\nu}^s) \end{aligned}$$

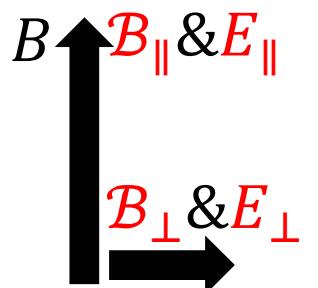
Background LLL $j_{(0)s}^\mu = (u + sb)^\mu \delta(\bar{p}_0 - s\bar{p}_3) f_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$

Perturbation expansion

$$j_s^\mu = j_{(0)s}^\mu + j_{as}^\mu + \sum_{\mathcal{A}s} j_{\mathcal{A}s}^\mu$$

$O(a)$ $O(\partial a)$

$$\mathcal{A}s = \mathcal{B}_{\parallel}^s, \mathcal{B}_{\perp}^s, E_{\parallel}^s, E_{\perp}^s$$



Perturbative solution: CMW

At $O(a)$ give CMW

$$j_{as}^\mu = (u + sb)^\mu \Delta\mu_s \delta(\bar{p}_0 - s\bar{p}_3) f'_s(\bar{p}_0) e^{\frac{p_T^2}{B}}$$

CMW propagating with the speed c in the limit $B \rightarrow \infty$

Kharzeev, Yee, PRD 2011

$$j_{(0)s}^\mu + j_{as}^\mu = j_{(0)s}^\mu (\mu_s \rightarrow \mu_s + \Delta\mu_s) \quad \text{shifted by } \Delta\mu_s$$

At $O(\partial a)$ collisionless limit \rightarrow non-dissipative $\rightarrow j_{E\parallel s}^\mu = 0$

$$j_{(0)s}^\mu + j_{B\parallel s}^\mu = j_{(0)s}^\mu (B \rightarrow B + \mathcal{B}_{\parallel}^s)$$

$$j_{(0)s}^\mu + j_{B\perp s}^\mu = j_{(0)s}^\mu (b^\mu \rightarrow b^\mu + \mathcal{B}_{\perp}^s / B)$$

$$j_{(0)s}^\mu + j_{E\perp s}^\mu = j_{(0)s}^\mu (u^\mu \rightarrow u^\mu + U_s^\mu / B)$$

shifted by $\mathcal{B}_{\parallel}^s, \mathcal{B}_{\perp}^s, U^s$

$$U_s^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} b_\nu f_{\rho\sigma}^{E\perp s}$$

Two-point functions from CKT in magnetized plasma

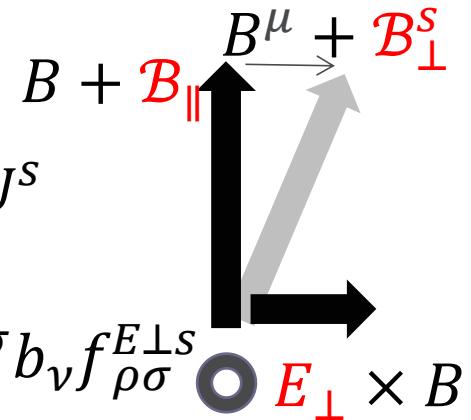
$$\Delta\mu_s \equiv \frac{\bar{q}_3 \bar{a}_0^s - \bar{q}_0 \bar{a}_3^s}{s \bar{q}_0 - \bar{q}_3}$$

redistribution of
the chiral plasma



$$\Delta\mu = \frac{\Delta\mu_R + \Delta\mu_L}{2}$$

$$\Delta\mu_5 = \frac{\Delta\mu_R - \Delta\mu_L}{2}$$



Constitutive relation of currents

At $O(1)$

$$J_{(0)}^\mu = \frac{\mu B}{2\pi^2} u^\mu + \frac{\mu_5 B}{2\pi^2} b^\mu$$

$$J_{(0)5}^\mu = \frac{\mu_5 B}{2\pi^2} u^\mu + \frac{\mu B}{2\pi^2} b^\mu$$

CSE

At $O(a)$

$$J_a^\mu = \frac{\Delta \mu B}{2\pi^2} u^\mu + \frac{\Delta \mu_5 B}{2\pi^2} b^\mu$$

$$J_{a5}^\mu = \frac{\Delta \mu_5 B}{2\pi^2} u^\mu + \frac{\Delta \mu B}{2\pi^2} b^\mu$$

At $O(\partial a)$

$$J_{\mathcal{A}}^\mu = \frac{\mu \mathcal{B}_{\parallel} + \mu_5 \mathcal{B}_{\parallel}^5}{2\pi^2} u^\mu + \frac{\mu U^\mu}{2\pi^2} + \frac{\mu_5 U_5^\mu}{2\pi^2} + \frac{\mu_5 (\mathcal{B}_\perp^\mu + \mathcal{B}_{\parallel}^\mu)}{2\pi^2} + \frac{\mu (\mathcal{B}_{\perp 5}^\mu + \mathcal{B}_{\parallel 5}^\mu)}{2\pi^2}$$

$$J_{\mathcal{A}5}^\mu = \frac{\mu_5 \mathcal{B}_{\parallel} + \mu \mathcal{B}_{\parallel}^5}{2\pi^2} u^\mu + \frac{\mu_5 U^\mu}{2\pi^2} + \frac{\mu U_5^\mu}{2\pi^2} + \frac{\mu (\mathcal{B}_\perp^\mu + \mathcal{B}_{\parallel}^\mu)}{2\pi^2} + \frac{\mu_5 (\mathcal{B}_{\perp 5}^\mu + \mathcal{B}_{\parallel 5}^\mu)}{2\pi^2}$$

No Ohm current in collisionless limit

Perturbation: longitudinal vorticity

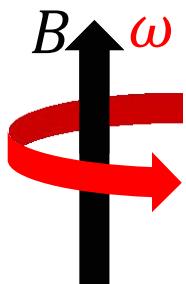
Vorticity from metric $\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho (u^\alpha g_{\alpha\sigma})$

chirality
right-handed

$$\begin{aligned}\Pi_\mu j^\mu &= 0 \\ \Delta_\mu j^\mu &= 0 \\ \Pi^\mu j^\nu - \Pi^\nu j^\mu &= -\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \Delta_\rho j_\sigma\end{aligned}$$

$$\left. \begin{array}{l} \text{gradient } \partial u^\rho \sim O(\partial) \\ \text{homogeneous } B^\sigma \sim O(1) \end{array} \right\} \rightarrow \partial F_{\mu\nu} \sim O(\partial) \rightarrow \Pi_\mu = p_\mu - \frac{1}{12} \frac{\partial^2}{\partial p_\nu \partial p_\lambda} \partial_\lambda F_{\mu\nu}$$

Perturbation expansion $j^\mu = j_{(0)}^\mu + j_\omega^\mu$ $j_\omega^\mu \propto \omega$ for $\omega^\mu = \omega b^\mu$



Response functions as functional derivatives

Response functions from generating functional Γ

$$\mathcal{G}_{VA}^{\mu\nu}(q)\delta^{(4)}(k-q) = \frac{\delta\mathcal{J}^\mu(k)}{\delta a_\nu^5(q)} = \frac{\delta^2\Gamma}{\delta a_\nu^5(q)\delta a_\mu(-k)}$$

$$\mathcal{G}_{VV}^{\mu\nu} \quad \mathcal{G}_{AA}^{\mu\nu} \quad \mathcal{G}_{AV}^{\mu\nu}$$

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_\mu(q)} = \frac{\delta^2\Gamma}{\delta a_\mu(q)\delta g_{\lambda\nu}(-k)}$$

$$\mathcal{G}_{VT}^{\mu,\lambda\nu} \quad \mathcal{G}_{TA}^{\lambda\nu,\mu} \quad \mathcal{G}_{AT}^{\mu,\lambda\nu}$$

Derivative symmetry $\mathcal{G}_{VA}^{\mu\nu}(q) = \mathcal{G}_{AV}^{\nu\mu}(-q)$

General form $\mathcal{G}_{ab}(q) = \mathcal{G}_{ba}(-q) \quad a, b = \mathcal{J}_V^\mu, \mathcal{J}_A^\mu, T^{\mu\nu} \dots$

CKT solutions satisfy derivative symmetry?

CKT with consistent & covariant anomaly

Consistent \mathcal{J} & covariant J anomaly

$$\partial_\mu \mathcal{J}^\mu = \partial_\mu J^\mu + \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}^5 = 0$$

$$\partial_\mu \mathcal{J}_5^\mu = \partial_\mu J_5^\mu = -\frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}$$

Bardeen, Zumino, Nucl. Phys. B 1984

Landsteiner, Phys. Pol. B 2016

in contrast to consistent anomaly

Carignano, Manuel, Torres-Rincon, PRD 2018

CKT with covariant anomaly

Son, Yamamoto, PRL 2012, PRD 2013

Manuel, Torres-Rincon, PRD 2014

Gorbar, Miransky, Shovkovy, Sukhachov, PRL 2017

$$\int_q \Delta_\mu j^\mu$$

Relation between consistent & covariant current

$$\mathcal{J}^\mu = J^\mu + \frac{1}{4\pi^2} \epsilon^{\mu\nu\rho\lambda} a_\nu^5 F_{\rho\lambda}$$

$$\mathcal{J}_5^\mu = J_5^\mu$$

$$G_{VA,a}^{\mu\lambda}(q) \neq G_{AV,a}^{\lambda\mu}(-q) \rightarrow \text{Chern-Simons term} \rightarrow G_{VA,a}^{\mu\lambda}(q) = G_{AV,a}^{\lambda\mu}(-q)$$

Photon polarization

Structures of response functions

$$G_{VV,a}^{\mu\lambda}(q) = G_{AA,a}^{\mu\lambda}(q) = \frac{B}{2\pi^2} \frac{\bar{q}_3^2 u^\mu u^\lambda + \bar{q}_0^2 b^\mu b^\lambda + \bar{q}_0 \bar{q}_3 u^{\{\mu} b^{\lambda\}}}{\bar{q}_0^2 - \bar{q}_3^2}$$

$$G_{VV,\mathcal{A}}^{\mu\lambda}(q) = G_{AA,\mathcal{A}}^{\mu\lambda}(q)$$

$$= \frac{i\mu_5}{2\pi^2} (\epsilon^{\mu\lambda\rho\sigma} \bar{q}_3 - b^{[\mu} \epsilon^{\lambda]\nu\rho\sigma} q_\nu^T) u_\rho b_\sigma - \frac{i\mu}{2\pi^2} (\epsilon^{\mu\lambda\rho\sigma} \bar{q}_0 + u^{[\mu} \epsilon^{\lambda]\nu\rho\sigma} q_\nu^T) u_\rho b_\sigma$$

CME

CSE

Effect on photon splitting and polarization

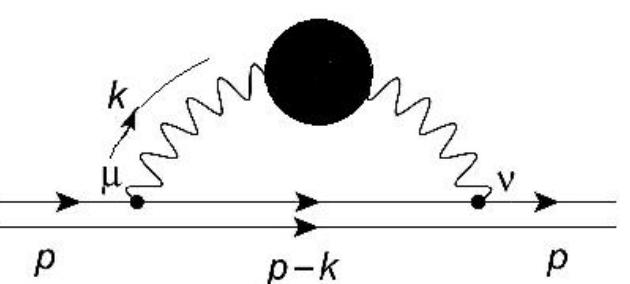
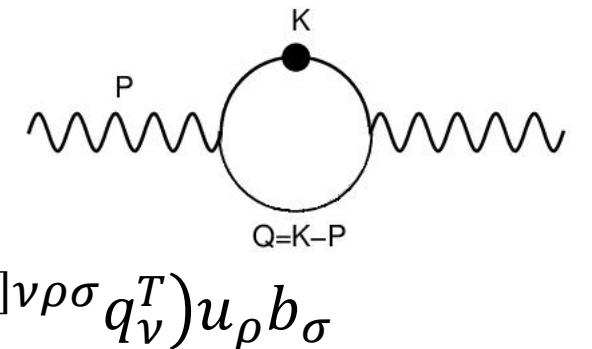
→ spin polarization of probe fermion

Talk by Lihua Dong

Correlators of consistent currents satisfy Onsager relations

$$\mathcal{G}_{ab}(q_0, \mathbf{q}, \tilde{\mathbf{B}}) = \gamma_a \gamma_b \mathcal{G}_{ba}(q_0, -\mathbf{q}, -\tilde{\mathbf{B}})$$

LLL photon polarization
Fukushima, PRD 2011



Summary

- CKT from Landau level basis gives covariant anomaly
- Response functions from CKT gives photon polarization in strong B

Outlook

- Finite interactions, collisional terms in strong B
- Effect on photon splitting and polarization of probe fermion

Thanks for your attention!

Consistent & covariant anomaly

Consistent anomaly & covariant anomaly

$$\partial_\mu \mathcal{J}_s^\mu = \pm \frac{1}{96\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s$$

$$\partial_\mu J_s^\mu = \pm \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu}^s F_{\rho\lambda}^s$$

Bardeen, Zumino, Nucl. Phys. B 1984

Bardeen counterterms



Landsteiner, Phys. Pol. B 2016

Consistent axial anomaly & covariant anomaly

$$\partial_\mu J^\mu + \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}^5 = \partial_\mu \mathcal{J}^\mu = 0$$

$$\partial_\mu J_5^\mu = \partial_\mu \mathcal{J}_5^\mu = - \frac{1}{8\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} f_{\rho\lambda}$$

CKT with free particle basis

$$O(1) : \text{spinless particle} \quad \partial_t f + \mathbf{v} \cdot \nabla_x f + (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p f = 0 \quad \delta(p^2)$$

$$O(\hbar) : \text{particle with Berry curvature } \Omega = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

$$(1 + \hbar \Omega \cdot \mathbf{B}) \partial_t f$$

$$+ [\mathbf{v} + \hbar (\mathbf{E} \times \Omega) + \hbar (\mathbf{v} \cdot \Omega) \mathbf{B}] \cdot \nabla_x f$$

$$+ [\mathbf{E} + \mathbf{v} \times \mathbf{B} + \hbar (\mathbf{E} \cdot \mathbf{B}) \Omega] \cdot \nabla_p f = 0$$

magnetic moment $\frac{\mathbf{p}}{2|\mathbf{p}|^2}$ $\delta(p^2) \rightarrow \delta(\tilde{p}^2)$

$\tilde{p}^2 \equiv p^2 + \hbar \frac{\mathbf{B} \cdot \mathbf{p}}{p_0}$

valid when $\sqrt{\hbar E}, \sqrt{\hbar B}, \hbar \partial_x \ll p$

$$O(\hbar^2) : \text{particle no longer on-shell, simple picture lost}$$

$$p_\mu G_{(0)}^\mu [f \delta(\tilde{p}^2)] + \frac{\hbar s}{2} \mathbf{G}^{(0)} \cdot \left\{ \frac{1}{p_0} \mathbf{G}^{(0)} \times [\mathbf{p} f \delta(\tilde{p}^2)] \right\} + \hbar^2 C(f) = 0$$

$\delta(p^2) \not\rightarrow \delta(\tilde{p}^2)$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang, PRD 2018

Noncommutable limits

Derivative symmetry in static limit

Shu Lin, LXY, JHEP 2021

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q)\delta^{(4)}(k-q) = \frac{\delta T^{\lambda\nu}(k)}{\delta a_\mu(q)}$$

$$\mathcal{G}_{VT}^{\mu,\lambda\nu}(q)\delta^{(4)}(k-q) = \frac{\delta J_\nu^\mu(k)}{\delta g_{\lambda\nu}(q)}$$

With limits $\bar{q}_3 \rightarrow 0$ before $\bar{q}_0 \rightarrow 0$ Son, Yamamoto, PRD 2013

$$\mathcal{G}_{TV}^{\lambda\nu,\mu}(q) = \mathcal{G}_{VT}^{\mu,\lambda\nu}(-q)$$

Two limits noncommutable, finite interactions needed Satow, Yee, PRD 2014

Suggestive form: LLL, on-shell

At $O(a)$

shifted by $\Delta\mu_s$

$$j_{(0)s}^\mu + j_{as}^\mu = j_{(0)s}^\mu (\mu_s \rightarrow \mu_s + \Delta\mu_s)$$

At $O(\partial a)$

in static limit, shifted by $\mathcal{B}_{||}^s, \mathcal{B}_\perp^s, U^s$

$$j_{(0)s}^\mu + j_{\mathcal{B}_{||s}}^\mu = j_{(0)s}^\mu (B \rightarrow B + \mathcal{B}_{||}^s)$$

magnitude enhanced

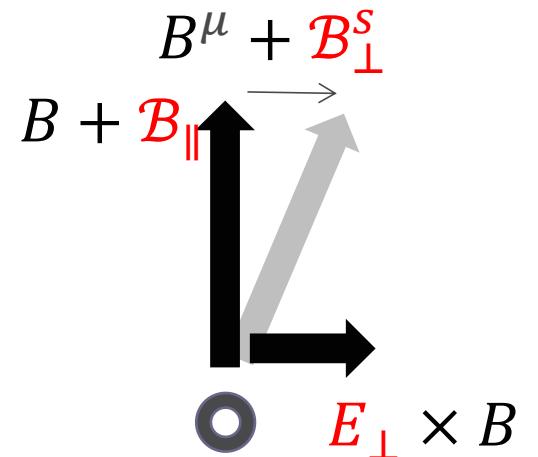
$$j_{(0)s}^\mu + j_{\mathcal{B}_{\perp s}}^\mu = j_{(0)s}^\mu (b^\mu \rightarrow b^\mu + \mathcal{B}_\perp^s / B)$$

tilted

$$j_{(0)s}^\mu + j_{E\perp s}^\mu = j_{(0)s}^\mu (u^\mu \rightarrow u^\mu + U_s^\mu / B)$$

boosted

$$U_s^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} b_\nu f_{\rho\sigma}^{E\perp s}$$



Phenomenology of LLL in HIC

Estimate parameters in QGP

$$eB = m_\pi^2 \sim 10m_\pi^2$$

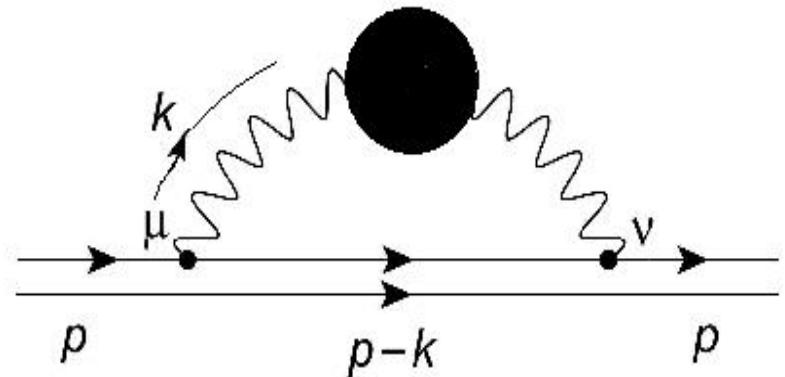
$$T = 350\text{MeV}$$

$$\frac{eB}{T^2} = 0.2 \sim 1.6$$

higher LLs to be included

UPC?

$$\frac{eB}{T^2} \gg 1 \quad \text{lower LLs summation}$$



Effect on photon splitting and polarization → spin polarization of probe fermion

Lihua Dong, Shu Lin, poster at Guangzhou & upcoming works