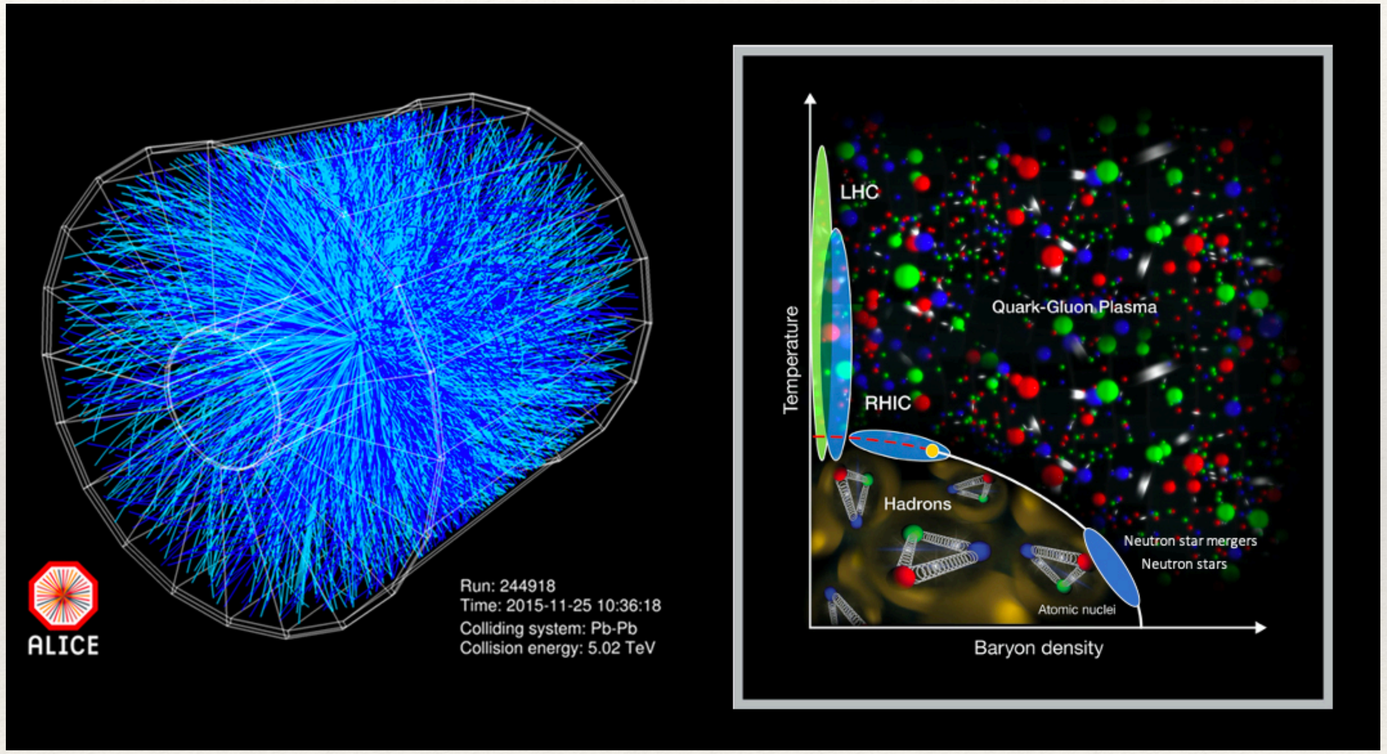
# A novel spectral sum rule of thermal QCD

Nan Su
Shandong Institute of Advanced Technology

7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions
July 19, 2023

# QCD deconfinement and quark-gluon plasma

- Mechanism of QCD deconfinement
  - $g \sim \mathcal{O}(1)$
  - Open question in Yang-Mills theory
- Nature of quark-gluon plasma (QGP)
  - Conventional thermal QCD theory
    - Weakly coupled gas — short-range correlations
  - Heavy-ion experiments
    - Strongly coupled liquid — long-range correlations
- How to tackle with thermal quantum field theory?
  - Strongly coupled QCD formalism
  - Physics & Math




https://www.innovationnewsnetwork.com/exploring-matter-after-the-big-bang/8120/

# Spectral function & spectral sum rule

- Spectral function $\rho(\omega, p)$ characterizes physical degrees of freedom and transport properties
- Sum rule of spectral function induced by quantum mechanics
  - Anti-commutation relation of fermions

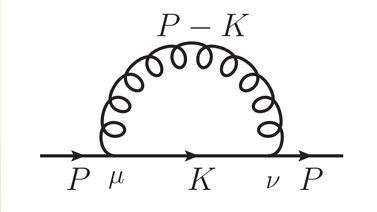
$$\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \rho_\psi(\omega, p) = \frac{1}{4} \int \mathrm{d}^4 x \, \delta(t) \, \exp\left(-i\mathbf{p} \cdot \mathbf{x}\right) \left\{ \psi_\alpha(x) \psi_\alpha^\dagger(0) + \psi_\alpha(0)^\dagger \psi_\alpha(x) \right\}_\beta = 1$$

  - Crucial consistency check of the underlying theory: whether the supposed fermions obey anti-commutation relation?!
  - Established for QED/weakly coupled QCD [Le Bellac, 96]
  - Not yet for strongly coupled QCD(!)

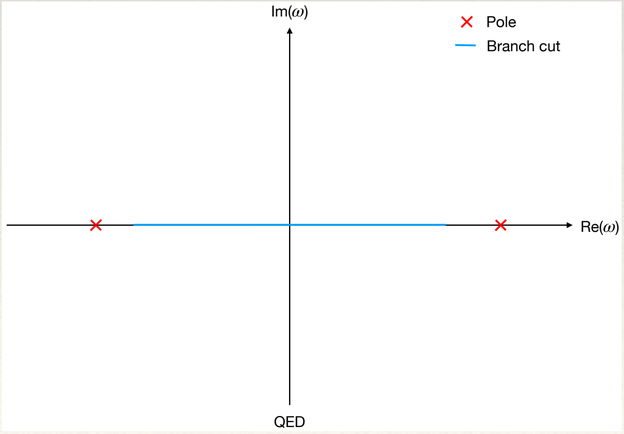
# QED case: dispersion relation & spectral sum rule

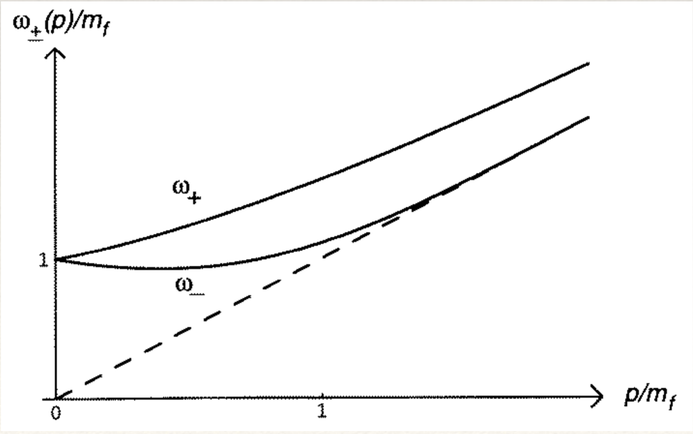
- Crucial measure of collective behaviors: screening masses, dispersion relations, spectral functions

$$\Sigma(P) = (ig)^2 C_F \sum\!\!\!\!\!\!\int_{\{K\}} \gamma^\mu S(K) \gamma^\nu D^{\mu\nu}(P-K)$$




- Spectral function: imaginary part of the propagator $S(\omega, p) = \dfrac{i}{\not{P} - \Sigma(\omega, p)}$
  - Non-analyticity of the propagator in the complex $\omega$ plane
- Non-analyticity of thermal QED: two poles (quasiparticles), one branch cut (Landau damping) — all on the real axis(!)
- Spectral sum rule: $\displaystyle\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \rho_f^{\mathrm{QED}}(\omega, p) = 1$



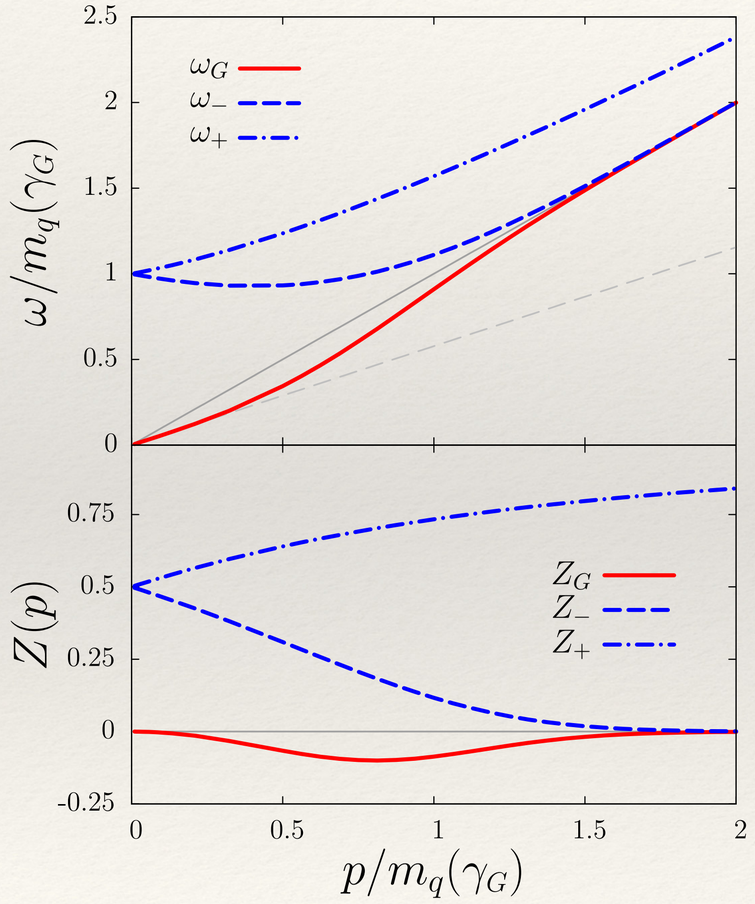
# A strongly coupled formalism of QCD: Gribov-Zwanziger quantization

- At finite temperature, Gribov-Zwanziger (GZ) action [Gribov 78; Zwanziger 89] provides a renormalizable framework to study confinement effects at the magnetic scale $g^2T$ — improvement over Faddeev-Popov action in the IR
  - Faddeev-Popov: $D^{\mu\nu}_{\rm FP}(P) = \frac{1}{P^2}\left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2}\right) \to$ IR divergent(!)
  - Gribov-Zwanziger: $D^{\mu\nu}_{\rm GZ}(P) = \frac{P^2}{P^4+\gamma_G^4}\left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2}\right) \to$ IR regular

    with $\gamma_G = \frac{d}{d+1}\frac{N_c}{4\sqrt{2}\pi} g^2T$ solved from Gribov gap equation [Zwanziger, PRD 76, 125014 (2007); Fukushima, NS, PRD 88, 076008 (2013)]
- Hard-thermal-loop (HTL) systematics: contributions from $P \ll K$ on the same order as tree-level ones, gauge invariant [Braaten, Pisarski, 90]
  - $iS^{-1}(P) = \not{P} - \Sigma(P) \equiv A_0\gamma_0 - A_S\hat{\boldsymbol{p}}\cdot\boldsymbol{\gamma}$

# GZ quantization: thermal quark dispersion relations & residues of poles

NS & Tywoniuk, PRL 114, 161601 (2015)



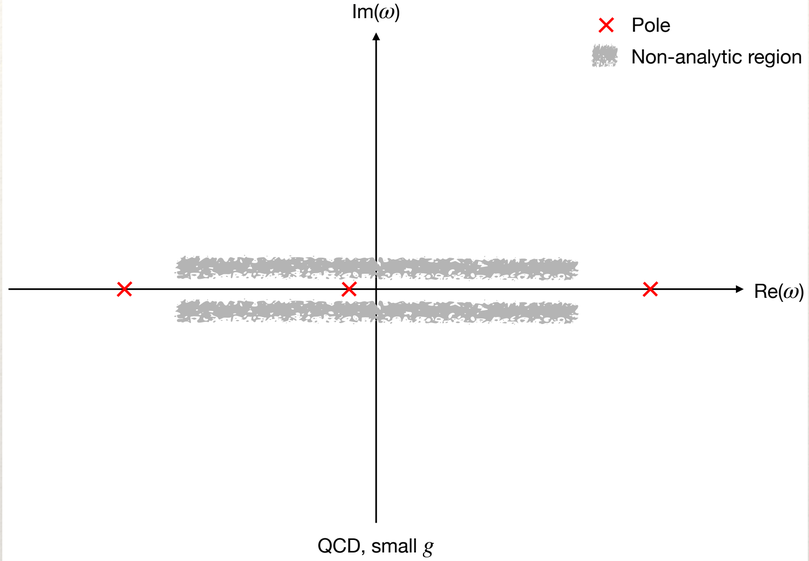

- Quasiparticles recovered — time-like; massless mode(!) — space-like
- Sound wave at small p: $\omega \sim v_s p$ with $v_s = 1/\sqrt{3}$
  - Hydrodynamic-like behaviors → long-range correlations incorporated
- Significance for heavy-ion phenomenology at RHIC and LHC energies
  - Viscosities: $\frac{\zeta}{\eta} \propto \left(\frac{1}{3} - c_s^2\right)$, strongly coupled liquid(!)

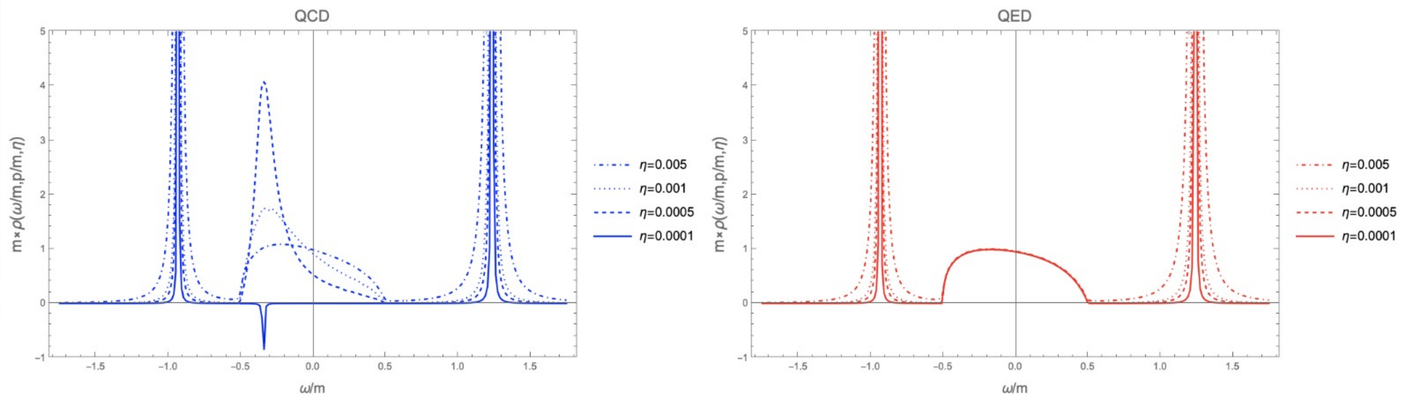
    [Florkowski, Ryblewski, NS & Tywoniuk, Acta Phys. Polon. B 47, 1833 (2016); PRC 94, 044904 (2016)]
- 1st evidence of positivity violation for thermal quarks
  - Genuine feature of non-Abelian theories, distinctive from QED(!)
  - Confinement effects ($\gamma_G \sim g^2 T$) indispensable for hot QCD at $T \to \infty$(!)
  - Strongly coupled QCD formalism(!)

# GZ quantization: non-analyticity & spectral sum rule

Du, NS & Tywoniuk, forthcoming

- GZ quantization enables tackling non-analyticity of QCD in the complex $\omega$ plane
  - Fundamentally different than QED
  - Massive & massless poles on the real axis — no conventional quasiparticles(?)
  - Continuous but non-analytic regions away from the real axis in the complex place
    - Cauchy-Riemann conditions — no branch cut(!), never seen before
- QCD spectral sum rule: $\oint \frac{d\tilde{\omega}}{2\pi} \rho_f^{\rm QCD}(\tilde{\omega}, p) = 1$, complexness of $\tilde{\omega}$(!)
  - Non-analyticity on the real axis not enough
    - $\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \rho_{\rm quark}(\omega, p) < 1$

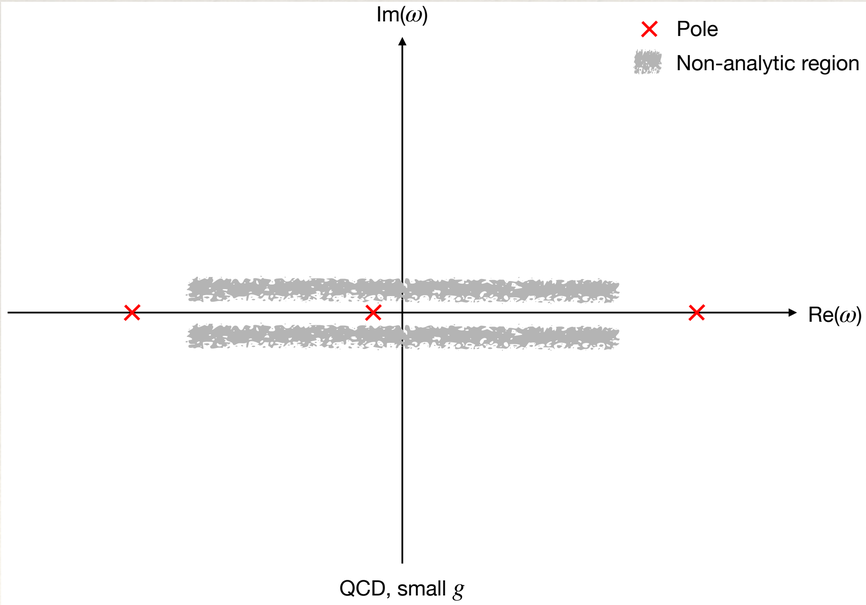


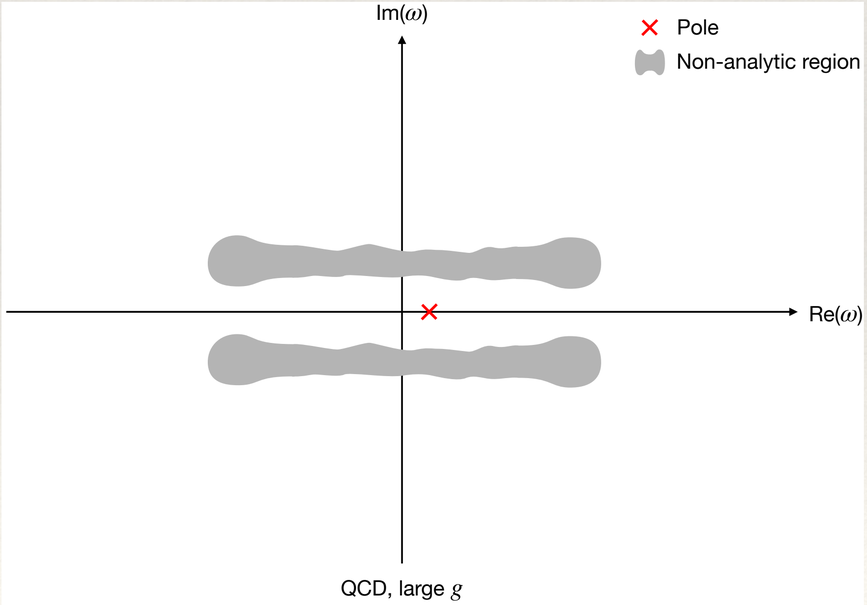



# An intrinsic transition from QCD spectral sum rule

Du, NS & Tywoniuk, forthcoming

- Spectral sum rule sets a standard of dispersion relations as $g$ varies
  - $\oint \frac{d\tilde{\omega}}{2\pi} \rho_f^{\mathrm{QCD}}(\tilde{\omega}, p) = 1$ contains an intrinsic transition: 3 modes (small $g$) — 1 mode (large $g$)
  - ?!QCD transition?!



# Conclusions and outlook

- Gribov-Zwanziger quantization provides a strongly coupled formalism of QCD
  - Massless mode & positivity violation: long-range correlations from confinement effects
  - Spectral sum rule: self-consistency, i.e. quarks are fermions
- Questions to address:
  - Physical meaning of the continuous and non-analytic regions
  - Microscopic picture of the intrinsic transition
  - GZ quantization is topological in nature, any impacts to heavy-ion collisions?