Closing the gap between functional QCD method and experimental observables

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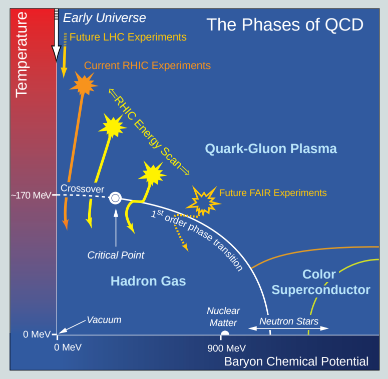
based on arXiv:2307.xxxx

at **Chirality 2023**, July 19th, 2023

The phenomena of QCD can be mapped into one phase diagram:

- The main character of QCD phase transition is chiral phase transition
- Fine structures to classify phases like inhomogeneous phase, chiral spin symmetric phase, color superconductivity phase
- The chiral phase transition driven by different combination of temperature and chemical potential, and connected with crossover at low density by critical end point (CEP)



[1]The frontiers of nuclear science, A long range plan[J]. 2008.

Dyson-Schwinger equations (DSEs) and functional renormailization group (fRG) approach are the nonperturbative approach in continuum QCD which contain the features of both confinement and chiral symmetry breaking.

DSEs are the equations of motion in quantum field theory:

$$\frac{\partial S}{\partial \phi} = J$$

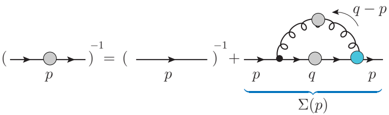
fRG is based on the idea of homotopy:

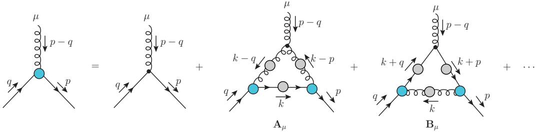
$$f(\lambda) = \int_0^\infty dx e^{-\lambda x^2} \to \frac{\partial f(\lambda)}{\partial \lambda} = -\int_0^\infty dx x^2 e^{-\lambda x^2} = -\frac{f(\lambda)}{2\lambda}$$

The truncation of the functional QCD methods

The truncation is required in functional QCD methods as the equations are not closed.

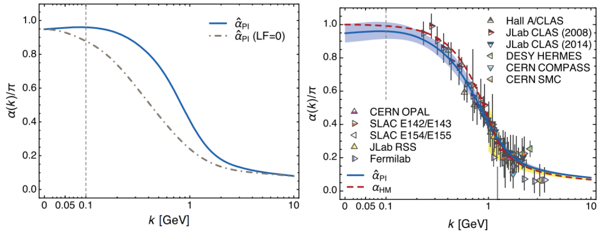
- How to generally evaluate the truncation?
- How to reduce the higher order correction and make the truncation controllable?





Effective Charge

The hints from effective charge:



D. Binosi et al, PRD96, 054026 (2017); A. Aguilar et al, PRD80, 085018 (2009).
A. Deur et al, Prog. Part.Nucl. Phys. 90, 1 (2016);

- The running of the coupling can be partly captured by the running of two point functions (STIs)
- The efforts beyond props are "perturbative", and can be captured by the correction of vertex.

fRG equation:

$$k\partial_k \Gamma_k = \frac{1}{2} Tr\left[k\partial_k R_k(p) \cdot G_k(p)\right] \equiv \beta_\Gamma$$

Identifying $k = 1/z$, the equation becomes Holographic equation:

$$\left(-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U_J(z)\right)\psi(z) = \mathcal{M}^2\psi(z)$$

$$U_J(z) = -\frac{z^{-1-\delta}}{\Gamma_z^{(n)}}\partial_z\left(z^\delta \beta_\Gamma^{(n)}\right)$$

[1] **FG**, Masatoshi Yamada, PRD 106, 126003(2022).

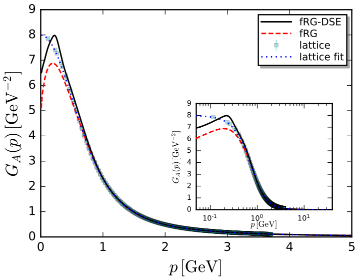
The hints from the relation between fRG and holographic equation:

- AdS/CFT correspondence is hold through the fixed point since only there the potential can be uniquely determined.
- The bound states spectrum (Regge trajectory, etc) can be studied through AdS/CFT correspondence, and it is also a useful tool for studying phase transition(universality class).

The fixed point simplifies the truncation:

- The fixed point defines an "perturbative" expansion in infrared.
- Only the running of propagator and vertex is relevant.
- It is possible to construct a minimal truncation in quark gap equation which can describe both the vacuum and the finite T and μ physics

The Yang-Mills sector is relatively separable. One can apply the data in vacuum:



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Compute the difference between finite T/μ and vacuum:

$$D^{-1}_{\mu\nu}(k)|_{T,\boldsymbol{\mu}} = D^{-1}_{\mu\nu}(k)|_{0,\mathbf{0}} + \Delta\Pi^{\mathrm{gauge}}_{\mu\nu}(k) + \Delta\Pi^{\mathrm{qrk}}_{\mu\nu}(k),$$

In Landau gauge:

$$\Gamma^{\mu}(q,-p) = \sum_{i=1}^{8} \lambda_i(q,-p) P^{\mu\nu}(q-p) \mathcal{T}_i^{\nu}(q,-p)\,,$$

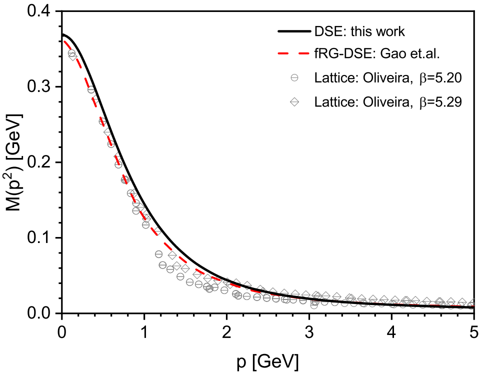
The optimised truncation:

$$\mathcal{T}_1(p,q) = -i\gamma^{\mu}\,, \mathcal{T}_4^{\mu}(p,q) = (\not{p}+\not{q})\gamma^{\mu}\,,$$

$$\lambda_1(p,q) = F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

$$\lambda_4(p,q) = \left[Z(k^2)\right]^{-1/2} \frac{B(p^2)-B(q^2)}{p^2-q^2}$$

With all quantities are expressed by the running of two point functions, The Quark Mass function:



QCD phase diagram

A first estimation of QCD phase transition line:

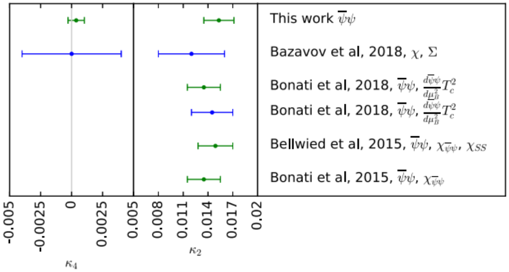
$$\frac{d(P_N - P_W)}{dT} = \left(\frac{\partial P_N}{\partial \mu} - \frac{\partial P_W}{\partial \mu}\right)\frac{\partial \mu}{\partial T} + \left(\frac{\partial P_N}{\partial T} - \frac{\partial P_W}{\partial T}\right) = 0\,.$$

Therefore, the phase transition line should bend down typically:

$$\frac{\partial \mu}{\partial T} = -\frac{s_N - s_W}{n_N - n_W} < 0\,.$$

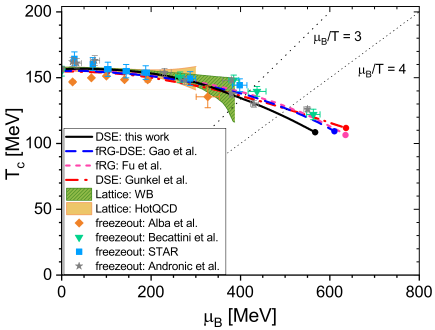
The line can be parametrized as:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$



QCD phase diagram

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

- $T_c = 157\,\mathrm{MeV}$ and $\kappa \sim 0.017$
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (550, 650)$ MeV
- $\sqrt{s_{NN}} \in 2 \sim 4$ GeV.

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and Jan M. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

- with no model parameters included
- subleading term: the hadron resonance channel

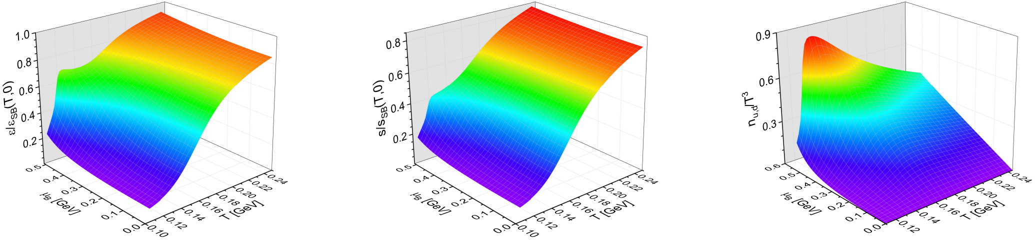
Currently, the functional QCD approaches can only calculate the quark potential directly, while the gluon sector still awaits further investigations.
One may incorporate the lattice QCD simulation at $\mu = 0$ here to combine the advantages of the two methods. One can calculate the quark number densities $\{n_q\}$ at finite chemical potential and obtain the pressure by:

$$P(T, \boldsymbol{\mu}) = P_{Latt.}(T, \mathbf{0}) + \sum_q \int_0^{\mu_q} n_q(T, \mu)\, d\mu$$

[1] private comm. with N. Wink and J. M. Pawlowski
[2] P. Isserstedt, C.S. Fischer and T. Steinert, PRD103 (2021) 054012
[3] **FG**, Yuxin Liu, PRD 94 (2016) 9, 094030
[4] H. Chen, M. Baldo, G. F. Burgio, and H.-J. Schulze, PRD86(2012)045006

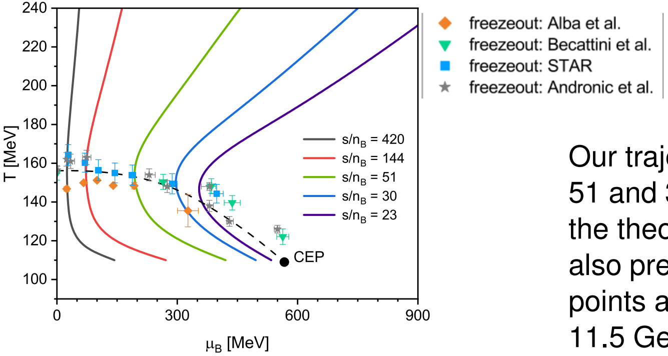
QCD thermodynamic properties

The calculated number density, entropy and energy density in the plane of temperature and chemical potential:



As getting closer to CEP, the slopes of the thermodynamics quantities become sharper.

isentropic trajectories in the up to date scheme:



Our trajectories for $s/n_B = 420$, 144, 51 and 30 which values are chosen in the theoretical studies,
also precisely meet with the freezeout points at $\sqrt{s_{\text{NN}}} = 200$, 62.4, 19.6 and 11.5 GeV, respectively.

Summary:

- A simple truncation which captures the main character of QCD and accessible for the thermodynamics quantities.
- CEP Estimation at around $(T, \mu_B) \sim (110, 600)$ MeV, and EoS that is consistent with the previous studies.

In the future:

- Incorporating the EoS of QCD into hydrodynamics simulations;
- Studying the global properties of QCD matter generated in HIC, for instance, the transport coefficients and the polarization structure.
- Investigating the spectral function of QCD states at finite T and μ.

Thank you!