

Light-nuclei production and QCD critical point

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In collaboration with **Koichi Murase², Shian Tang³, Shujun Zhao³, Huichao Song³**

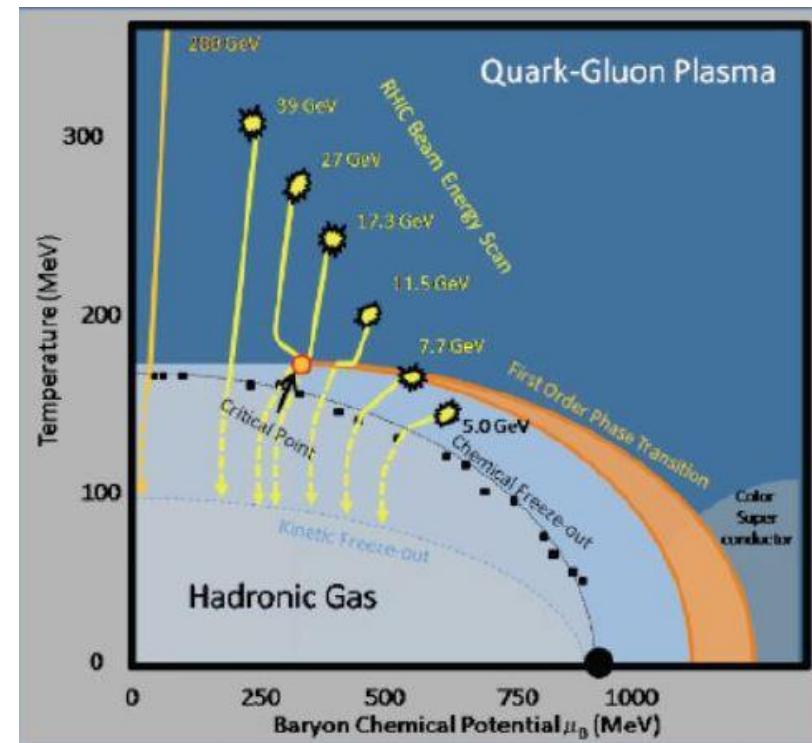


Lanzhou Univ.¹ , YITP, Kyoto Univ.², Peking Univ.³

The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion
Collisions, Jul 15-19, 2023@UCAS

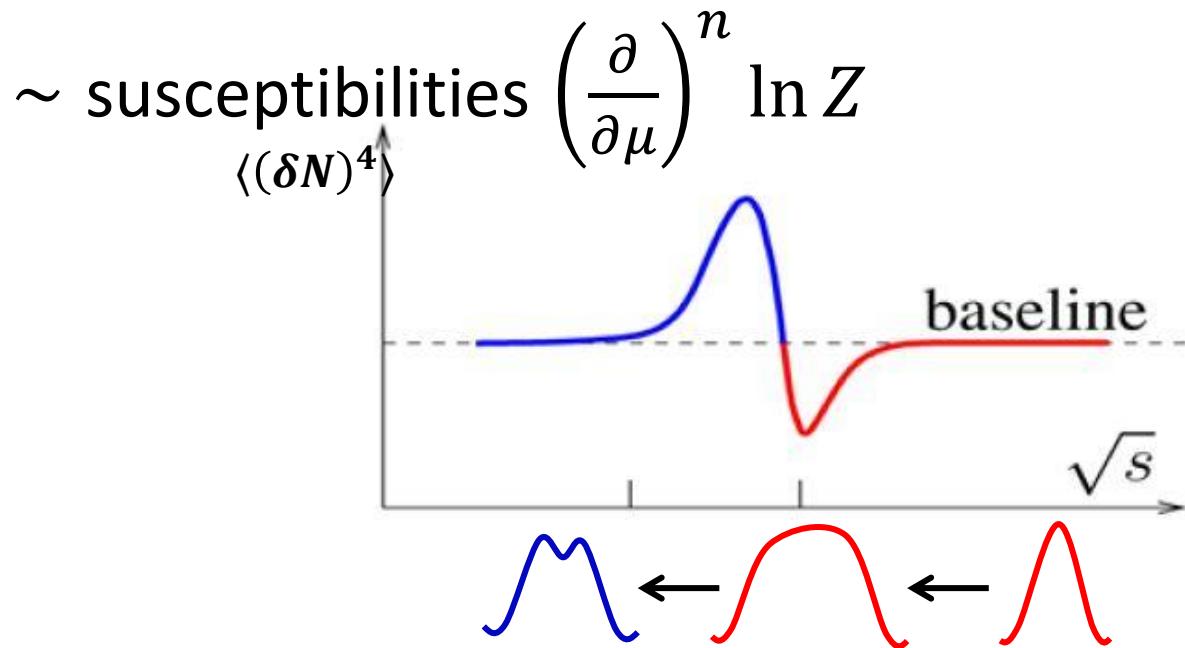
QCD phase diagram

- **Lattice QCD** (small μ_B finite T):
 - Crossover
 - **Effective models** (large μ_B)
 - 1st order phase trans.
- **Critical point**
- Lattice QCD: sign problem at large μ_B
 - Effective models: parameters dependent
- **Heavy-ion collisions :**
- tuning $\sqrt{s_{NN}}$, mapping $T - \mu$ phase diagram:
RHIC(BES),NICA,FAIR,J_PARC....

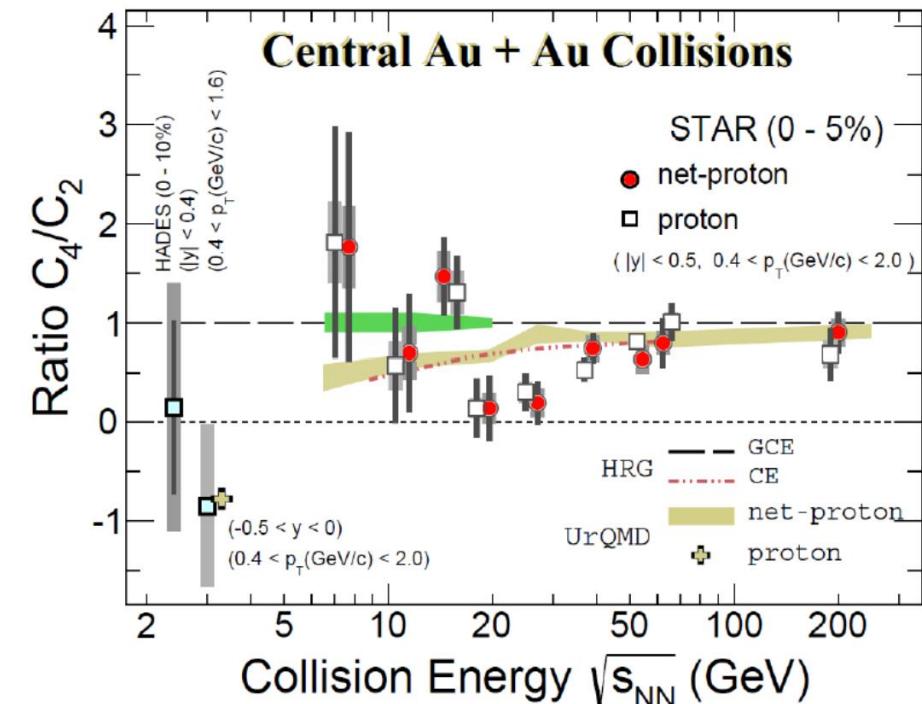


Net-proton fluctuations near critical point

- Characteristic feature of critical point:
 - long range correlation
 - large fluctuations
- Non-monotonicity** of Net-Proton Cumulant



M.Stephanov, PRL 107,052301

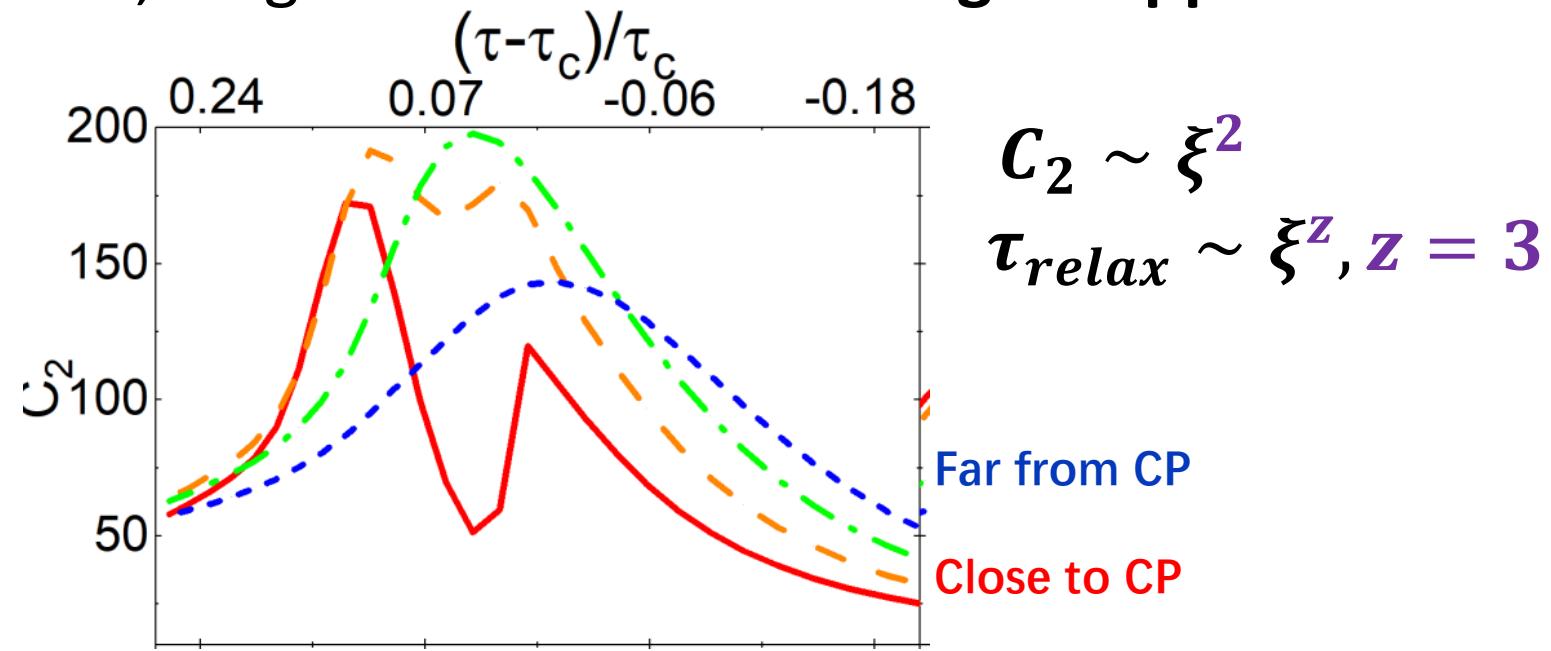
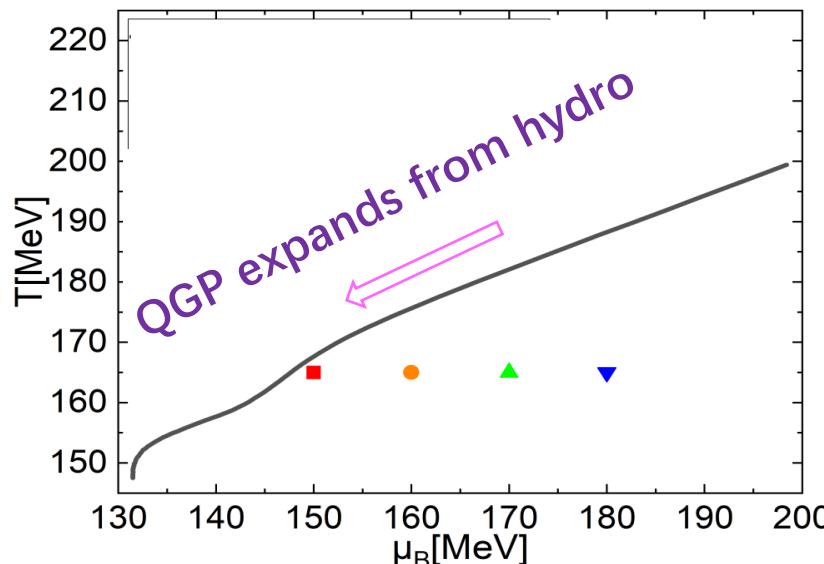


STAR, PRL 126,092301
STAR,PRL 128,202303

Fluctuations is non-trivial in expanding QGP

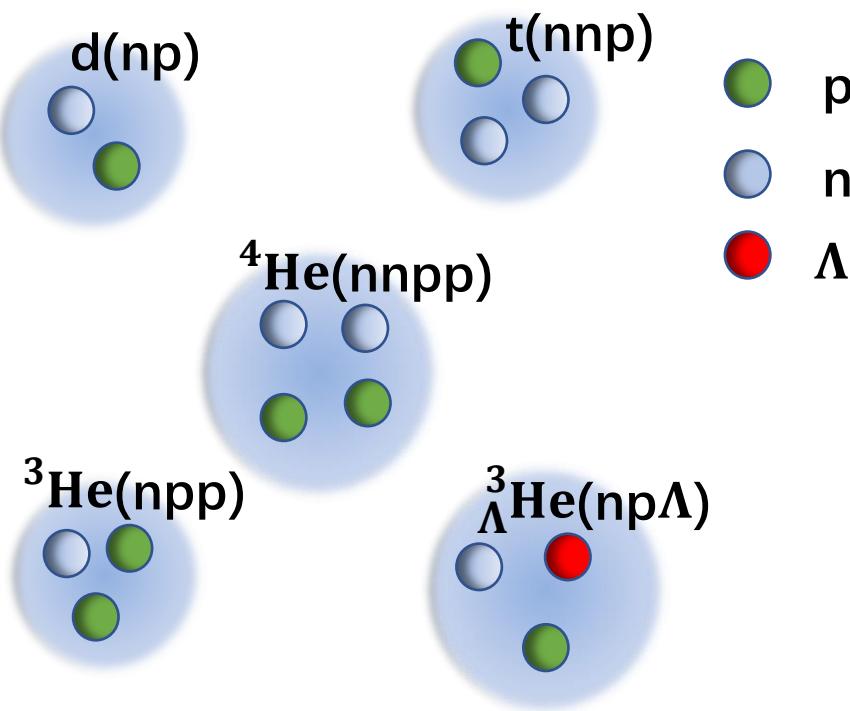
S.Tang, SW, H.Song, 2303.15017

- Hydro background cools down => **Critical Slowing Down**.
- Critical slowing down effects suppress the fluctuations
- Fireball closer to critical point, Larger fluctuations but **larger suppression**

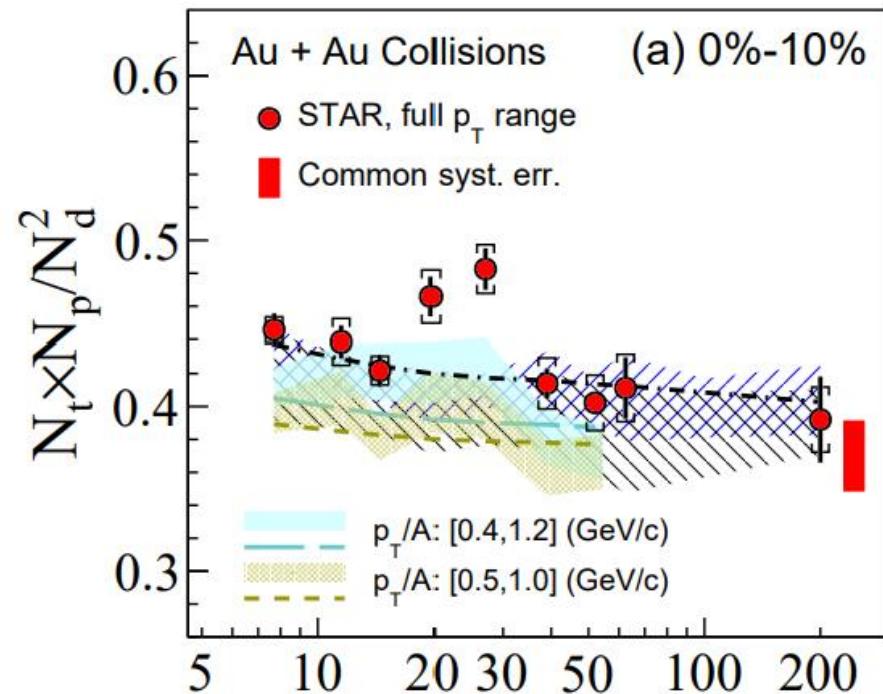


Other observable: Light Nuclei?

Light Nuclei Production

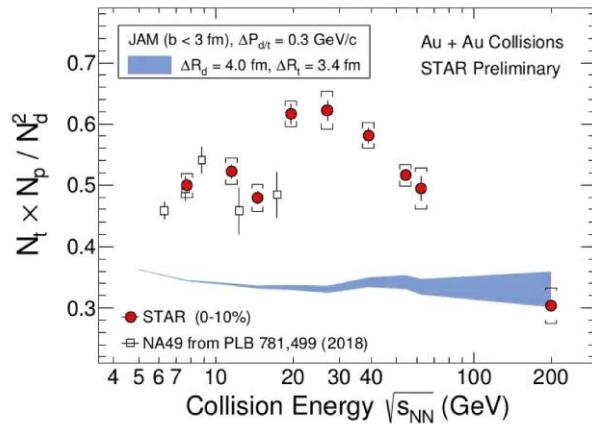


STAR Collaboration, PRL 130.202301

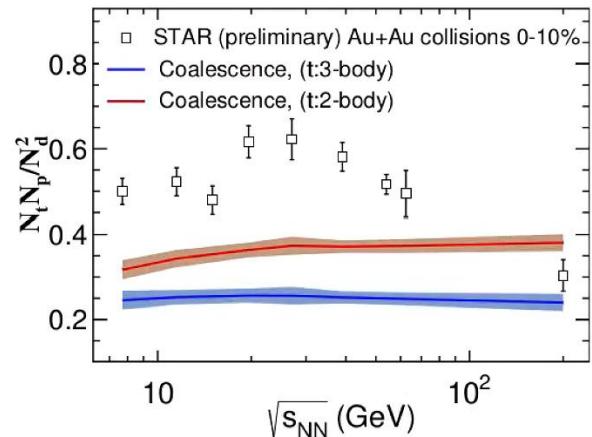


- Light nuclei produced at **late stage** of heavy-ion collisions
- **Non-monotonic behavior** also been observed

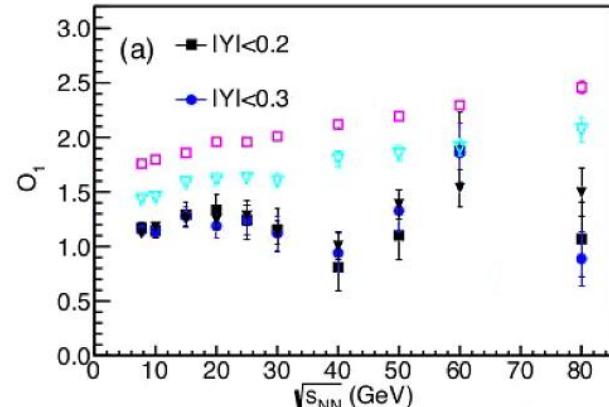
Dynamical models on Light-Nuclei



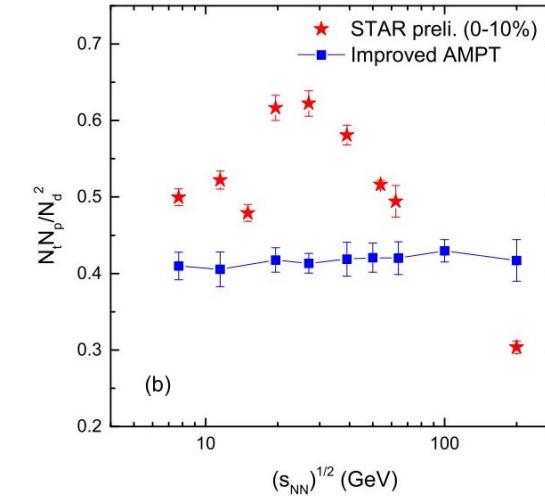
Hui Liu et al., PLB (2020)



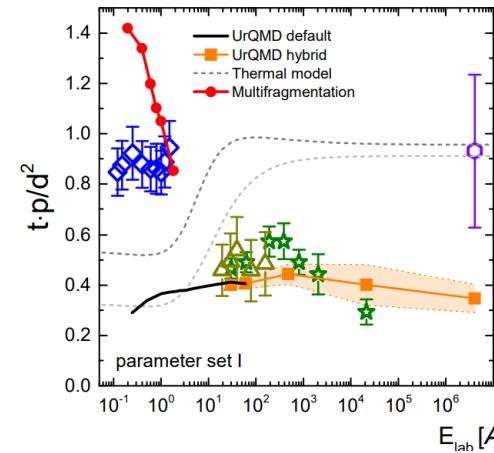
W. Zhao et al., PRC (2018)



X.Deng et al., PLB (2020)



K-J.Sun et al., PRC (2021) ...
K-J.Sun et al., Phys. Lett. B, 781:499–504(2018)
K-J.Sun et al., Phys. Lett. B, 774:103–107(2017)

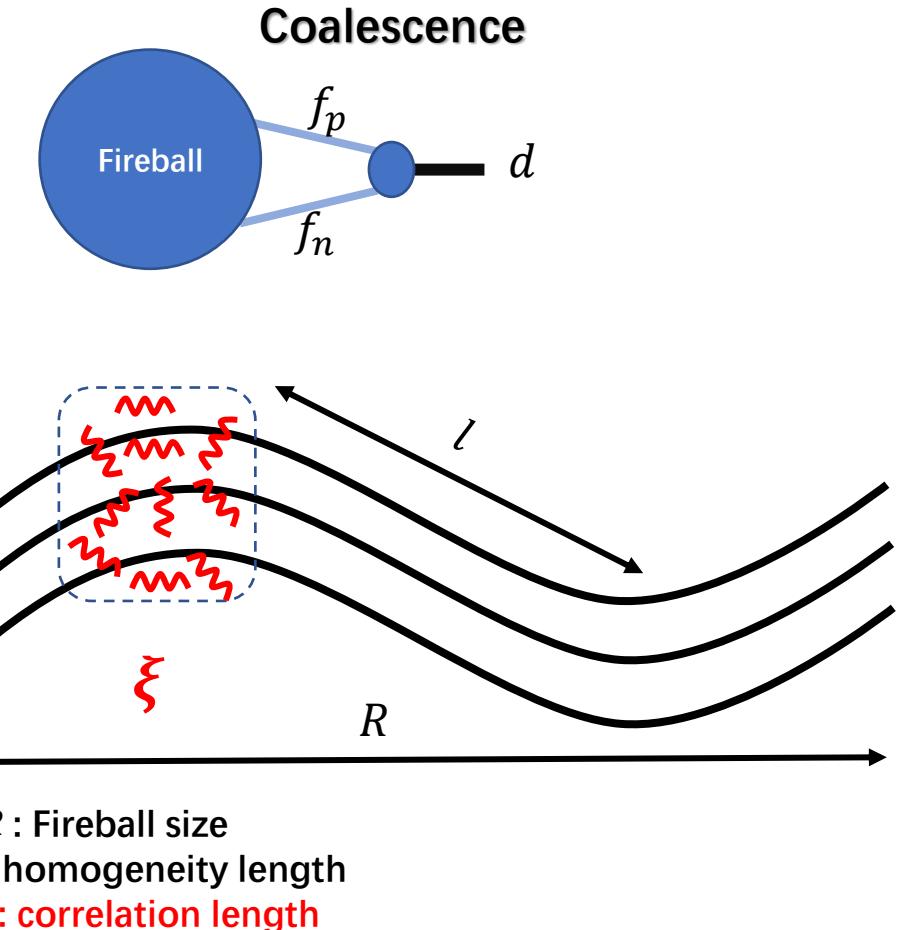


P.Hillmann et al., 2109.05972

And others....

Can light nuclei detect critical effects?

- **Light-nuclei production:**
phase-space, nucleons interaction
Fireball size R , homogeneity length l



- **Homogeneity:**
Nucleons close to each other in r space have similar momentum p
 \Rightarrow Homogeneity length $l \sim 1/\partial_\mu u^\mu$

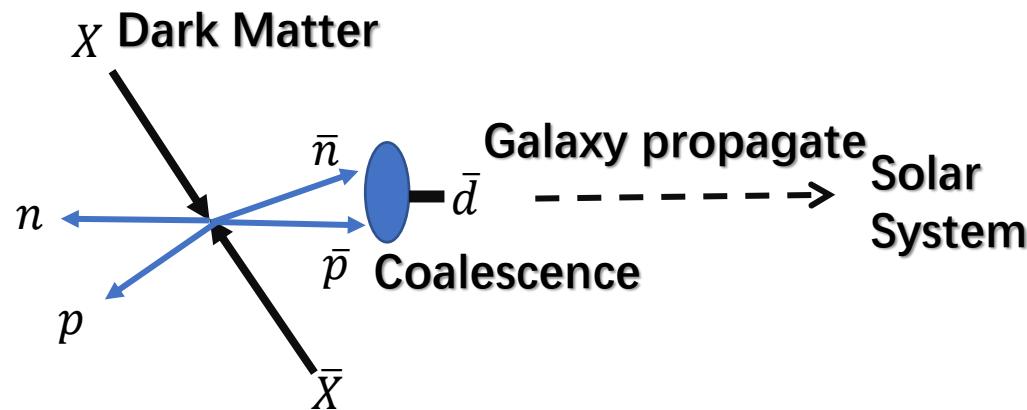
R.Scheibl, U.Heinz, PRC 59, 1585

- **When not so close to critical point:**
 - Fireball size R , homogeneity length $l \gg \xi$
 - **Background is large**, comparing critical signal

Light Nuclei Yield Ratio (Background+~~Critical~~):

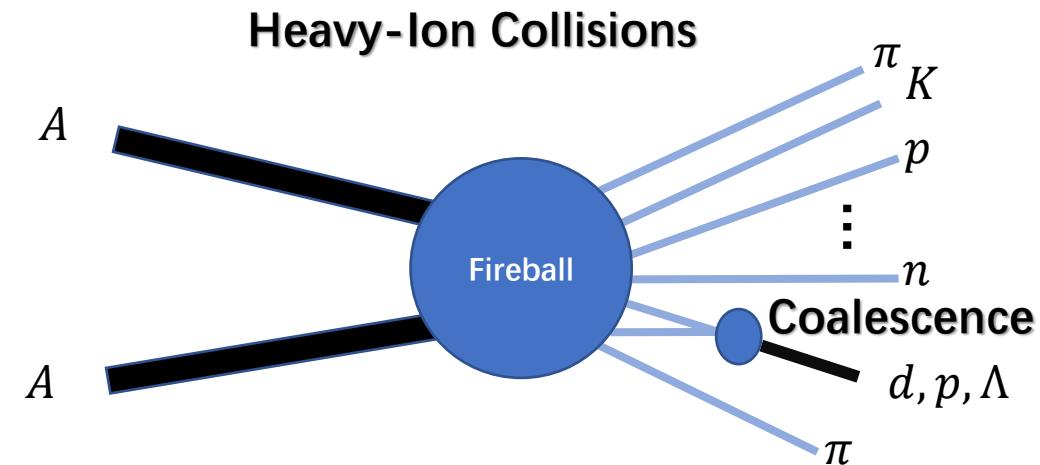
Suppress the background

Coalescence is widely used model



Anti Light nuclei as Indirect
detection of Dark Matter

See N.Fornengo et al., JCAP 09
(2013) 031 for review



Coalescence in Heavy-Ion Collisions

- quark + quark \rightarrow hadron
- S quark \rightarrow Lambda polarization
- nucleon + nucleon \rightarrow light nuclei

R.J.Fries et al., PRC 68.044902

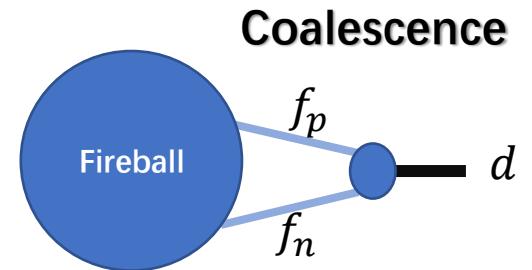
L.-W.Chen et al., PRC 68.017601

X.-L. Sheng et al., PRD 102. 056013

Coalescence model

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

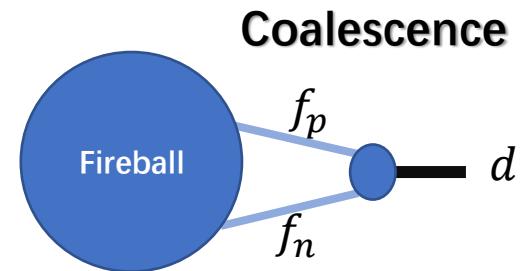
$$N_A = g_A \int \left[\prod_i^A d^3\mathbf{r}_i d^3\mathbf{p}_i f(\mathbf{r}_i, \mathbf{p}_i) \right] W_A(\{\mathbf{r}_i, \mathbf{p}_i\}_{i=1}^A)$$



Coalescence model

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

$$N_A = g_A \int \begin{array}{|c|c|} \hline \text{Phase-space distribution} & \text{Wigner function} \\ \hline \end{array}$$



- Two ingredients in Coalescence model:
 - Constituent particle distribution
 - Wigner func.(probability to produce the light nuclei): Only depends on the relative distance in phase space $x_p - x_n$ NOT $(x_p + x_n)/2$

Phase-space dis. to particle production

Example 1: Gaussian

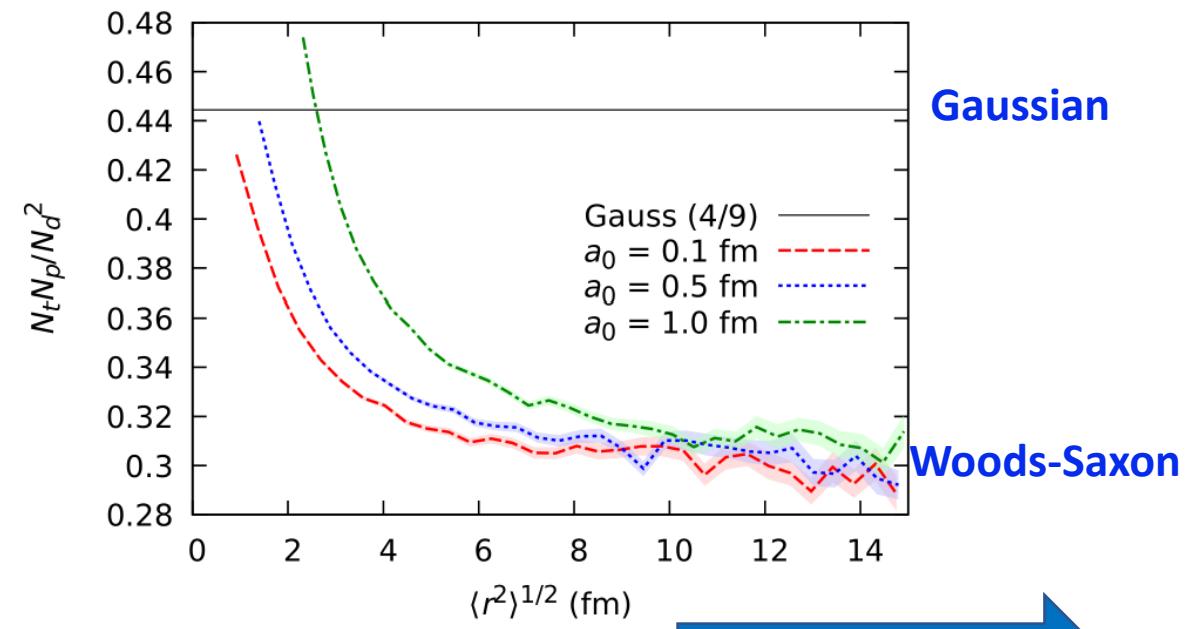
$$f(\mathbf{r}, \mathbf{p}) = \frac{\rho_0}{(2\pi mT)^{3/2}} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right) \exp\left(-\frac{\mathbf{p}^2}{2mT}\right)$$

$$N_A = g_A \int \left[\prod_i^A d^3\mathbf{r}_i d^3\mathbf{p}_i f(\mathbf{r}_i, \mathbf{p}_i) \right] W_A(\{\mathbf{r}_i, \mathbf{p}_i\}_{i=1}^A)$$

Example 2: Woods-Saxon

$$f(\mathbf{r}, \mathbf{p}) = \frac{\rho_{WS}}{1 + \exp \frac{r - R_0}{a_0}} \cdot \frac{1}{(2\pi mT)^{3/2}} \exp\left(-\frac{\mathbf{p}^2}{2mT}\right)$$

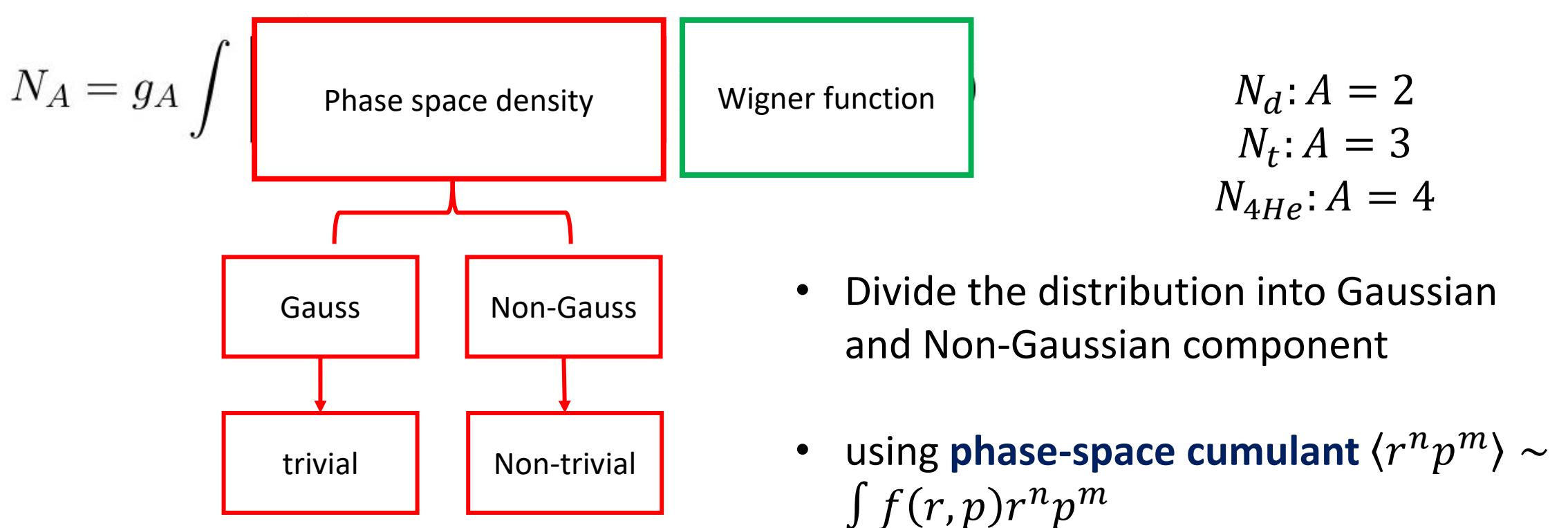
SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



Gaussian form distribution of nucleon phase-space is trivial

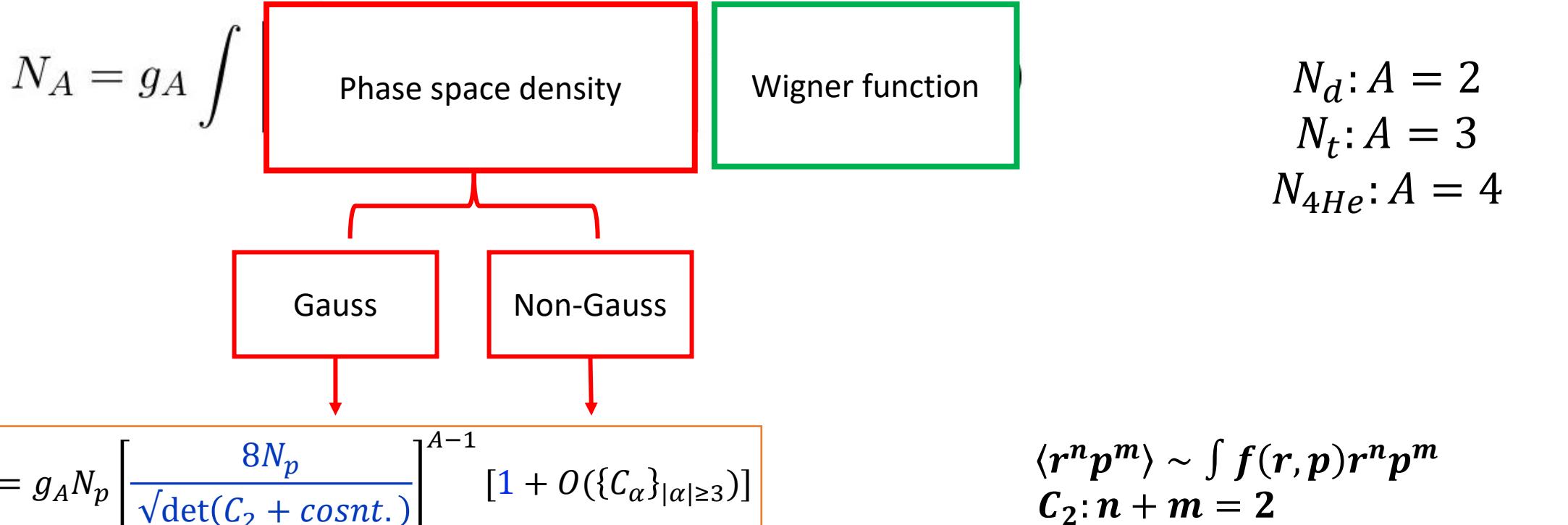
Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



N_d, N_t, N_{4He} have similar behavior in case of Gaussian phase-space density

Similar result with: R.Scheibl, U.Heinz, PRC 59, 1585; K.Blum, M.Takimoto, PRC 99, 044913

Phase-space cumulant in light nuclei

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

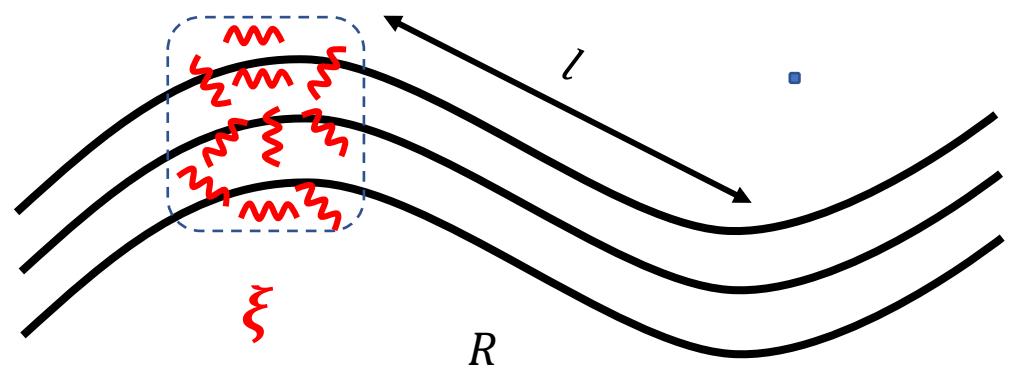
$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(C_2 + \text{const.})}} \right]^{A-1} [1 + O(\{C_\alpha\}_{|\alpha| \geq 3})]$$

phase-space cumulant $\langle r^n p^m \rangle \sim \int f(r, p) r^n p^m$

2nd phase-space cumulant

$$C_2 = 2 \begin{pmatrix} \#\langle rr^T \rangle & \langle rp^T \rangle \\ \langle pr^T \rangle & \#\langle pp^T \rangle \end{pmatrix}$$

$$\sim 2 \begin{pmatrix} \#R_{\text{fireball}}^2 & \#l_{\text{homoge}} \\ \#l_{\text{homoge}} & \#T_{fo} \end{pmatrix}$$



R : Fireball size

l : homogeneity length

ξ : correlation length

Relevant scales in light-nuclei yield N_A : Fireball size R_{fireball} , homogeneity length l_{homoge} and freeze-out temperature T_{fo}

Example: Anisotropic flow (Blast-Wave)

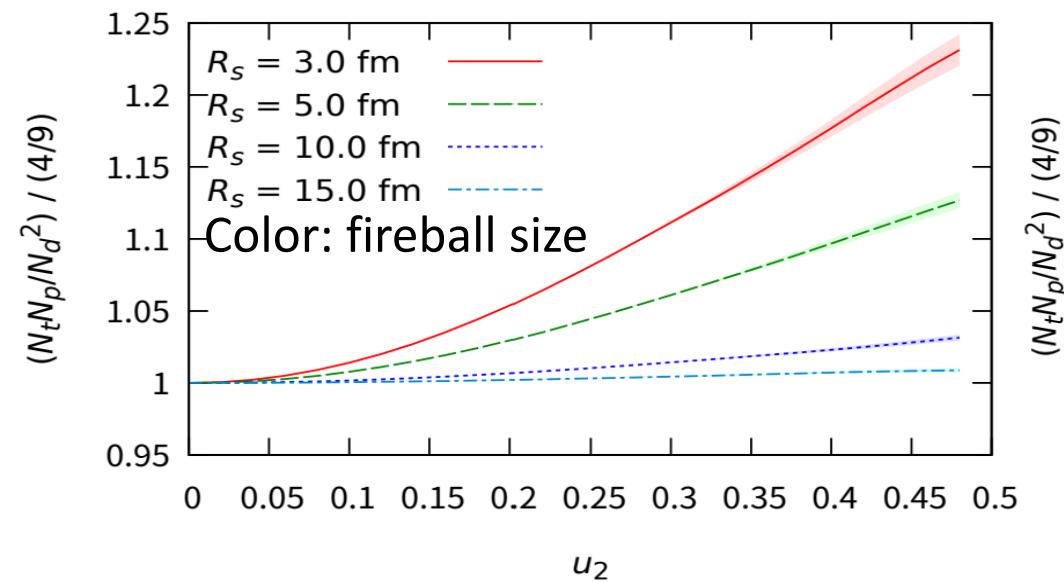
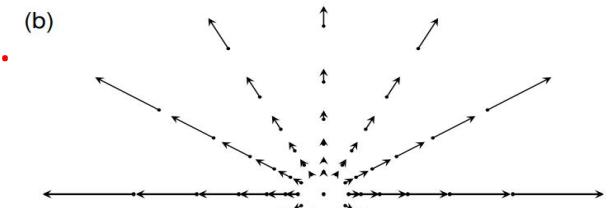
SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905

phase-space distribution

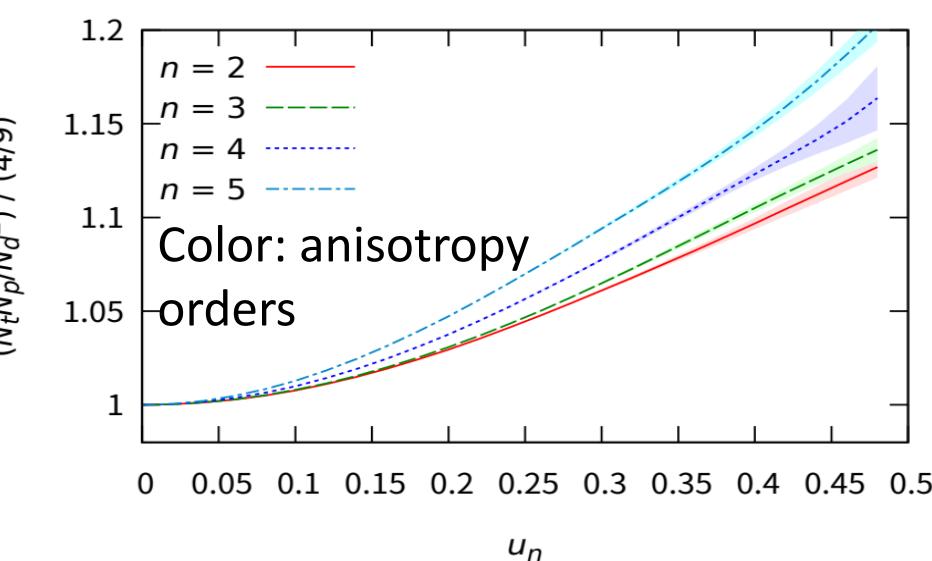
Blast-wave flow P.Huovinen et al, PLB 503, 58(2001)

$$f(\mathbf{r}, \mathbf{p}) = \frac{\rho_0}{(2\pi mT)^{3/2}} e^{-\frac{\mathbf{r}^2}{2R_s^2}} \exp\left(-\frac{m}{2T} \left[\frac{\mathbf{p}}{m} - \mathbf{v}(\mathbf{r})\right]^2\right)$$

$$\mathbf{v}(\mathbf{r}) = \frac{1}{R_s} (r_x, r_y, 0)^T (1 + 2u_2 \cos 2\phi_s)$$



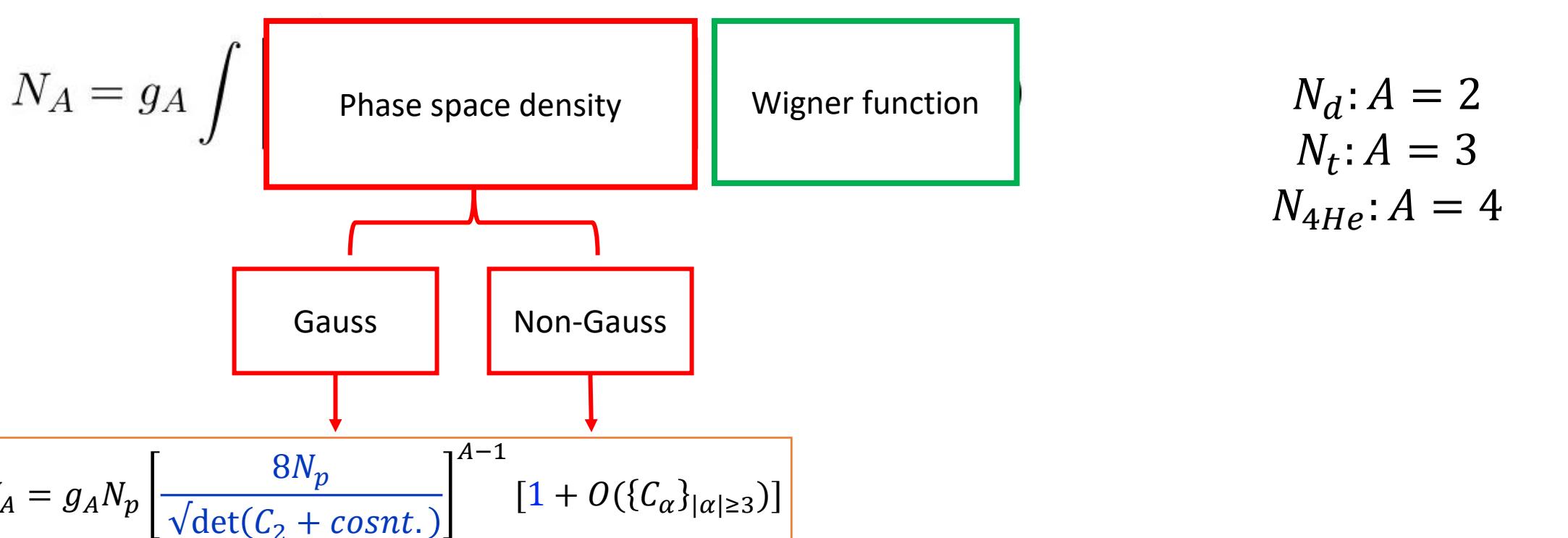
Anisotropic effects is negligible when fireball size is large



Momentum anisotropy increase the ratio

Light-nuclei yield (Background)

SW, K.Murase, S.Tang, H.Song, Phys.Rev.C.106.034905



$$\begin{aligned} N_d &: A = 2 \\ N_t &: A = 3 \\ N_{^4He} &: A = 4 \end{aligned}$$

$N_d, N_t, N_{^4He}$ have similar behavior in case of Gaussian phase-space density

Light Nuclei Ratio Near QCD Critical Point: (Background+Critical)

Critical fluctuations δf in light nuclei

SW, K.Murase, S.Zhao, H.Song, in preparation

Introduce critical fluctuations δf

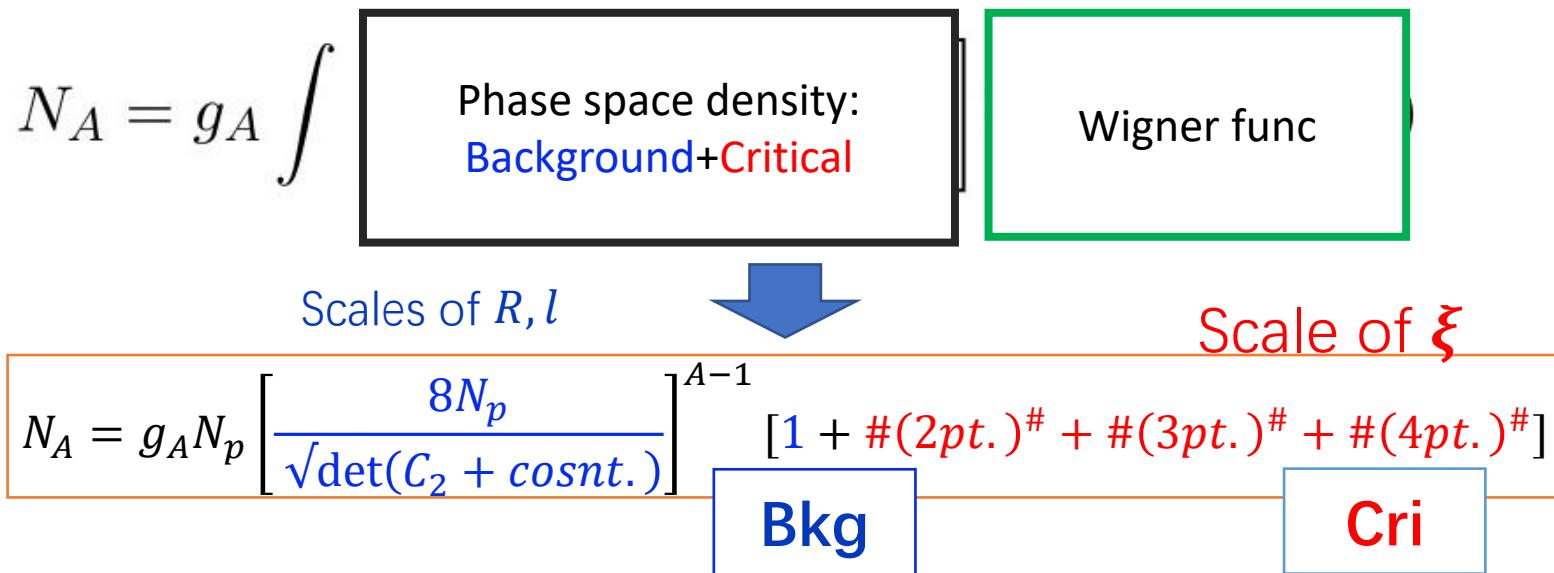
$$N_A \sim \langle (f_0 + \delta f)^A \rangle_\sigma \sim f_0^A + \langle (\delta f)^2 \rangle_\sigma^{\beta_2} + \langle (\delta f)^3 \rangle_\sigma^{\beta_3} + \langle (\delta f)^4 \rangle_\sigma^{\beta_4} + \dots$$

Bkg	2-point correlator	3-point correlator	4-point correlator
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- N_A : includes contribution from **2, 3, ... A-point critical correlator**
- **Contribution hierarchy:** $f_0^A \gg \langle (\delta f)^2 \rangle_\sigma^{\beta_2} \gg \langle (\delta f)^3 \rangle_\sigma^{\beta_3} \gg \dots \gg \langle (\delta f)^A \rangle_\sigma^{\beta_A}$

Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, in preparation



N_A share a analogous structure $N_A \propto [\dots]^{A-1} [Bkg + Cri]$ => Construct ratios of N_A suppress *Bkg* and highlight *Cri*

$$\begin{aligned} \tilde{R}(A, B) &= \text{Ratio}(N_t, N_d)\text{-statistical factor} \\ &\sim \mathcal{O}(\xi) \end{aligned}$$

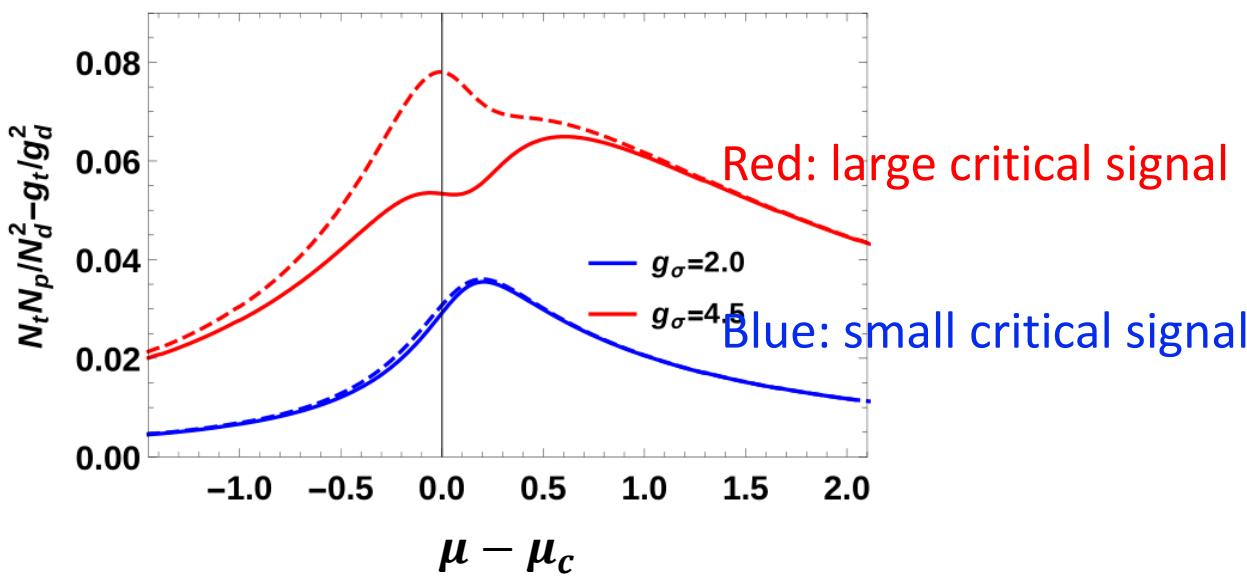
$$\begin{aligned} \tilde{R}(A, B, C) &= \text{Ratio}(N_t, N_d)\text{-}\#\text{Ratio}(N_t, N_d, N_{4He}) \\ &\sim \mathcal{O}(\xi) \end{aligned}$$

Example: near critical regime

SW, K.Murase, S.Zhao, H.Song, in preparation

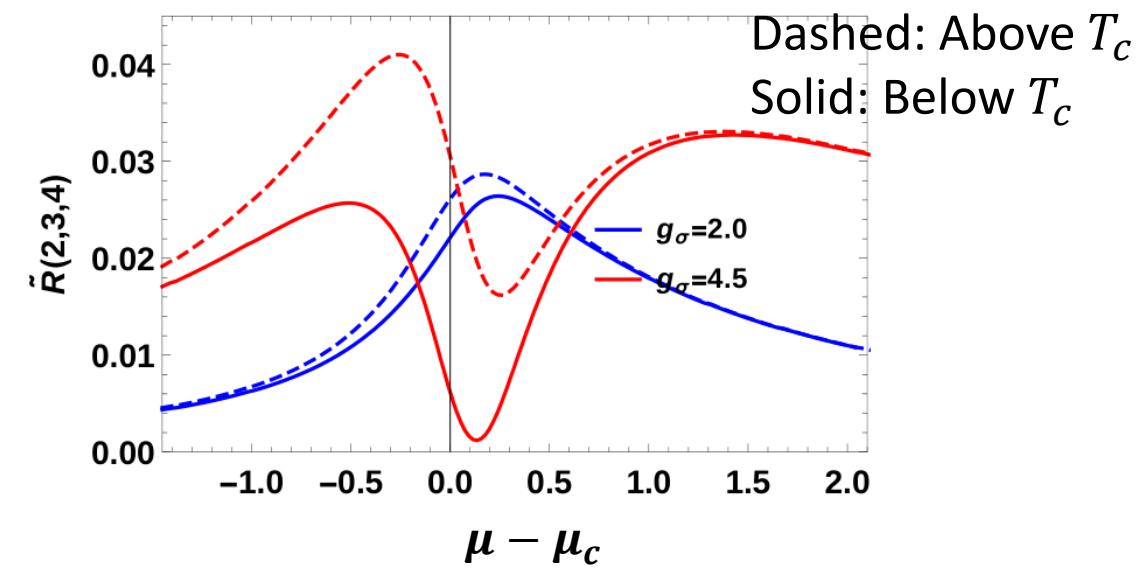
Ratio 1

Ratio(N_t, N_d)-statistical factor
 $\sim 2\text{pt} - 3\text{pt} - (2\text{pt})^2$



Ratio 2

Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})
 $\sim 2\text{pt} - 4 (2\text{pt})^2$



Light nuclei ratios have a peak near critical point μ_c , also have double peak because of $(2\text{pt.})^2$ when the critical effect is large

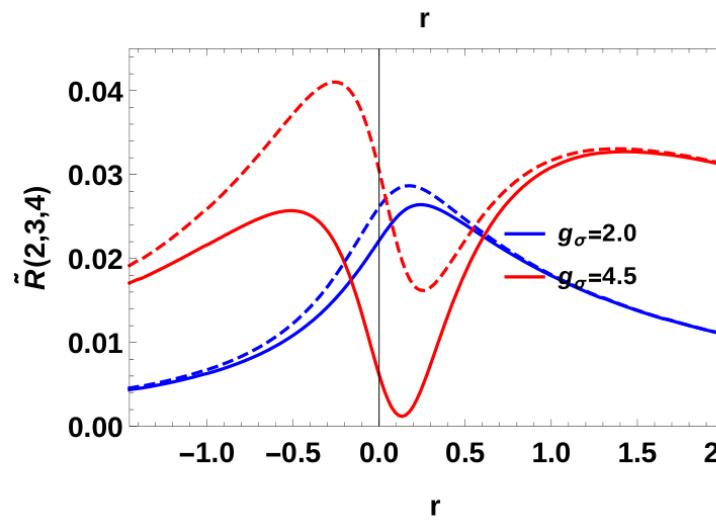
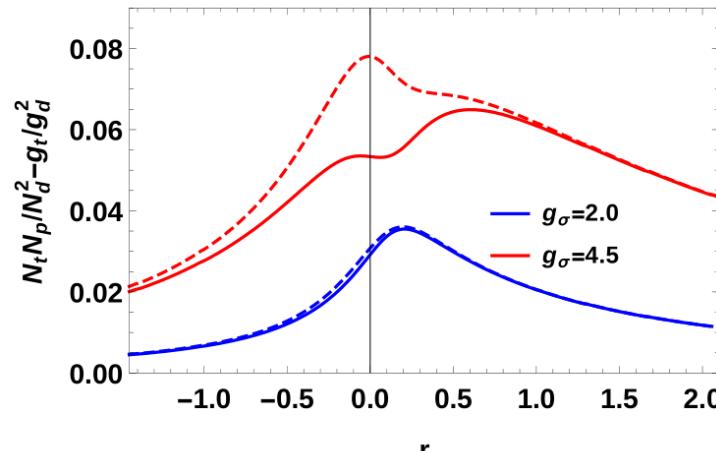
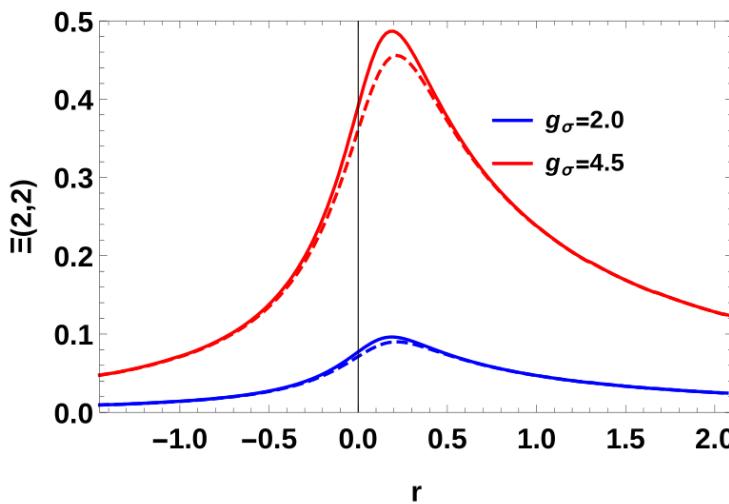
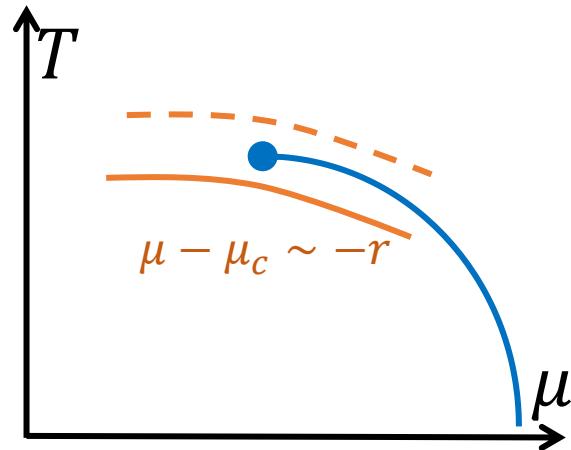
Conclusion and Outlook

- $N_d, N_t, N_{^4He}$ depends on fireball size, homogeneity length, freeze temperature in analogous way when nucleon distribution close to Gaussian, because Wigner function depends on relative distance
- Construct the ratios to suppress the background effects
- Long range correlation results a peak, and the square of 2-point correlation induces a double peak
- Non-critical EbyE in light-nuclei: K.Murase, ATHIC2023

Backup

Example: in the Ising critical regime

SW, K.Murase, S.Zhao, H.Song, in preparation



Ratio(N_t, N_d)-statistical factor

$\sim 2\text{pt} - 3\text{pt} - (2\text{pt})^2$

Ratio(N_t, N_d)-Ratio(N_t, N_d, N_{4He})

$\sim 2\text{pt} - 4 (2\text{pt})^2$