



華中師範大學
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Drag force in a rotating background

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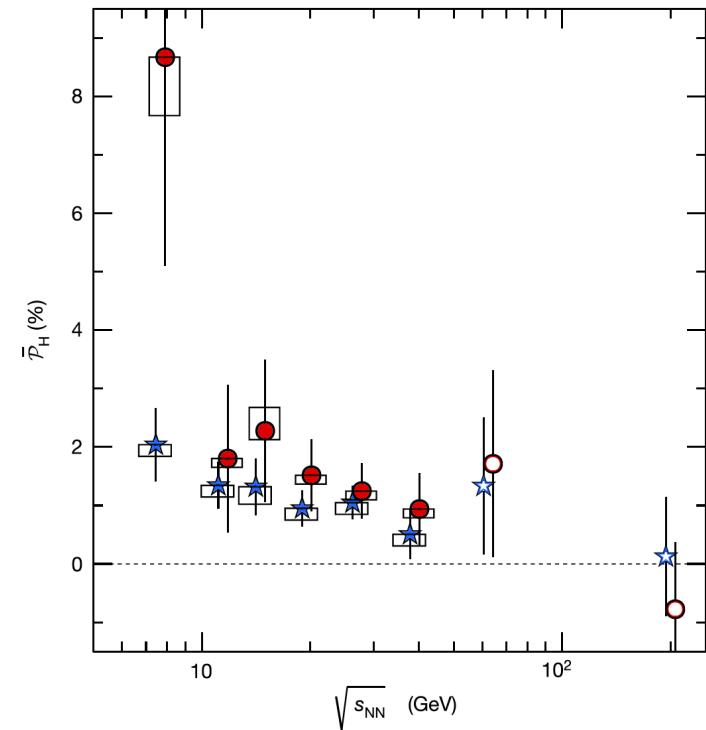
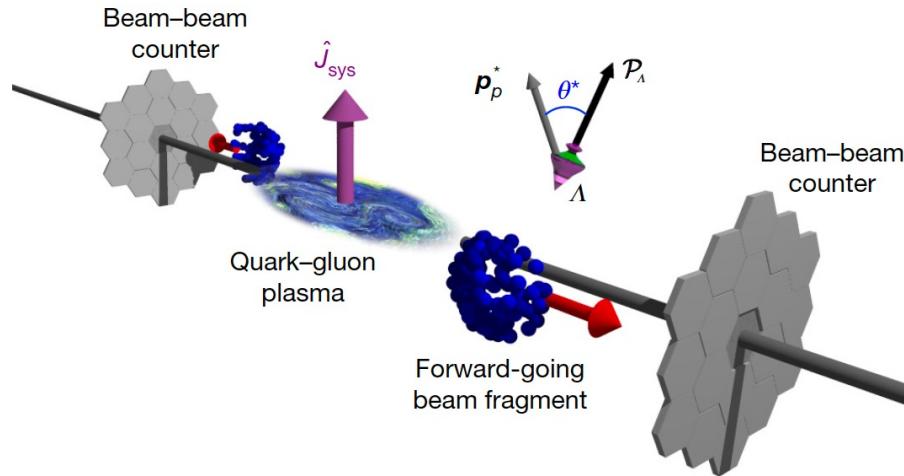
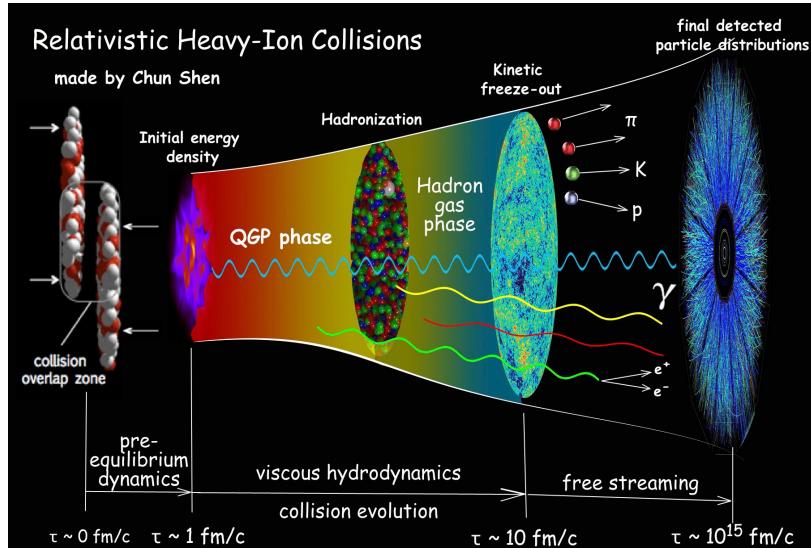
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Motivation

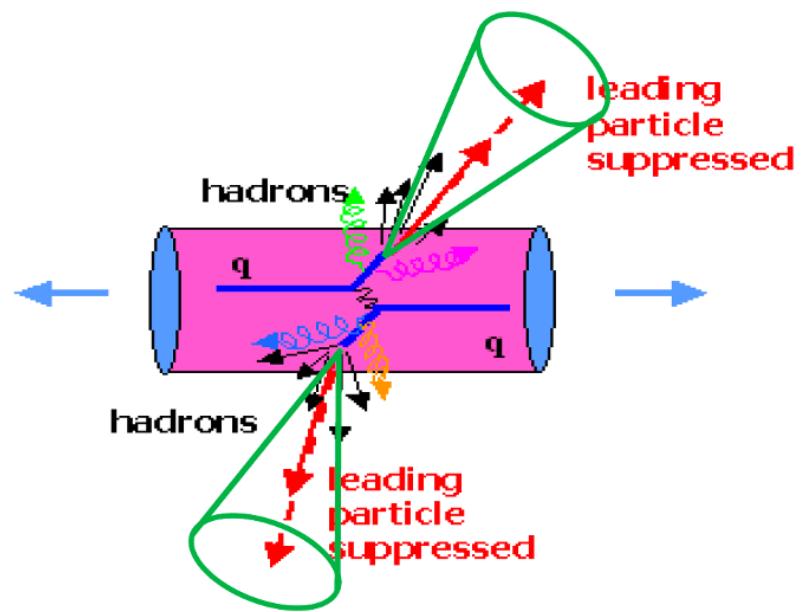


Strong vorticity fields exist in relativistic heavy-ion collisions $\omega = (9 \mp 1) \times 10^{21} \text{ s}^{-1}$

[1] L. Adamczyk et al. (STAR), Nature 548, 62 (2017), arXiv:1701.06657 [nucl-ex]

Drag force

When a heavy quark traverses a hot QGP, its momentum and energy are dissipated through friction with medium. Such a friction is quantified by a drag force.



$$\frac{dp_\mu}{dt} = -F_\mu^{drag}$$



AdS/CFT duality

The Large N limit of superconformal field theories and supergravity

Juan Martin Maldacena (Harvard U.) (Nov, 1997)

Published in: *Int.J.Theor.Phys.* 38 (1999) 1113-1133 (reprint), *Adv.Theor.Math.Phys.* 2 (1998) 231-252 • e-Print: [hep-th/9711200](#) [hep-th]

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18,700 citations

The strongly coupled gauge theory in four-dimensional space-time is dual to the weakly coupled string theory on $AdS_5 \times S^5$.

Several methods

➤ Local Lorentz transformation

$$t \rightarrow \frac{t + vx}{\sqrt{1 - v^2}}$$

$$x \rightarrow \frac{x + vt}{\sqrt{1 - v^2}}$$

$$v = \omega l_0$$

$$x = l_0 \phi$$



$$t \rightarrow \frac{1}{\sqrt{1 - (\omega l_0)^2}} (t + \omega l_0^2 \phi)$$

$$\phi \rightarrow \frac{1}{\sqrt{1 - (\omega l_0)^2}} (\phi + \omega t)$$

This method can only describe a small neighbourhood around l_0 .

$$\text{Period } T = 2\pi \sqrt{1 - (\omega l_0)^2} \leq 2\pi$$

Phase diagram, free energy, entropy and so on.

➤ Global transformation $\phi \rightarrow \phi + \omega t$

The angular velocity of QGP $\omega = 1 \times 10^{22} s^{-1} = 0.007 GeV$

radius $8 fm = 40.5 GeV^{-1}$

the linear velocity $v \simeq 0.3$

➤ Kerr-AdS₅, rotating string

- [1] Y.-Q. Zhao, S. He, D. Hou, et, al, JHEP 04, 115 (2023), arXiv:2212.14662 [hep-ph].
- [2] X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, (2020), arXiv:2010.14478 [hep-ph].



Background geometry

The Schwarzschild metric in cylindrical coordinates, (t, l, ϕ, z, r) are coordinates of AdS₅.

$$ds^2 = -\frac{r^2}{R^2}f(r)dt^2 + \frac{r^2}{R^2}(dl^2 + l^2d\phi^2 + dz^2) + \frac{1}{f(r)}\frac{R^2}{r^2}dr^2$$

Global rotation
 $\phi \rightarrow \phi + \omega t$

Azimuth angle



$$ds^2 = \frac{r^2}{R^2}[-(f(r) - \omega^2 l^2)dt^2 + l^2d\phi^2 + 2\omega l^2 dtd\phi + dl^2 + dz^2] + \frac{1}{f(r)}\frac{R^2}{r^2}dr^2$$

Hawking temperature

$$T = \frac{r_t}{\pi R^2}$$



Nambu-Goto action

Taking $\sigma^\alpha = (t, r)$ as the string world-sheet coordinates, and assume

$$z = z(t, r), \phi = \phi(t, r), l = l(t, r)$$

Nambu-Goto action $S = \frac{1}{2\pi\alpha'} \int dt dr \sqrt{-g}$ → the determinant of the induced metric

$$g_{\alpha\beta} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}$$

$$= \frac{r^2}{R^2} \begin{pmatrix} \omega^2 l^2 - f + 2\omega l^2 \dot{\phi} + l^2 \dot{\phi}^2 & \dot{l}^2 + \dot{z}^2 & \omega l^2 \phi' + l^2 \dot{\phi} \phi' + \dot{l} l' + \dot{z} z' \\ \omega l^2 \phi' + l^2 \dot{\phi} \phi' + \dot{l} l' + \dot{z} z' & l^2 \phi'^2 + l'^2 + z'^2 + \frac{R^4}{r^4 f} \end{pmatrix}$$

$\downarrow \frac{d}{dt}$ $\downarrow \frac{d}{dr}$



Euler-Lagrange equations

Euler-Lagrange equations

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{z}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial z'} \right) - \frac{\partial \sqrt{-g}}{\partial z} = 0$$

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{\phi}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial \phi'} \right) - \frac{\partial \sqrt{-g}}{\partial \phi} = 0$$

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{l}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial l'} \right) - \frac{\partial \sqrt{-g}}{\partial l} = 0$$

Static gauge when $\omega = 0$, a heavy quark moving along z direction

$$\boxed{\phi = C, l = C, z = vt + \xi(r)}$$

Solution

Euler-Lagrange equations

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{z}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial z'} \right) - \frac{\partial \sqrt{-g}}{\partial z} = 0$$

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{\phi}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial \phi'} \right) - \frac{\partial \sqrt{-g}}{\partial \phi} = 0$$

$$\frac{d}{dt} \left(\frac{\partial \sqrt{-g}}{\partial \dot{l}} \right) + \frac{d}{dr} \left(\frac{\partial \sqrt{-g}}{\partial l'} \right) - \frac{\partial \sqrt{-g}}{\partial l} = 0$$

Static gauge for small ω

$$z = vt + \xi_0(r) + \omega^2 \xi_1(r)$$

$$l = l_0 + \omega^2 l_1(r)$$

$$\phi = \phi_0 + \omega \phi_1(r)$$

**Solution
without
rotation**

$$\xi'_1 = 0$$

$$\phi'_1 = \frac{R^2 r_t^2}{r^4 f}$$



$$l'_1 = \frac{R^4}{r^4} \frac{r_c - f}{f - v^2} l_0$$

[1] I. Y. Arefeva, A. A. Golubtsova, and E. Gourgoulhon, JHEP 04, 169 (2021), arXiv:2004.12984[hep-th].



Drag force

Components of drag force

$$\frac{dp_\mu}{dt} = -\frac{1}{2\pi\alpha'} \frac{\partial \mathcal{L}}{\partial(\frac{\partial X^\mu}{\partial r})}$$

Lagrangian density of the Nambu-Goto action

Azimuthal $\frac{dp_\phi}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\omega l_0^2 \frac{1}{\sqrt{1-v^2}}$

Drag force of μ component

Radial $\frac{dp_l}{dt} = -\omega^2 l_0 \frac{1}{\sqrt{1-v^2}} \left[\frac{T\sqrt{\lambda}}{2(1-v^2)^{\frac{1}{4}}} - m_{rest} \right]$ cut off $r_m = 2\pi\alpha' m_{rest}$

Longitudinal $\frac{dp_z}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2} \frac{v}{\sqrt{1-v^2}} \left(1 + \frac{\omega^2 l_0^2}{2} \frac{1}{1-v^2} \right) \propto \omega^2$

$\boxed{\omega = 0} \rightarrow \frac{dp_\phi}{dt} = 0, \quad \frac{dp_l}{dt} = 0, \quad \frac{dp_z}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2} \frac{v}{\sqrt{1-v^2}}$

[1] S. S. Gubser, Phys. Rev. D 74, 126005 (2006), arXiv:hep-th/0605182.

[2] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, JHEP 07, 013 (2006), arXiv:hep-th/0605158.



Conclusion

- The rotation enhances the drag force along the direction of motion and thereby cause additional energy loss. The quark also experiences forces perpendicular to its trajectory.

Thanks for your attention



Kerr-AdS black hole

The five-dimensional Kerr-AdS metric with two non-zero rotational parameters in the Boyer-Lindquist coordinates is

$$\begin{aligned} ds^2 = & -\frac{\Delta}{\rho^2} \left(dt - \frac{a \sin^2 \theta}{\Xi_a} d\phi - \frac{b \cos^2 \theta}{\Xi_b} d\psi \right)^2 + \frac{\Delta_\theta \sin^2 \theta}{\rho^2} \left(adt - \frac{r^2 + a^2}{\Xi_a} d\phi \right)^2 \\ & + \frac{\Delta_\theta \cos^2 \theta}{\rho^2} \left(bdt - \frac{r^2 + b^2}{\Xi_b} d\psi \right)^2 + \frac{\rho^2}{\Delta} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \\ & + \frac{1 + \frac{r^2}{l^2}}{r^2 \rho^2} \left[abdt - \frac{b(r^2 + a^2) \sin^2 \theta}{\Xi_a} d\phi - \frac{a(r^2 + b^2) \cos^2 \theta}{\Xi_b} d\psi \right]^2, \end{aligned}$$

where

$$\begin{aligned} \Delta &= \frac{1}{r^2} (r^2 + a^2)(r^2 + b^2) \left(1 + \frac{r^2}{l^2} \right) - 2M, & \rho^2 &= r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \\ \Delta_\theta &= 1 - \frac{a^2}{l^2} \cos^2 \theta - \frac{b^2}{l^2} \sin^2 \theta, & \Xi_a &= 1 - \frac{a^2}{l^2} > 0, & \Xi_b &= 1 - \frac{b^2}{l^2} > 0, \end{aligned}$$



Kerr-AdS black hole

Hawking temperature

$$T = \frac{\Delta'_+ r_+^2}{4\pi(r_+^2 + a^2)(r_+^2 + b^2)}$$

Angular velocities

$$\omega = \frac{\Xi_a a}{r_+^2 + a^2} \quad \varpi = \frac{\Xi_b b}{r_+^2 + b^2}$$

Conformal boundary $r \rightarrow \infty$:

The boundary metric = $\left(\frac{r}{l}\right)^2 \times (\text{the metric of } R \times S^3)$

To describe the physics in Minkowski spacetime $R \times R^3$

$$Tl \gg 1$$

$$\frac{\omega}{T} \ll 1 \quad \& \quad \frac{\varpi}{T} \ll 1 \quad \& \quad \frac{M}{l^2} \gg \infty$$

Apply Witten's large mass scaling to Kerr-AdS

$$ds^2 = \left(\frac{r}{l}\right)^2 (-fd\tau^2 + d\varrho^2 + \varrho^2 d\phi^2 + dz^2) + \frac{l^2}{r^2 f} dr^2 \quad f = 1 - \frac{l^4}{r^2}$$

- Kerr-AdS may be useful for small angular velocity;
- Rotation effect may intertwine with the artifact of the spatial curvature.