

CHIRAL CONDENSATE WITH THE EFFECT OF ROTATION AND ACCELERATION

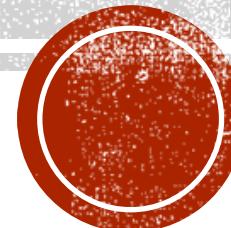
—PHASE DIAGRAM, FUNCTIONAL RENORMALIZATION GROUP APPROACH, NJL MODEL

arXiv:2306.08362

Speaker: Zhi-Bin Zhu

Collaborator: Hao-Lei Chen, Xu-Guang Huang

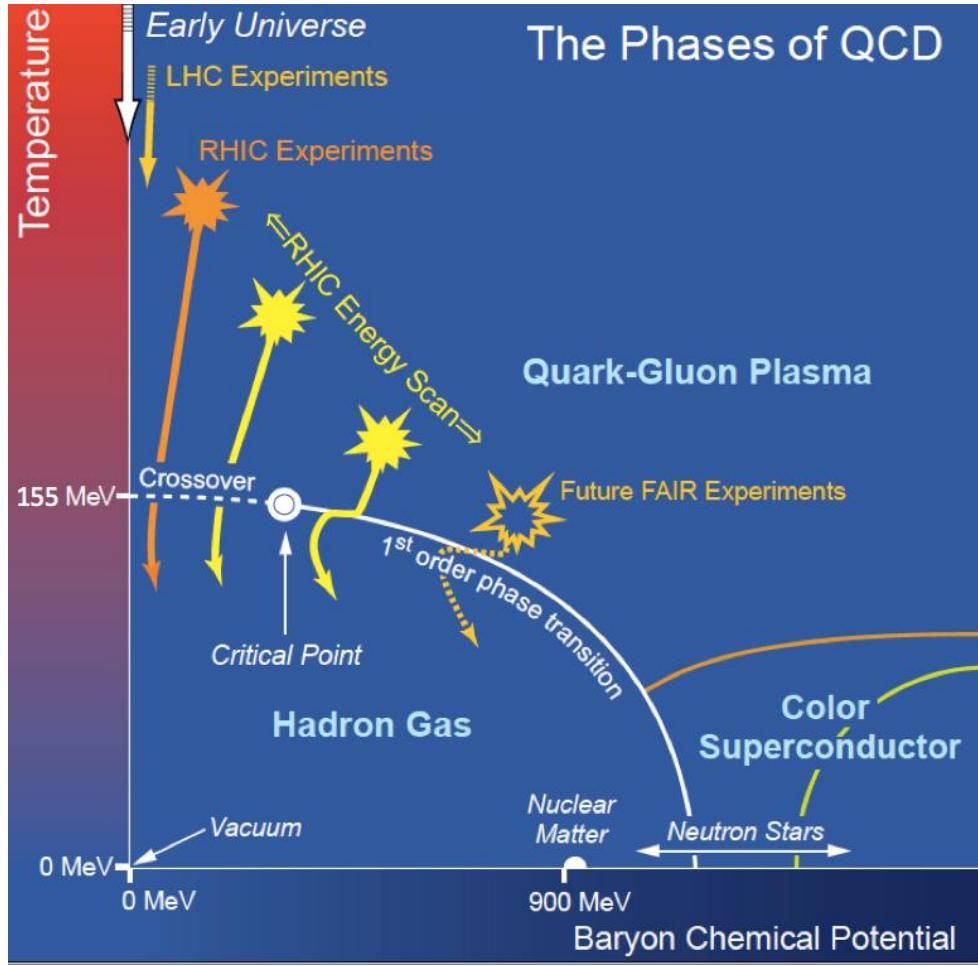
Acknowledgement: thanks Kenji Fukushima for fruitful discussions in the early stages



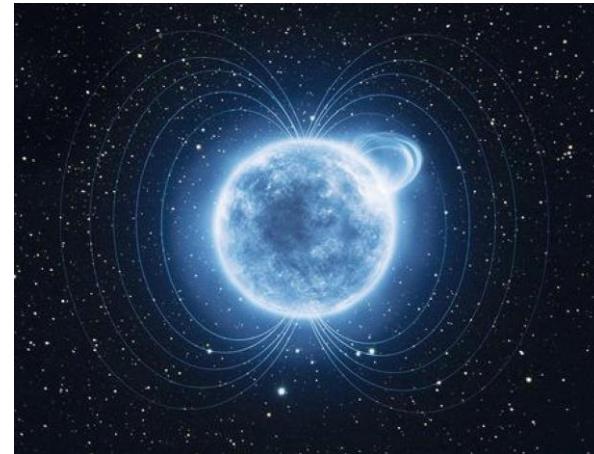
OUTLINE

- **QCD PHASE TRANSITION**
- **FUNCTIONAL RENORMALIZATION APPROACH**
- **QUARK-MESON MODEL AND FRG FLOW EQUATION**
- **NUMERICAL RESULTS FROM THE FRG FLOW EQUATION**
- **THE EFFECT OF ACCELERATION**
- **OUTLOOK**

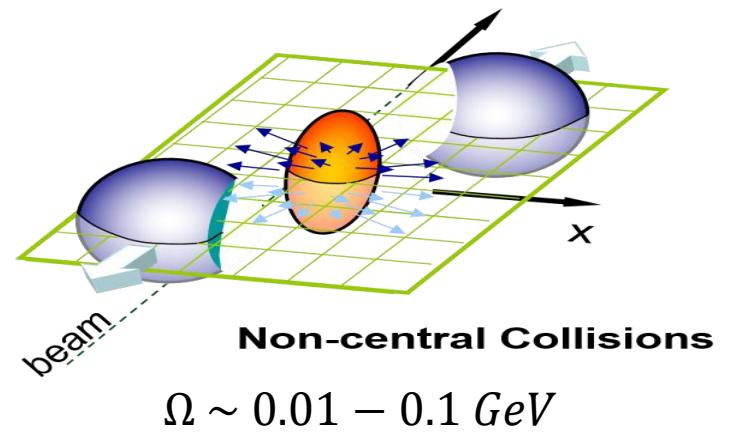
QCD PHASE TRANSITION



The Hot QCD White Paper (2015)

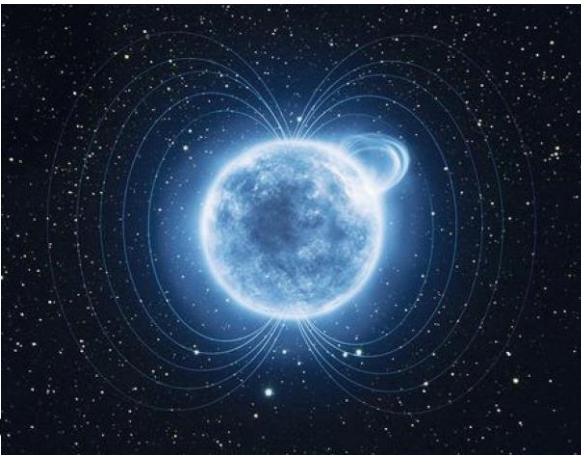
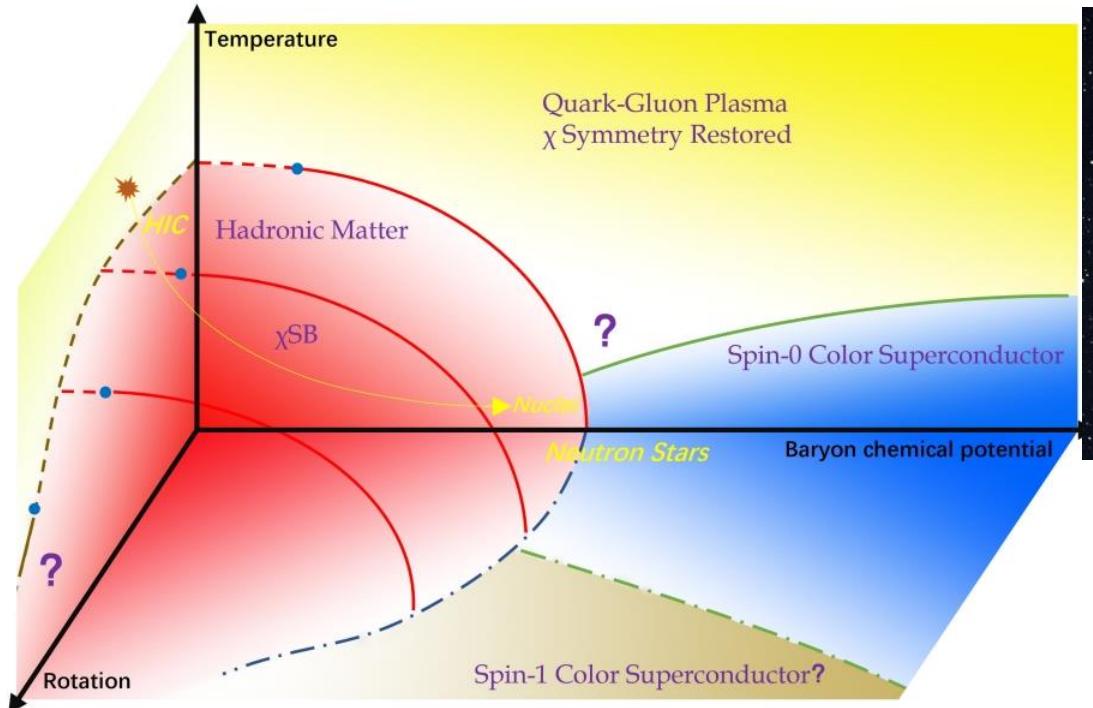


neutron star

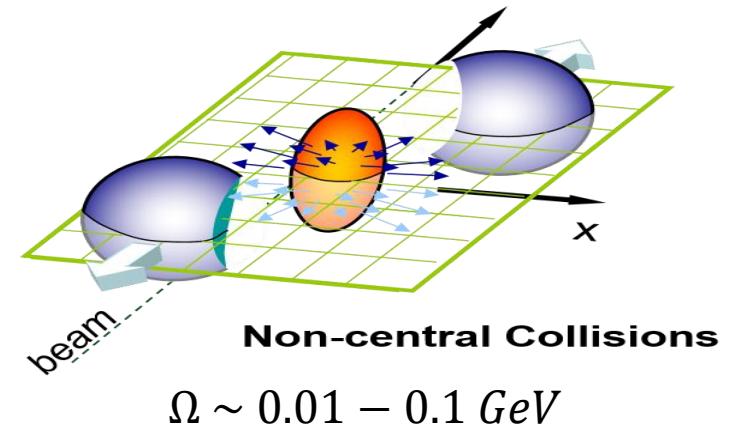


Chiral condensate $\langle \bar{\psi} \psi \rangle$
QGP phase $\langle \bar{\psi} \psi \rangle = 0$
Hadronic Phase $\langle \bar{\psi} \psi \rangle = \text{finite}$

QCD PHASE TRANSITION



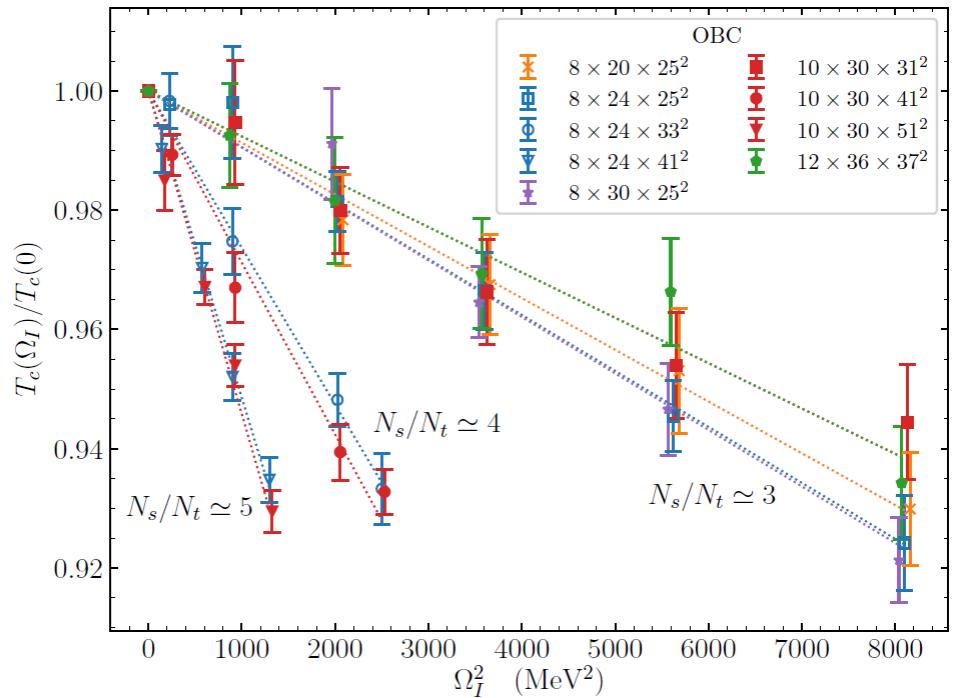
neutron star



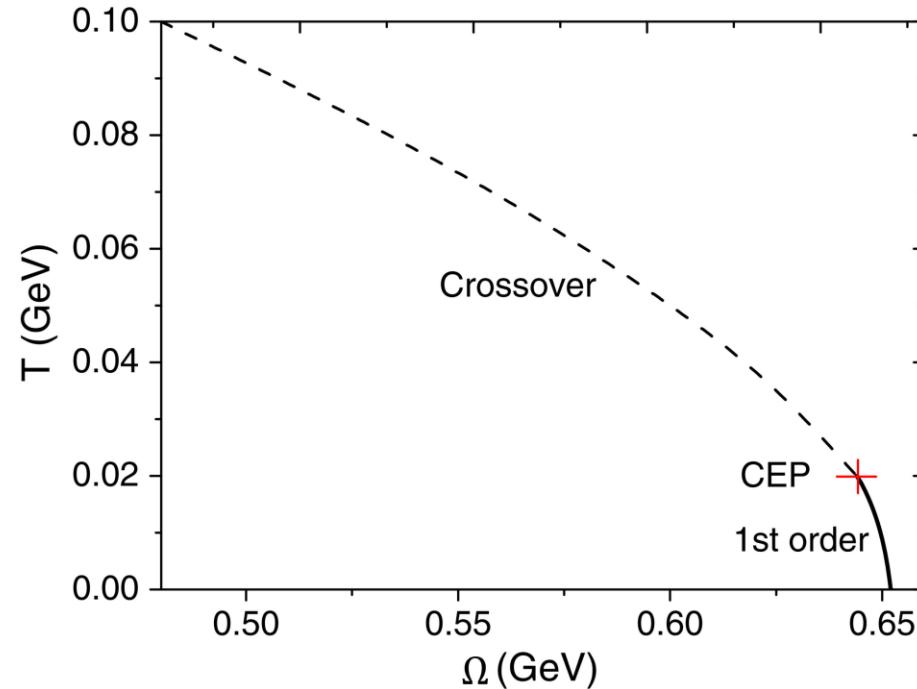
A schematic phase diagram of QCD matter in the 3-dimensional parameter space spanned by the temperature–baryon chemical potential–rotation axes

Chiral condensate $\langle \bar{\psi}\psi \rangle$
QGP phase $\langle \bar{\psi}\psi \rangle = 0$
Hadronic Phase $\langle \bar{\psi}\psi \rangle = \text{finite}$

QCD PHASE TRANSITION



Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.



Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016)
[arXiv:1606.03808 [hep-ph]].

- Imaginary angular velocity suppress the pseudocritical temperature, if performs the analytical to real rotation which will mean T increasing with real angular velocity (Lattice QCD)
- Real angular velocity suppress the pseudocritical temperature(NJL model)

FUNCTIONAL RENORMALIZATION GROUP APPROACH

Functional integral with an IR regulator

$$Z_k[J] = \int D\chi e^{-S[\chi] + \int_x \chi(x)J(x) - \Delta S_k[\chi]}$$

regulator

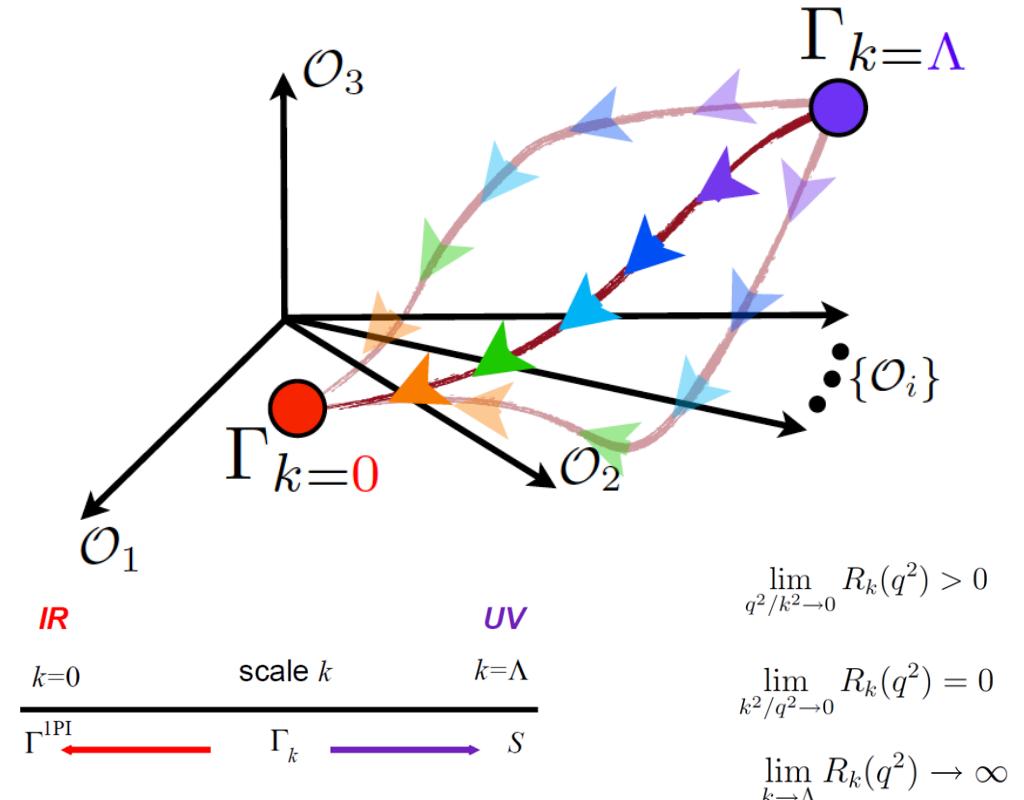
$$\Delta S_k[\chi] = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \chi^*(q) R_k(q) \chi(q)$$

Legendre transformation:

$$\Gamma_k[\phi] = -W_k[J] + \int_x \phi(x) J(x) + \Delta S_k[\phi]$$

flow equation

$$\partial_k \hat{\Gamma}_k[\Phi, \bar{\psi}, \psi] = \frac{1}{2} S \text{Tr} \left[\left(\hat{\Gamma}_k^{(2)}[\Phi, \bar{\psi}, \psi] + \hat{R}_k \right)^{-1} \partial_k \hat{R}_k \right]$$



$$\partial_k \Gamma_k = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2,0)} + R_k^B} \partial_k R_k^B \right] - \text{Tr} \left[\frac{1}{\Gamma_k^{(0,2)} + R_k^F} \partial_k R_k^F \right]$$

ROTATING FRAME

The QM model Lagrangian in Euclidean spacetime with rotation:

$$L = \phi \left[-(-\partial_\tau + \Omega \hat{L}_z)^2 - \nabla^2 \right] \phi + U(\phi) \\ + \bar{q} [\gamma^0 (\partial_\tau - \Omega \hat{J}_z) - i \gamma^i \partial_i + g (\sigma + i \vec{\pi} \cdot \vec{\tau} \gamma^5)] q$$

$$U(\phi) = \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 - c\sigma \quad \hat{L}_z = (-i \vec{r} \times \nabla)_z \\ \hat{J}_z = \hat{L}_z + \hat{S}_z$$

Effective action for mesons:

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2 \Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Gamma_k^B = \frac{1}{2} l n d e t \left[-(-\partial_\tau + \Omega \hat{L}_z)^2 - \nabla^2 + \frac{\partial^2 U}{\partial \phi_i \partial \phi_j} \right] \\ = \frac{1}{2} \int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi} \frac{1}{2\pi} \sum_{l,i} \frac{1}{N_{l,i}^2} \text{tr} \ln \left[-(i\omega_n + \Omega ll)^2 + p_{l,i}^2 + p_z^2 + R_{\phi,k} + \frac{\partial^2 U}{\partial \phi_i \partial \phi_j} \right] J_l (p_{l,i} r)^2$$

effective action for fermion:

$$\Gamma_k^F = -\frac{1}{2} l n d e t (D_k \gamma^5 D_k^\dagger \gamma^5) \\ = -\frac{1}{2} \text{tr} \ln \left[-(\partial_\tau - \Omega \hat{J}_z)^2 - \nabla^2 + \hat{R}_{q,k}^2 + g M M^\dagger \right] \\ = -\frac{1}{2} \int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi} \frac{1}{2\pi} \sum_{l,i} \frac{1}{N_{l,i}^2} 2 N_c N_f l n [(\nu_n + i\Omega j)^2 + \tilde{p}^2 + R_{\phi,k} + g^2 \phi^2] [J_l (\tilde{p}_{l,i} r)^2 + J_{l+1} (\tilde{p}_{l,i} r)^2]$$

ROTATING FRAME

The QM model Lagrangian in Euclidean spacetime with rotation:

$$\Gamma_k = \int d^4x \frac{1}{2} \phi [-(-\partial_\tau + \Omega L_z)^2 - \nabla^2] \phi + \bar{q} [\gamma^0 (\partial_\tau - i\Omega \hat{J}_z) - i\gamma^i \partial_i + g(\sigma + i\vec{\tau} \cdot \vec{\pi})] q + U_k(\rho)$$

Effective action for mesons:

$$\begin{aligned}\Gamma_k^B &= \frac{1}{2} l n d e t \left[-(-\partial_\tau + \Omega \hat{L}_z)^2 - \nabla^2 + \frac{\partial^2 U}{\partial \phi_i \partial \phi_j} \right] \\ &= \frac{1}{2} \int d^4x_E T \sum_n \int \frac{dp_z}{2\pi} \frac{1}{2\pi} \sum_{l,i} \frac{1}{N_{l,i}^2} \text{tr} \ln \left[-(i\omega_n + \Omega ll)^2 + p_{l,i}^2 + p_z^2 + R_{\phi,k} + \frac{\partial^2 U}{\partial \phi_i \partial \phi_j} \right] J_l(p_{l,i} r)^2\end{aligned}$$

effective action for fermion:

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QUARK-MESON MODEL AND FRG FLOW EQUATION

Regulator for mesons:

$$R_{\phi,k} = (k^2 - p^2)\theta(k^2 - p^2)$$

Flow equation for mesons:

$$\partial_k \Gamma_k^B = \frac{1}{2} \int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi} \frac{1}{2\pi} \sum_{l,i} \frac{1}{N_{l,i}^2} \text{tr} \frac{2k\theta(k^2 - p^2)}{-(i\omega_n + \Omega l)^2 + k^2 + \frac{\partial^2 U}{\partial \phi_i \partial \phi_j}} J_l(p_{l,i} r)^2$$

Regulator for fermions:

$$\hat{R}_{\psi,k} = -i\gamma^i \partial_i \left(\frac{k}{\sqrt{-\nabla^2}} - 1 \right) \theta(k^2 + \nabla^2)$$

Flow equation for fermions:

$$\partial_k \Gamma_k^F = -\frac{1}{2} \int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi} \frac{1}{2\pi} \sum_{l,i} \frac{1}{\tilde{N}_{l,i}^2} 2N_c N_f \frac{\partial_k R_{\phi,k}}{(\nu_n + i\Omega j)^2 + \tilde{p}^2 + R_{\phi,k} + g^2 \phi^2} [J_l(\tilde{p}_{l,i} r)^2 + J_{l+1}(\tilde{p}_{l,i} r)^2]$$

Flow equation:

$$\begin{aligned} \partial_k U_k &= \frac{1}{\beta V} (\partial_k \Gamma_k^B + \partial_k \Gamma_k^F) \\ &= \frac{1}{\beta V} \int d^4 x_E \frac{1}{(2\pi)^2} \left\{ \sum_{l,i} \frac{1}{N_{l,i}^2} \text{tr} \frac{k \sqrt{k^2 - p_{l,i}^2}}{\varepsilon_\phi} \frac{1}{2} \left[\coth \frac{\beta(\varepsilon_\phi + \Omega l)}{2} + \coth \frac{\beta(\varepsilon_\phi - \Omega l)}{2} \right] J_l(p_{l,i} r)^2 \right. \\ &\quad \left. - \sum_{l,i} \frac{1}{\tilde{N}_{l,i}^2} 2N_c N_f \frac{k \sqrt{k^2 - \tilde{p}_{l,i}^2}}{\varepsilon_q} \frac{1}{2} \left[\tanh \frac{\beta(\varepsilon_q + \Omega j)}{2} + \tanh \frac{\beta(\varepsilon_q - \Omega j)}{2} \right] [J_l(\tilde{p}_{l,i} r)^2 + J_{l+1}(\tilde{p}_{l,i} r)^2] \right\} \end{aligned}$$

$$\varepsilon_\sigma = \sqrt{k^2 + 2\bar{U}' + 4\rho\bar{U}''} = \sqrt{k^2 + \partial_\sigma^2 \bar{U}},$$

$$\varepsilon_\pi = \sqrt{k^2 + 2\bar{U}'} = \sqrt{k^2 + \partial_\sigma \bar{U}/\sigma},$$

$$\varepsilon_q = \sqrt{k^2 + g^2 \rho},$$

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

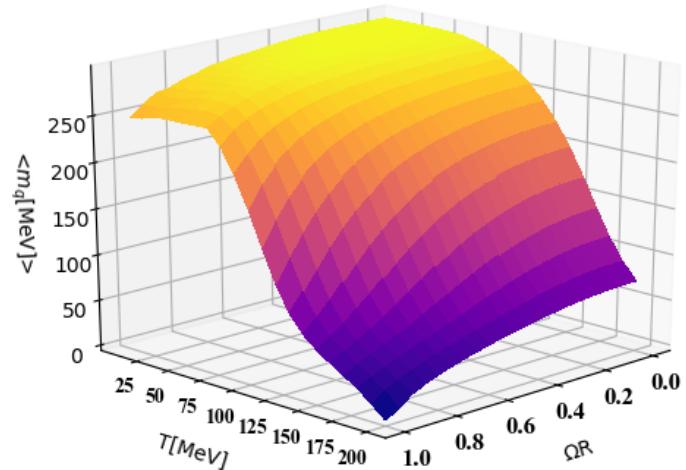


FIG. 1. The quark mass m_q as a function of Ω and T at $r = 0.9R$ in QM model.

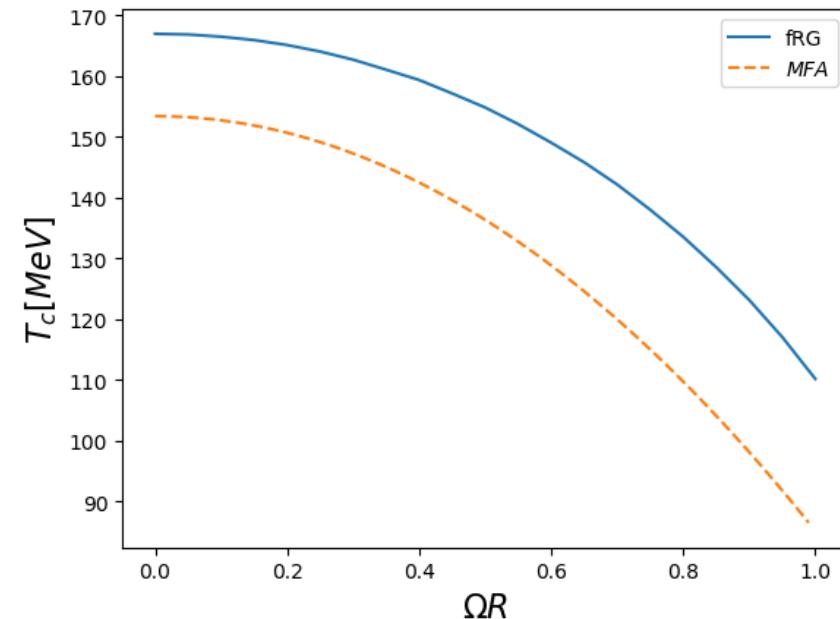


FIG. 2. The pesudo-critical temperature T_c as a function of Ω at $r = 0.9R$ from fRG and MFA in QM model.

- **initial condition:**
$$U_\Lambda = \frac{m_\Lambda^2}{2} \phi^2 + \frac{\lambda_\Lambda}{4} \phi^4 - c\sigma$$
- **Parameters chosen as:**
$$\begin{aligned} m_\Lambda &= 0.794\Lambda \\ \lambda_\Lambda &= 2 \\ c &= 0.00175\Lambda^{-3} \end{aligned}$$
- **Using the grid method**
- **Rotation effect almost invisible in low temperature**
- **Rotation effect suppress the chiral condensate**

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

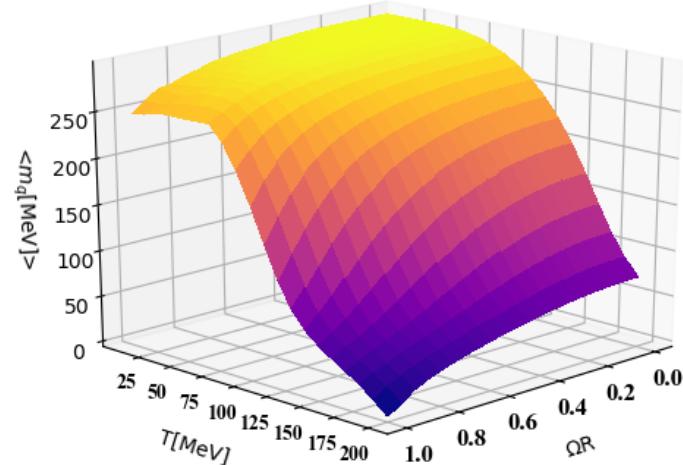


FIG. 1. The quark mass m_q as a function of Ω and T at $r = 0.9R$ in QM model.

- Using the grid method
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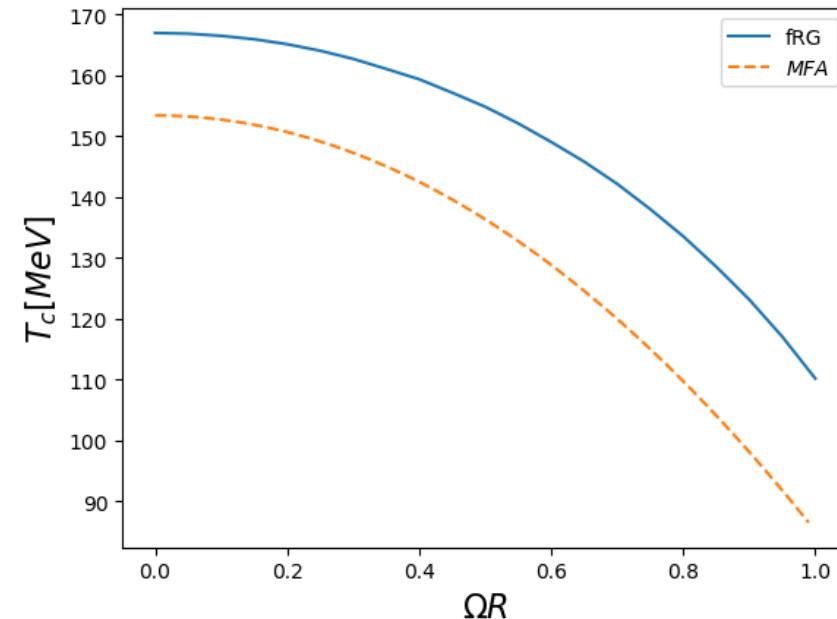


FIG. 2. The pesudo-critical temperature T_c as a function of Ω at $r = 0.9R$ from fRG and MFA in QM model.

- T_c defined in where chiral susceptibility reach maximum
- T_c decreases with increasing rotation Ω

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

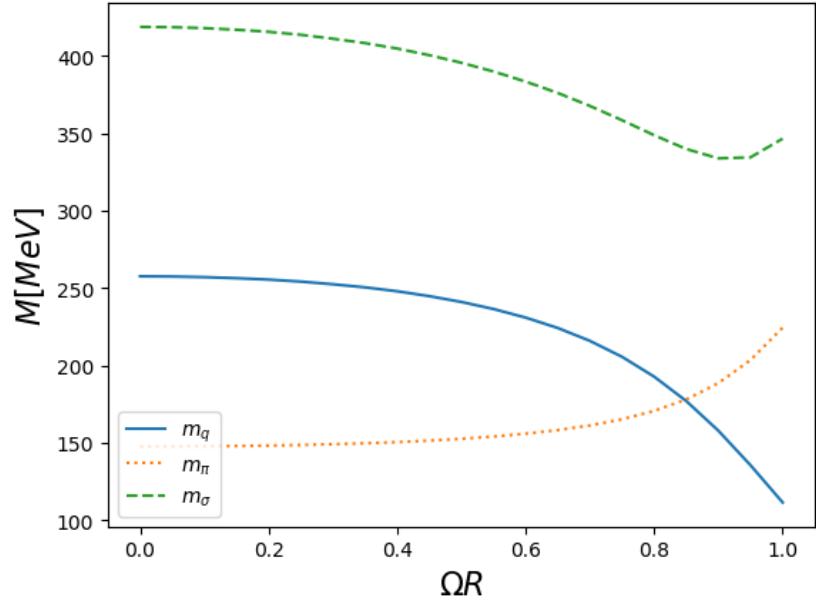


FIG. 3. Meson masses and quark mass as functions of Ω at $T = 120$ MeV from QM model FRG.

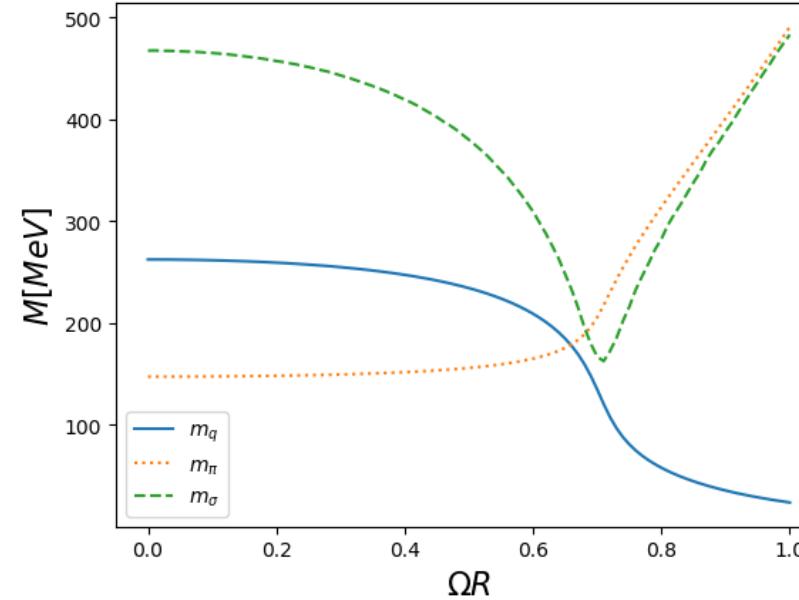


FIG. 4. Meson masses and quark mass as functions of Ω at $T = 120$ MeV from QM model MFA.

- **m_π and m_σ have a tendency become degenerate**
- **rotational effect is milder in our FRG calculation compared to the mean-field approximation**

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

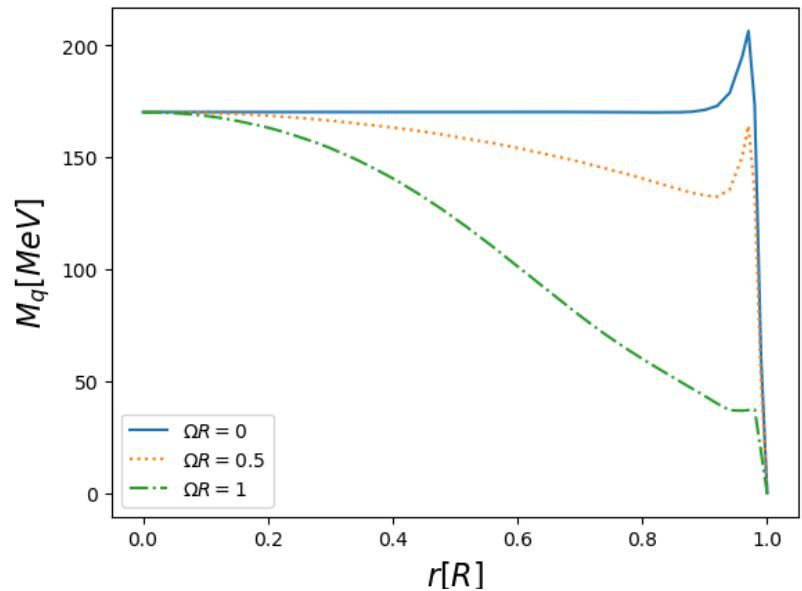


FIG. 5. The quark mass as a function of the radius r at different Ω at $T = 160$ MeV.

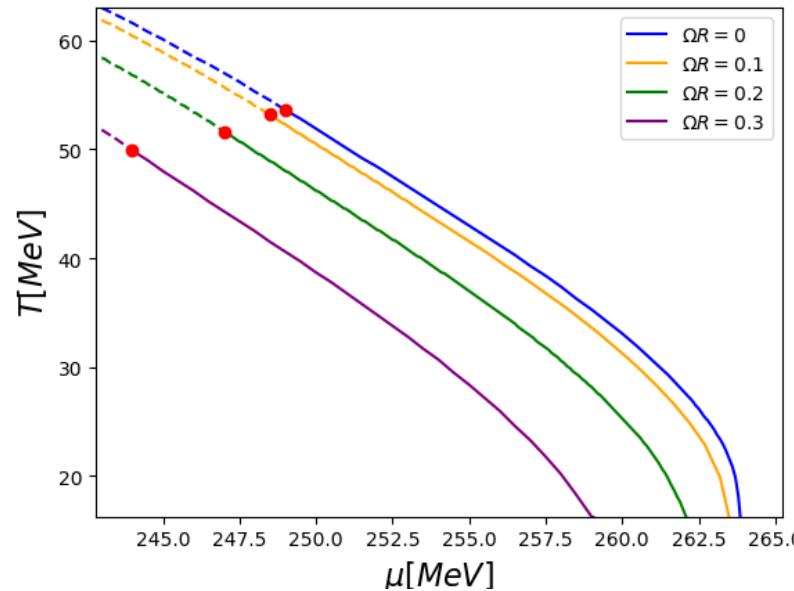


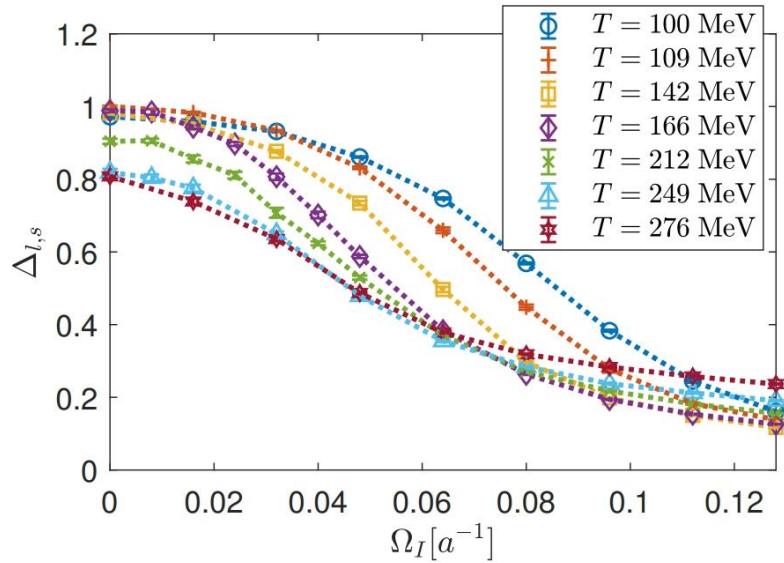
FIG. 6. $T - \mu$ phase diagram near the critical end point at different angular velocity.

- **Fig 6 choose the parameters:**

$$\begin{aligned} \Lambda &= 500\text{MeV}, \\ m_\Lambda &= 0, \\ \lambda_\Lambda &= 10, \\ c &= 0. \end{aligned}$$

- **Rotational suppression is strong near the boundary**
- **The fermions feel an effective chemical potential of $\Omega/2$**
- **The CEP shifting into low T and μ with increasing angular velocity**

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION



Ji-Chong Yang and Xu-Guang Huang
arxiv:2307.05755

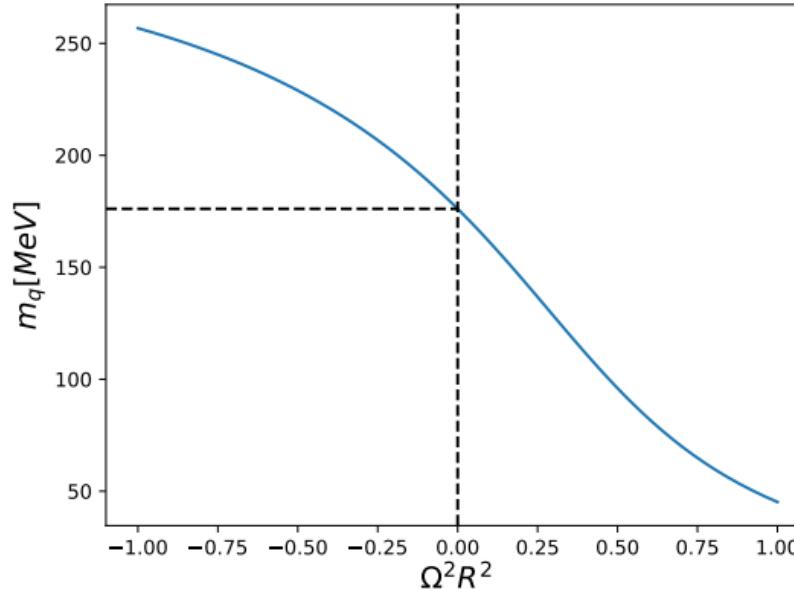


FIG. 7. The quark mass as a function of the square of the rotating angular velocity at $T = 160$ MeV and $r = 0.9R$ from fRG calculation

- Quark mass is a smooth function of Ω^2 which provide a condition for analytic continuation
- The behavior with increasing imaginary angular velocity is contrary

NJL MODEL UNDER ROTATION AND ACCELERATION

$$g_{\mu\nu} = \begin{pmatrix} (1 + az)^2 - \omega^2 r^2 & \omega y & -\omega x & 0 \\ \omega y & -1 & 0 & 0 \\ -\omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

NJL model action in rotation and acceleration frame:

$$S = \int d^4x \left\{ \bar{\psi} \left[i\gamma^{\hat{\mu}} \partial_{\mu} + ia \cdot x \gamma^{\hat{i}} \partial_i + \frac{i}{2} \mathbf{a} \cdot \boldsymbol{\gamma} + \gamma^{\hat{0}} \boldsymbol{\omega} \cdot \mathbf{J} - m\phi \right] \psi + G\phi \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right] \right\}$$

The gap equation:

$$1 = \frac{G}{2} \sum_{l,k,\pm} \int d\Omega \frac{1}{4(\Omega \pm \frac{i}{2}a)} \left\{ \tanh \left(\frac{(\Omega \pm \frac{i}{2}a) + (\omega j \pm \frac{i}{2}a)}{2T} \right) + \tanh \left(\frac{(\Omega \pm \frac{i}{2}a) - (\omega j \pm \frac{i}{2}a)}{2T} \right) \right\}$$

$$\frac{1}{N_{\Omega}^{\mp 2}} K_{i\Omega \mp \frac{1}{2}}^2 (\alpha \phi_1) \frac{1}{N_{l,k}^2} [J_l^2(p_{l,k}r) + J_{l+1}^2(p_{l,k}r)] 2N_c N_f$$

RESULT FROM GAP EQUATION

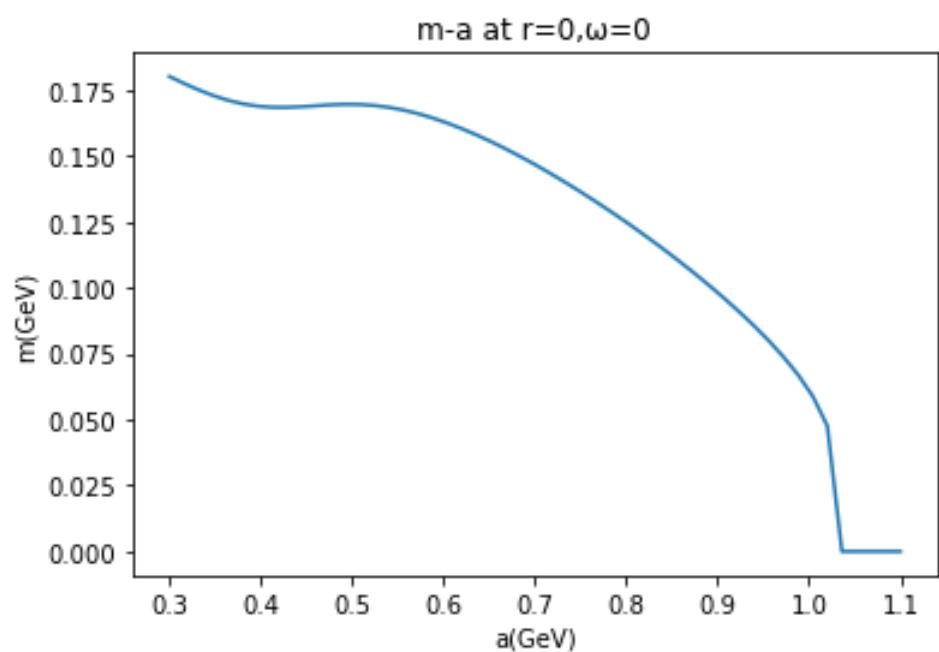


FIG. 9. chiral condensate as a function of acceleration obtained form gap equation

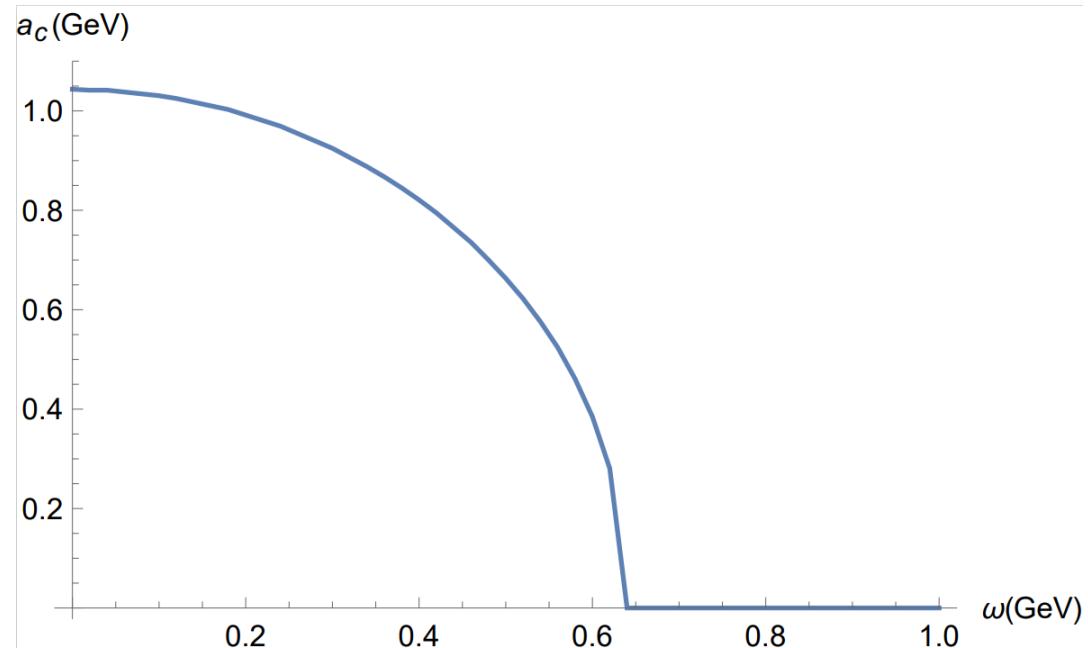


FIG. 10. Critical acceleration a_c as a function of angular velocity ω

- **Acceleration suppress the chiral condensate at large value**
- **Critical acceleration suppressed with increasing angular velocity**

SUMMARY AND OUTLOOK

- Calculate the chiral condensate using FRG approach under rotating QM model
 - Rotation will suppress chiral condensate at non-zero temperature which agree with the NJL model calculation
 - Work out chiral condensate as function of acceleration
-
- Outlook:
 - Consider the contribution from gluon
 - Studying the effect by using rotation PNJL model
 - Calculate some current in rotation and acceleration frame

THANKS !