Examination of nucleon distribution with Bayesian imaging for isobar collisions

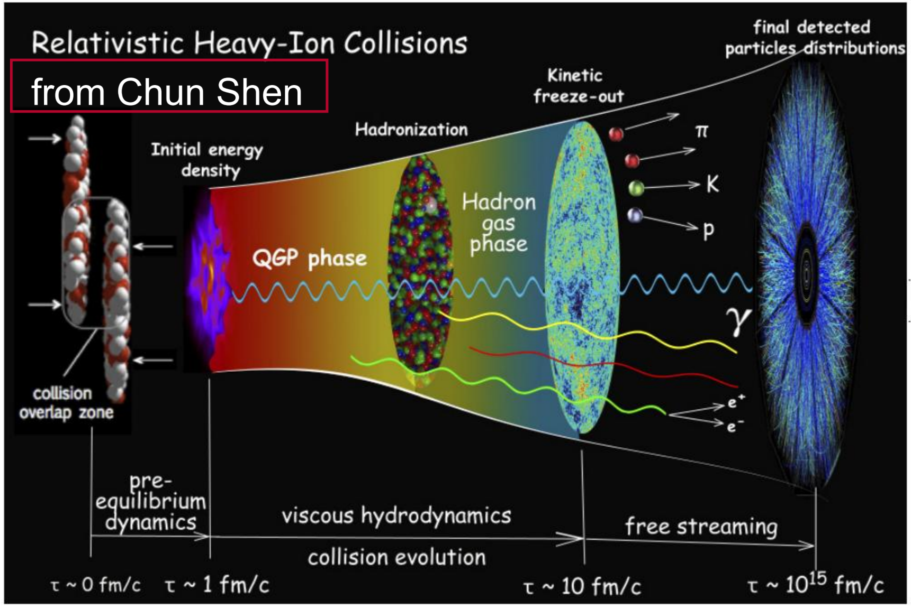
based on Phys. Rev. C 107, 064909

Yilin Cheng

Co-authors: Kai Zhou, Shuzhe Shi, Yugang Ma, Horst Stöcker

Motivation: -Understanding Heavy-ion collisions

-Mapping between the initial and final stage



Observables of final stage have a strong relationship with the initial state

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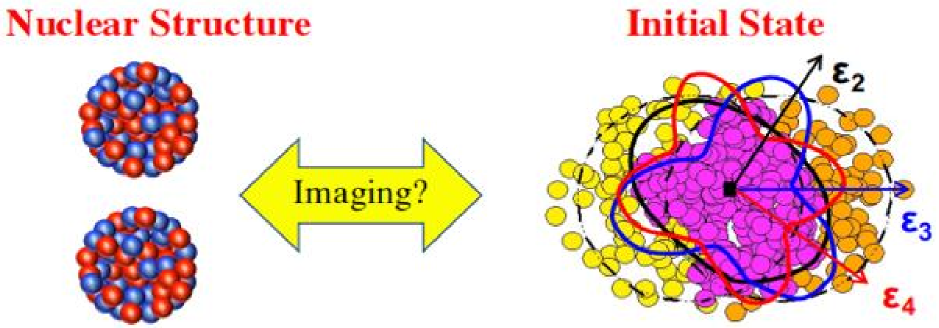

Slides from ATHIC2021, Chunjian Zhang's talk

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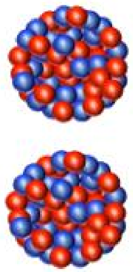
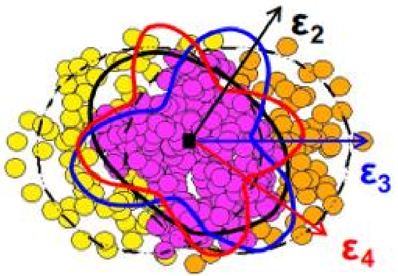
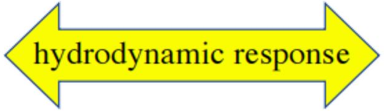
Nuclear Structure

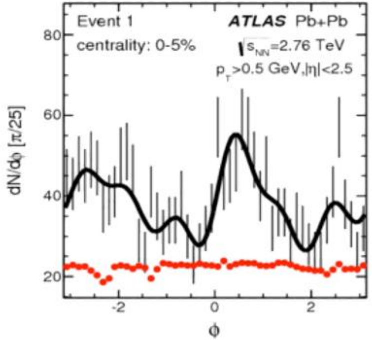


Initial State





Produced Particle Flow



Radial Flow Anisotropic Flow

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n e^{-in\phi}\right)$$

D. Teaney and L. Yan, Phys. Rev. C 86, 044908

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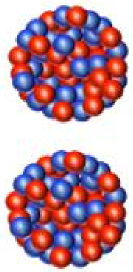
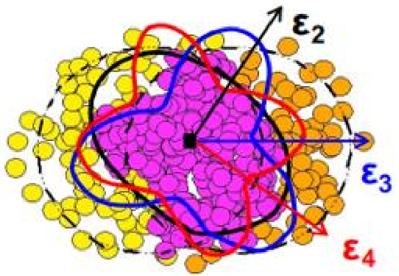


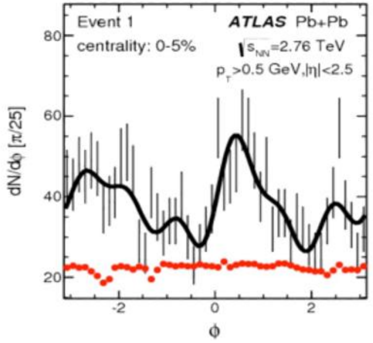
Imaging?

Initial State



hydrodynamic response

Produced Particle Flow



Radial Flow   Anisotropic Flow

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n e^{-in\phi}\right)$$

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Approximate linear response in each event:

$$\boxed{\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n}$$

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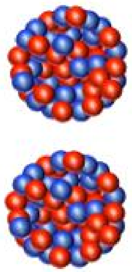

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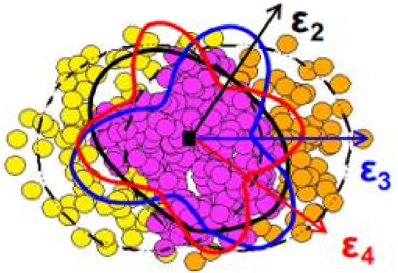

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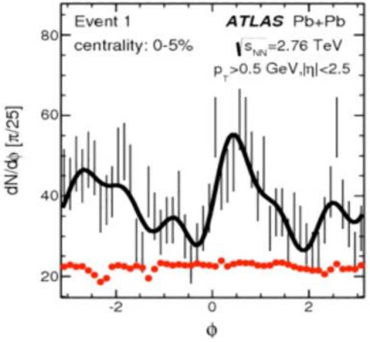

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + e^{(r - R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi)))/a_0}}$$

Initial Size

$$R_\perp^2 \propto \langle r_\perp^2 \rangle$$

Initial Shape

$$\mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$$

?

$$R_0 \quad a_0 \quad \beta_n$$

$\beta_2 \rightarrow$ Quadrupole deformation

$\beta_3 \rightarrow$ Octupole deformation

$a_0 \rightarrow$ Surface diffuseness

$R_0 \rightarrow$ Nuclear size

Radial Flow   Anisotropic Flow

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$\downarrow$ Monte-Carlo Glauber as Estimator

Initial state $\boxed{E,\ \varepsilon_2,\ \varepsilon_3,\ d_\perp}$

$\Downarrow$

Final state $\boxed{N_{ch},\ v_2\{2\},\ v_3\{2\},\ \langle p_T \rangle}$

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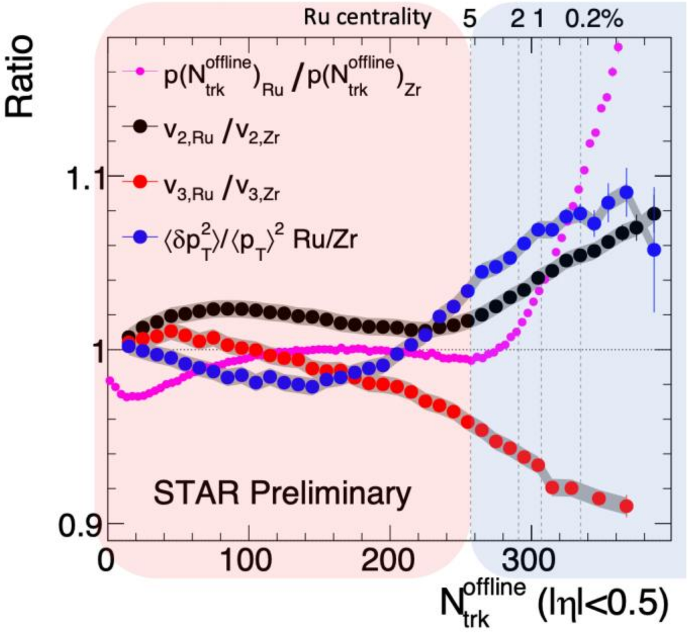

Plot from Chunjian Zhang's talk

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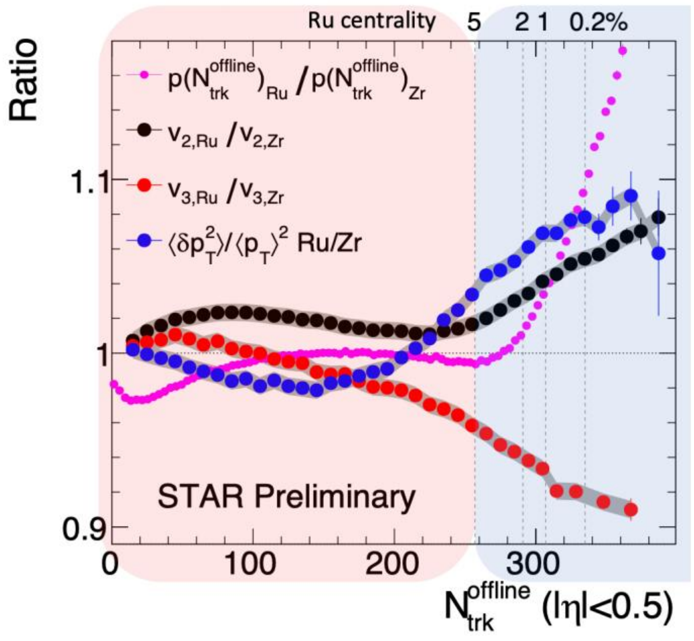

Plot from Chunjian Zhang's talk

Infer the nuclear structure from final state observables

- In single collision system ?
- Simultaneously for isobar systems with ratios ?

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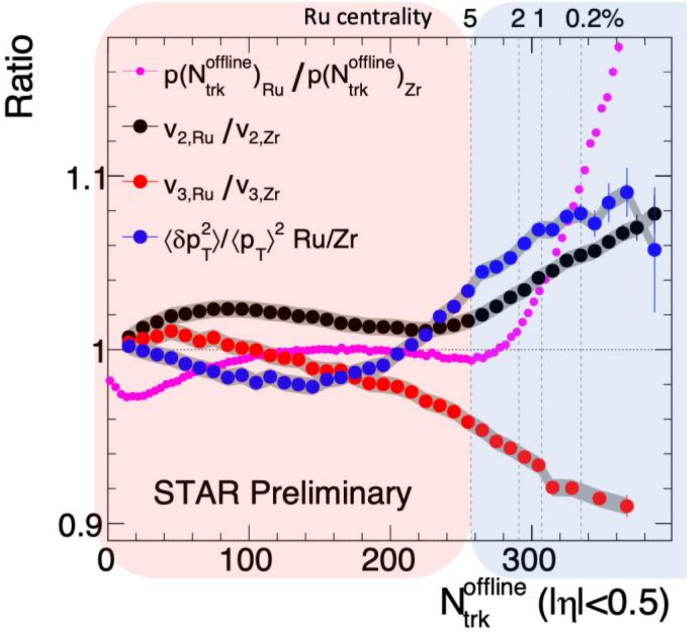

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Bayesian inference

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$$P(\theta|\mathcal{D}) \propto P(\mathcal{D}|\theta)P(\theta)$$

Bayesian inference

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nuclear structure

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nuclear structure **heavy-ion collision observables**

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Bayesian inference

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nuclear structure

heavy-ion collision observables

$$P(\boxed{\theta}|\boxed{\mathcal{D}}) \propto P(\boxed{\mathcal{D}}|\boxed{\theta})P(\boxed{\theta})$$

$$P(\mathcal{D}|\theta) = \frac{1}{\sqrt{(2\pi)^m det\Sigma}} exp(-\frac{1}{2}[\boxed{y_m(\theta)} - \boxed{y_e}]^T\Sigma^{-1}[y_m(\theta) - y_e])$$

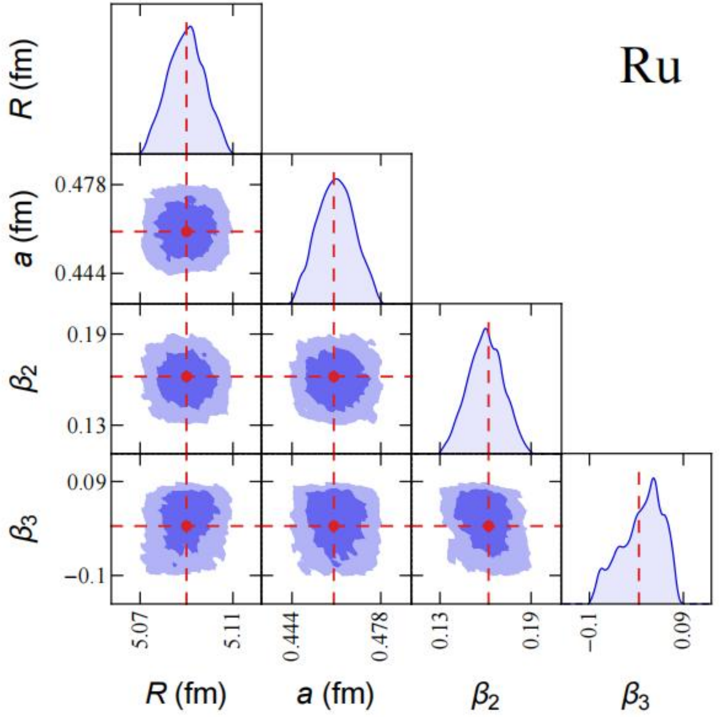
model

experiment

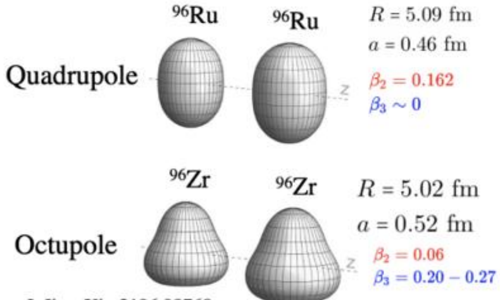
MCMC

Bayesian inference : Single system construction

It is possible to reconstruct the nuclear structure from the final state observablses in heavy-ion collisions



$$\boldsymbol{y}_{\mathrm{Ru}} \equiv \left\{ P_a^{\mathrm{Ru}}, \varepsilon_{2,a}^{\mathrm{Ru}}, \varepsilon_{3,a}^{\mathrm{Ru}}, d_{\perp,a}^{\mathrm{Ru}} \right\}_{a=1,\cdots,40}$$



J. Jia, aXiv:2106.08768

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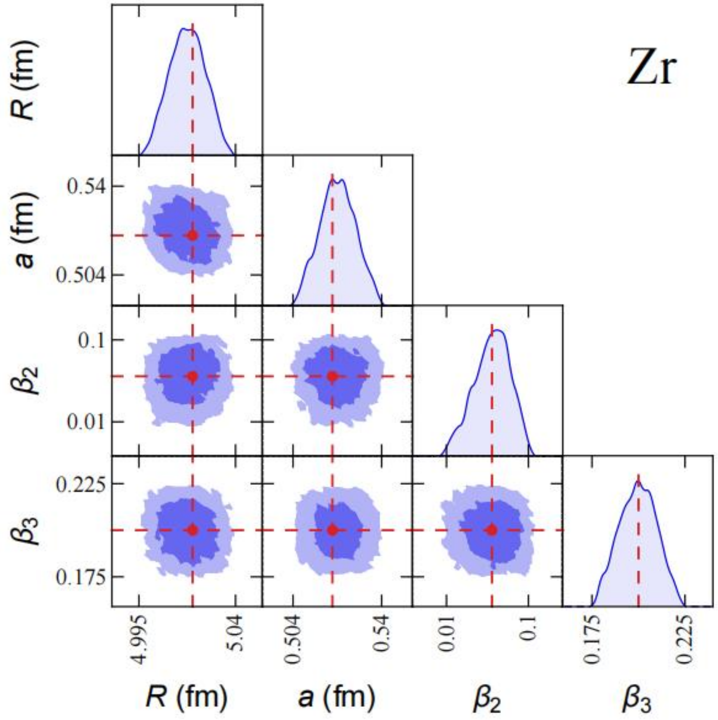
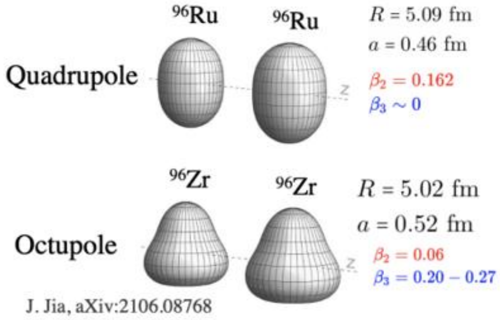
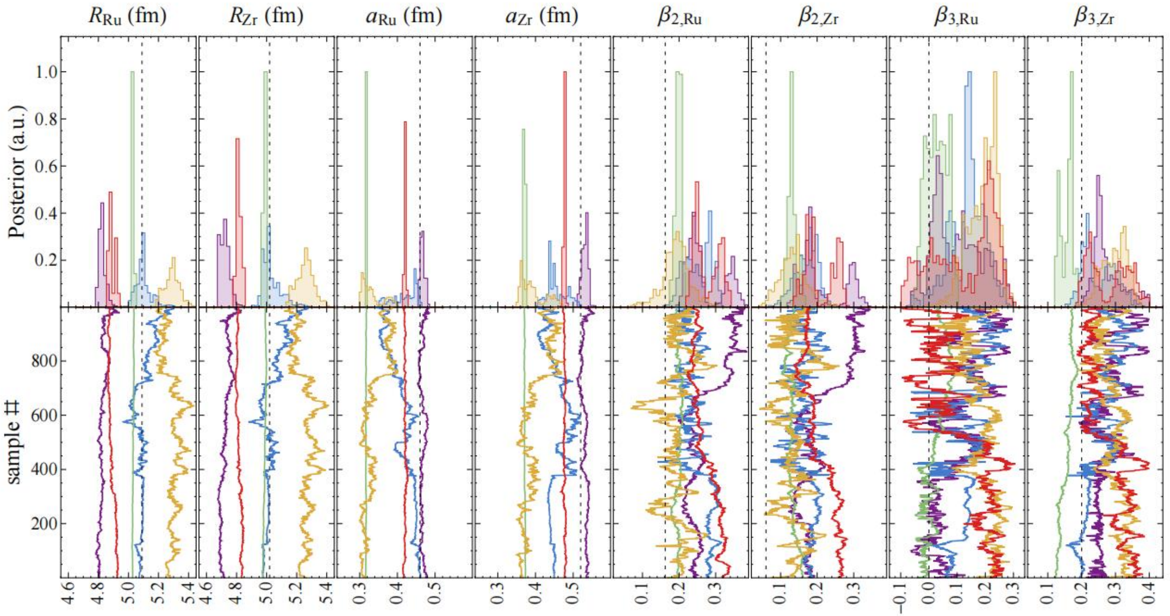


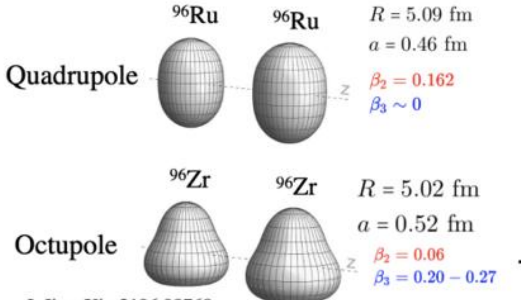


Bayesian inference : Simultaneous Reconstruction for Isobar Systems

Starting from purely the ratios, one can not simultaneously determine the nuclear structure s of the two isobar systems.



$$\boldsymbol{y}_{\mathrm{r},1} \equiv \left\{\boxed{R_{P,a}}, R_{\varepsilon_2,a}, R_{\varepsilon_3,a}, R_{d_\perp,a}\right\}_{a=1,\cdots,40}$$



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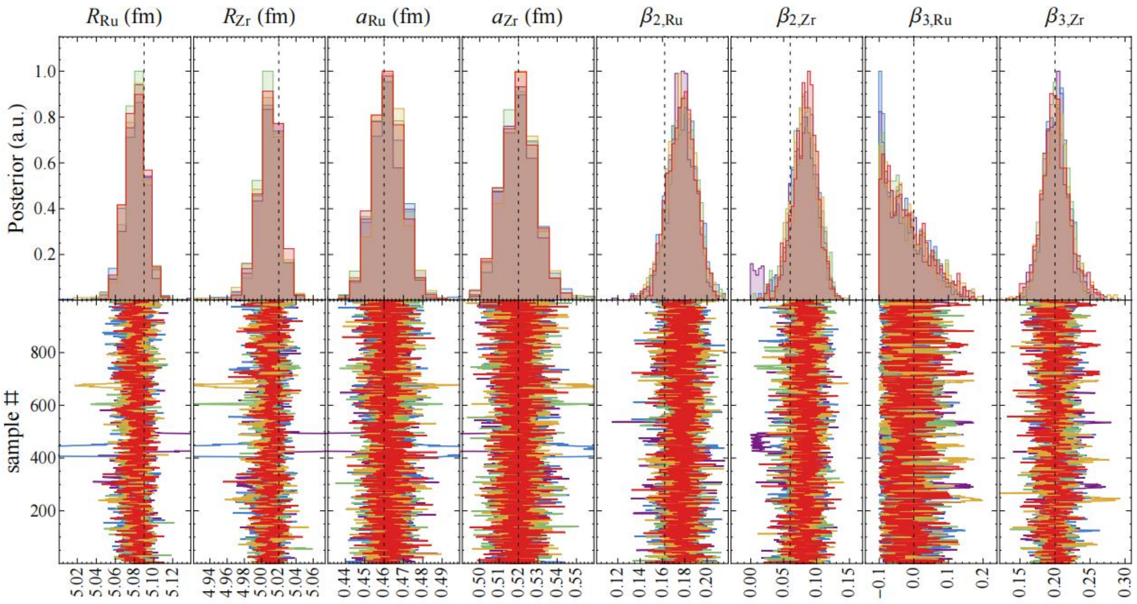
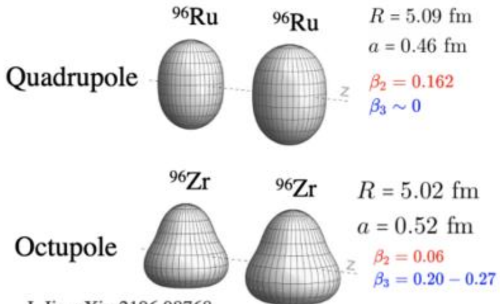
Bayesian inference : Simultaneous Reconstruction for Isobar Systems

Taking the multiplicity distributions of the two isobar systems together with the ratios of ε2, ε3, and d⊥, one can infer the isobar nuclear structures to very high precision.



$$\boldsymbol{y}_{r,2} \equiv \left\{ \boxed{P_a^{Ru}, P_a^{Zr},} R_{\varepsilon_2,a}, R_{\varepsilon_3,a}, R_{d_\perp,a} \right\}_{a=1,\cdots,40}$$



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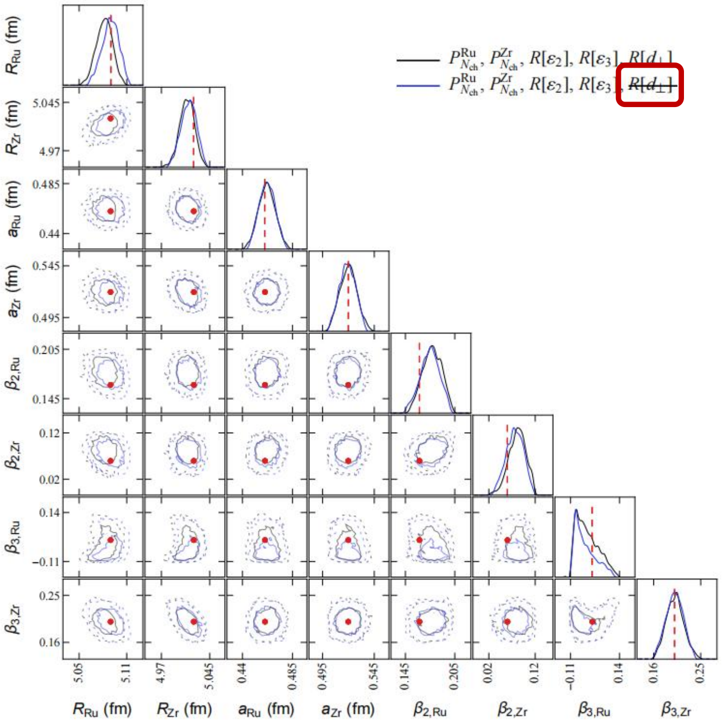
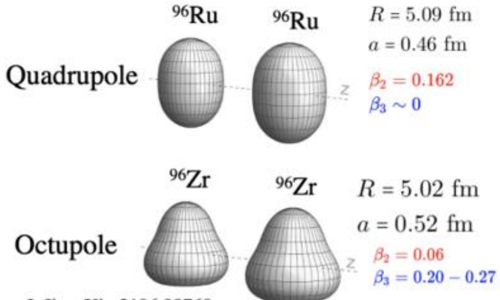
Bayesian inference : Simultaneous Reconstruction for Isobar Systems

Radial flow (<pT >) ,which can be estimated by $d_\perp$, carries redundant information as the ratios of elliptic/triangular flows.



$$\boldsymbol{y}_{\mathrm{r},3} \equiv \{P_a^{\mathrm{Ru}}, P_a^{\mathrm{Zr}}, R_{\varepsilon_2,a}, R_{\varepsilon_3,a}\}_{a=1,\cdots,40}$$



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Summary and outlook

- Infer nuclear structure from final observables： paves the way for precise predict non-CME background
  - In single systems:

    works well
  - In isobar systems:

    only from all of the ratios can not work

    single-system multiplicity distributions are provided can work

    ratio of radial flow is found to be nonessential

- Outlook: more realistic model needed; AMPT-based in progress.

Thank you!

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$$\delta O_a^{\mathrm{rel}} \equiv \sqrt{\frac{1}{d}\sum_{i=1}^{d}\left(\frac{O_{a,i}^{\mathrm{pred}} - O_{a,i}^{\mathrm{truth}}}{O_{a,i}^{\mathrm{truth}}}\right)^2}.$$

Comparison of relative difference between the ground truth and predicted values using Gaussian Processor with linear(green), quadratic(blue), 4th-order(red) and RBF(orange) kernels.
As references, gray curves represent the differences due to the PCA truncation. Statistical errors over the mean value in the MC-Glauber modeling are also presented as black curves.