Heavy Quark Diffusion coefficients in Magnetised Medium

Aritra Bandyopadhyay

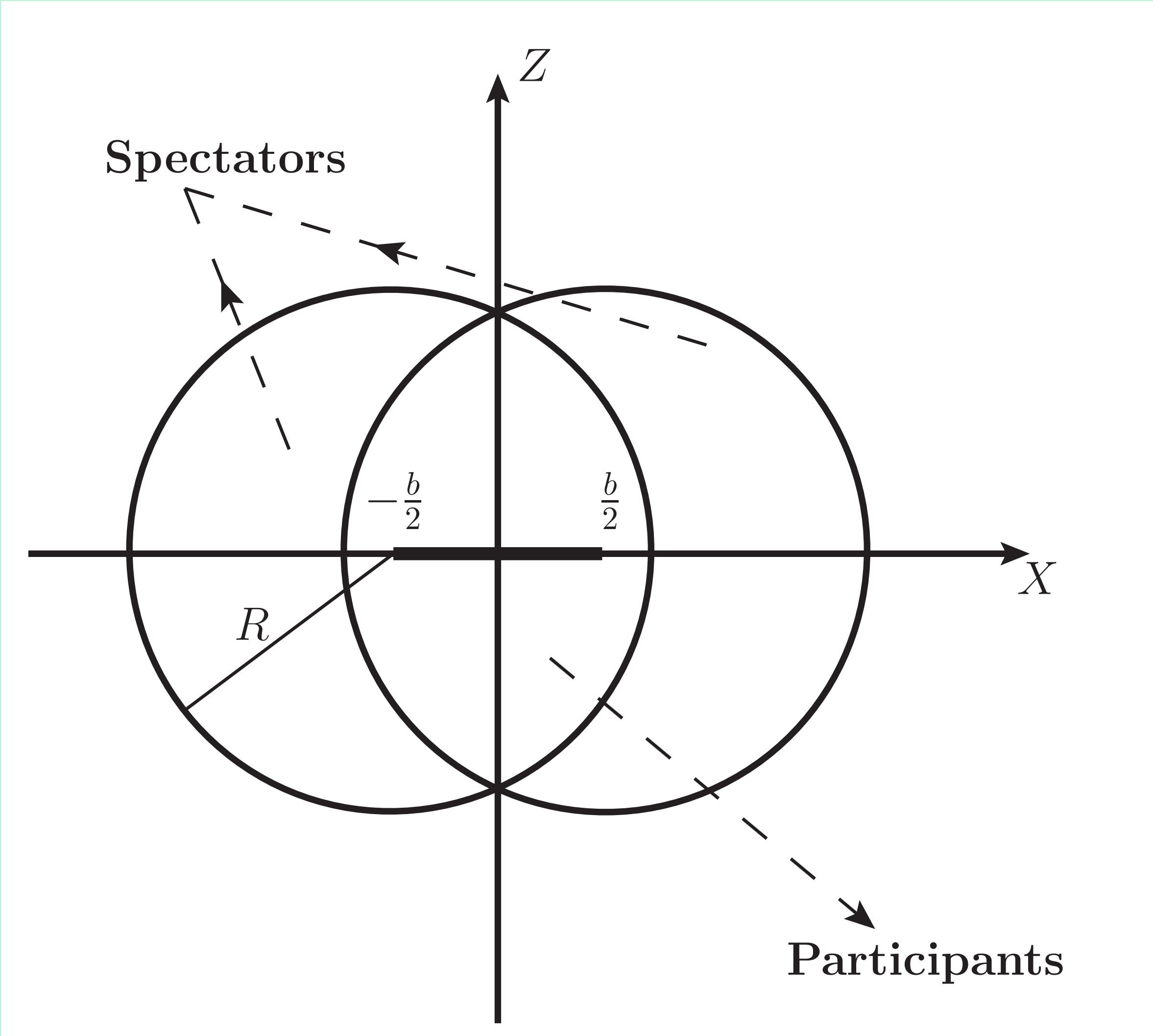
UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

Unterstützt von / Supported by
Alexander von Humboldt Stiftung / Foundation

Talk prepared for 7th international conference on chirality, vorticity and magnetic field in Heavy Ion Collisions

International Conference Center, UCAS, Beijing, 2023

Magnetised medium



- **Strong magnetic fields are present in some stellar objects and non-central heavy ion collisions (e.g. $eB \sim \hat{O}(10)m_\pi^2$ at LHC).**

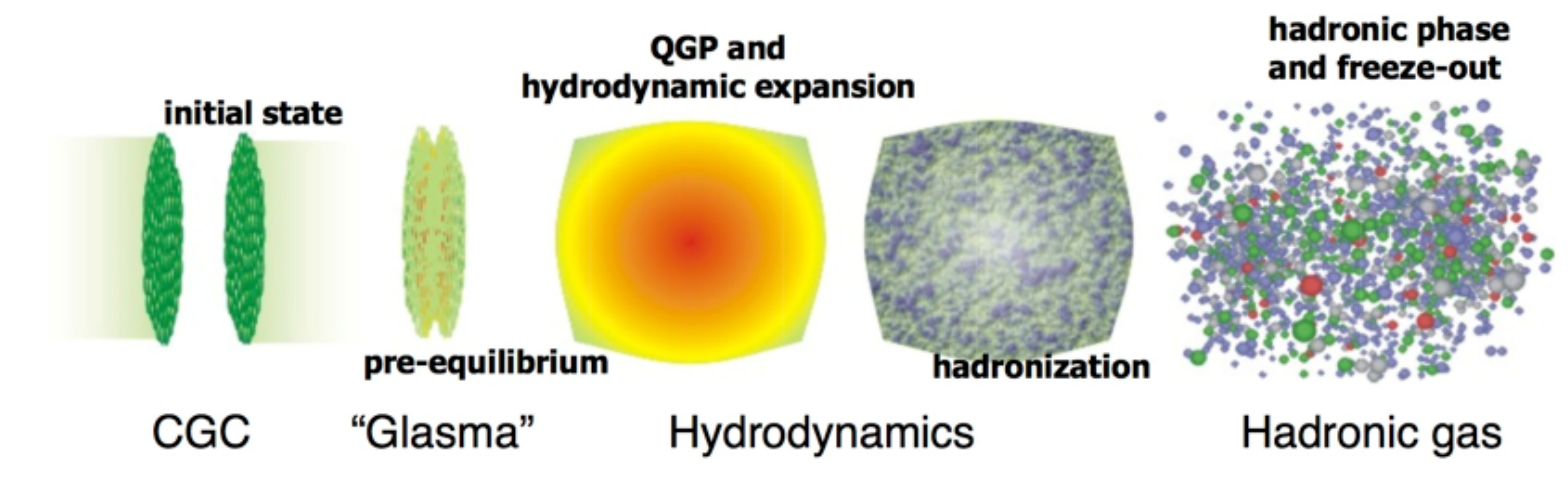
- **Introduces extra scale eB in the medium in addition to T, μ $\rightarrow$ triggers significant interest in theoretical understanding of the properties of a magnetised medium.**

Novel phenomena in magnetised medium

1. **Chiral Magnetic Effect** *[Kharzeev, McLerran, Warringa - NPA 803]*
2. **Chiral Magnetic Wave** *[Burnier, Kharzeev, Liao, Yee - PRL 107]*
3. **Charge dependent directed flow** *[Gursoy, Kharzeev, Rajagopal - PRC 89]*
4. **Enhancement in Dilepton production rate** *[Das, AB, Islam - PRD 106]*
5. **Inverse Magnetic Catalysis** *[Bali, Bruckmann, Endrodi et al. - JHEP 1202]*

And many more …………

Heavy quark as a QGP signature



$$\left.\begin{array}{l}\text{• Large mass compared to } T \\ \text{• External to the bulk medium.}\end{array}\right\} \rightarrow \text{Less Contamination}$$

- Generated at the early stage $\rightarrow$ More Information

In this talk: Heavy quark momentum diffusion coefficients

Heavy quark diffusion

- HQs experience drag forces as well as random kicks from the bulk medium.
- A widely adopted approach is to use the Langevin equations for describing HQ in-medium evolution.
- Essential theoretical inputs : HQ momentum diffusion coefficients → influence phenomenological modelings of predictions for experimental observables. (R_{AA} and v_2)

QGP Signatures

Static properties

(e.g. Heavy quark diffusion coefficients)

Dynamic properties

(e.g. Heavy quark spectra)

Approaches

Usually two approaches are taken in the literature to incorporate the theoretical modifications due to a magnetised medium

A. Through modification of the Debye mass :

- **Debye mass is related to the temporal part of the gluon self energy** Π_{00}
- $B = 0 \rightarrow$ **Most of the HQ observable can be expressed in such a way where the sole medium effect lies within the Debye mass** $m_D(T)$
- $B \neq 0 \rightarrow$ **Replace** $m_D(T)$ **by magnetised medium modified** $m'_D(T, eB)$

B. Through structural changes of the correlation functions :

- **Employing general structure of the gluon self energy** $\Pi_{\mu\nu}$ **for** $B \neq 0$
- **Evaluating the corresponding coefficients / form factors required**
- **Computing the HQ observable with the** eB **modified gluon CFs**

Static and Dynamic limits of Heavy Quark

$B = 0$

- **Static limit :** $M \gg T$

$$\langle \xi_i(t)\xi_j(t') \rangle = \kappa \; \delta_{ij}\delta(t - t')$$

Single diffusion coefficient κ

- **Dynamic limit :** $\gamma v \lesssim 1 \rightarrow p \lesssim M,\, M \gtrsim p \gg T$

$$\langle \xi_i(t)\xi_j(t') \rangle = \kappa_{ij}(\overrightarrow{p}) \;\delta(t - t')$$

where $$\kappa_{ij}(\overrightarrow{p}) = \kappa_L(p)\; \hat{p}_i\hat{p}_j + \kappa_T(p)\Big(\delta_{ij} - \hat{p}_i\hat{p}_j\Big)$$

Longitudinal (κ_L **) and Transverse (** κ_T **) diffusion coefficients.**

Static and Dynamic limits of Heavy Quark

$B \neq 0$

- **Static limit :** $M \gg (\sqrt{eB}, T)$

  Anisotropy given by $\overrightarrow{v}$ is now replaced by $\overrightarrow{B}$

  Longitudinal (κ_L) and Transverse (κ_T) diffusion coefficients.

- **Dynamic limit :** $M \gtrsim p > (\sqrt{eB}, T)$

  Two anisotropic directions - $\overrightarrow{v}$ and $\overrightarrow{B}$

  Case 1 : $\overrightarrow{v} \parallel \overrightarrow{B}$ **Diffusion coefficients** $\rightarrow \kappa_L,\ \kappa_T$

  Case 2 : $\overrightarrow{v} \perp \overrightarrow{B}$ **Diffusion coefficients** $\rightarrow \kappa_1,\ \kappa_2,\ \kappa_3$

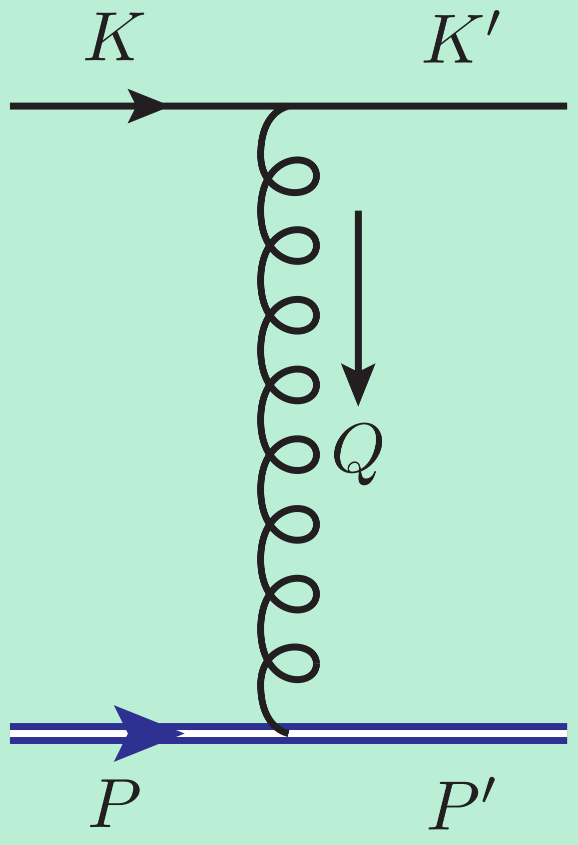
Scattering / Interaction rate

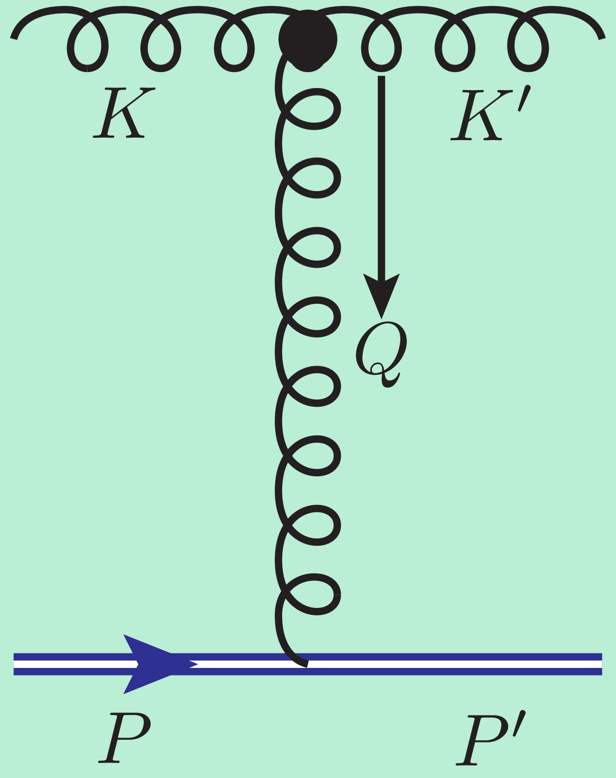
$$\kappa_i(p) = \int d^3q \frac{d\ \Gamma(v)}{d^3q}\ q_i^2$$

- $2 \leftrightarrow 2$ **scattering processes in a finite temperature medium**

$$qH \to qH \text{ and } gH \to gH \ (q \to \text{quark}, g \to \text{gluon and } H \to \text{HQ}).$$

- **At leading order in strong coupling, these processes are dominated by t-channel gluon exchange. (Compton scattering is suppressed by a factor $Q^2/PK \equiv T/M$)**

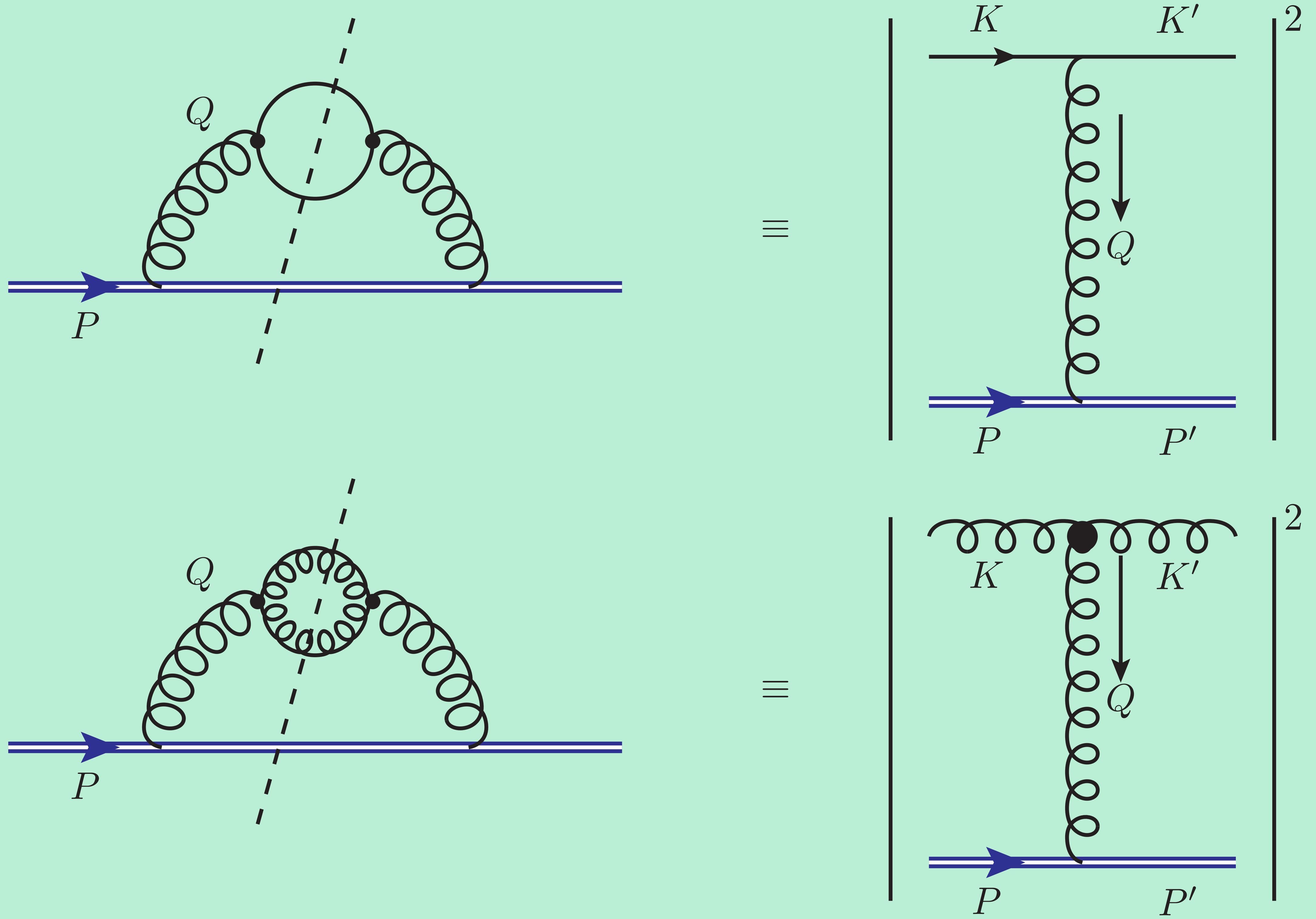




Scattering / Interaction rate

An effective way of expressing Γ is in terms of the cut/imaginary part of the HQ self energy $\Sigma(P)$

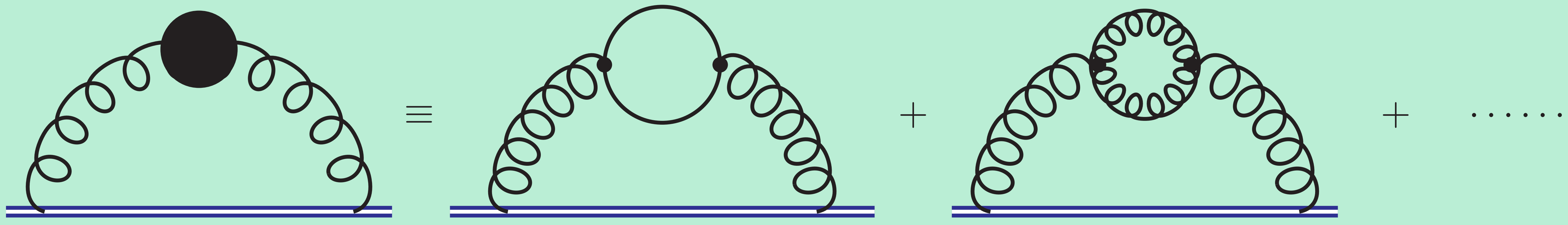
$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\,\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$



Scattering / Interaction rate

An effective way of expressing Γ is in terms of the cut/imaginary part of the HQ self energy $\Sigma(P)$

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\,\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$



$$\Sigma(P) = ig^2 \int \frac{d^4Q}{(2\pi)^4} \mathscr{D}^{\mu\nu}(Q)\ \gamma_\mu\ S^s_m(P-Q)\ \gamma_\nu$$

Scattering / Interaction rate

$$\Gamma(P \equiv E, \mathbf{v}) = -\frac{1}{2E}\frac{1}{1+e^{-E/T}}\mathrm{Tr}\left[(\gamma_\mu P^\mu + M)\ \mathrm{Im}\ \Sigma(p_0 + i\epsilon, \mathbf{p})\right]$$

$$\Sigma(P) = ig^2 \int \frac{d^4Q}{(2\pi)^4}\mathscr{D}^{\mu\nu}(Q)\ \gamma_\mu\ S_m(P-Q)\ \gamma_\nu$$

$$S_m(K) = e^{-\frac{k_\perp^2}{|q_fB|}}\sum_{l=0}^{\infty}\frac{(-1)^l D_l(q_fB,K)}{K_\parallel^2 - M^2 - 2lq_fB},$$

$$D_l(q_fB,K) = (\gamma_\mu K_\parallel^\mu + M)\left((1-i\gamma^1\gamma^2)L_l\left(\frac{2k_\perp^2}{q_fB}\right) - (1+i\gamma^1\gamma^2)L_{l-1}\left(\frac{2k_\perp^2}{q_fB}\right)\right) - 4(\gamma\cdot k)_\perp L_{l-1}^1\left(\frac{2k_\perp^2}{q_fB}\right),$$

Karmakar, AB, Haque, Mustafa : 1804.11336

$$\mathscr{D}^{\mu\nu}(Q) = \frac{\xi Q^\mu Q^\nu}{Q^4} + \frac{(Q^2-d_3)\Delta_1^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{\Delta_2^{\mu\nu}}{Q^2-d_2} + \frac{(Q^2-d_1)\Delta_3^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2} + \frac{d_4\Delta_4^{\mu\nu}}{(Q^2-d_1)(Q^2-d_3)-d_4^2}$$

where $\Delta_1^{\mu\nu} = \frac{1}{\bar{u}^2}\bar{u}^\mu\bar{u}^\nu,\ \Delta_2^{\mu\nu} = g_\perp - \frac{Q_\perp^\mu Q_\perp^\nu}{Q_\perp^2},\ \Delta_3^{\mu\nu} = \frac{\bar{n}^\mu\bar{n}^\nu}{\bar{n}^2},\ \Delta_4^{\mu\nu} = \frac{\bar{u}^\mu\bar{n}^\nu + \bar{u}^\nu\bar{n}^\mu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}},$

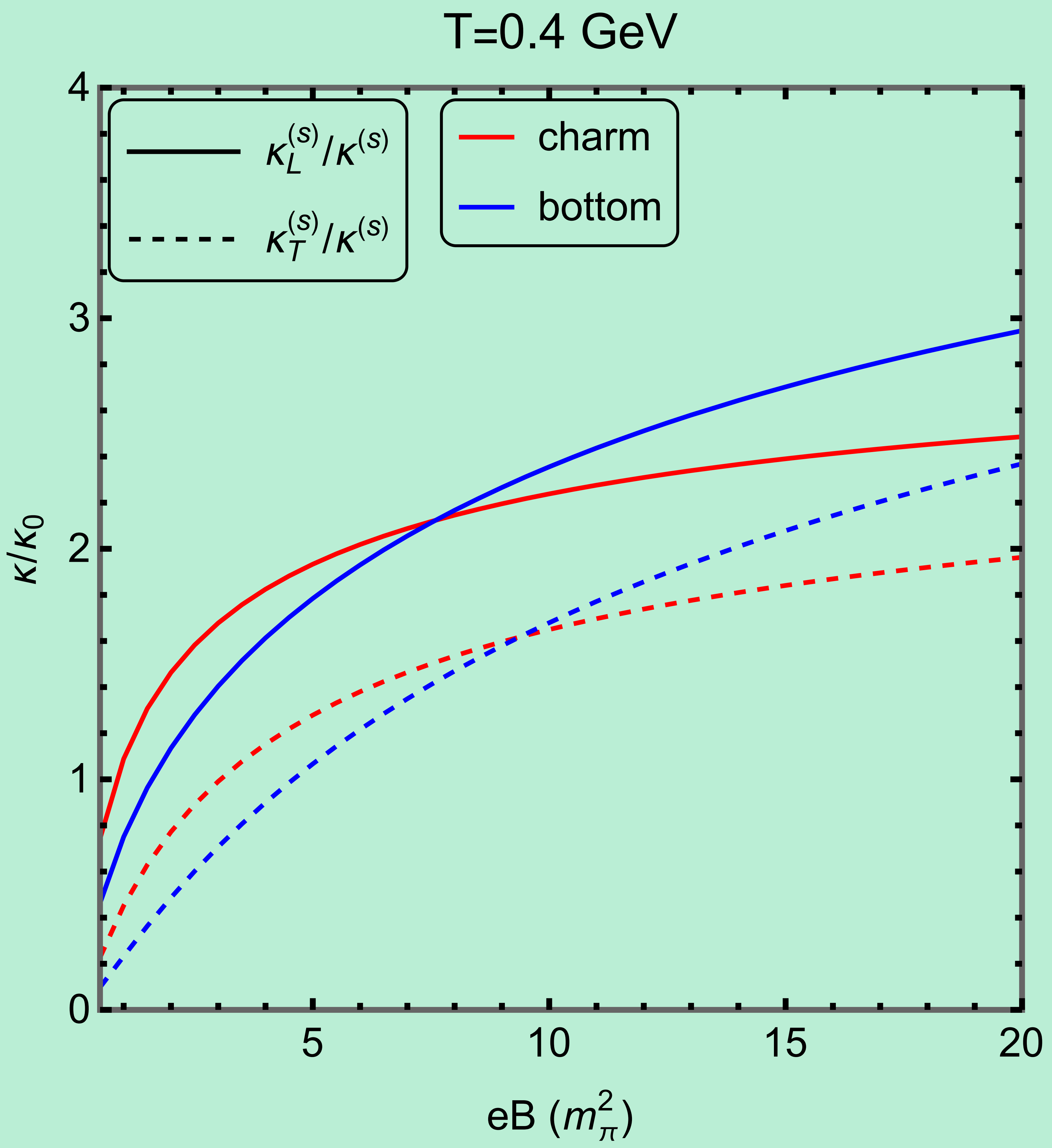
with $d_1(Q) = \Delta_1^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_2(Q) = \Delta_2^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_3(Q) = \Delta_3^{\mu\nu}\Pi_{\mu\nu}(Q),\ d_4(Q) = \frac{1}{2}\Delta_4^{\mu\nu}\Pi_{\mu\nu}(Q)$

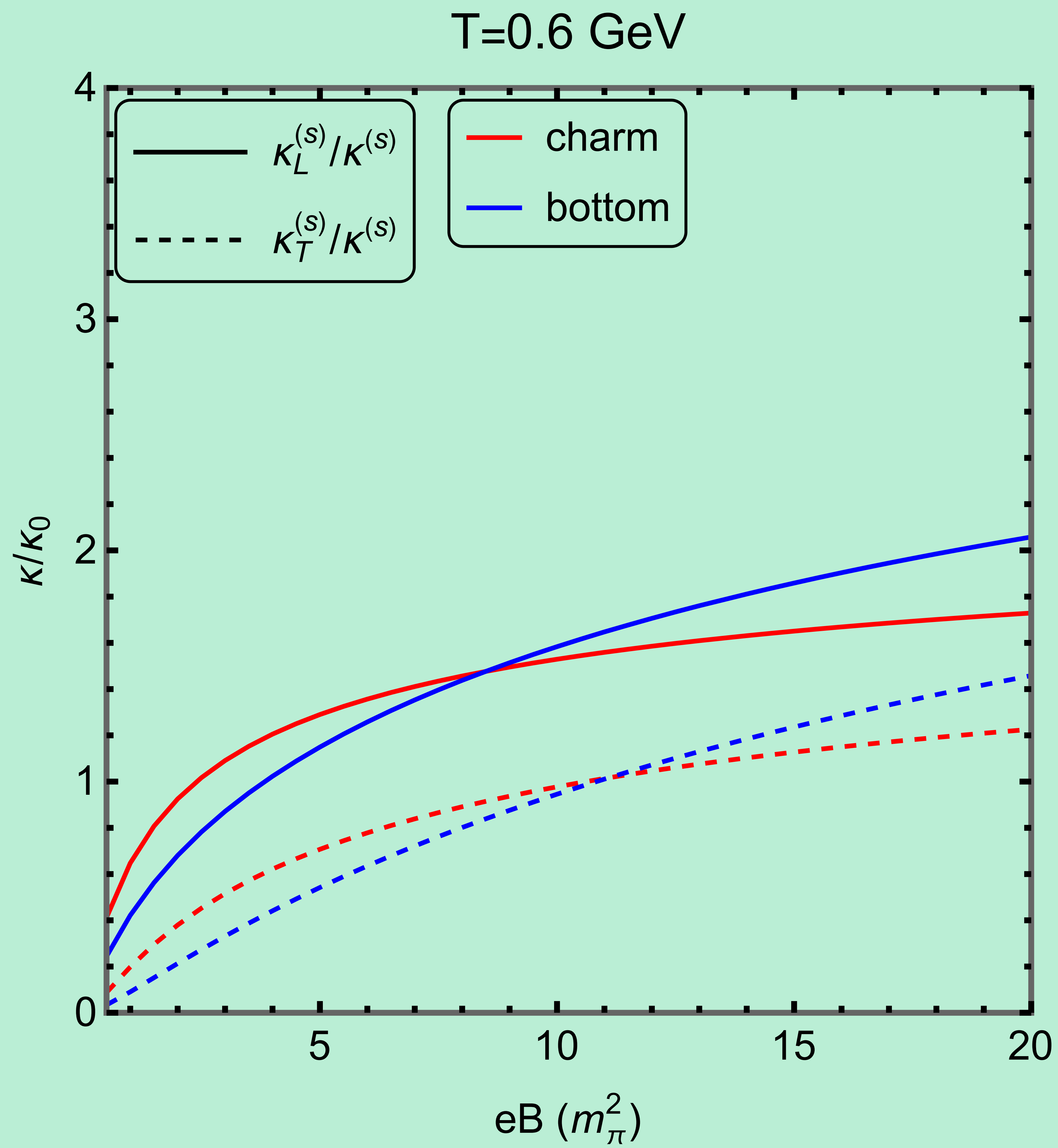
$B \neq 0$, static limit result

$$\kappa_L^{(s)} = \sum_{l=0}^{\infty} \frac{(-1)^l\, 2\pi g^2 TM}{\sqrt{M^2 + 2l\,|\,q_f B\,|}} \int \frac{d^3q}{(2\pi)^3} q_3^2 e^{-q_\perp^2/|q_f B|} \left[\frac{(m_D^g)^2 (L_l(\xi_q^\perp) - L_{l-1}(\xi_q^\perp))}{2q(q^2 + (m_D')^2)^2} \right]$$

$$\kappa_T^{(s)} = \sum_{l=0}^{\infty} \frac{(-1)^l \pi g^2 TM}{\sqrt{M^2 + 2l\,|\,q_f B\,|}} \int \frac{d^3q}{(2\pi)^3} q_\perp^2 e^{-q_\perp^2/|q_f B|} \left[\frac{\left(\frac{1}{q}(m_D^g)^2 + \delta(q_3) \sum_f \delta m_{D,f}^2\right)(L_l(\xi_q^\perp) - L_{l-1}(\xi_q^\perp))}{2(q^2 + (m_D')^2)^2} \right]$$

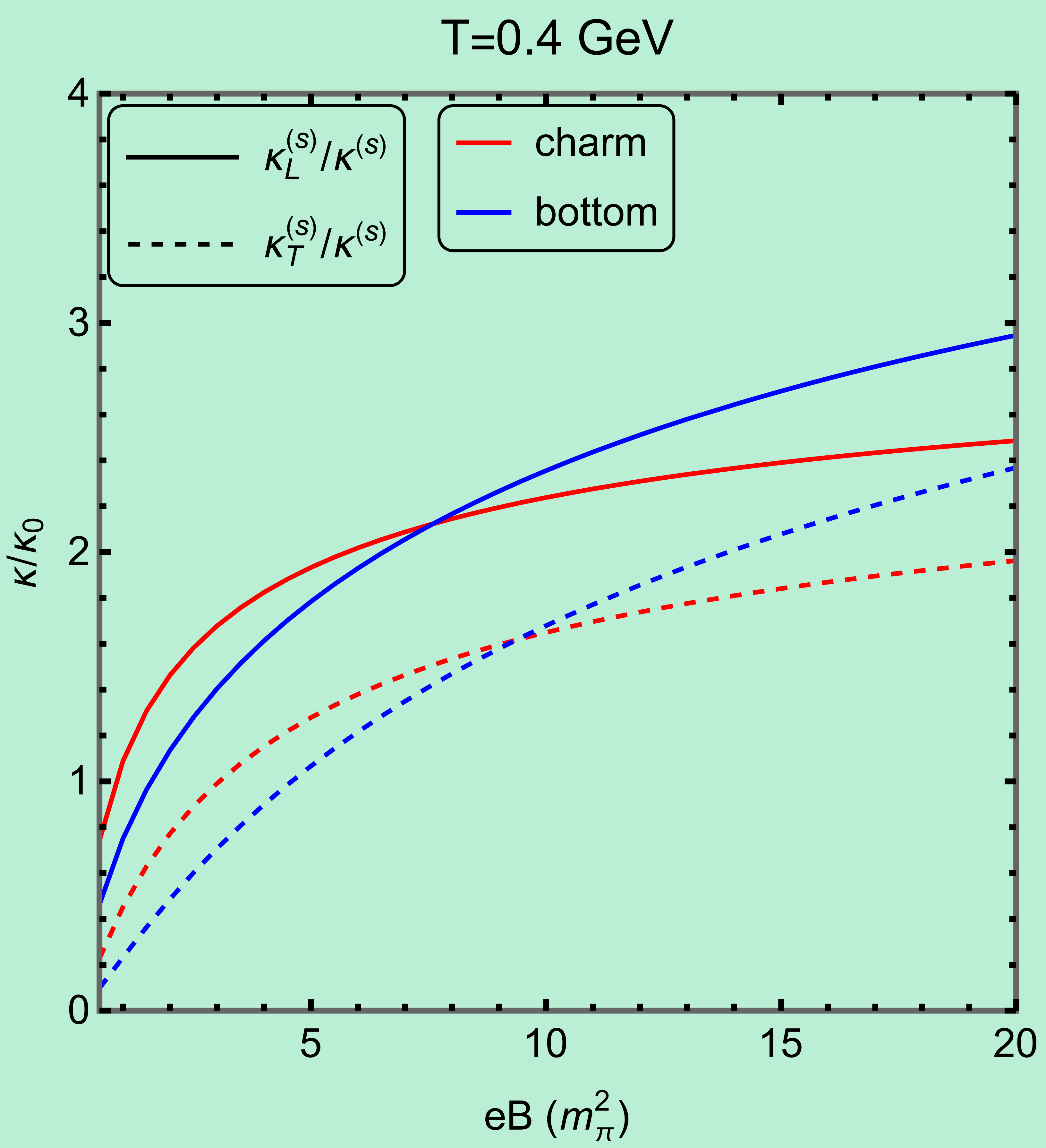
$B \neq 0$, static limit result

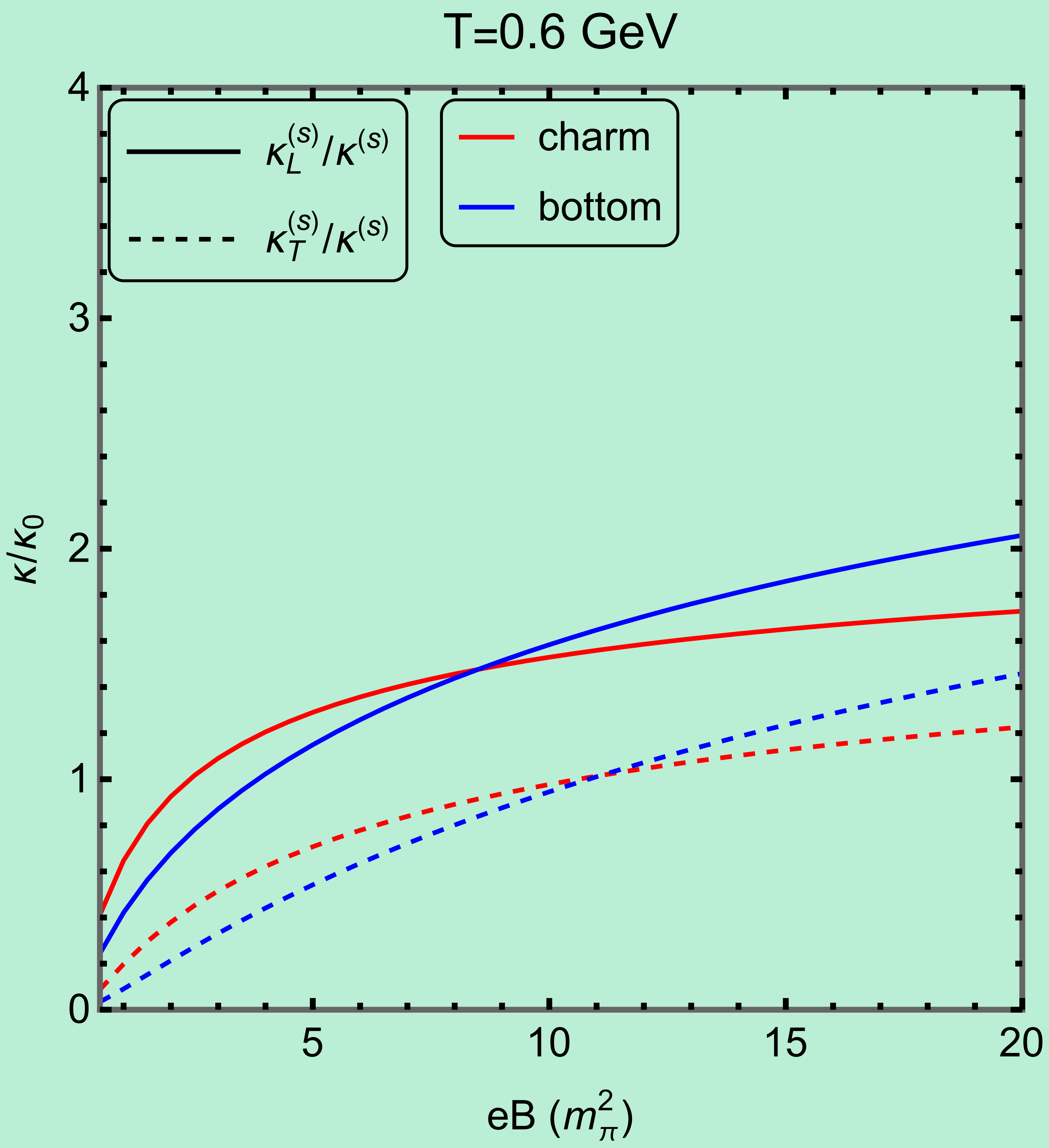




The magnetised medium modified exact results (κ) has been scaled with respect to the $eB = 0$ result (κ_0), variation of which with respect to eB has been shown for longitudinal (solid lines) and transverse (dashed lines) HQ momentum diffusion coefficients within the static limit of both charm (red curves) and bottom (blue curves) quarks.

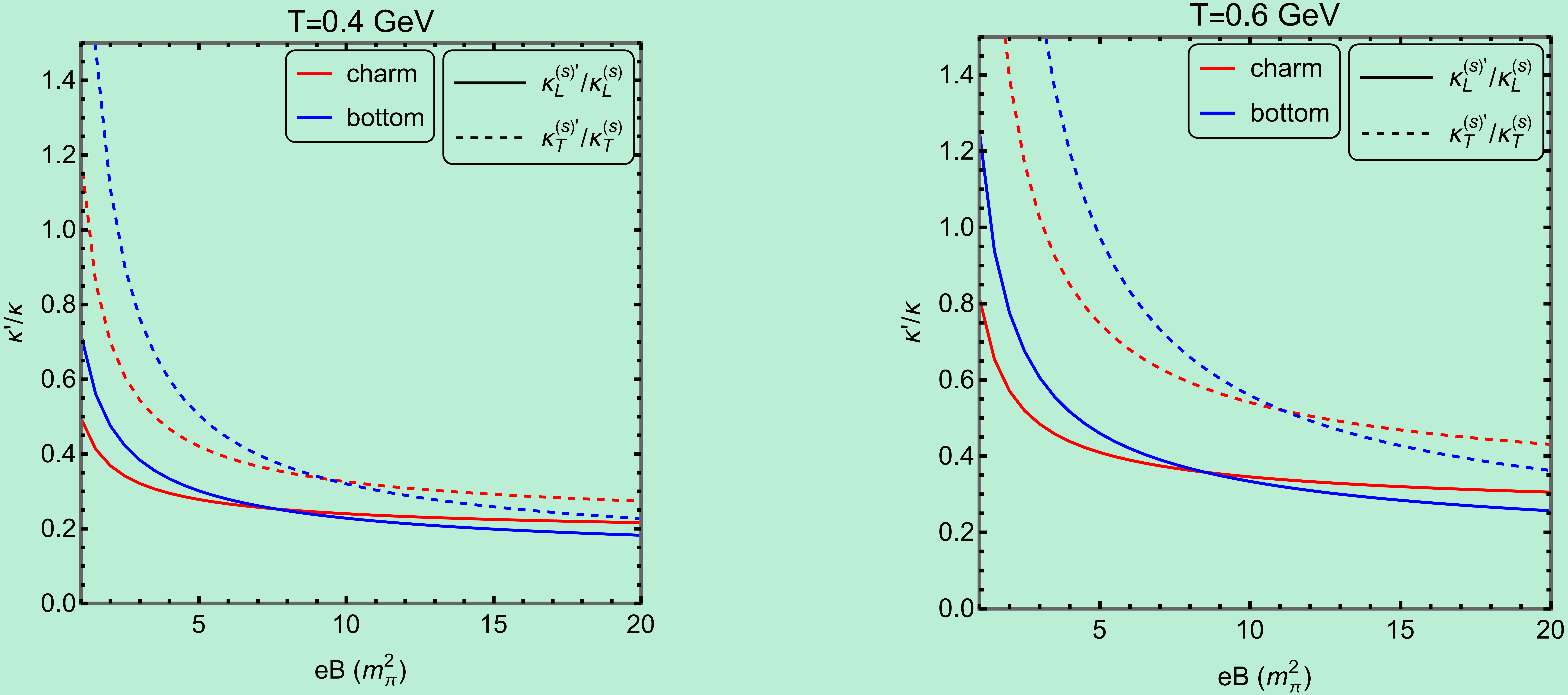
$B \neq 0$, static limit result





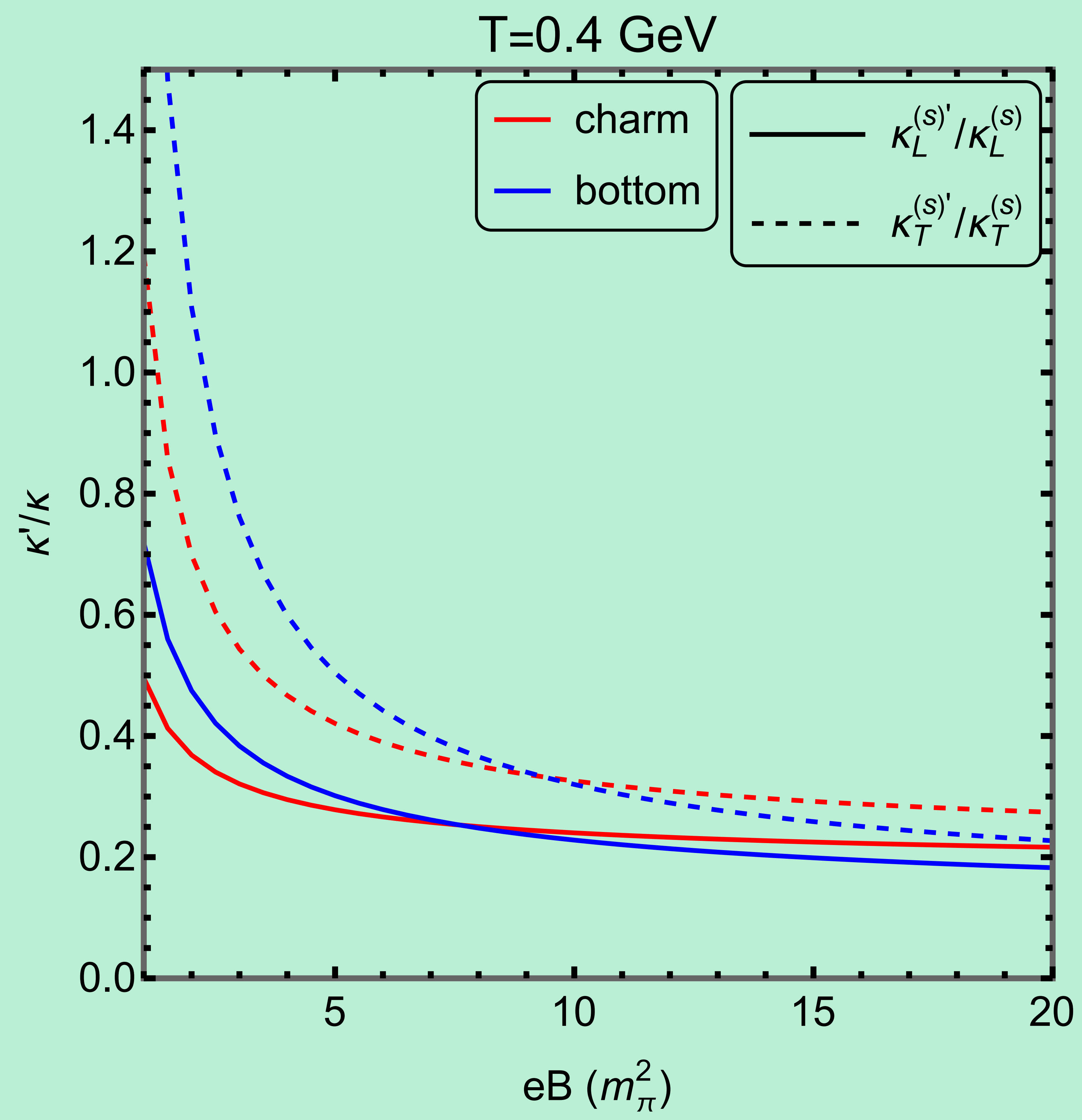
- Rate of increase for $\kappa_L/\kappa_T \rightarrow$ Low $eB >$ High eB. (More evident for charm quarks)
- $\kappa_L > \kappa_T \rightarrow$ dominant gluonic contribution in the t-channel scatterings

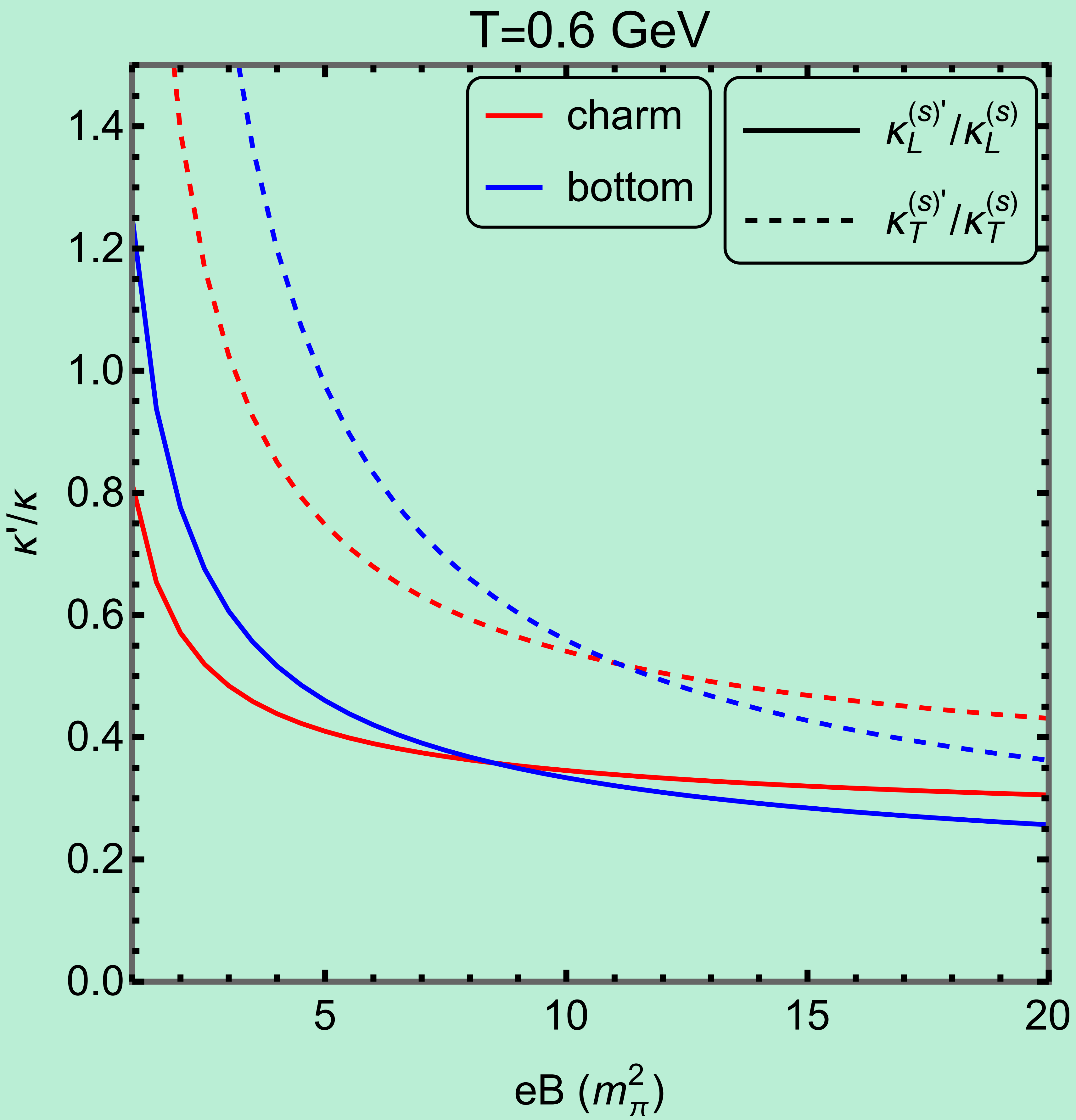
Comparison between two approaches



Variation of the ratio between the Debye mass approximated results (κ') and the exact results (κ) with respect to eB has been shown for longitudinal (solid lines) and transverse (dashed lines) HQ momentum diffusion coefficients within the static limit of both charm (red curves) and bottom (blue curves) quarks.

Comparison between two approaches





- Debye mass approximated results underestimate the exact results for larger values of eB and overestimate them for smaller values of eB. (More prominent in the case of bottom quarks)

Conclusion

- We attempt to study the HQ dynamics with arbitrary values of the external magnetic field, for the first time in literature.

- eB dependence of κ is rapidly increasing for lower values of eB, whereas it becomes saturated for relatively higher values of eB.

- Even without the quark contributions, κ_L dominates over κ_T within the static limit of HQ.

- By comparing the results of an alternate approximated procedure with our exact results, we clearly emphasise the importance of employing the general structure of the gluon two-point correlation functions in a hot magnetised medium.

THANK YOU FOR YOUR ATTENTION.