



中国科学院大学
University of Chinese Academy of Sciences

**The 7th international
Conference on Chirality,
Vorticity and Magnetic field
in Heavy Ion Collisions**

Mass spectra of neutral mesons K_0, π_0, η, η' at finite magnetic field, temperature and quark chemical potential

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Outline

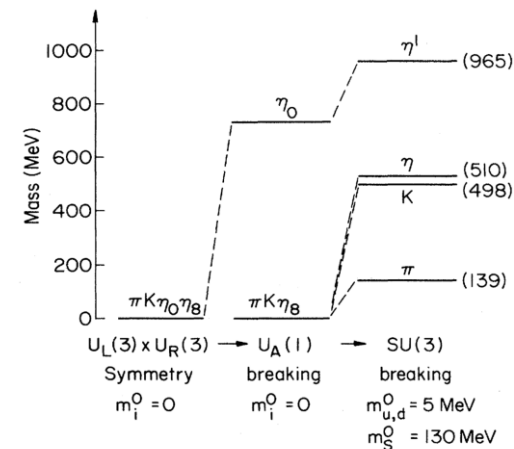
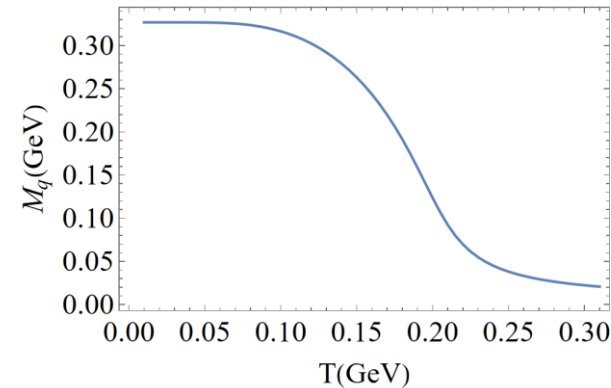
- Introduction
- Calculation and Analysis
- Numerical Results
- Summary

Introduction

Spontaneous chiral symmetry breaking is one of the most important features in QCD. Approximately 95% of the mass in the universe is believed to originate from the spontaneous breaking of chiral symmetry. Correspondingly, the restoration of chiral symmetry at extreme condition is also an important phase transition in QCD.

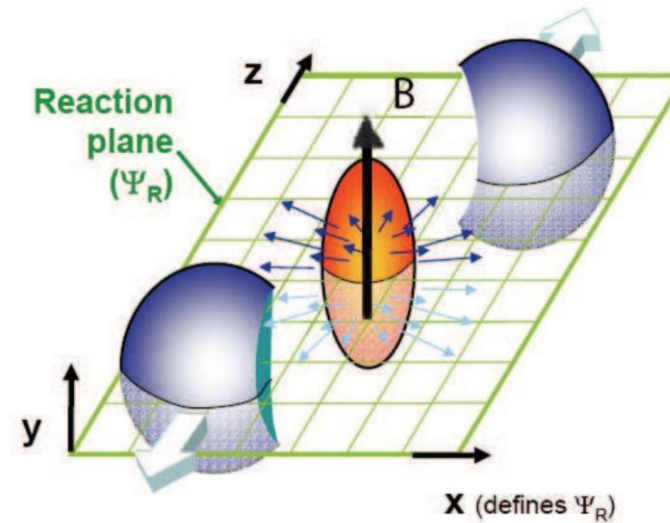
Probes to study chiral symmetry restoration:

- **Order parameter:** chiral condensate/effective quark mass.
- **Goldstone boson:** $\pi_0, K_0, \eta, \dots$



Introduction

- An intense magnetic field can be generated in the early stage of heavy ion collision.
- Many interesting features will occur when a magnetic field is induced: IMC, CME.....



RHIC: 0.03GeV^2 LHC: 0.3GeV^2

Magnetic field breaks the isospin symmetry between u and d quarks, and charged pions and kaons are no longer goldstone bosons.

In this work we focus on masses of **neutral mesons** K_0, π_0, η, η' and their responses to **magnetic field, temperature** and **quark chemical potential**.

Calculation and Analysis

- In SU(3) Nambu-Jona-Lasinio model with a uniform magnetic field, the Lagrangian reads

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G \sum_{\alpha=0}^8 [(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2],$$

$$\mathcal{L}_{KMT} = -K[\det \bar{\psi}(1 + \gamma_5)\psi + \det \bar{\psi}(1 - \gamma_5)\psi].$$

- The thermodynamic potential

$$\Omega_{\text{mf}} = 2G(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \Omega_q,$$

$$\Omega_q = -3 \sum_{f=u,d,s} \frac{|Q_f B|}{2\pi} \sum_l \alpha_l \int \frac{dp_z}{2\pi} \left[E_f + T \ln \left(1 + e^{-\frac{E_f + \mu}{T}} \right) + T \ln \left(1 + e^{-\frac{E_f - \mu}{T}} \right) \right]$$

with quark energy

$$E_f = \sqrt{p_z^2 + 2l|Q_f B| + m_f^2}$$

- Get the chiral condensates by solving the gap equation

$$\partial\Omega_{\text{mf}}/\partial\sigma_i = 0, \quad i = u, d, s,$$

- Use Pauli-Villars regularization scheme

Calculation and Analysis

- In NJL model, quarks are treated in mean field level and mesons are the quantum fluctuations constructed from the quark bubble.

$$\text{---}\text{---}\text{---} \approx \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \text{---}\text{---}\text{---} + \dots = \frac{\text{---}\text{---}\text{---}}{1 - \text{---}\text{---}\text{---}}$$

- For **neutral kaon**, its pole mass can be obtained by solving the pole equation

$$1 - 2K_6^+ \Pi_{K_0 K_0}^P(m_{K_0}, \vec{0}) = 0.$$

with the polarization function

$$\Pi_{M'M}^P(k) = i\text{Tr} \left[\Gamma_{M'}^* S \left(p + \frac{1}{2}k \right) \Gamma_M S \left(p - \frac{1}{2}k \right) \right], \quad \Gamma_M = \begin{cases} i\gamma_5 \lambda_0, & M = \eta_0 \\ i\gamma_5 \lambda_3, & M = \pi_0 \\ i\gamma_5 (\lambda_6 \pm i\lambda_7) / \sqrt{2}, & M = K_0, \bar{K}_0 \\ i\gamma_5 \lambda_8, & M = \eta_8 \end{cases}$$

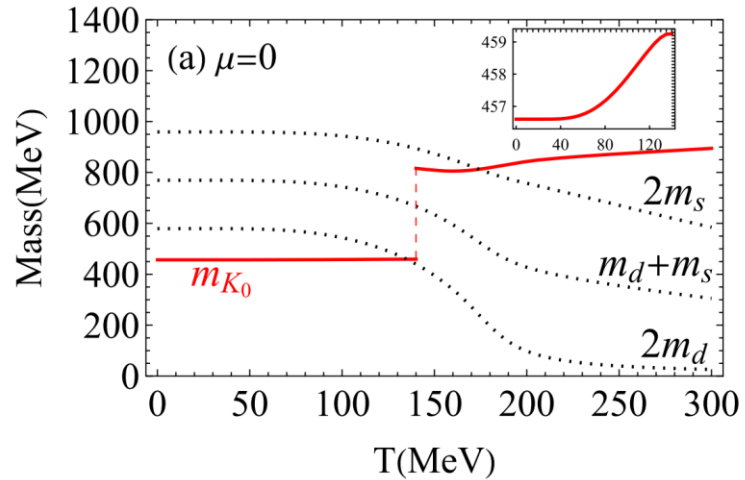
- For **neutral π_0 , η and η' mesons**, because of the mixing effect, their masses should be solved altogether.

$$\det [\mathcal{M}^{-1}(k_0, \vec{0})] = 0.$$

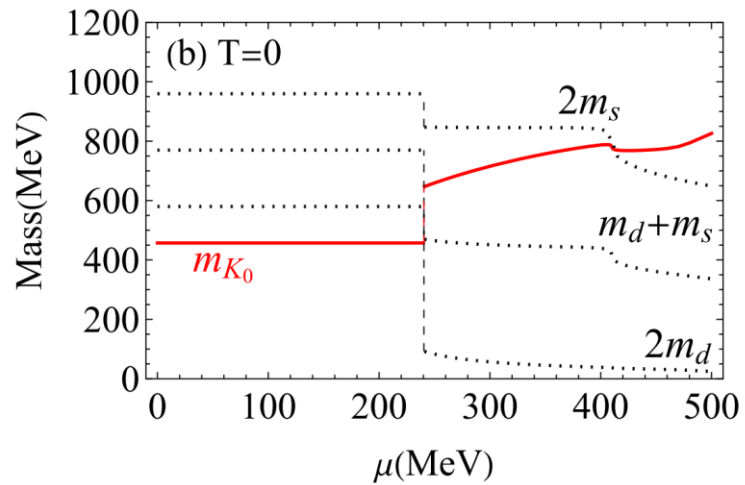
$$\mathcal{M} = 2K^+ (1 - 2\Pi^P K^+)^{-1}$$

$$K^+ = \begin{pmatrix} K_0^+ & K_{03}^+ & K_{08}^+ \\ K_{30}^+ & K_3^+ & K_{38}^+ \\ K_{80}^+ & K_{83}^+ & K_8^+ \end{pmatrix}, \quad \Pi^P = \begin{pmatrix} \Pi_0^P & \Pi_{03}^P & \Pi_{08}^P \\ \Pi_{30}^P & \Pi_3^P & \Pi_{38}^P \\ \Pi_{80}^P & \Pi_{83}^P & \Pi_8^P \end{pmatrix}.$$

Calculation and Analysis



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Meson mass
jump induced by
the jump of
constituent quark

Calculation and Analysis

- If the mass of certain meson can pass the mass sum of its corresponding constituent quark continuously. The relation

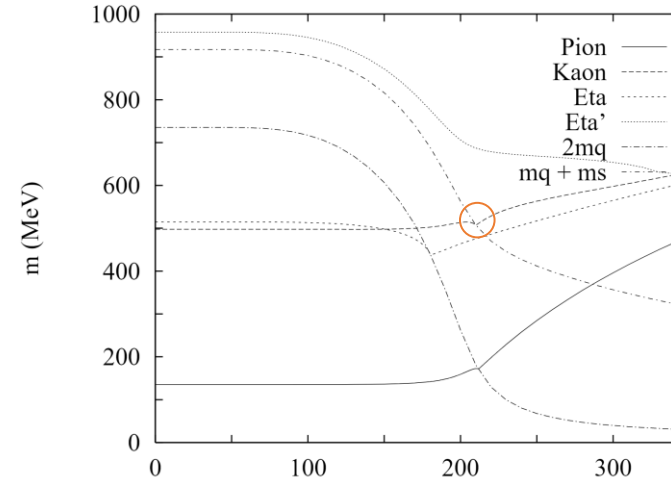
$$1 - 2K_6^+ \Pi_{K_0 K_0}^P(m_d + m_s, \vec{0}) = 0.$$

must be satisfied. (using neutral kaon as an example)

$$\Pi_{K_0 K_0}^P(k_0, \vec{0}) = J_1^{(d)} + J_1^{(s)} + 2((m_d - m_s)^2 - k_0^2)J_2^{(ds)}(k_0^2),$$

$$J_1^{(f)} = 3 \sum_l \alpha_l \frac{|Q_f B|}{2\pi} \int \frac{dp_z}{2\pi} \frac{\tanh \frac{E_f + \mu}{2T} + \tanh \frac{E_f - \mu}{2T}}{2E_f}, J_2^{(ds)}(k_0^2) = -3 \sum_l \alpha_l \frac{|Q_f B|}{2\pi} \int \frac{dp_z}{2\pi} \frac{1}{8E_s E_d} \\ \times \left[\frac{1}{E_s + E_d + k_0} \left(\tanh \frac{E_s - \mu}{2T} + \tanh \frac{E_d + \mu}{2T} \right) + \frac{1}{E_s - E_d + k_0} \left(\tanh \frac{E_d - \mu}{2T} - \tanh \frac{E_s - \mu}{2T} \right) \right. \\ \left. + \frac{1}{E_s + E_d - k_0} \left(\tanh \frac{E_d - \mu}{2T} + \tanh \frac{E_s + \mu}{2T} \right) + \frac{1}{E_s - E_d - k_0} \left(\tanh \frac{E_d + \mu}{2T} - \tanh \frac{E_s + \mu}{2T} \right) \right].$$

- When magnetic field is turned on, the integral term $\frac{1}{E_s + E_d - k_0}$ diverges when $k_0 = M_d^{(l)} + M_s^{(l)}$, with $M_q^{(l)}$ being the effective quark mass of l -th Landau level. This infrared divergence will lead to the (multiple) **mass jumps** of meson at the Mott transition.

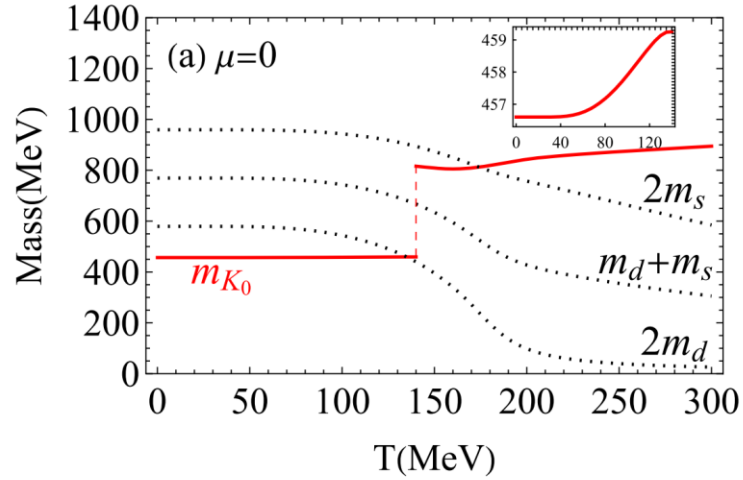


Rehberg, P., S. P. Klevansky, and J. Hüfner.
Phys. Rev. C 53 (1): 410–29(2002).

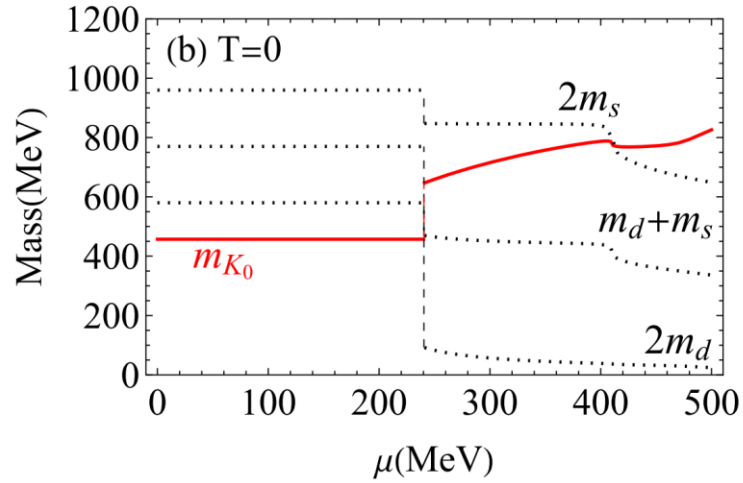
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Numerical Results

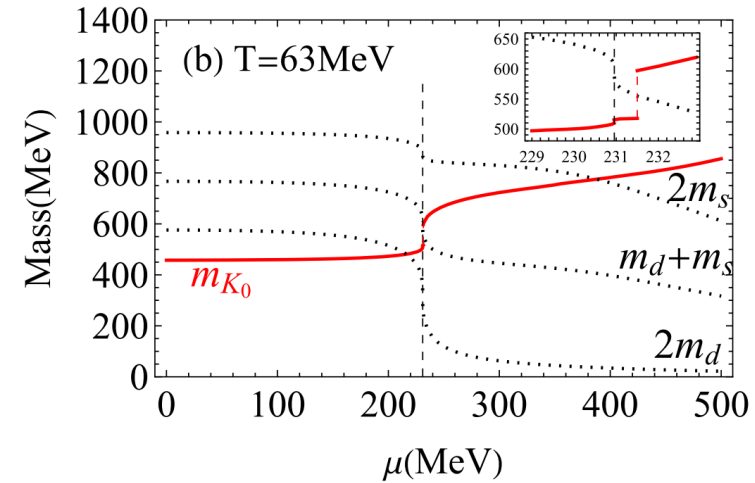
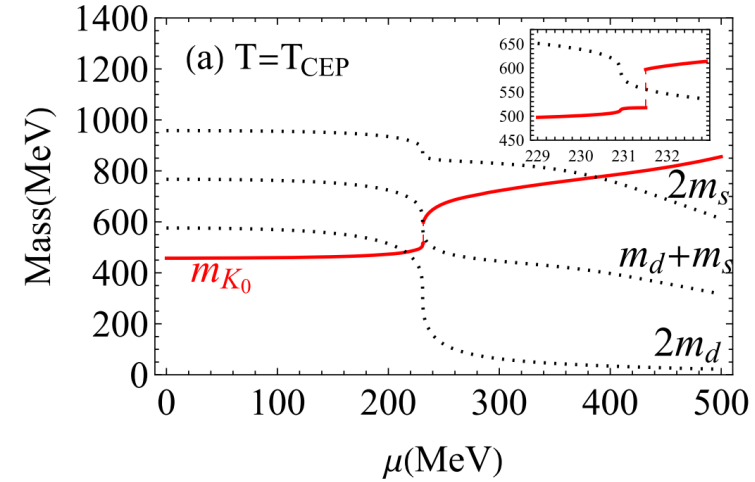
$$(T_{\text{CEP}}, \mu_{\text{CEP}}) = (63.3, 230.9 \text{ MeV}), eB = 20m_\pi^2$$



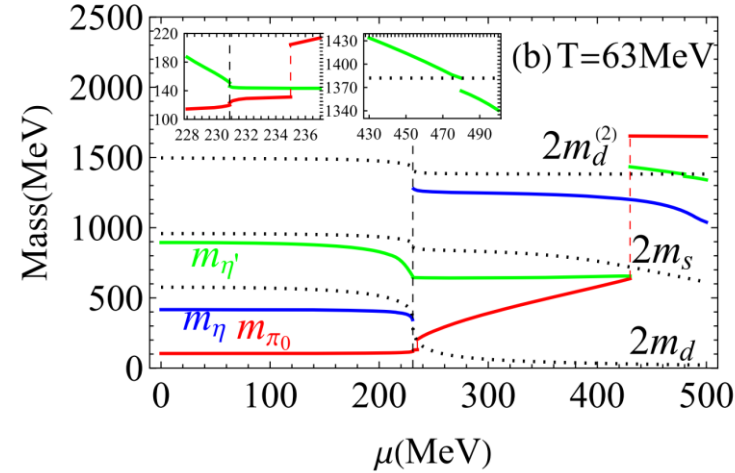
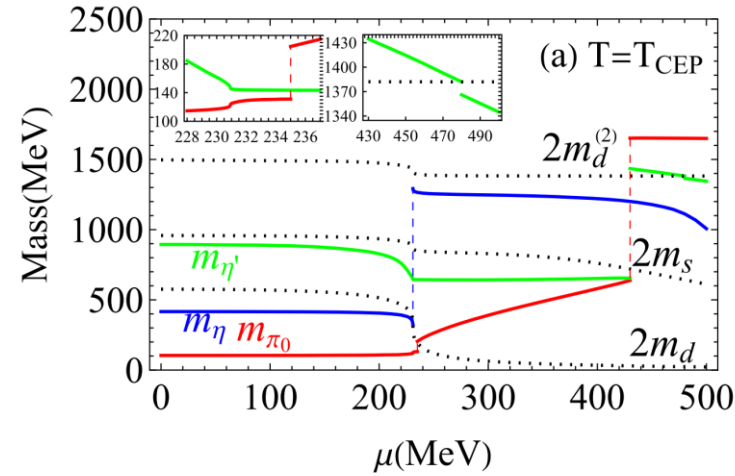
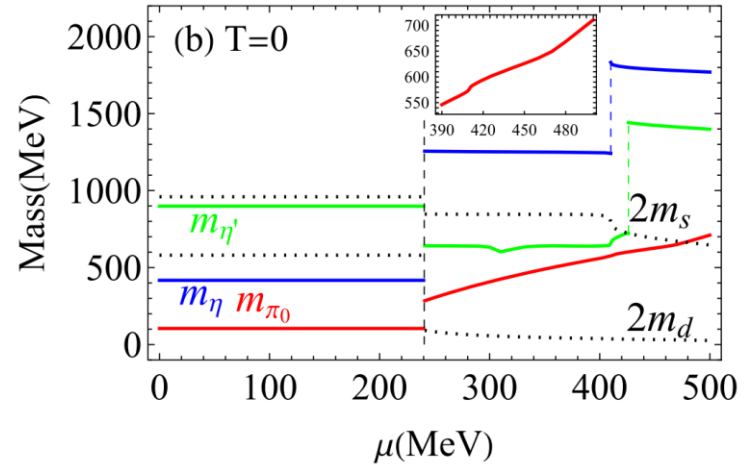
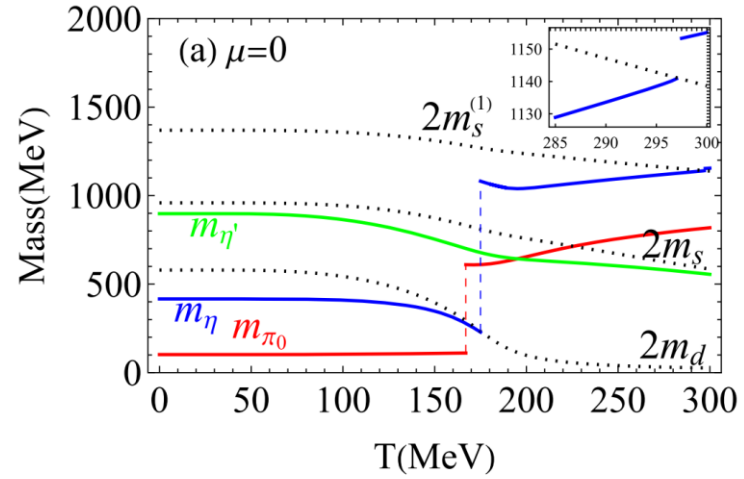
Meson mass
jump induced by
magnetic field



Meson mass
jump induced by
the jump of
constituent quark



Numerical Results



Multiple mass jumps will occur when the meson mass is large enough.

Summary

- When a uniform magnetic field is turned on, an associated meson mass jump occurs at the Mott transition.
- When the system approaches to CEP or the meson mass is large enough, multiple meson mass jumps can be observed.

Thank you!