The 7th International Conference on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

The quark-meson model under magnetic fields within FRG approach

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Based on: Rui Wen, Shi Yin, Wei-jie Fu, Mei Huang arXiv:2306.04045

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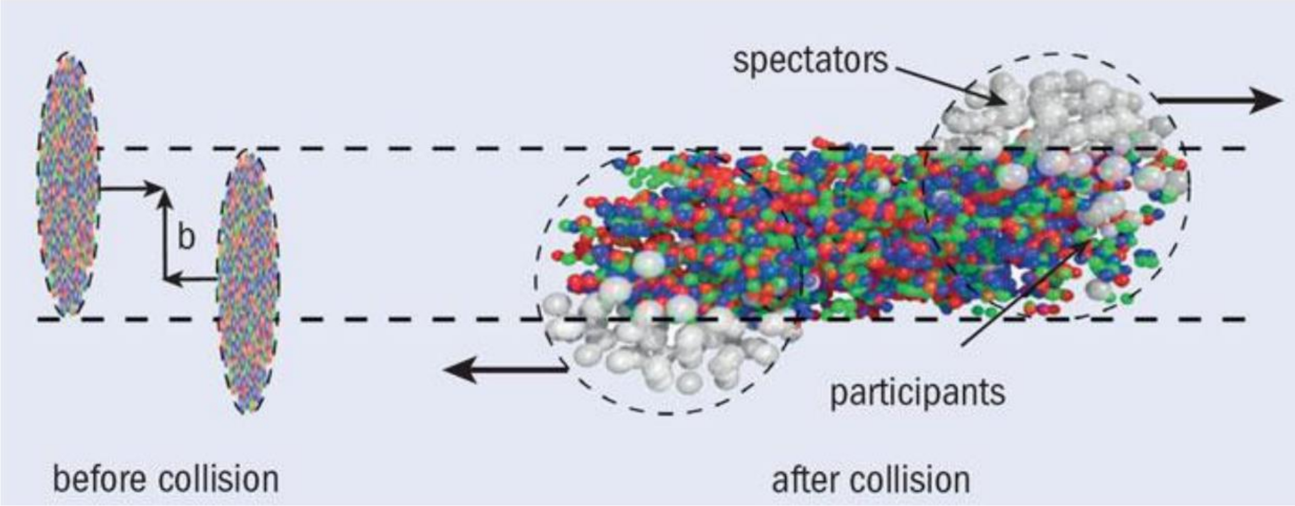
Introduction



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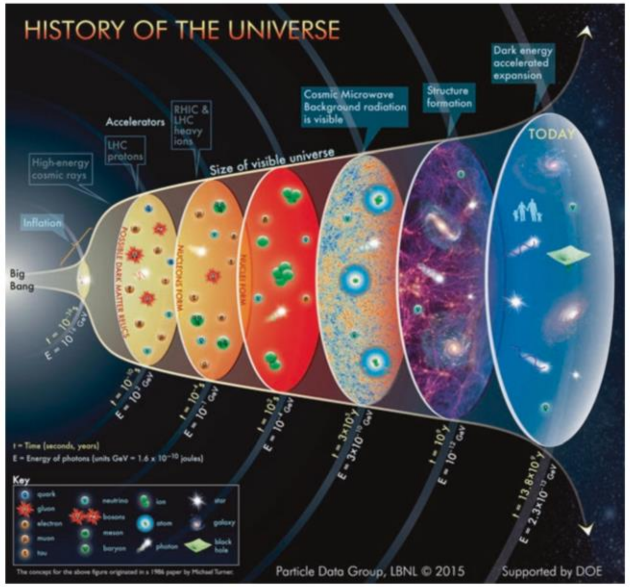
$$B \sim 10^{18}\ \text{Gauss}$$



early universe

magnetars

The effective action

The effective action of two-flavor quark-meson model in Euclidean space

$$\Gamma_k = \int_x \bar{q}\gamma_\mu(\partial_\mu - iQA_\mu)q + \mathrm{Tr}(D_\mu\phi \cdot D_\mu\phi^\dagger)$$
$$+ h\bar{q}(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi})q + V_k(\rho) - c\sigma,$$

The quark fields

$$q = (u, d)^T \qquad Q = diag(2/3, -1/3)e$$

The meson fields

$$\phi = T^0\sigma + \vec{T}\cdot\vec{\pi} = \frac{1}{2}\begin{pmatrix} \sigma + \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & \sigma - \pi^0 \end{pmatrix}$$

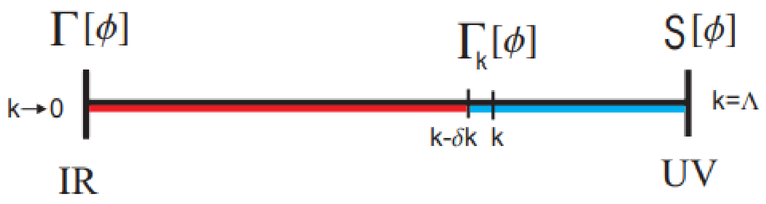
The covariant derivative of meson fields

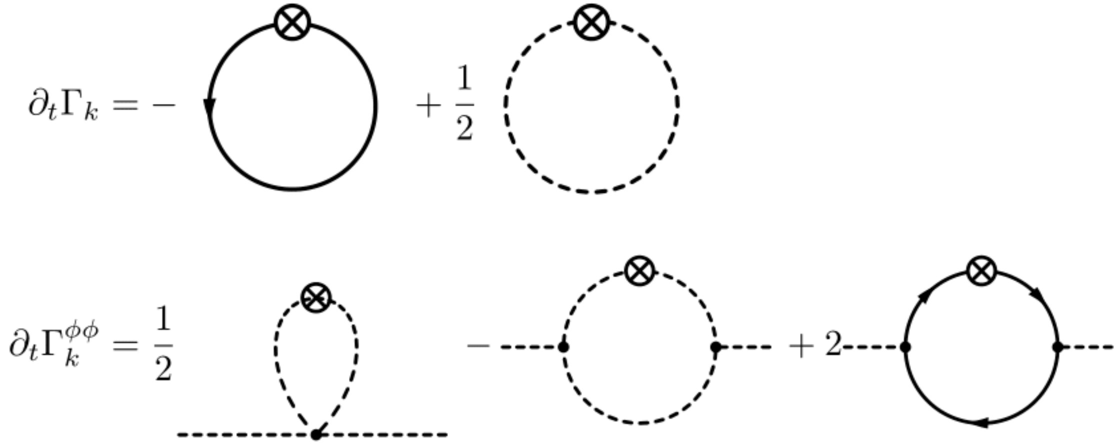
$$D_\mu\phi = \partial_\mu - iA_\mu[Q, \phi]$$

magnetic field along z-direction

$$A_\mu \ = \ (0, 0, xB, 0)$$

Functional renormalization group and flow equations





Wetterich equations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathbf{STr}[G_k \cdot \partial_t R_k]$$

$$m^2_{\phi,\mathrm{cur}} = \Gamma^{(2)}_{\phi\phi}(p_0 = 0, \vec{p} = 0)$$

The regulators

$$R_k^B = p_\parallel^2 r_B(p_\parallel^2/k^2) \qquad r_B(x) = \left(\frac{1}{x} - 1\right)\Theta(1-x)$$

$$R_k^F = i p_\parallel \cdot \gamma_\parallel r_F(p_\parallel^2/k^2) \qquad r_F(x) = \left(\frac{1}{\sqrt{x}} - 1\right)\Theta(1-x)$$

$$p_\perp = (p_1, p_2) \text{ and } p_\parallel = (p_0, p_3)$$

The quark propagator

Landau levels representation

$$\tilde{G}_k^q(p) = \exp\left(-\frac{p_\perp^2}{|q_f B|}\right) \sum_{n=0}^{\infty} \frac{(-1)^n D_n(p_{\parallel,R_F}, p_\perp)}{p_{\parallel,R_F}^2 + 2n|q_f B| + m_f^2}$$

weak-field expansion

$$\begin{aligned}
&\tilde{G}_k^q(p) \\
&= \frac{-i p_{\mu,R_F}\gamma_\mu + m_f}{p_{R_F}^2 + m_f^2} + i\frac{\gamma_1\gamma_2(m_f - i\gamma_\parallel p_{\parallel,R_F})}{(p_{R_F}^2 + m_f^2)^2} q_f B \\
&+ 2\frac{p_\perp^2(m_f - i\gamma_\parallel p_{\parallel,R_F}) + i\gamma_\perp p_\perp(m_f^2 + p_{\parallel,R_F}^2)}{(p_{R_F}^2 + m_f^2)^4}(q_f B)^2 \\
&+ \mathcal{O}(q_f B)^3.
\end{aligned}$$

The charged meson propagators

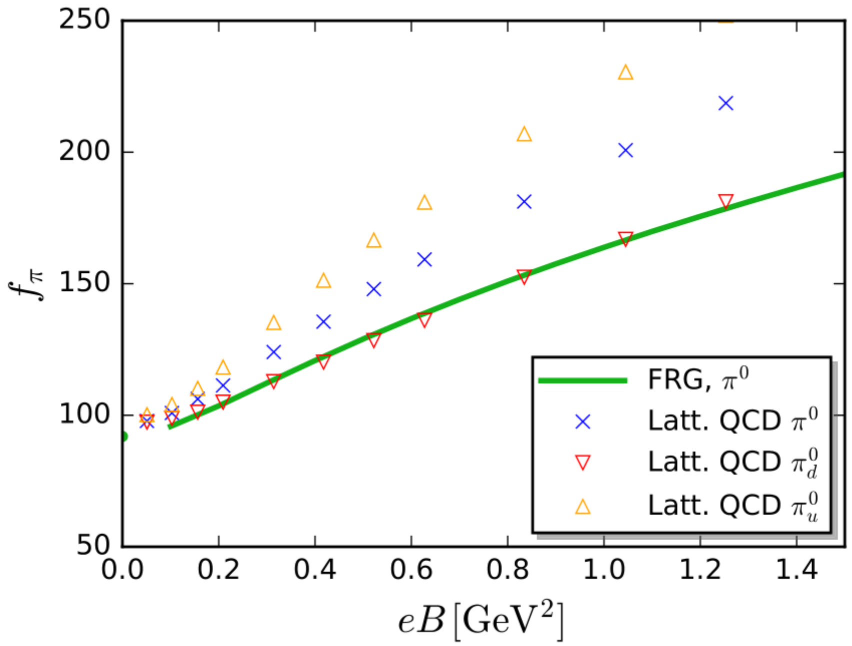
Landau levels representation

weak-field expansion

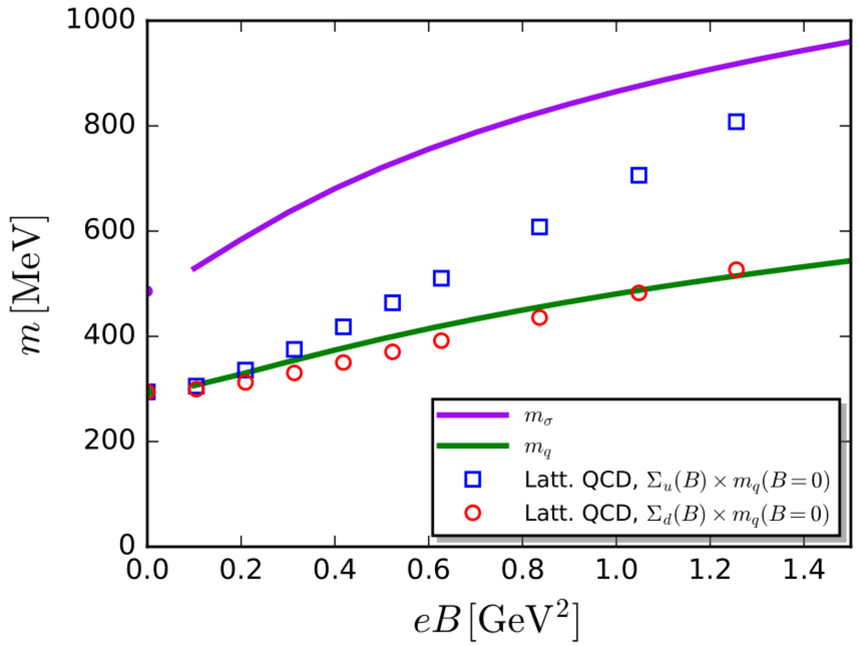
Numerical results

$\lambda_1\,[\mathrm{MeV}]^2$	λ_2	h	$c\,[\mathrm{MeV}]^3$	m_π [MeV]	m_σ [MeV]	m_q [MeV]	f_π [MeV]
$(740)^2$	-5.0	6.4	4.5×10^6	220	475	295	92
$(775)^2$	6.0	6.4	1.6×10^7	416	675	295	92

Pion decay constant



Quark mass and σ meson mass



magnetic catalysis

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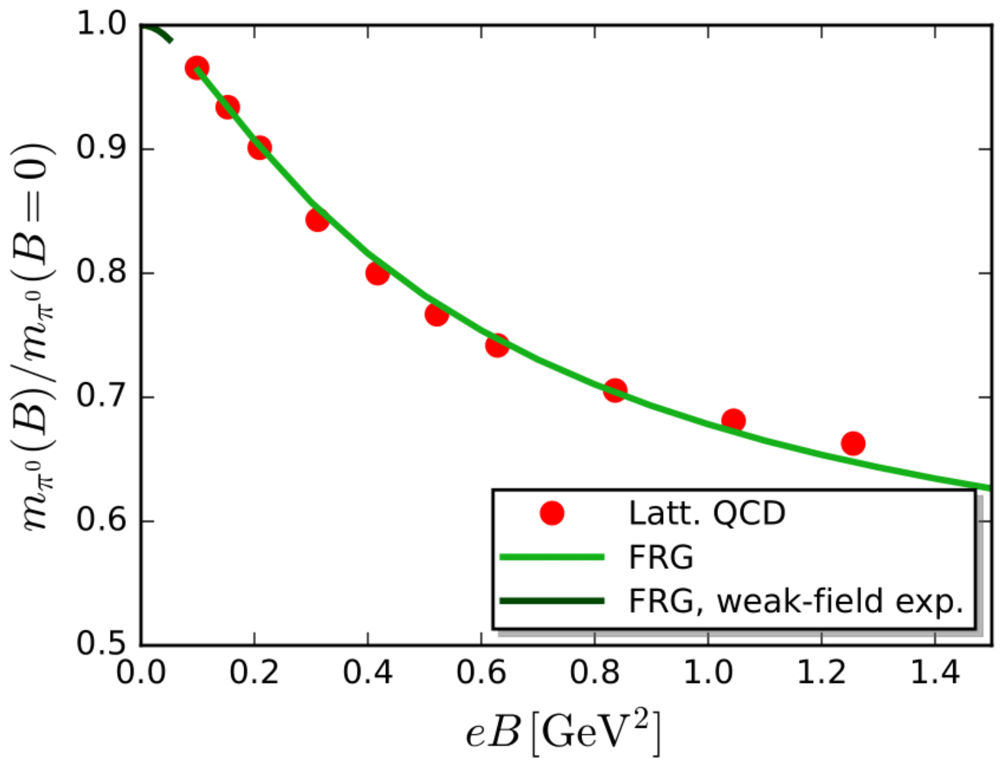
The neutral pion mass m_{π^0}

$$m^2_{\phi,\mathrm{cur}} = \Gamma^{(2)}_{\phi\phi}(p_0 = 0, \vec{p} = 0)$$



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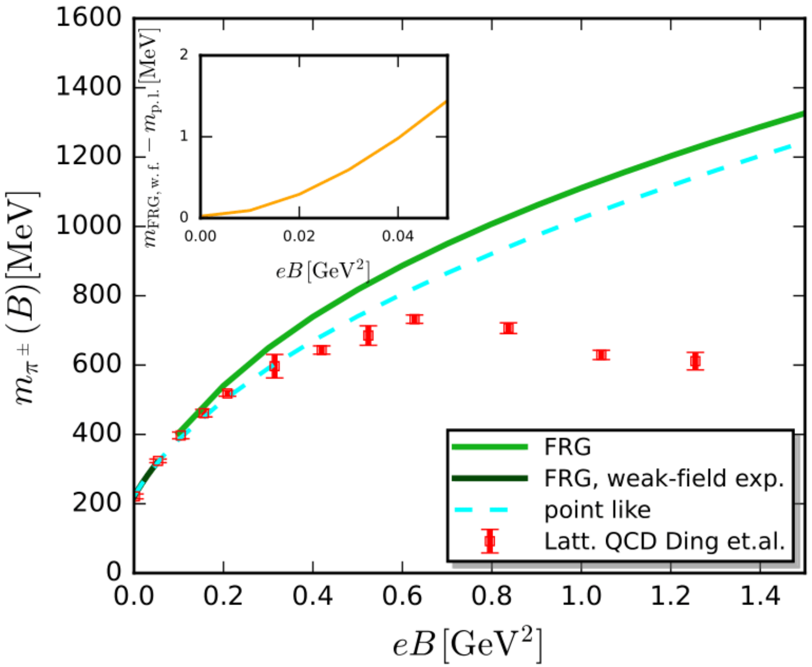
Charged pion mass $m_{\pi^\pm}$

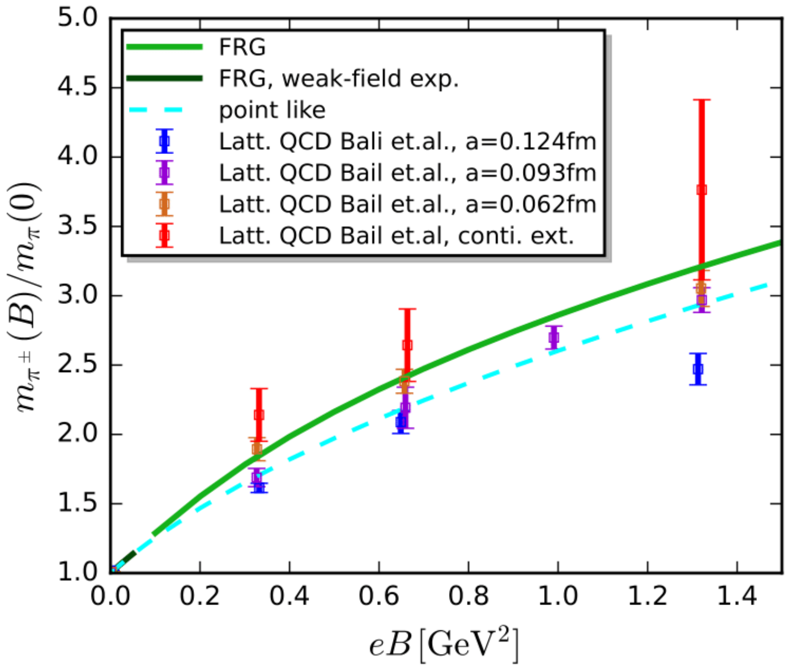
$$m_{\pi^\pm}(B) = \sqrt{\Gamma^{(2)}_{\pi^\pm\pi^\pm}(p_\parallel = 0, p_\perp = 0) + eB}.$$

$m_\pi(0)$ = 220 MeV



$a \simeq 0.117$ fm

$m_\pi(0)$ =416 MeV



H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang, and Y. Zhang Phys. Rev. D 105, 034514 (2022)
G. S. Bali, B. B. Brandt, G. Endrödi, and B. Gläßle Phys. Rev. D 97, 034505 (2018)

Summary and outlook

1.The neutral pion mass and pion decay constant are quantitatively in agreement with the lattice QCD results especially in the range of eB $< 1.2\ GeV^2$.

2. No non-monotonic mass behavior for charged pion has been observed in this framework. This needs further investigation from both lattice QCD and functional methods.

We will go beyond the LPA truncation and include the strange quark and vector meson in future work.

Thanks for your attention

Backup

The quark propagator

$$
\begin{aligned}
&D_n(p_{\|}, p_{\perp}) \\
=&(-i\gamma_{\|}p_{\|} + m_f)\left[(1 + i\gamma_1\gamma_2 s_{\perp})\mathcal{L}_n\left(\frac{2p_{\perp}^2}{|q_f B|}\right)\right. \\
&\left. - (1 - i\gamma_1\gamma_2 s_{\perp})\mathcal{L}_{n-1}\left(\frac{2p_{\perp}^2}{|q_f B|}\right)\right] \\
&+ 4i\gamma_{\perp}p_{\perp}\mathcal{L}_{n-1}^1\left(\frac{2p_{\perp}^2}{|q_f B|}\right).
\end{aligned}
$$

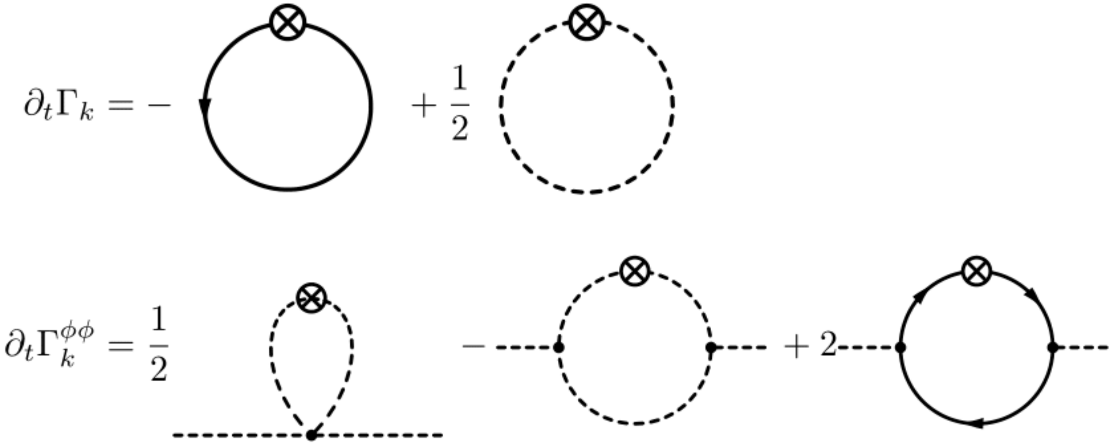
The meson propagator

Landau levels representation

$$\tilde{G}_k^{\phi}(p) = 2\exp(-\frac{p_\perp^2}{|q_\phi B|}) \times \sum_{n=0}^{\infty} \frac{(-1)^n \mathcal{L}_n\left(\frac{2p_\perp^2}{|q_\phi B|}\right)}{p_{\parallel,R_B}^2 + (2n+1)|q_\phi B| + m_\phi^2}.$$

weak-field expansion

$$\tilde{G}_k^{\phi}(p) = \frac{1}{p_{R_B}^2 + m_\phi^2} + \frac{p_\perp^2 - p_{\parallel,R_B}^2 - m_\phi^2}{(p_{R_B}^2 + m_\phi^2)^4}(q_\phi B)^2 + \mathcal{O}(q_\phi B)^4.$$



$$
\begin{aligned}
\mathcal{J}_{2F}(u,d) = &-\frac{k^4 N_c}{2\pi^2}\Bigg[\frac{\Lambda_\perp^2}{(k^2+m_f^2)(k^2+m_f^2+\Lambda_\perp^2)} \\
&+\Big(\frac{1}{4(k^2+m_f^2)^3}+\frac{5k^2+5m_f^2+8\Lambda_\perp^2}{12(k^2+m_f^2+\Lambda_\perp^2)^4}\Big)(q_u B)(q_d B)\Bigg] \\
&+\mathcal{O}(B)^4.
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\mathcal{J}_{2F}(q_f) = &-\frac{k^4 N_c}{2\pi^2}\Bigg[\frac{\Lambda_\perp^2}{(k^2+m_f^2)(k^2+m_f^2+\Lambda_\perp^2)} \\
&+\Big(\frac{1}{4(k^2+m_f^2)^3}+\frac{5k^2+5m_f^2+8\Lambda_\perp^2}{12(k^2+m_f^2+\Lambda_\perp^2)^4}\Big)(q_f B)^2\Bigg] \\
&+\mathcal{O}(B)^4.
\end{aligned}
$$