



中山大學
SUN YAT-SEN UNIVERSITY

Photon polarization in the vorticity and magnetic field

Lihua Dong
In collaboration with
Shu Lin
Sun Yat-Sen University

Based on

L. Dong, and S. Lin, EPJA, 58 (2022) 9, 176
L. Dong, and S. Lin, to appear

Outline

1

Background Introduction

2

Photon polarization in the vorticity

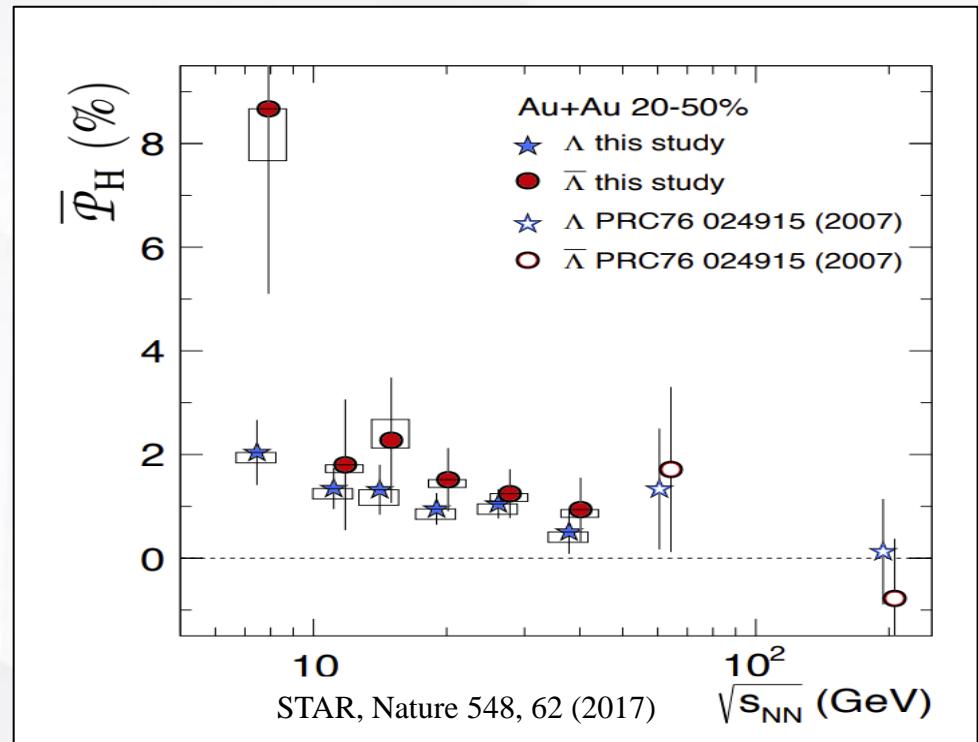
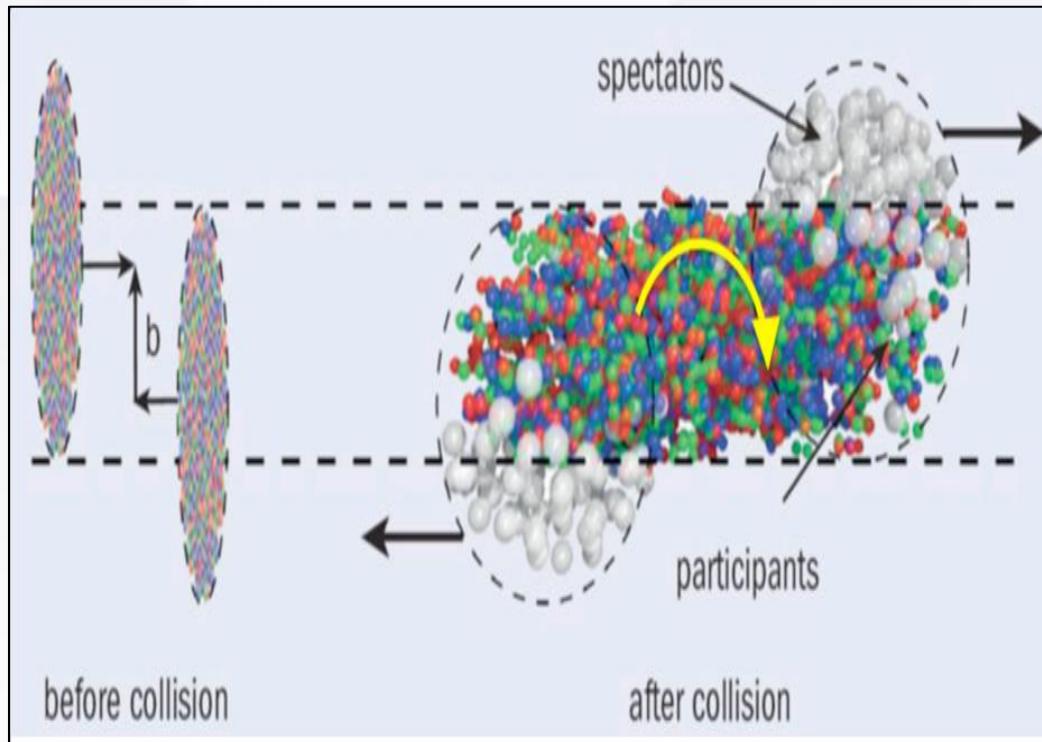
3

Photon polarization in the magnetic field

4

Summary

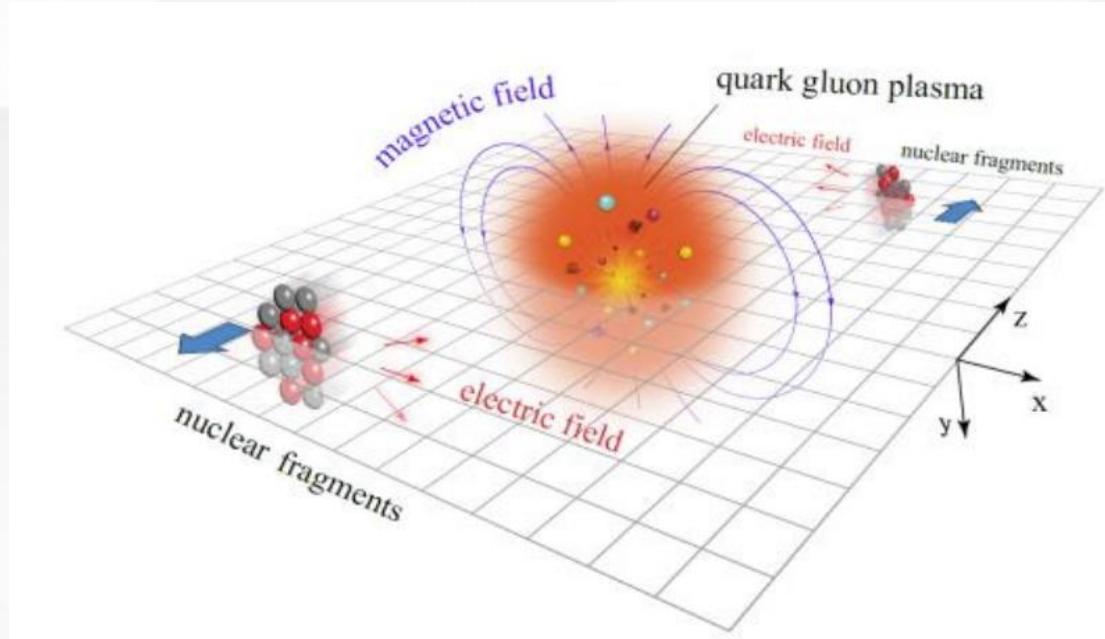
1. Background Introduction



- ◆ Off-central HIC can produce huge global orbital angular momentum.
- ◆ By spin-orbital coupling, global orbital angular momentum can lead to the polarizations of Λ hyperons and vector mesons.

$\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$
QGP is the most vortical fluid
Solar subsurface flow $10^{-7} s^{-1}$
Superfluid nanodroplets $10^7 s^{-1}$

1. Background Introduction



The strongest E & B field is created in HIC, 10^{19} Gauss!

W. Deng and X. Huang, PRC85,044907 (2012)

Pulsar: 10^{13} Gauss

Magnetar: 10^{15} Gauss

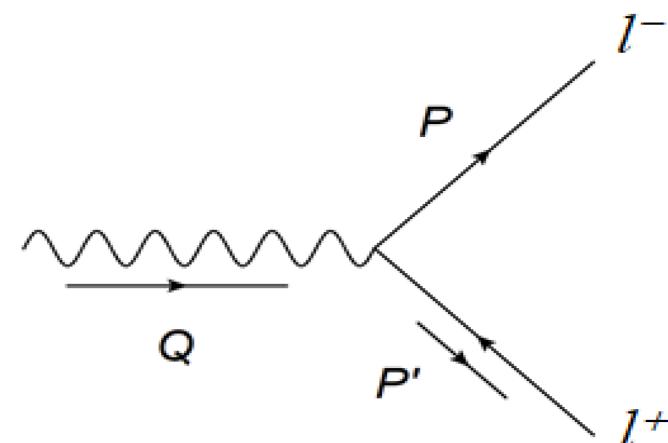
2. Photon polarization in the vorticity

Photon polarization (dilepton helical production) in a vortical QGP massless quarks and dileptons

New spin-polarized observable for dilepton production

$$\frac{dN(Q)}{d^4Q} = \sum_{P+P'=Q} (N_R(P, P') - N_L(P, P')) (P' - P) \cdot \hat{\mathbf{n}}$$

weighted difference



$N_R(P, P')$: right-handed lepton pairs

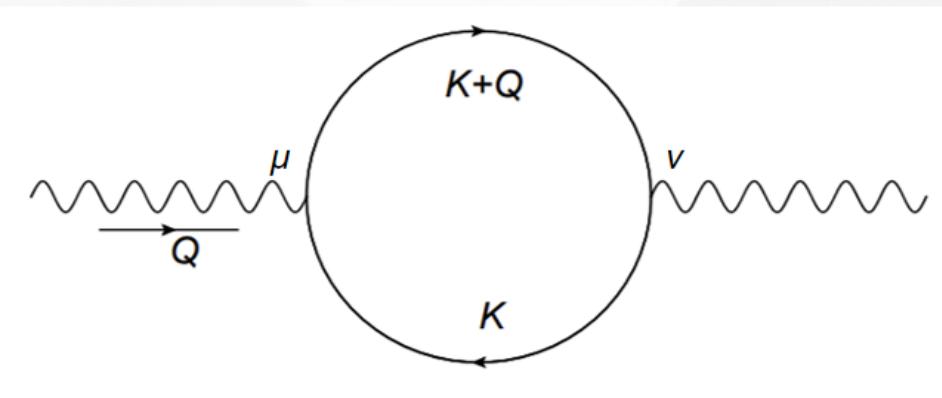
$N_L(P, P')$: left-handed lepton pairs

$\hat{\mathbf{n}}$: auxiliary spacelike unit vector

$(P' - P) \cdot \hat{\mathbf{n}}$: weight factor

Photon polarization (dilepton helical production) in a vortical QGP

- Photon self-energy



Propagator in local equilibrium:

$$S^{<(0)}(K) = -2\pi K_\mu \gamma^\mu \delta(K^2) \epsilon(k_0) \tilde{f}(k_0)$$

$$S^{>(0)}(K) = 2\pi K_\mu \gamma^\mu \delta(K^2) \epsilon(k_0) (1 - \tilde{f}(k_0))$$

$\tilde{f}(k_0)$: Fermi-Dirac distribution function

$\epsilon(k_0)$: sign function

Vortical correction to propagator

$$S^{<(1)}(K) = S^{>(1)}(K) = -2\pi \frac{1}{2} K^\mu \tilde{\Omega}_{\mu\nu} \gamma^\nu \gamma^5 \delta(K^2) \epsilon(k_0) \tilde{f}'(k_0)$$

$$\tilde{\Omega}_{\mu\nu} = \omega_\mu u_\nu - \omega_\nu u_\mu \quad \text{ ω_μ : vorticity} \quad u_\mu: \text{fluid velocity}$$

J. Gao, J. Pang and Q. Wang,
PRD.100.016008(2019)
R. Fang, L. Pang, Q. Wang and
X. Wang, PRC.94.024904(2016)

Lowest-order vortical correction

$$\Pi^{ij<(1)}(Q) = -\frac{i}{2\pi^2} \left(N_c \sum_{u,d,s} e_q^2 \right) \epsilon^{ijk} ((\hat{q} \cdot \omega) \hat{q}_k \mathcal{C}_1 + \omega_k \mathcal{C}_2)$$

Anti-symmetric

$$\Pi^{0i<(1)}(Q) = -\frac{i}{2\pi^2} \left(N_c \sum_{u,d,s} e_q^2 \right) \epsilon^{ijk} \hat{q}_j \omega_k \mathcal{C}_3$$

$\mathcal{C}_1/\mathcal{C}_2/\mathcal{C}_3$: the integral of k

Photon polarization (dilepton helical production) in a vortical QGP

- Dilepton helical rate

$$\frac{dN}{d^4Q} = \frac{e^4}{Q^2} \frac{1}{16\pi^4} \left(N_c \sum_{u,d,s} e_q^2 \right) H_{\mu\nu} \Pi^{\mu\nu<(1)}(Q)$$

$$H_{\mu\nu} = \int \frac{d^3p}{2E(2\pi)^3} \frac{d^3p'}{2E'(2\pi)^3} (2\pi)^4 \delta^{(4)}(Q - P - P') h_{\mu\nu}$$

vanish

even

Exchange $P \leftrightarrow P'$

odd

$$h^{\mu\nu} = l_R^{\mu\nu} - l_L^{\mu\nu} = 4i\epsilon^{\mu\nu\rho\sigma} P_\rho P'_\sigma$$

$l_R^{\mu\nu}/l_L^{\mu\nu}$: right-handed/left-handed polarization tensor

Introduce a weighted factor
 $(P' - P) \cdot \hat{n}$

\hat{n} : auxiliary spacelike unit vector

Weighted difference phase space integration

$$H_{\mu\nu} = \int \frac{d^3p}{2E(2\pi)^3} \frac{d^3p'}{2E'(2\pi)^3} (2\pi)^4 \delta^{(4)}(Q - P - P') (P' - P) \cdot \hat{n} h_{\mu\nu} = -\frac{iQ^2}{12\pi} \epsilon_{\mu\nu\rho\sigma} Q^\rho \hat{n}^\sigma$$

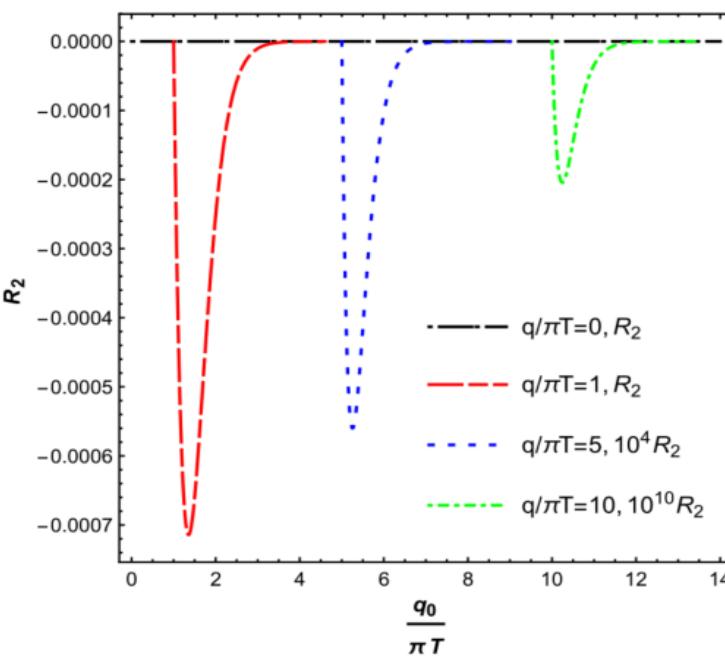
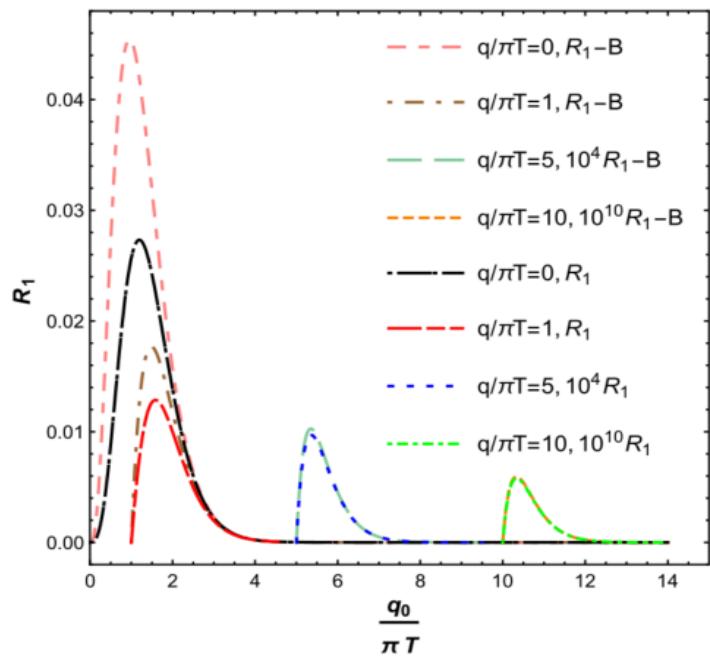
Photon polarization (dilepton helical production) in a vortical QGP

- Dilepton helical rate

$$\frac{dN}{d^4Q} = \frac{\alpha^2}{Q^2} \frac{1}{12\pi^4} \left(N_c \sum_{u,d,s} e_q^2 \right) ((\boldsymbol{\omega} \cdot \hat{\mathbf{n}}) R_1 + (\hat{\mathbf{q}} \cdot \hat{\mathbf{n}})(\hat{\mathbf{q}} \cdot \boldsymbol{\omega}) R_2)$$

R_1 / R_2 : the rotational invariant function

The q_0 dependence of R_1 and R_2 with different q



- ◆ R_1 - B : Boltzmann approximation results
- ◆ Large q_0 , R_1 - B becomes accurate
- ◆ Small q_0 , R_1 - B tends to overestimate
- ◆ R_1 is positive, and R_2 is negative
- ◆ R_1 is about twenty times that of R_2

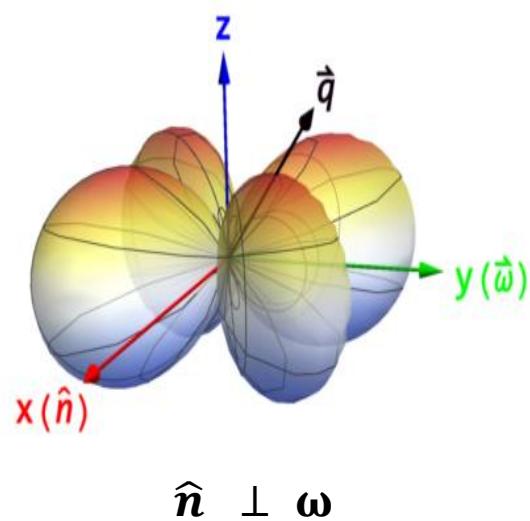
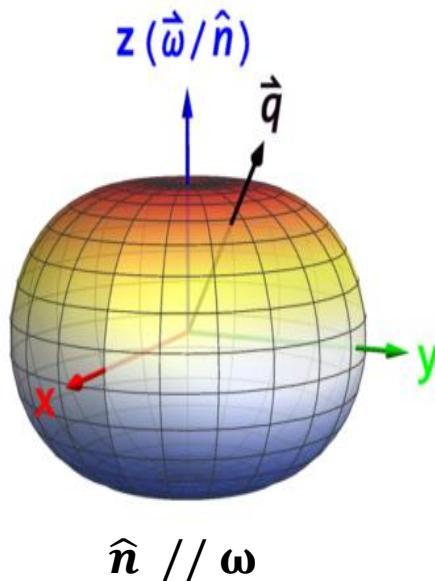
Photon polarization (dilepton helical production) in a vortical QGP

- Dilepton helical rate

$$\frac{dN}{d^4Q} = \frac{\alpha^2}{Q^2} \frac{1}{12\pi^4} \left(N_c \sum_{u,d,s} e_q^2 \right) ((\boldsymbol{\omega} \cdot \hat{\mathbf{n}}) R_1 + (\hat{\mathbf{q}} \cdot \hat{\mathbf{n}})(\hat{\mathbf{q}} \cdot \boldsymbol{\omega}) R_2)$$

$R_{1/2}$: the rotational invariant function

The schematic plots of the angular distribution



$\hat{n} // \omega$

- ◆ R_1 gives a spherical angular distribution.
- ◆ The correction R_2 deforms the sphere into an approximate **oblate ellipsoid**.

$\hat{n} \perp \omega$

- ◆ Taking \hat{n} and ω along the x and y axis respectively, the resulting angular distribution $\propto \sin^2 \theta \cos \phi \sin \phi$.

3. Photon polarization in the magnetic field

Photon polarization in a strong background magnetic field

Photon Self-energy: $\Pi_{\mu\nu} = \Pi_L Q_{\mu\nu} + \Pi_T R_{\mu\nu} + \Pi_P P_{\mu\nu}$

J F, Nieves, and Palash B. Pal ,
Phys. Rev. D 40, 2148 (1989)

$$Q_{\mu\nu} = \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}^2}, R_{\mu\nu} = \tilde{g}_{\mu\nu} - Q_{\mu\nu}$$

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}$$

$$P_{\mu\nu} = \frac{i}{\kappa} \epsilon_{\mu\nu\alpha\beta} k^\alpha u^\beta,$$

$$\kappa = \sqrt{(\omega^2 - K^2)}, \quad \omega = K \cdot u$$

Resummed photon propagator: $D_{\mu\nu} = -\frac{A_{\mu\nu}}{K^2 - \Pi_T - \Pi_P} - \frac{B_{\mu\nu}}{K^2 - \Pi_T + \Pi_P} - \frac{Q_{\mu\nu}}{k^2 - \Pi_L}$

$$A_{\mu\nu} = 1/2(R_{\mu\nu} + P_{\mu\nu}), B_{\mu\nu} = 1/2(R_{\mu\nu} - P_{\mu\nu})$$

In a **charged plasma**, the photon self-energy in the LLL approximation:

$$\Pi_R^{\mu\nu} = \frac{B}{2\pi^2} \frac{q_3^2 u^\mu u^\nu + q_0^2 b^\mu b^\nu + q_0 q_3 u^{\{\mu} b^{\nu\}}}{{q_0^2 - q_3^2}} - \frac{i \mu}{2\pi^2} (q_0 \epsilon^{\mu\nu\rho\sigma} + u^{[\mu} \epsilon^{\nu]\lambda\rho\sigma} q_\lambda^T) u_\rho b_\sigma$$

μ : chemical potential

u_μ : fluid velocity

b_μ : the direction of the magnetic field

Photon polarization in a strong background magnetic field

- Resummed photon propagator $D_{\mu\nu}^{rr}$ (in ra basis in Coulomb gauge)

$$D_{\mu\nu}^{rr} = -2\pi i \epsilon(q_0) (S_{\mu\nu} + A_{\mu\nu}) \left(\frac{1}{2} + f(Q) \right) \left(\frac{\delta(q_0^2 - x_1^2)}{q_0^2 - x_2^2} + \frac{\delta(q_0^2 - x_2^2)}{q_0^2 - x_1^2} \right)$$

$S_{\mu\nu}$: symmetry tensor

$A_{\mu\nu}$: anti-symmetry tensor

- ◆ Space-like mode: $\delta(q_0^2 - x_1^2)$

$$x_1^2 = \frac{1}{2} \left(q_1^2 + q_2^2 + 2q_3^2 + \mu 1^2 - \sqrt{(q_1^2 + q_2^2 + \mu 1^2)^2 + 4 q_3^2 \mu 1^2} \right)$$

- ◆ Time-like mode: $\delta(q_0^2 - x_2^2)$

$$x_2^2 = \frac{1}{2} \left(q_1^2 + q_2^2 + 2q_3^2 + \mu 1^2 + \sqrt{(q_1^2 + q_2^2 + \mu 1^2)^2 + 4 q_3^2 \mu 1^2} \right)$$

$$\mu 1 = \mu / 2\pi$$

- ◆ One mode related to B

4. Summary

- In the vorticity field, we have proposed an observable counting weighted difference between right-handed and left-handed lepton pairs.
- The helical rate is sensitive to the vorticity. It is maximized when the $\hat{\mathbf{n}} \parallel \boldsymbol{\omega}$, in which case, it has a nearly spherical ellipsoidal distribution.
- In the strong background magnetic field, considering a charged plasma, the photon splits into two modes due to the anti-symmetric component of the photon self-energy.

THANKS

Thanks for your attention !

Back up!

Dilepton rate is maximized when the $\hat{n} // \omega$

$$q = 0, \quad (\mathbf{P}' - \mathbf{P}) \cdot \hat{\mathbf{n}} = 2\hat{\mathbf{p}} \cdot \hat{\mathbf{n}}$$

For right-handed lepton, it is proportional to the spin projected onto $\hat{\mathbf{n}}$.

For left-handed lepton, it is proportional to minus the spin projected onto $\hat{\mathbf{n}}$.

Since spin tends to align with vorticity.

Lowest-order vortical correction

$$\Pi^{\mu\nu<} (Q) = \int \frac{d^4 K}{(2\pi)^4} \text{tr}(\gamma^\alpha S^{<(1)}(K+Q)\gamma^\beta S^{>(0)}(K) + \gamma^\alpha S^{<(0)}(K+Q)\gamma^\beta S^{>(1)}(K))$$

$$R_1 = q_0 \mathcal{C}_2 + q \mathcal{C}_3$$

$$R_2 = q_0 \mathcal{C}_1 - q \mathcal{C}_3$$

\mathcal{C}_n : the integral of k

Resummed equation

$$\begin{pmatrix} D^{rr} & D^{ra} \\ D^{ar} & 0 \end{pmatrix}_{\mu\nu} = \begin{pmatrix} D_0^{rr} & D_0^{ra} \\ D_0^{ar} & 0 \end{pmatrix}_{\mu\nu} + \begin{pmatrix} D_0^{rr} & D_0^{ra} \\ D_0^{ar} & 0 \end{pmatrix}_{\mu\alpha} \begin{pmatrix} 0 & \Pi^{ra} \\ \Pi^{ar} & \Pi^{aa} \end{pmatrix}^{\alpha\beta} \begin{pmatrix} D^{rr} & D^{ra} \\ D^{ar} & 0 \end{pmatrix}_{\beta\nu}$$

$$q_T^\mu = q^\mu - (q \cdot u)u^\mu + (q \cdot b)b^\mu$$