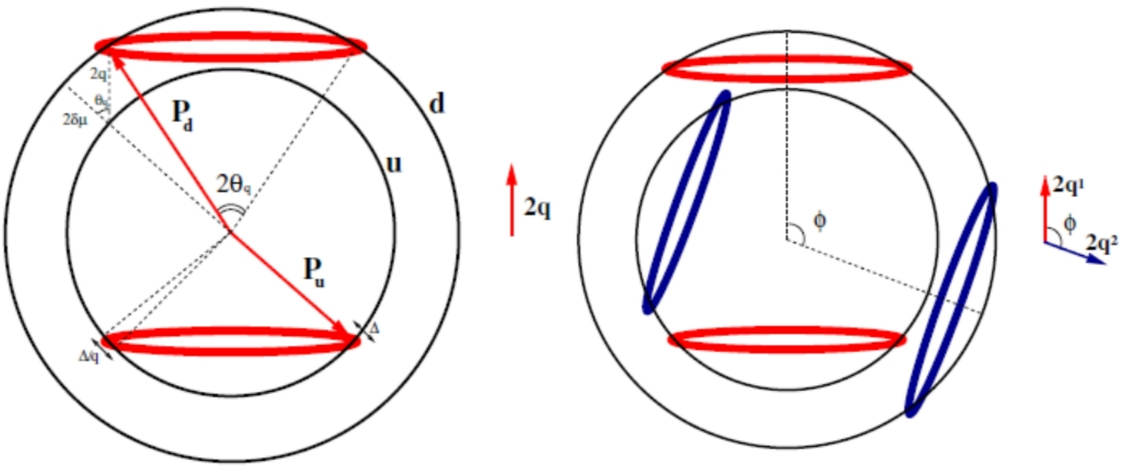
The competition between chiral density wave and two-flavor LOFF phase of color superconductivity in the NJL model

Chengfu Mu, Huzhou Normal University
Collaborated with M. Hosein Gholami, Technische Universitat Darmstadt
Prof. Michael Buballa, Technische Universitat Darmstadt

Outline

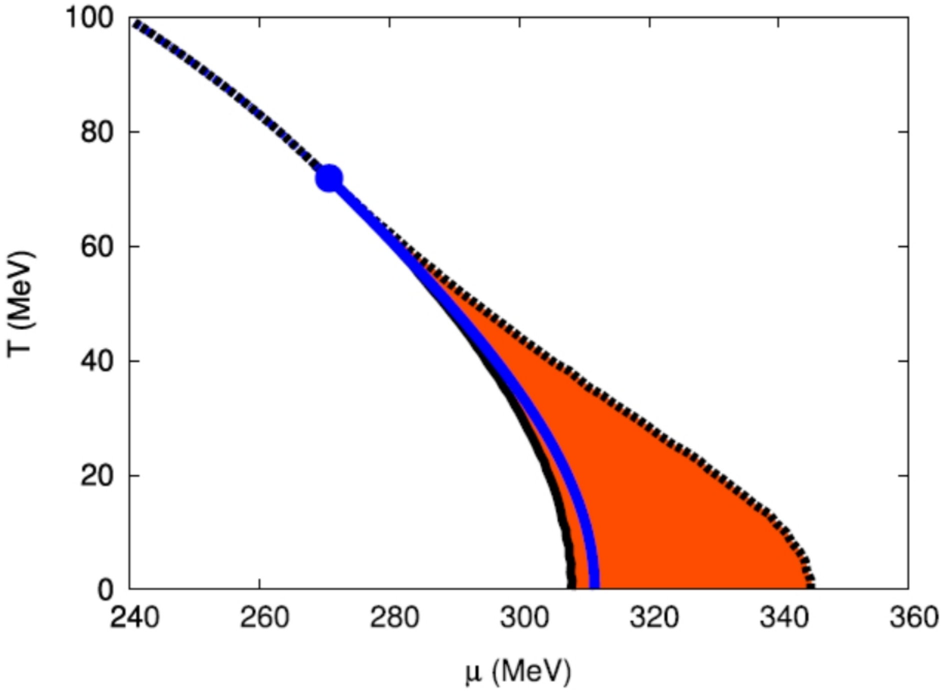
- Motivation
- The NJL model with diquarks
- The $T - \mu$ phase diagram
- The $\mu - \delta\mu$ phase diagram
- Summary and Outlook

Motivation: two kinds of inhomogeneous phases



Pictorial description of the LOFF pairing,
Which carries the nonzero total momentum

$$\Delta_A(x) = -2G_D \left\langle \bar{\psi}^C i\gamma_5 \tau_2 \lambda_A \psi \right\rangle \ , \quad \Delta_A^*(x) = -2G_D \left\langle \bar{\psi} i\gamma_5 \tau_2 \lambda_A \psi^C \right\rangle$$



Phase diagram for the NJL model, allowing for chiral density wave (CDW)-type modulations

$$\sigma(x) = -M \cos \vec{q} \cdot \vec{x}, \qquad \pi_a(x) = -M \delta_{a3} \sin \vec{q} \cdot \vec{x}.$$

The NJL Lagrangian

$$\mathcal{L}_{\mathrm{NJL}+\Delta} = \mathcal{L}_{\mathrm{NJL}} + \mathcal{L}_{\Delta}\,,$$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} + \hat{\mu}\gamma_0)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

$$\mathcal{L}_{\Delta} = G_D\left(\bar{\psi}_c i\gamma_5\tau_2\lambda_A\psi\right)\left(\bar{\psi}i\gamma_5\tau_2\lambda_A\psi_c\right)\,,$$

$$\Delta_A(x) = -2G_D\left\langle\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi\right\rangle\,, \quad \Delta_A^*(x) = -2G_D\left\langle\bar{\psi}i\gamma_5\tau_2\lambda_A\psi^C\right\rangle$$

Under the mean field approximation

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}\Bigg[\bar{\psi}(i\partial\!\!\!/ + \hat{\mu}\gamma_0 + \sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})\psi + \bar{\psi}_c(i\partial\!\!\!/ - \hat{\mu}\gamma_0 + \sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}^T)\psi_c$$

$$+ \Delta_A^*(x)\left(\bar{\psi}_c i\gamma_5\tau^2\lambda^A\psi\right) + \Delta_A(x)\left(\bar{\psi}i\gamma_5\tau^2\lambda^A\psi_c\right) - \frac{\sigma^2+\vec{\pi}^2}{2G_S} - \frac{|\Delta_A|^2}{2G_D}\Bigg]$$

$$\sigma(x) = -M\cos\vec{q}\cdot\vec{x}, \qquad \pi_a(x) = -M\,\delta_{a3}\sin\vec{q}\cdot\vec{x}.$$

Making the chiral field transformation

$$\psi'(x) = \exp(\frac{i}{2}\gamma_5\tau_3\vec{q}\cdot\vec{x})\psi(x),$$
$$\psi'_C(x) = \exp(\frac{i}{2}\gamma_5\tau_3\vec{q}\cdot\vec{x})\psi_C(x),$$
$$\bar{\psi}'(x) = \bar{\psi}(x)\exp(\frac{i}{2}\gamma_5\tau_3\vec{q}\cdot\vec{x}),$$
$$\bar{\psi}'_C(x) = \bar{\psi}_C(x)\exp(\frac{i}{2}\gamma_5\tau_3\vec{q}\cdot\vec{x}),$$

$$\Delta_A(x) = \Delta_A e^{2i\vec{q}'\cdot\vec{x}}, \qquad \Delta_A^*(x) = \Delta_A e^{-2i\vec{q}'\cdot\vec{x}}.$$

introduce the auxiliary quark fields

$$\chi(x) = \psi'(x)e^{-i\vec{q'}\cdot\vec{x}}, \quad \chi^C(x) = \psi'_C(x)e^{i\vec{q'}\cdot\vec{x}},$$

$$\begin{aligned}
\psi'(x) &= \chi(x)e^{i\vec{q'}\cdot\vec{x}}, \\
\psi'_C(x) &= \chi_C(x)e^{-i\vec{q'}\cdot\vec{x}}, \\
\bar{\psi}'(x) &= \bar{\chi}(x)e^{-i\vec{q'}\cdot\vec{x}}, \\
\bar{\psi}'_C(x) &= \bar{\chi}_C(x)e^{i\vec{q'}\cdot\vec{x}},
\end{aligned}$$

The partition function can be obtained by integrating over the fermionic fields

$$\mathcal{Z} = \exp\left[-\int_x \left(\frac{M^2}{4G_S} + \frac{|\Delta|^2}{4G_D}\right) + \frac{1}{2}\ln\det S^{-1}\right].$$

$$\mathcal{S}_1^{-1} = \gamma^0 (i\omega_n - H_{1\mathrm{NG}})$$

$$\mathcal{S}_2^{-1} = \gamma^0 (i\omega_n - H_{2\mathrm{NG}})$$

$$H_{1\mathrm{NG}} = \begin{pmatrix} \mu_d + \gamma^0\vec{\gamma}\cdot(\vec{p}+\vec{q}') + \frac{1}{2}\gamma^0\gamma_5\vec{\gamma}\cdot\vec{q} + M\gamma^0 & i\gamma^0\gamma_5\Delta \\ i\gamma^0\gamma_5\Delta & -\mu_u + \gamma^0\vec{\gamma}\cdot(\vec{p}-\vec{q}') - \frac{1}{2}\gamma^0\gamma_5\vec{\gamma}\cdot\vec{q} + M\gamma^0 \end{pmatrix},$$

$$H_{2\mathrm{NG}} = \begin{pmatrix} -\mu_u + \gamma^0\vec{\gamma}\cdot(\vec{p}+\vec{q}') - \frac{1}{2}\gamma^0\gamma_5\vec{\gamma}\cdot\vec{q} + M\gamma^0 & i\gamma^0\gamma_5\Delta \\ i\gamma^0\gamma_5\Delta & \mu_d + \gamma^0\vec{\gamma}\cdot(\vec{p}-\vec{q}') + \frac{1}{2}\gamma^0\gamma_5\vec{\gamma}\cdot\vec{q} + M\gamma^0 \end{pmatrix},$$

$\mathcal{E}_i$ are the 16 eigenvalues of $H_{1\mathrm{NG}}$ and $H_{2\mathrm{NG}}$.

$$\ln Z_{15,24} = \frac{1}{2}\ln\mathrm{Det}(\beta G^{-1}_{15,24}) = \ln\left[\mathrm{Det}\mathcal{S}_1^{-1}\mathrm{Det}\mathcal{S}_2^{-1}\right] = \sum_{i=1}^{16} T \sum_n \ln(i\omega_n + \mathcal{E}_i).$$

$$T\sum_n \ln\left(\frac{i\omega_n + \mathcal{E}_i}{T}\right) = \frac{\mathcal{E}_i}{2} + T\ln\left(1 + \mathrm{e}^{-\mathcal{E}_i/T}\right).$$

$$\Omega(T,\mu,\delta\mu,M,\Delta,q,q') = \frac{M^2}{4G_S}+\frac{\Delta^2}{4G_D}-\sum_{i=1}^{16}\int\frac{d^3p}{(2\pi)^3}\left[\frac{\mathcal{E}_i}{2}+T\ln\left(1+\mathrm{e}^{-\mathcal{E}_i/T}\right)\right] -\frac{1}{2}\sum_{i=1}^{16}\int\frac{d^3p}{(2\pi)^3}\left[\frac{\mathcal{E}_i}{2}+T\ln\left(1+\mathrm{e}^{-\mathcal{E}_i/T}\right)\right]_{\Delta=0,\vec{q'}=0,\text{for blue quarks}}$$

subtract the unphysical divergence term proportional to $\Lambda^2 q^2$

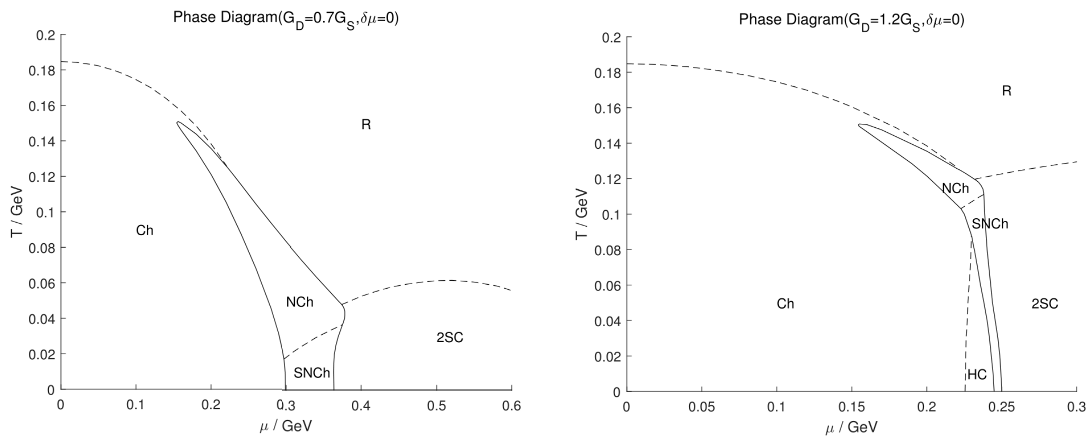
$$\begin{aligned}&\Omega(T,\mu,\delta\mu,M,\Delta,\vec{q},\vec{q'})\\ &= \frac{M^2}{4G_S}+\frac{\Delta^2}{4G_D}+\frac{M^2F_\pi^2}{2M_0^2}(\vec{q})^2-\frac{3}{2}\sum_{j=1}^{16}\int^\Lambda\frac{d^3p}{(2\pi)^3}\left[\frac{1}{2}\mathcal{E}_{j,\Delta=0,\vec{q}=0,\vec{q'}=0,\text{for blue quarks}}\right]\\ &-\sum_{i=1}^{16}\int^\Lambda\frac{d^3p}{(2\pi)^3}\left[\frac{\mathcal{E}_i}{2}+T\ln\left(1+\mathrm{e}^{-\mathcal{E}_i/T}\right)\right]+\sum_{i=1}^{16}\int^\Lambda\frac{d^3p}{(2\pi)^3}\left[\frac{1}{2}\mathcal{E}_{i,\Delta=0,\vec{q'}=0,\text{for blue quarks}}\right]\\ &-\frac{1}{2}\sum_{i=1}^{16}\int^\Lambda\frac{d^3p}{(2\pi)^3}\left[T\ln\left(1+\mathrm{e}^{-\mathcal{E}_i/T}\right)\right]_{\Delta=0,\vec{q'}=0,\text{for blue quarks}}\end{aligned}$$

For LOFF part, we apply He's subtraction scheme

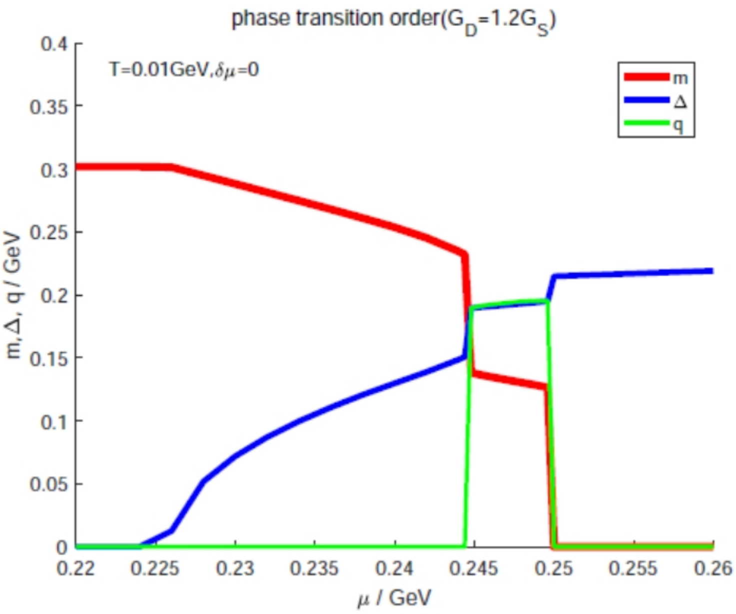
$$\Omega_{\mathrm{sub}}(T,\mu,\delta\mu,M,\Delta,\vec{q},\vec{q}') = \Omega(T,\mu,\delta\mu,M,\Delta,\vec{q},\vec{q}') - \Omega(T,\mu,\delta\mu,M=0,\Delta=0,\vec{q}=0,\vec{q}') + \Omega(T,\mu,\delta\mu,M=0,\Delta=0,\vec{q}=0,\vec{q}'=0).$$

- The chiral symmetry-restored phase (R): $M=0,\ q=0,\ \Delta=0, q'=0$.
- The homogeneous chiral symmetry-broken phase (Ch): $M\neq 0,\ q=0,\ \Delta=0, q'=0$.
- The inhomogeneous chiral phase (NCh): $M\neq 0,\ q\neq 0,\ \Delta=0, q'=0$.
- The 2SC phase (2SC): $M=0,\ q=0,\ \Delta\neq 0, q'=0$.
- The coexistence phase (SNCh): $M\neq 0,\ q\neq 0,\ \Delta\neq 0, q'=0$.
- The HC phase denotes the phase with $M\neq 0, \Delta\neq 0,\ q=0,\ q'=0$.
- The LOFF phase denotes the phase with $M=0, \Delta\neq 0,\ q=0,\ q'\neq 0$.

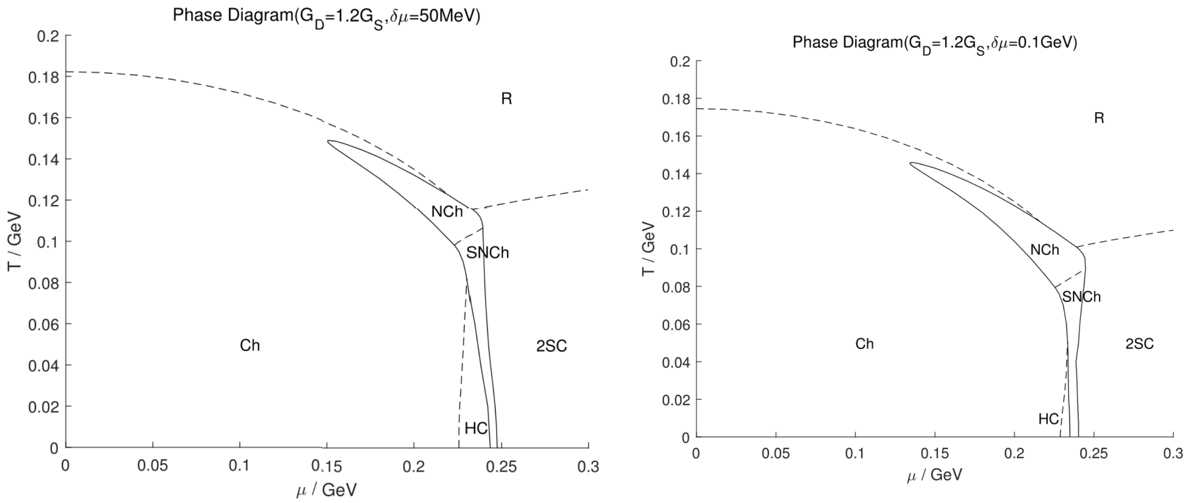
The T－ μ phase diagram



The T－ μ phase diagrams for GD = 0.7GS and GD = 1.2GS at δ μ = 0

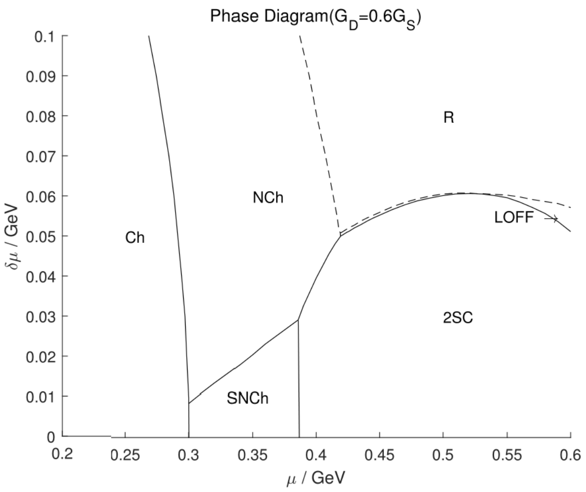


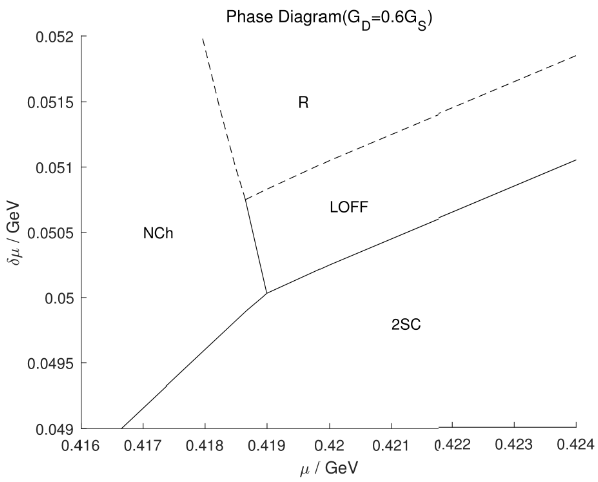
The order parameters as function of μ for G_D = 1.2GS at δ μ = 0 and q′ = 0.



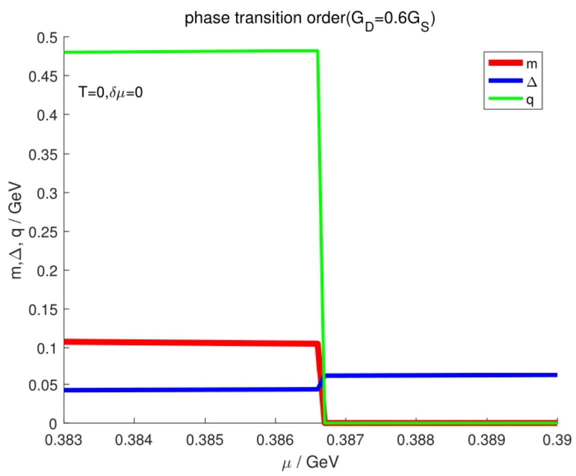
The T－μ phase diagram for $G_D = 1.2G_S$ at $\delta\mu = 0.05$GeV and $\delta\mu = 0.1$GeV

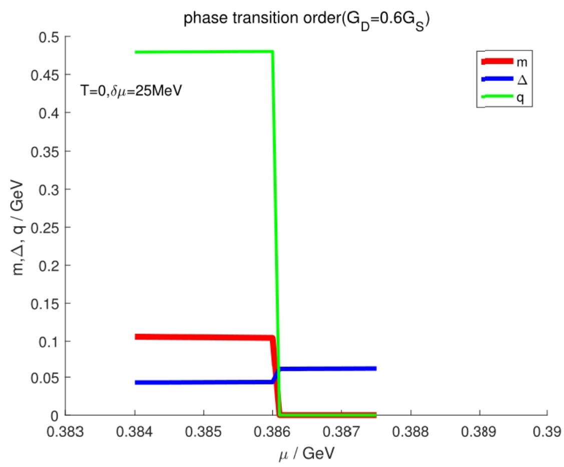
The $\mu - \delta\mu$ phase diagram



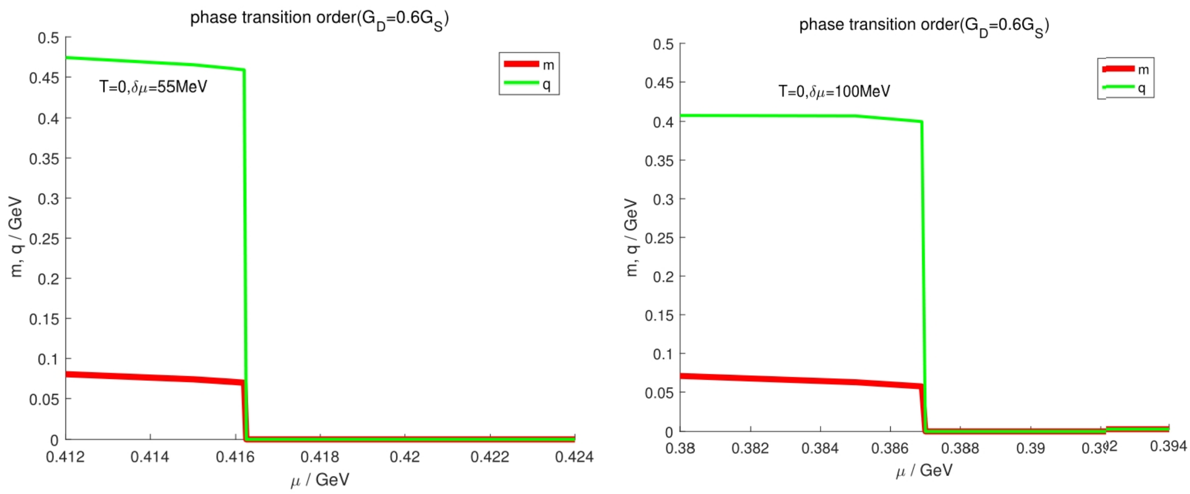


The $G_D = 0.6G_S$.

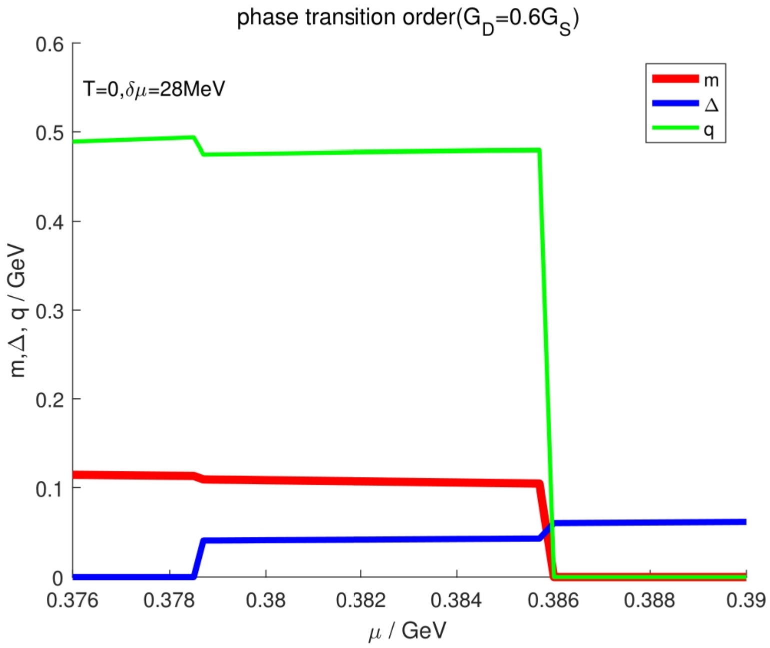




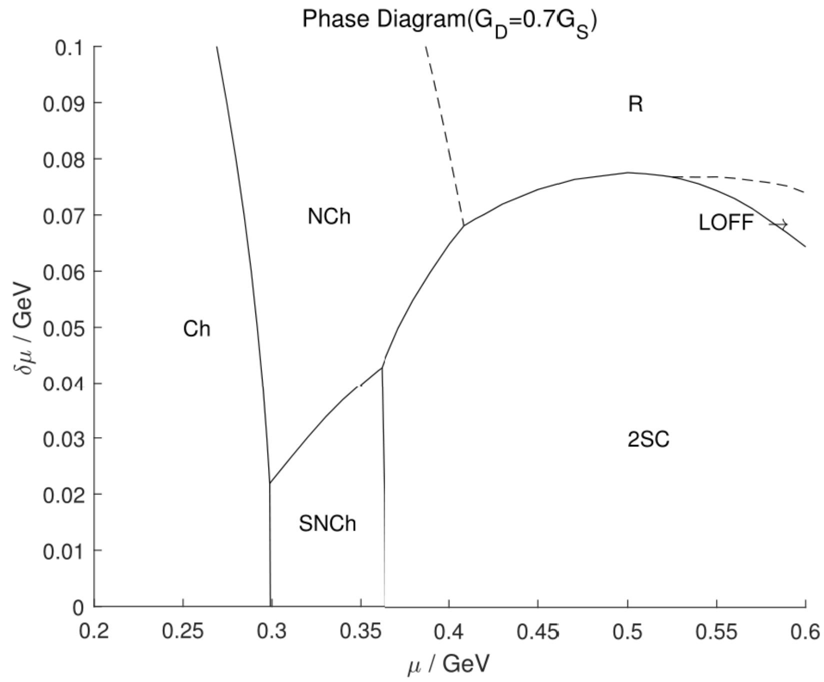
The transition between SNCh and 2SC phase is first order



The transition between NCh and R phase is first order



The transition between Ch and NCh phase is first order



Phase Diagram(G_D=0.7G_S)
R
NCh
LOFF
Ch
2SC
SNCh
$\delta\mu$ / GeV
μ / GeV
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6

The $G_D = 0.7G_S$

summary

We have studied the competition between the CDW and LOFF phases in the framework of two flavor NJL model in chiral limit at finite temperature and density. We find the HP phase with $M \neq 0$, $\Delta \neq 0$, $\vec{q} = 0$, $\vec{q}' = 0$ will occur between the chiral phase and SNCh phase if GD is large enough.

A first order phase transition connects the chiral density wave and LOFF phase if the ratio of GD/GS is less than 0.68, which is independent of the relative direction of chiral wave vector $\vec{q}$ and LOFF pair momentum $\vec{q}'$. There are two tricritical points in which chiral density wave, LOFF, 2SC and restored phase coincide, respectively.

When G_D/G_S is approximately larger than 0.68, the inhomogeneous chiral condensate is separated with the LOFF by 2SC and restored phase. There is no physical tetracritical point in the $\mu - \delta\mu$ phase diagram

Outlook

- Other regularization schemes

For example: Pauli-Villars schemes, proper-time scheme, four momentum cutoff

Thank you !