The Study of aQGC and nTGC

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遼寧師範大學
Liaoning Normal University

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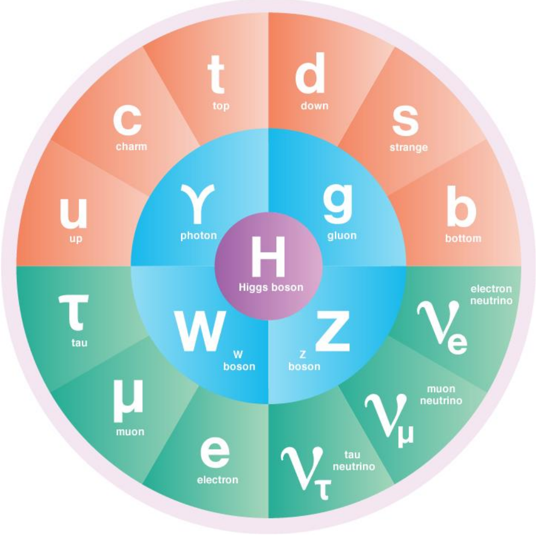
I. Introductions

The Standard Model

particles

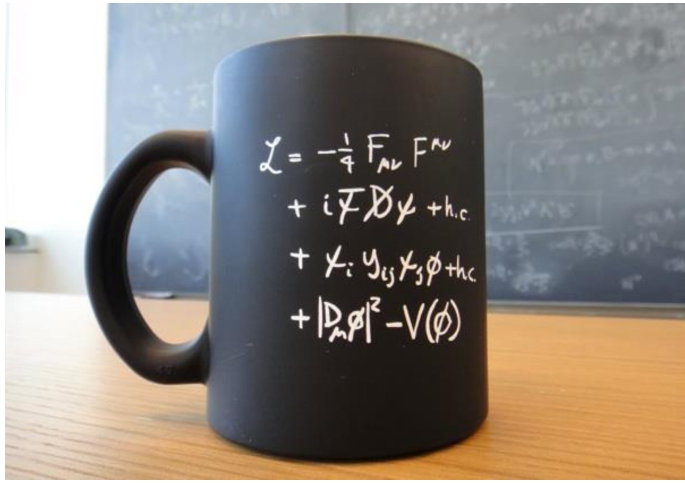
Quarks, leptons, Gauge bosons, Higgs.



Simple and powerful

yet unnatural, incomplete…

interactions

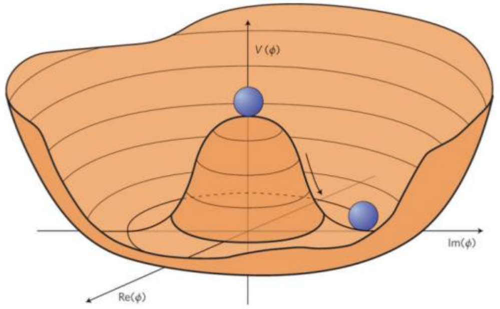


Gauge interactions:

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Higgs mechanism

The Higgs vacuum expectation value (vev) breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$, and gives particles masses.



But we have no idea what the new physics is...

The Standard Model Effective Field Theory

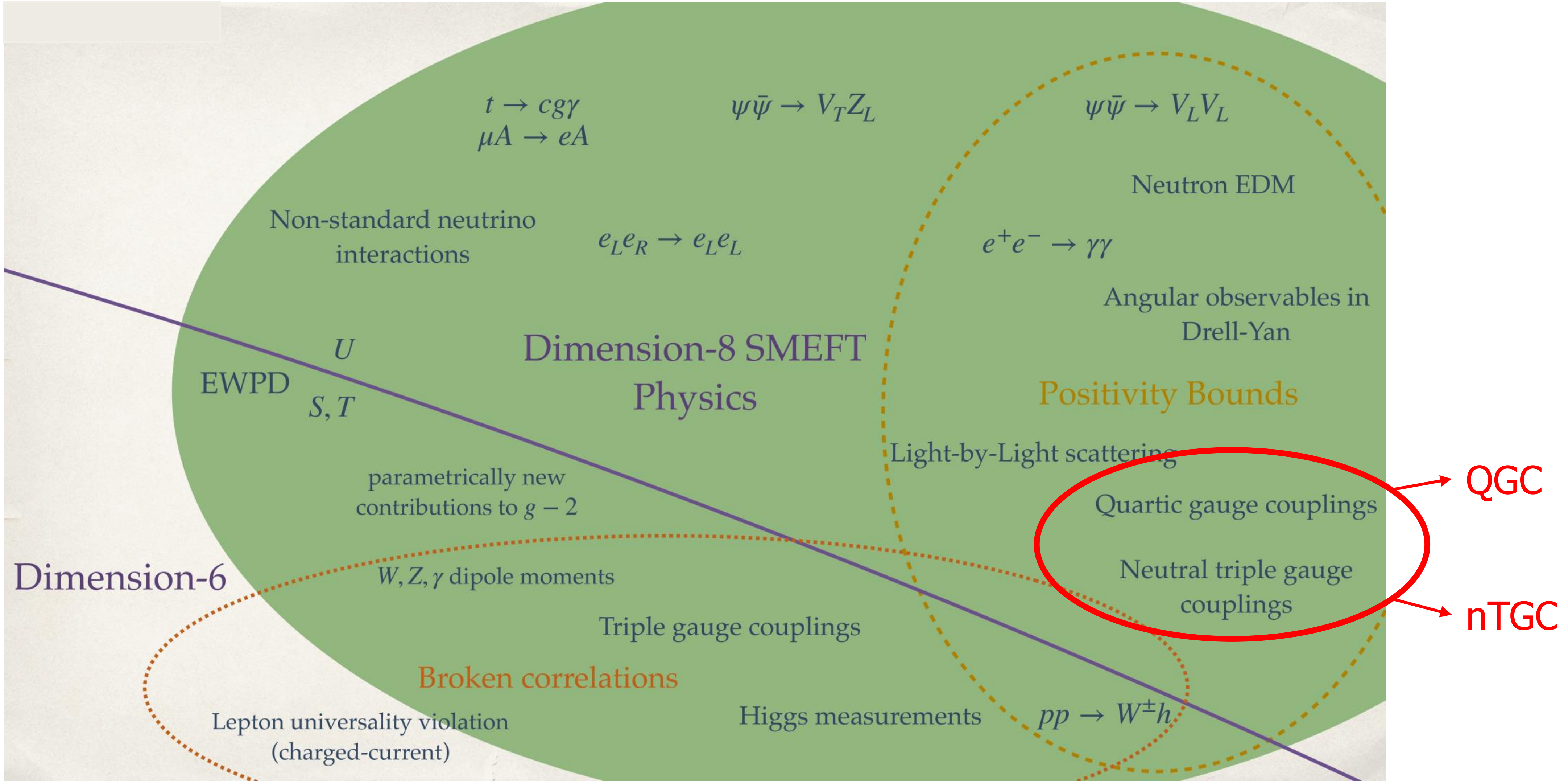
$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \cdots,$$

$$\mathcal{L}^{(d)} = \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} \quad , \quad d > 4$$

SMEFT: a more powerful way to analyze the data

- Assume the SM Lagrangian is correct but incomplete
- Look for additional interactions between SM particles
- Most efficient way to extract information from LHC and other experiments
- Model-independent way to look for physics beyond the Standard Model (BSM)

Dimension-8 Operators



Dimension-8 Operators affecting aQGC

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_{6i}}{\Lambda^2}\mathcal{O}_{6i} + \sum_j \frac{C_{8j}}{\Lambda^4}\mathcal{O}_{8j} + \dots$$

$$\mathcal{L}_{aQGC} = \sum_{i=0}^{2} \frac{f_{S_i}}{\Lambda^4} O_{S_i} + \sum_{j=0}^{7} \frac{f_{M_j}}{\Lambda^4} O_{M_j} + \sum_{k=0}^{9} \frac{f_{T_k}}{\Lambda^4} O_{T_k}$$

$$\begin{aligned}
O_{S_0} &= \left[(D_\mu \Phi)^\dagger D_\nu \Phi\right] \times \left[(D^\mu \Phi)^\dagger D^\nu \Phi\right], \\
O_{S_1} &= \left[(D_\mu \Phi)^\dagger D_\mu \Phi\right] \times \left[(D^\nu \Phi)^\dagger D^\nu \Phi\right], \\
O_{S_2} &= \left[(D_\mu \Phi)^\dagger D_\nu \Phi\right] \times \left[(D^\nu \Phi)^\dagger D^\mu \Phi\right],
\end{aligned}$$

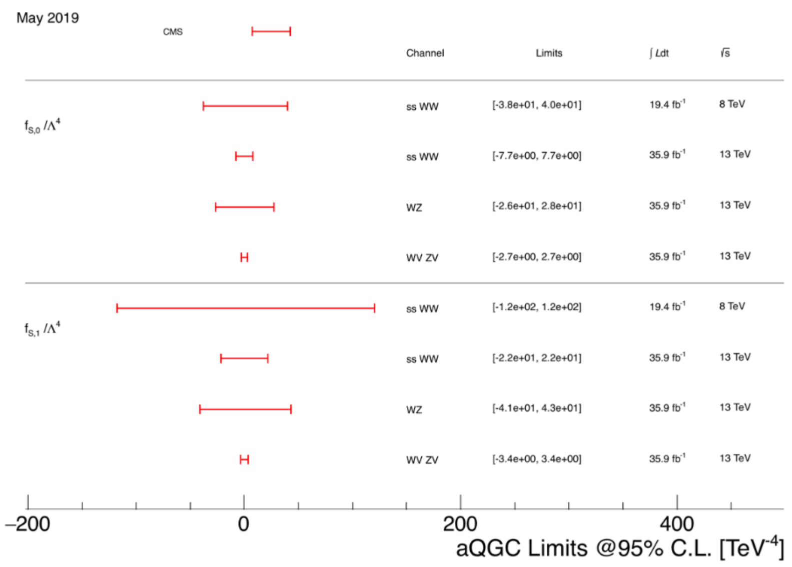
$$\begin{aligned}
O_{M_0} &= \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[\left(D^\beta \Phi\right)^\dagger D^\beta \Phi\right], \\
O_{M_1} &= \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[\left(D^\beta \Phi\right)^\dagger D^\mu \Phi\right], \\
O_{M_2} &= \left[B_{\mu\nu}B^{\mu\nu}\right] \times \left[\left(D^\beta \Phi\right)^\dagger D^\beta \Phi\right], \\
O_{M_3} &= \left[B_{\mu\nu}B^{\nu\beta}\right] \times \left[\left(D^\beta \Phi\right)^\dagger D^\mu \Phi\right], \\
O_{M_4} &= \left[(D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} D^\mu \Phi\right] \times B^{\beta\nu}, \\
O_{M_5} &= \left[(D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu} D_\nu \Phi\right] \times B^{\beta\mu} + h.c., \\
O_{M_7} &= (D_\mu \Phi)^\dagger \widehat{W}_{\beta\nu}\widehat{W}_{\beta\mu} D_\nu \Phi,
\end{aligned}$$

$$\begin{aligned}
O_{T_0} &= \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right], \\
O_{T_1} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right], \\
O_{T_2} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right], \\
O_{T_5} &= \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times B_{\alpha\beta}B^{\alpha\beta}, \\
O_{T_6} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times B_{\mu\beta}B^{\alpha\nu}, \\
O_{T_7} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times B_{\beta\nu}B^{\nu\alpha}, \\
O_{T_8} &= B_{\mu\nu}B^{\mu\nu} \times B_{\alpha\beta}B^{\alpha\beta}, \\
O_{T_9} &= B_{\alpha\mu}B^{\mu\beta} \times B_{\beta\nu}B^{\nu\alpha},
\end{aligned}$$

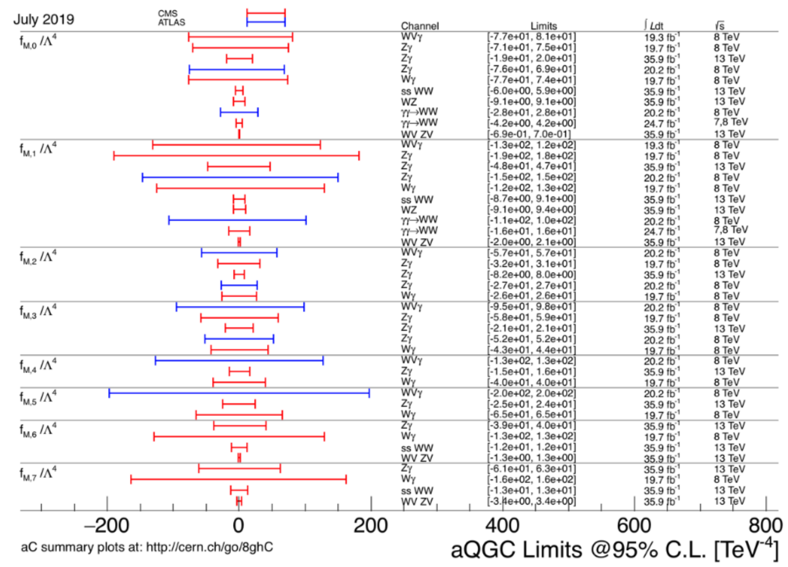
[Eboli, Gonzalez-Garcia, Mizukoshi, Phys. Rev. D 74 (2006) 073005]
[Eboli, Gonzalez-Garcia, Phys. Rev. D 93 (2016) 093013]

Limits on Dimension-8 Operators contributing to aQGC

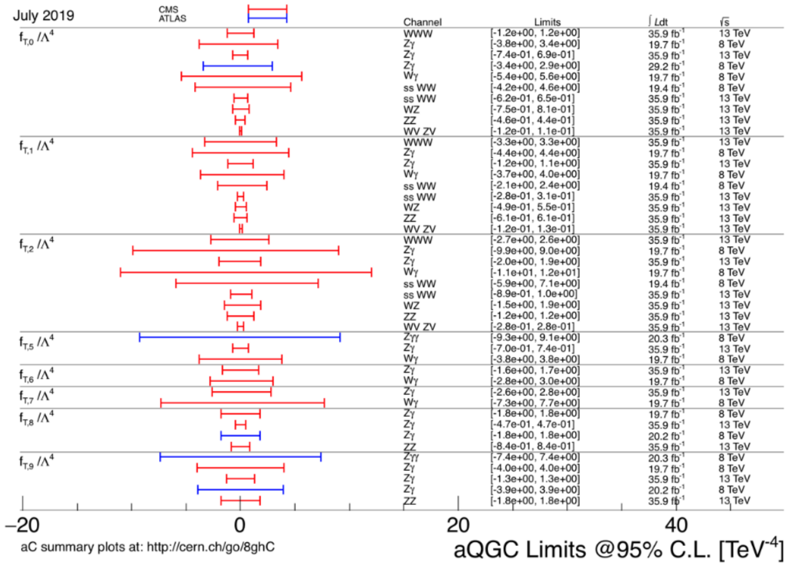
Scalar/longitudinal parameters $f_{S,i}$



Mixed transverse and longitudinal parameters $f_{M,i}$



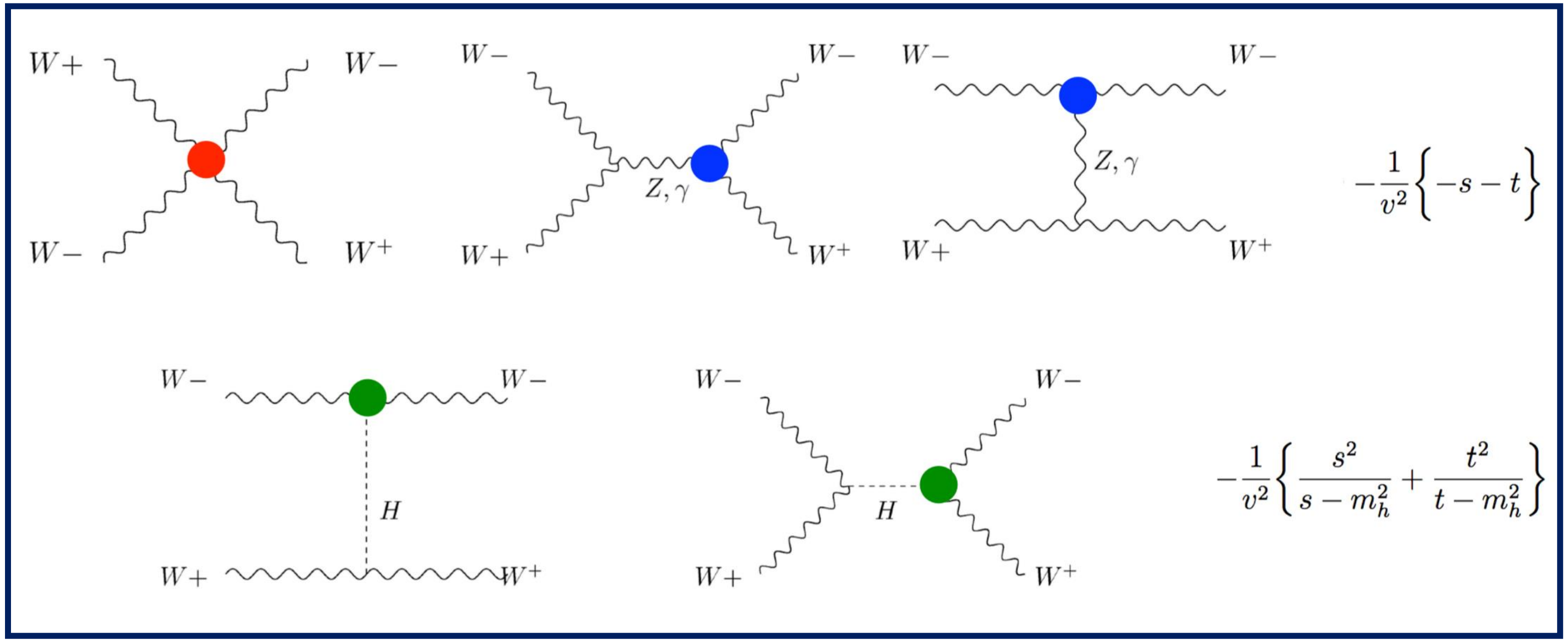
Transverse parameters $f_{T,i}$



https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC#aQGC_Results

Chinese experimentalists have made great contributions!

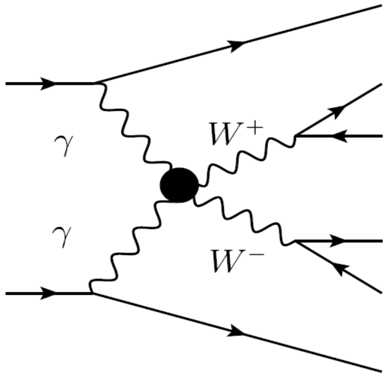
QGC in VBS



- Deviations in triple gauge boson couplings (TGC), Higgs couplings, or quartic gauge boson couplings (QGC) lead to ~s energy growth.
- QGC is unique in VBS

II. Study aQGC at the LHC

- The exclusive $\gamma\gamma \rightarrow W^+W^-$

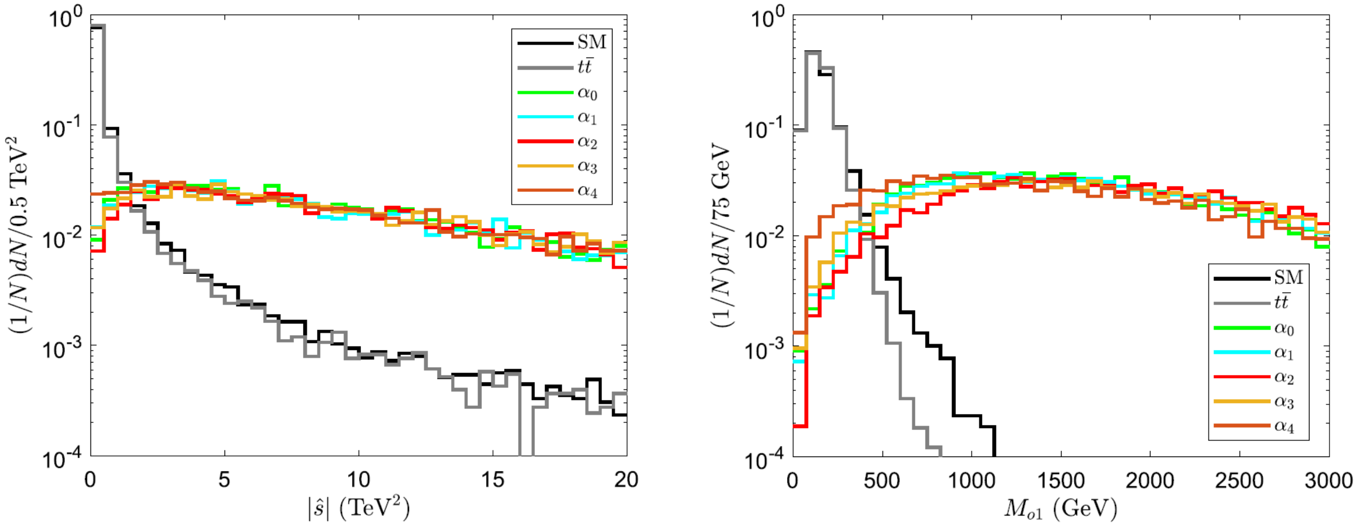


$$\mathcal{L}_{AAWW} = \sum_{i=0}^{4} \alpha_i V_{AAWW,i}$$

$$V_{AAWW,0} = F_{\mu\nu}F^{\mu\nu}W^{+\alpha}W^-_\alpha ,$$
$$V_{AAWW,1} = F_{\mu\nu}F^{\mu\alpha}W^{+\nu}W^-_\alpha ,$$
$$V_{AAWW,2} = F_{\mu\nu}F^{\mu\nu}W^+_{\alpha\beta}W^{-\alpha\beta},$$
$$V_{AAWW,3} = F_{\mu\nu}F^{\nu\alpha}W^+_{\alpha\beta}W^{-\beta\mu},$$
$$V_{AAWW,4} = F_{\mu\nu}F^{\alpha\beta}W^+_{\mu\nu}W^{-\alpha\beta},$$

The Traditional Approach

Nucl. Phys. B 961 (2020) 115222



[Barr et al. , Phys.Rev.D 84 (2011) 095031.; Kalinowski et al. , Eur. Phys. J. C 78 (2018) 403]

The constraints on the vertices at $\sqrt{s} = 14$ TeV with $\mathcal{L} = 3\ \mathrm{ab}^{-1}$.

Constraint	$SS \leq 2$	$SS \leq 3$	$SS \leq 5$
$\alpha_0(\mathrm{TeV}^{-2})$	[−0.0026, 0.0024]	[−0.0031, 0.0030]	[−0.0041, 0.0040]
$\alpha_1(\mathrm{TeV}^{-2})$	[−0.0080, 0.0092]	[−0.010, 0.011]	[−0.013, 0.015]
$\alpha_2(\mathrm{TeV}^{-4})$	[−0.16, 0.098]	[−0.18, 0.13]	[−0.23, 0.17]
$\alpha_3(\mathrm{TeV}^{-4})$	[−0.75, 0.38]	[−0.87, 0.50]	[−1.06, 0.70]
$\alpha_4(\mathrm{TeV}^{-4})$	[−0.50, 0.48]	[−0.62, 0.59]	[−0.80, 0.78]

The exclusive $\gamma\gamma \rightarrow WW$ production is sensitive to the O_{Mi} operators.

The Machine Learning approach

JHEP 09, 085 (2021), Phys. Rev. D 104, 035021 (2021)

What can Machine Learning do?

What methods have been used?

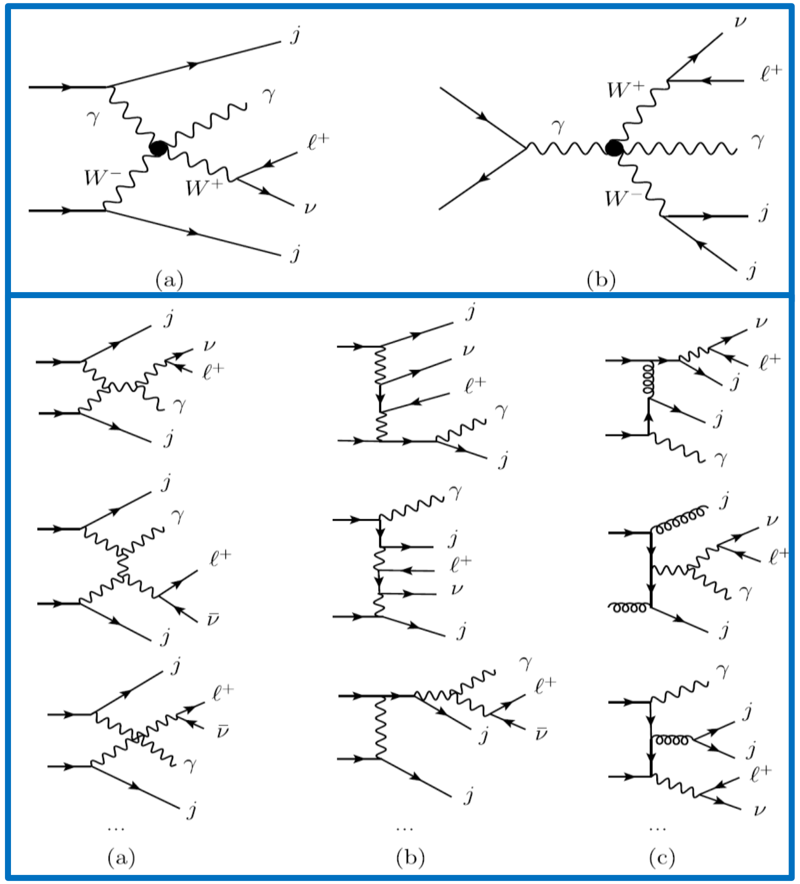
For more details please follow Parallel Session VIII (1):

"Using machine learning methods to study aQGCs and nTGCs "

by Ji-Chong Yang

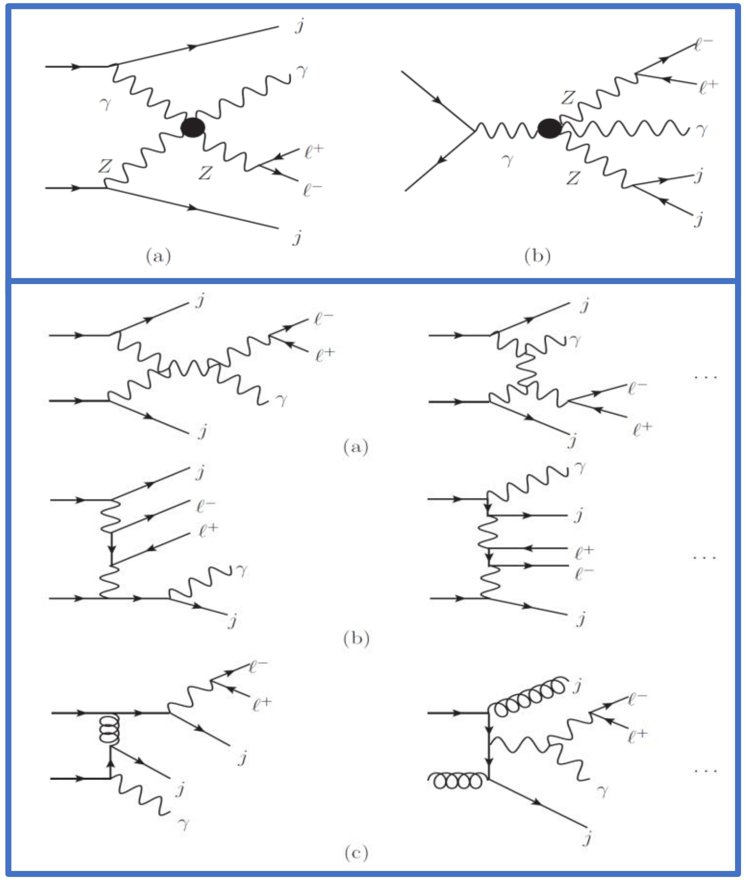
• $W\gamma jj$ and $Z\gamma jj$ production

$pp \rightarrow W\gamma jj \rightarrow l\nu\gamma jj$



Chin. Phys. C 44 (2020) 12, 123105

$pp \rightarrow Z\gamma jj \rightarrow ll\gamma jj$



Phys. Rev. D 104 (2021) 035015

Unitarity Bound

Strength of Weak Interactions at Very High Energies and the Higgs Boson Mass

Benjamin W. Lee, C. Quigg,* and H. B. Thacker
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 28 February 1977)

It is shown that if the Higgs boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$ partial-wave unitarity is not respected by the tree diagrams for two-body reactions of gauge bosons, and the weak interactions must become strong.

- scattering amplitudes are harder at high energies than allowed by unitarity
- model-independent upper bound on scale of Higgs

Partial-wave Unitarity

In the two-to-two scattering of electroweak gauge bosons $V_{1,\lambda_1} V_{2,\lambda_2} \to V_{3,\lambda_3} V_{4,\lambda_4}$

the helicity amplitude can be expanded in partial waves as

$$\mathcal{M}(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J (J + \frac{1}{2}) \sqrt{1 + \delta_{V_{1\lambda_1}}^{V_{2\lambda_2}}} \sqrt{1 + \delta_{V_{3\lambda_3}}^{V_{4\lambda_4}}} \, d^J_{\lambda\mu}(\theta, \varphi) \, e^{iM\varphi} \times T^J(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4})$$

Partial-wave unitarity requires

$$|T^J(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4})| \leq 2.$$

[Corbett, Eboli and Gonzalez-Garcia, Phys. Rev. D 91 (2014) 035014]
[Corbett, Eboli and Gonzalez-Garcia, Phys. Rev. D 96 (2017) 035006]

The strongest limit of $WV \to W\gamma$ and $VV \to Z\gamma$ subprocesses

$$\tilde{s}(f_{M_2}) \leq \sqrt{\frac{s_W^2 256\pi M_W^2 \Lambda^4}{c_W^2 e^2 v^2 |f_{M_2}|}},$$

$$\tilde{s}(f_{M_3}) \leq \sqrt{\frac{384\pi s_W^2 M_W^2 \Lambda^4}{c_W^2 e^2 v^2 |f_{M_3}|}},$$

$$\tilde{s}(f_{M_4}) \leq \sqrt{\frac{512\pi M_W M_Z s_W^2 \Lambda^4}{e^2 v^2 |f_{M_4}|}},$$

$$\tilde{s}(f_{M_5}) \leq \sqrt{\frac{384\pi M_W M_Z s_W \Lambda^4}{c_W e^2 v^2 |f_{M_5}|}},$$

$$\tilde{s}(f_{T_5}) \leq \sqrt{\frac{40\pi \Lambda^4}{c_W^2 |f_{T_5}|}},$$

$$\tilde{s}(f_{T_6}) \leq \sqrt{\frac{32\pi \Lambda^4}{c_W^2 |f_{T_6}|}},$$

$$\tilde{s}(f_{T_7}) \leq \sqrt{\frac{64\pi \Lambda^4}{c_W^2 |f_{T_7}|}}.$$

$$\hat{s}^{O_{M_0}} \leq \sqrt{\frac{32\sqrt{2}\pi c_W s_W M_Z^2 \Lambda^4}{|f_{M_0}| e^2 v^2}}, \quad \hat{s}^{O_{M_1}} \leq \sqrt{\frac{64\sqrt{2} c_W s_W M_Z^2 \Lambda^4}{|f_{M_1}| e^2 v^2}},$$

$$\hat{s}^{O_{M_2}} \leq \sqrt{\frac{16\sqrt{2}\pi c_W s_W M_Z^2 \Lambda^4}{|f_{M_2}| e^2 v^2}}, \quad \hat{s}^{O_{M_3}} \leq \sqrt{\frac{32\sqrt{2}\pi c_W s_W M_Z^2 \Lambda^4}{|f_{M_3}| e^2 v^2}},$$

$$\hat{s}^{O_{M_4}} \leq \sqrt{\frac{64\sqrt{2}\pi c_W^2 s_W^2 M_Z^2 \Lambda^4}{|f_{M_4}|(c_W^2 - s_W^2) e^2 v^2}}, \quad \hat{s}^{O_{M_5}} \leq \sqrt{\frac{192\pi s_W^2 M_W M_Z \Lambda^4}{|f_{M_5}| e^2 v^2}},$$

$$\hat{s}^{O_{M_7}} \leq \sqrt{\frac{128\sqrt{2}\pi c_W s_W M_Z^2 \Lambda^4}{|f_{M_7}| e^2 v^2}}, \quad \hat{s}^{O_{T_0}} \leq \sqrt{\frac{6\sqrt{2}\pi \Lambda^4}{5|f_{T_0}| c_W^3 s_W}},$$

$$\hat{s}^{O_{T_1}} \leq \sqrt{\frac{6\sqrt{2}\pi \Lambda^4}{5|f_{T_1}| c_W^3 s_W}}, \quad \hat{s}^{O_{T_2}} \leq \sqrt{\frac{8\sqrt{2}\pi \Lambda^4}{3|f_{T_2}| c_W^3 s_W}},$$

$$\hat{s}^{O_{T_5}} \leq \sqrt{\frac{\pi \Lambda^4}{|f_{T_5}| c_W^2 s_W^2}}, \quad \hat{s}^{O_{T_6}} \leq \sqrt{\frac{4\pi \Lambda^4}{|f_{T_6}|(c_W^2 - s_W^2)}},$$

$$\hat{s}^{O_{T_7}} \leq \sqrt{\frac{8\sqrt{2}\pi \Lambda^4}{3|f_{T_7}|(c_W^2 - s_W^2) c_W s_W}}, \quad \hat{s}^{O_{T_8}} \leq \sqrt{\frac{3\pi \Lambda^4}{10|f_{T_8}| c_W^2 s_W^2}},$$

$$\hat{s}^{O_{T_9}} \leq \sqrt{\frac{2\pi \Lambda^4}{3|f_{T_9}| c_W^2 s_W^2}}.$$

But $\tilde{s}$ is a distribution, not a limit.

How can we use Unitarity bound at LHC?

Select Event with Unitary bounds

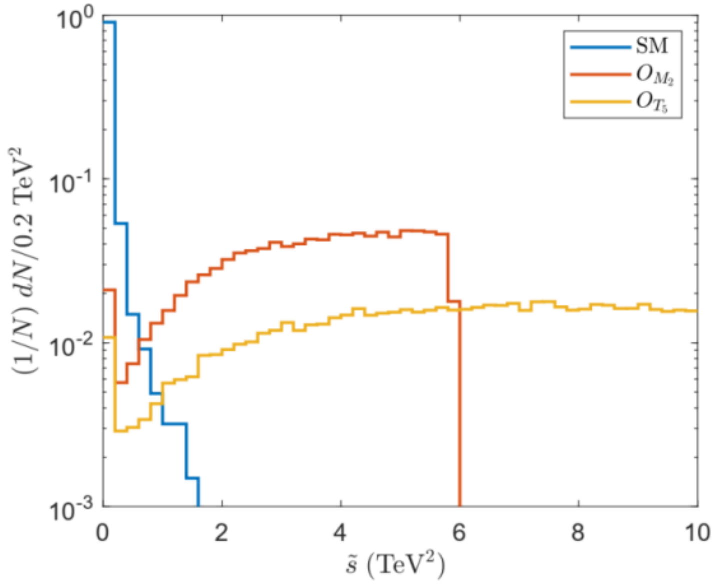


Table 4. Cross sections of SM backgrounds and signals for various operators after N_{j,γ,ℓ^+}, $\Delta\phi_{\ell m}$, $|\vec{p}_T^{\ell}|$, $|\vec{p}_T^{\text{miss}}|$, and $\bar{s}_U$ cuts. The maximum $\bar{s}$ used in the $\bar{s}_U$ cuts are obtained using the upper bounds of f_X/Λ^4 in Table 1 and Eq. (17).

Channel/fb	no cut	N_{j,γ,ℓ^+}	$\Delta\phi_{\ell m}$	$\lvert\vec{p}_T^{\ell}\rvert$	$\lvert\vec{p}_T^{\text{miss}}\rvert$	$\bar{s}_U$
SM	9520.8	3016.6	211.7	65.1	40.6	–
O_{M_2}	6.353	4.06	3.51	3.45	3.43	0.93
O_{M_3}	21.05	13.62	12.13	11.95	11.90	2.19
O_{M_4}	7.39	4.81	4.06	3.94	3.92	1.03
O_{M_5}	25.23	16.73	14.75	14.49	14.42	4.05
O_{T_5}	2.71	1.77	1.28	1.25	1.22	0.72
O_{T_6}	16.92	11.19	8.94	8.36	8.26	3.06
O_{T_7}	7.47	4.97	3.97	3.69	3.65	1.43

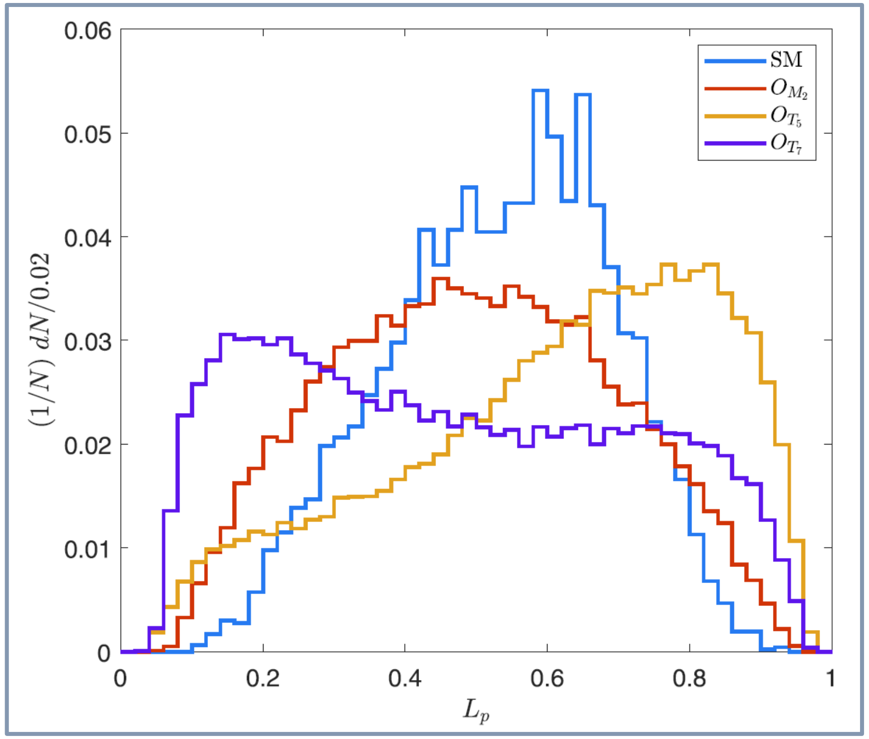
Polarization Feature

$\mathcal{M}(\gamma_+ W_0^+ \to \gamma_- W_0^+)$	$-\frac{f_{M_0}}{\Lambda^4}\frac{e^2 e^{i\varphi} v^2 \sin^4\left(\frac{\theta}{2}\right)}{8M_W^2}\hat{s}^2$ $\frac{f_{M_1}}{\Lambda^4}\frac{e^2 e^{i\varphi} v^2 \sin^4\left(\frac{\theta}{2}\right)}{32M_W^2}\hat{s}^2$
$\mathcal{M}(\gamma_+ W_0^+ \to \gamma_+ W_0^+)$	$\frac{f_{M_1}}{\Lambda^4}\frac{e^2 e^{i\varphi} v^2 (\cos(\theta)+1)}{32M_W^2}\hat{s}^2$
$\mathcal{M}(\gamma_+ W_+^+ \to \gamma_- W_-^+)$	$2\frac{f_{T_0}}{\Lambda^4}s_W^2\sin^4\left(\frac{\theta}{2}\right)\hat{s}^2$ $\frac{1}{2}\frac{f_{T_1}}{\Lambda^4}s_W^2\left(\sin^4\left(\frac{\theta}{2}\right)+\left(\frac{\cos(\theta)+3}{2}\right)^2\right)\hat{s}^2$ $\frac{1}{2}\frac{f_{T_2}}{\Lambda^4}s_W^2\sin^4\left(\frac{\theta}{2}\right)\hat{s}^2$
$\mathcal{M}(\gamma_+ W_-^+ \to \gamma_- W_+^+)$	$2\frac{f_{T_0}}{\Lambda^4}e^{2i\varphi}s_W^2\sin^4\left(\frac{\theta}{2}\right)\hat{s}^2$ $\frac{1}{2}\frac{f_{T_2}}{\Lambda^4}e^{2i\varphi}s_W^2\sin^4\left(\frac{\theta}{2}\right)\hat{s}^2$
$\mathcal{M}(\gamma_- W_-^+ \to \gamma_- W_-^+)$	$\frac{f_{T_1}}{\Lambda^4}s_W^2\hat{s}^2$ $\frac{1}{2}\frac{f_{T_2}}{\Lambda^4}s_W^2\hat{s}^2$
$\mathcal{M}(\gamma_+ W_-^+ \to \gamma_+ W_-^+)$	$\frac{f_{T_1}}{\Lambda^4}e^{2i\varphi}s_W^2\cos^4\left(\frac{\theta}{2}\right)\hat{s}^2$ $\frac{1}{2}\frac{f_{T_2}}{\Lambda^4}e^{2i\varphi}s_W^2\cos^4\left(\frac{\theta}{2}\right)\hat{s}^2$

When p_{T_W} is large, $\cos(\theta^*) \approx 2(L_p - 1)$ with $L_p = \frac{\mathbf{p}_T^\ell \cdot \mathbf{p}_T^W}{|\mathbf{p}_T^W|^2}$

[C. M. S. Collaboration, Phys. Rev. Lett. 107 (2011) 021802]

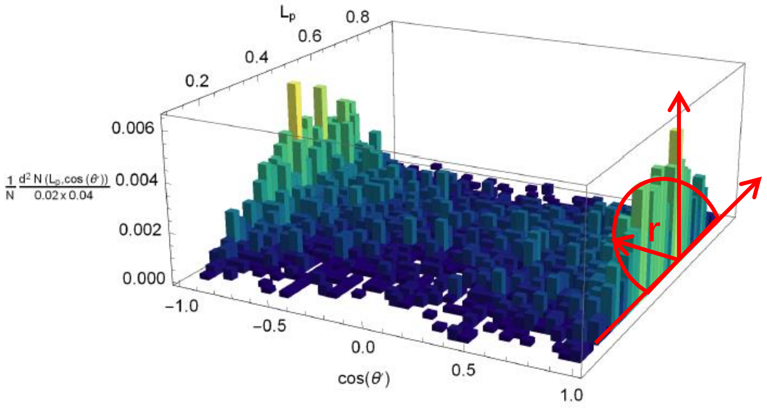
*4 polarization fraction patterns : **SM, O_{M_i}, $O_{T_{0,5}}$, $O_{T_{1,2,6,7}}$***



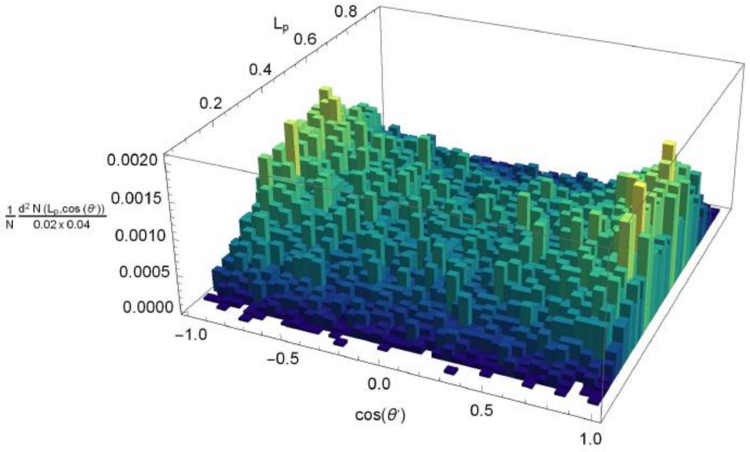
O_{Mi} : *longitudinal W^+ bosons are dominant*

O_{Ti} : *both left-handed and right-handed W^+ bosons dominate.*

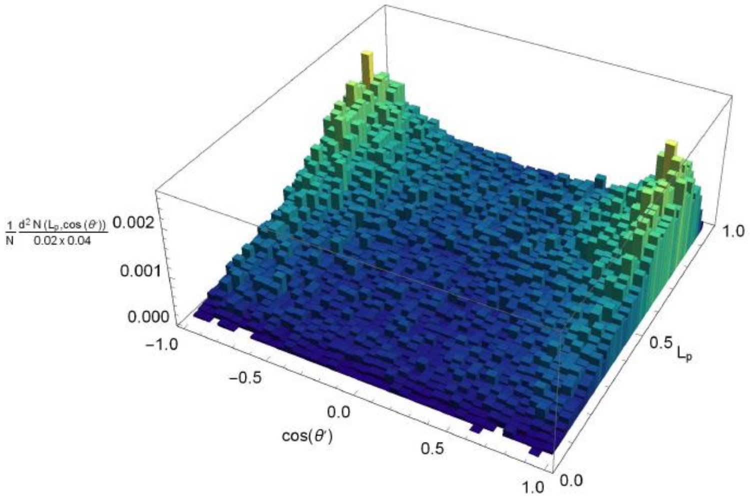
Angular Distribution



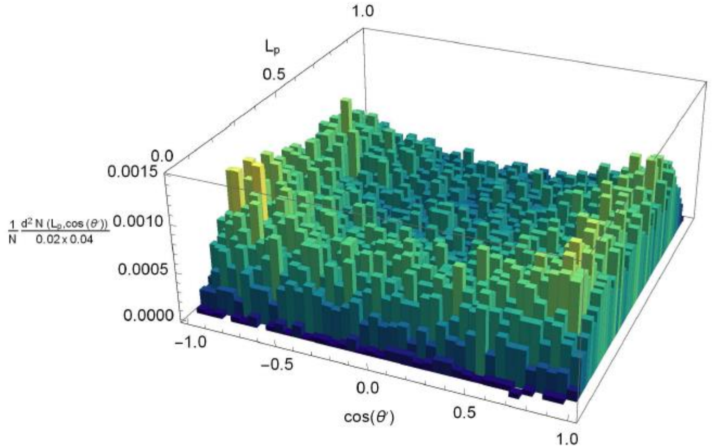
(a) SM



(b) O_{M_2}



(c) O_{T_5}

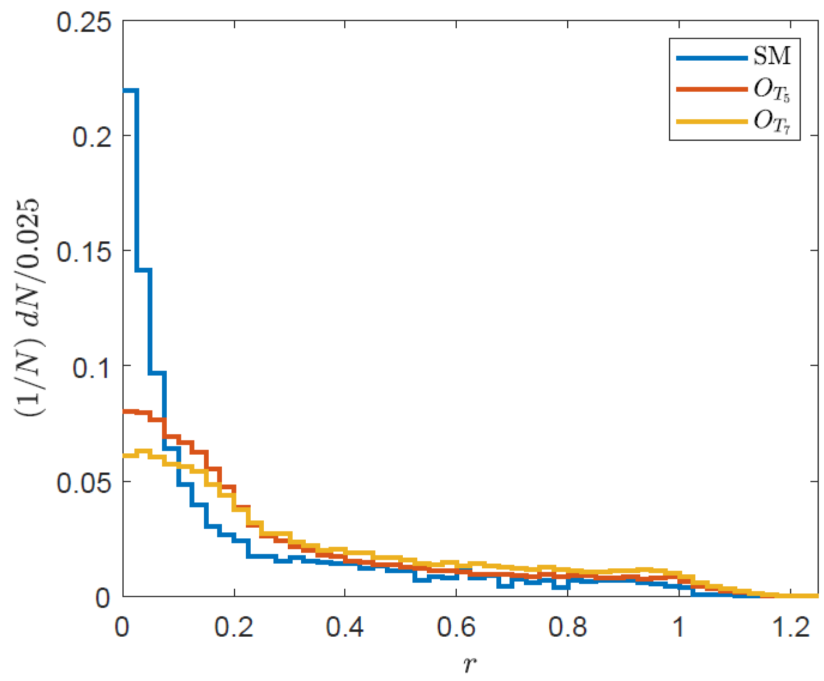


(d) O_{T_7}

Normalized distributions of L_p and $\cos\theta'$.

we define

$$r = \left(1 - |\cos(\theta')|\right)^2 + \left(\frac{1}{2} - L_p\right)^2$$



Constraints on Dimension-8 Operators

$W\gamma jj$ production

Table 7. Constraints on operators at LHC with $\mathcal{L} = 137.1\ \text{fb}^{-1}$.

Coefficients	$S_{\text{stat}} > 2$	Coefficients	$S_{\text{stat}} > 2$
f_{M_2}/Λ^4	$[-2.05, 2.0]$	f_{T_5}/Λ^4	$[-0.525, 0.37]$
f_{M_3}/Λ^4	$[-10.5, 5.25]$	f_{T_6}/Λ^4	$[-0.4, 0.425]$
f_{M_4}/Λ^4	$[-11.25, 4.0]$	f_{T_7}/Λ^4	$[-0.65, 0.7]$
f_{M_5}/Λ^4	$[-6.25, 6.0]$		

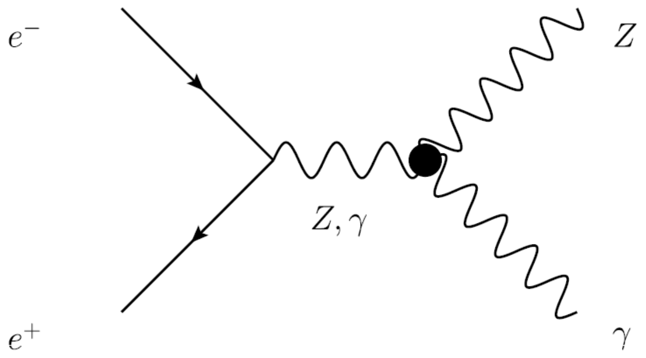
$Z\gamma jj$ production

	$300\ \text{fb}^{-1}$	$3\ \text{ab}^{-1}$
f_{M_4}/Λ^4	$[-15.0, 16.0]$	$[-1.8, 4.0]$
f_{M_5}/Λ^4	$[-12.5, 10.0]$	$[-3.0, 4.0]$
f_{T_5}/Λ^4	$[-0.40, 0.37]$	$[-0.09, 0.15]$
f_{T_6}/Λ^4	$[-1.0, 0.9]$	$[-0.4, 0.43]$
f_{T_7}/Λ^4	$[-1.7, 1.4]$	$[-0.7, 0.7]$
f_{T_9}/Λ^4	$[-0.55, 0.50]$	$[-0.15, 0.15]$

We thank *Jian Wang* and *Cen Zhang* for useful discussions.

III. Study nTGC and aQGC at Future Lepton Colliders

- nTGC in the $e^+ e^- \to Z\gamma$ process



$$\mathcal{L}_{\rm nTGC} = \frac{\text{sign}(c_{\tilde{B}W})}{\Lambda^4_{\tilde{B}W}}\mathcal{O}_{\tilde{B}W} + \frac{\text{sign}(c_{B\tilde{W}})}{\Lambda^4_{B\tilde{W}}}\mathcal{O}_{B\tilde{W}} + \frac{\text{sign}(c_{\tilde{W}W})}{\Lambda^4_{\tilde{W}W}}\mathcal{O}_{\tilde{W}W} + \frac{\text{sign}(c_{\tilde{B}B})}{\Lambda^4_{\tilde{B}B}}\mathcal{O}_{\tilde{B}B},$$

$$\mathcal{O}_{\tilde{B}W} = iH^\dagger \tilde{B}_{\mu\nu} W^{\mu\rho}\{D_\rho, D^\nu\} H + h.c.,$$
$$\mathcal{O}_{B\tilde{W}} = iH^\dagger B_{\mu\nu} \tilde{W}^{\mu\rho}\{D_\rho, D^\nu\} H + h.c.,$$
$$\mathcal{O}_{\tilde{W}W} = iH^\dagger \tilde{W}_{\mu\nu} W^{\mu\rho}\{D_\rho, D^\nu\} H + h.c.,$$
$$\mathcal{O}_{\tilde{B}B} = iH^\dagger \tilde{B}_{\mu\nu} B^{\mu\rho}\{D_\rho, D^\nu\} H + h.c.,$$

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Probing the scale of new physics in the $ZZ\gamma$ coupling at e^+e^- colliders*

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Probing new physics in dimension-8 neutral gauge couplings at e^+e^- colliders

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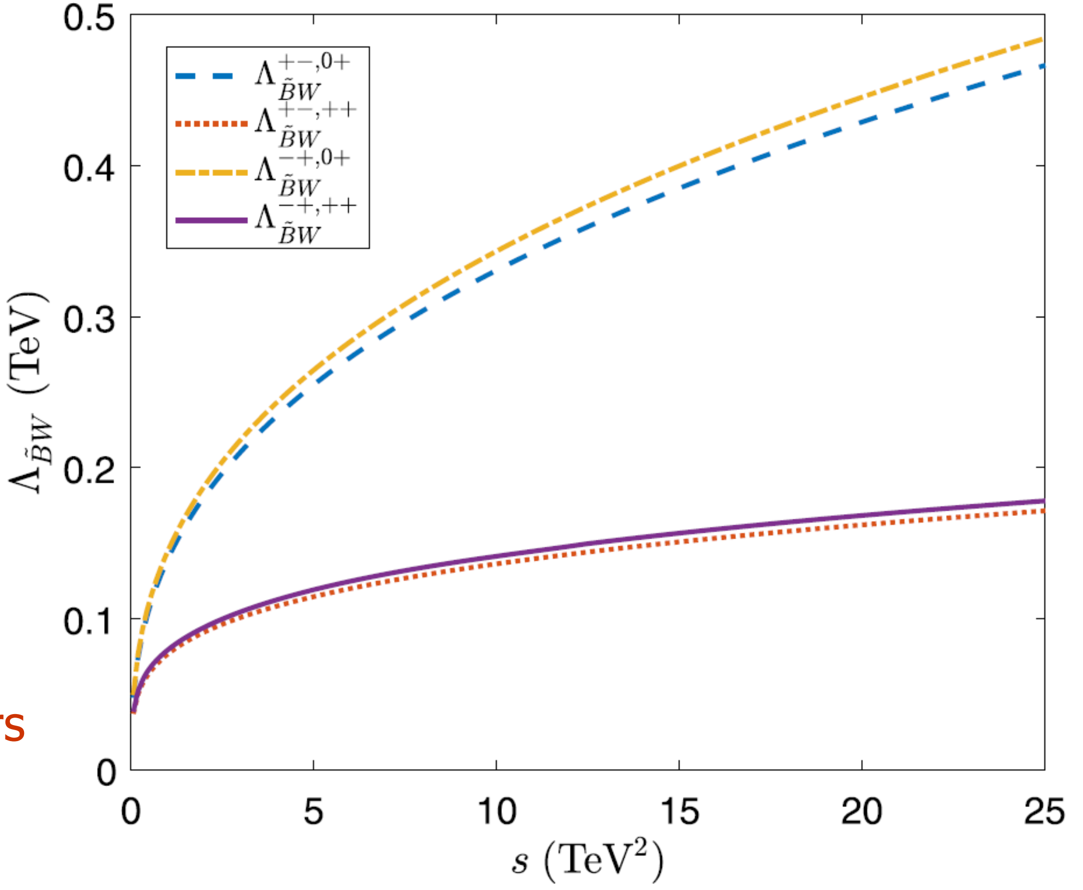
The Partial Wave Unitarity Bound

$$\Lambda_{\tilde{B}W}^{+-,0+} \geq \left(\frac{e^2\sqrt{s}v^2\left(s-M_Z^2\right)}{48\sqrt{2}\pi M_Z c_W^2}\right)^{\frac{1}{4}}, \quad \Lambda_{\tilde{B}W}^{+-,++} \geq \left(\frac{e^2 v^2\left(s-M_Z^2\right)}{48\sqrt{2}\pi c_W^2}\right)^{\frac{1}{4}}$$

$$\Lambda_{\tilde{B}W}^{-+,0+} \geq \left(\frac{e^2\sqrt{s}\left(1-2s_W^2\right)v^2\left(s-M_Z^2\right)}{96\sqrt{2}\pi M_Z s_W^2 c_W^2}\right)^{\frac{1}{4}},$$

$$\Lambda_{\tilde{B}W}^{-+,++} \geq \left(\frac{e^2\left(2s_W^2-1\right)v^2\left(M_Z^2-s\right)}{96\sqrt{2}\pi s_W^2 c_W^2}\right)^{\frac{1}{4}},$$



Unitary bounds can constrain operators directly at lepton colliders

The constraints on $\Lambda_{\tilde{B}W}$ from unitarity bounds.

$\sqrt{s}$ (GeV)	250	500	1000	3000	5000
$\Lambda_{\tilde{B}W}$ (GeV)	> 49.4	> 85.4	> 144.5	> 330.0	> 484.2

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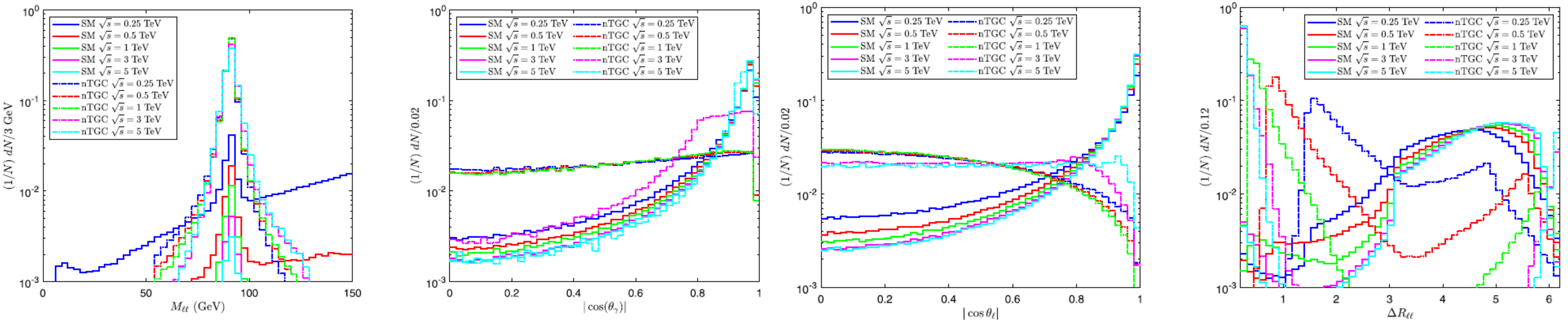
Constraints for Dimension-8 Operators



(a) (b) (c) (d)

The expected constraints on $\text{sign}(c_{\tilde{B}W})/\Lambda^4_{\tilde{B}W}$ (TeV^{-4}) at $\mathcal{L}=2$ ab^{-1} for hadronic Z decays.

$\mathcal{S}_{stat}$	$\sqrt{s}$ (GeV)				
	250 G	500	1000	3000	5000
2	[−10.5, 76.9]	[−1.0, 14.8]	[−0.35, 1.3]	[−0.030, 0.064]	[−0.013, 0.013]
3	[−14.9, 81.3]	[−1.5, 15.2]	[−0.48, 1.4]	[−0.040, 0.074]	[−0.016, 0.016]
5	[−22.7, 89.1]	[−2.3, 16.1]	[−0.69, 1.6]	[−0.055, 0.089]	[−0.020, 0.020]

Inspiration from nTGC research in $e^+e^- \to Z\gamma$

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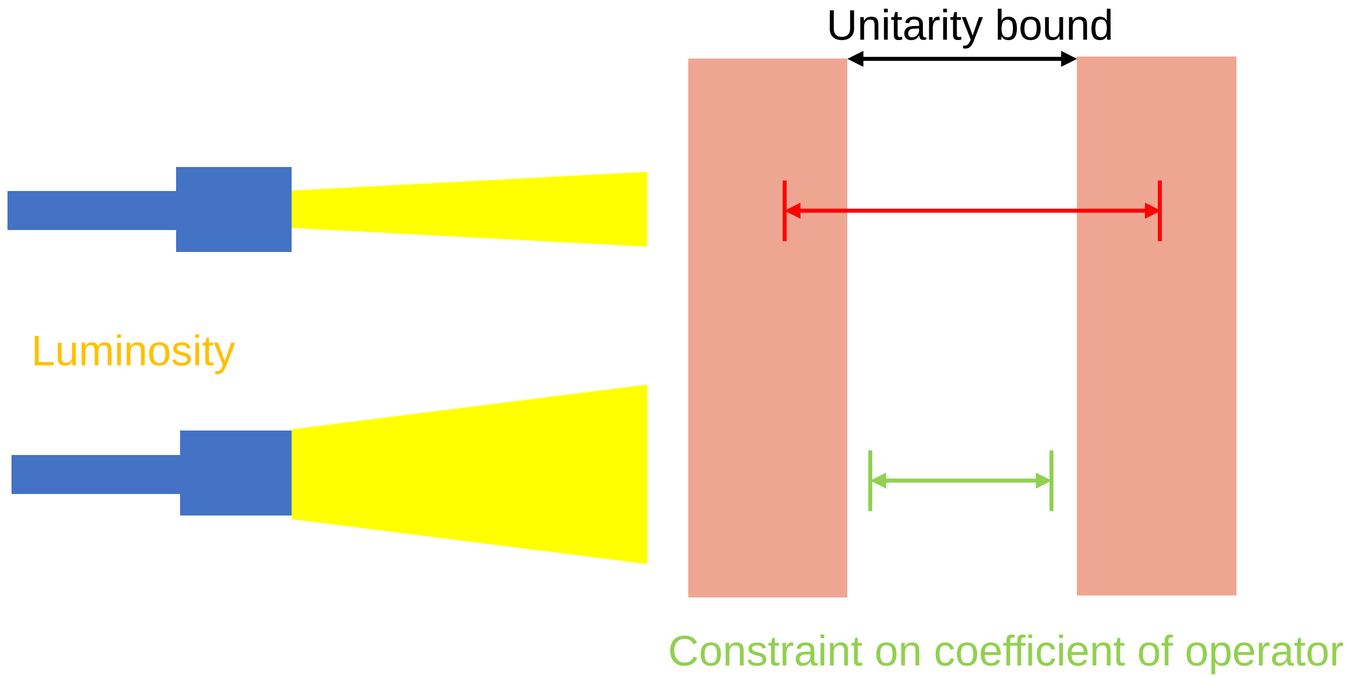
The unitarity bounds tell us the minimum integrated luminosity required to study nTGC and aQGC



Unlike VBS, the diboson induced by nTGC and the triphoton induced by aQGC are suppressed by a propagator for large s, so the unitarity constraint is only relevant at very low luminosity.

• Tri-photon at Future Muon Colliders

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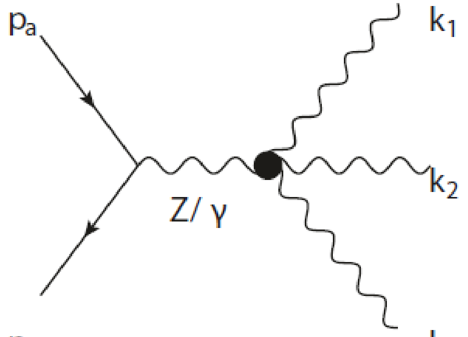
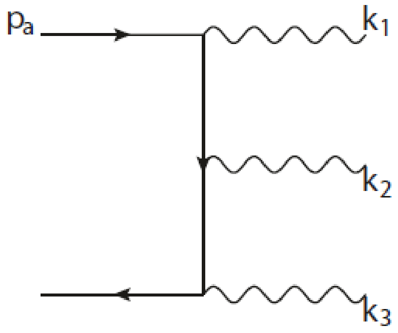
$$\mu^+\mu^- \to Z^*/\gamma^* \to \gamma\gamma\gamma$$

Advantages of the muon collider:

- High energy
- High integrated luminosity
- Cleaner environment
- Enhances the annihilation process than pp collider



(aQGC)



(SM)

Only O_{Ti} operators are relevant of tri-photon

$$O_{T_0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right], \quad O_{T_1} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right],$$

$$O_{T_2} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right], \quad O_{T_5} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times B_{\alpha\beta}B^{\alpha\beta},$$

$$O_{T_6} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times B_{\mu\beta}B^{\alpha\nu}, \quad O_{T_7} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times B_{\beta\nu}B^{\nu\alpha},$$

$$O_{T_8} = B_{\mu\nu}B^{\mu\nu} \times B_{\alpha\beta}B^{\alpha\beta}, \quad O_{T_9} = B_{\alpha\mu}B^{\mu\beta} \times B_{\beta\nu}B^{\nu\alpha},$$

Compare Annihilation Process with VBS Processes

$$\mu^+\mu^- \to V_1V_2V_3 \quad \text{(annihilation)}\,,$$
$$\mu^+\mu^- \to f f' V_1V_2 \quad \text{(VBS)}\,,$$

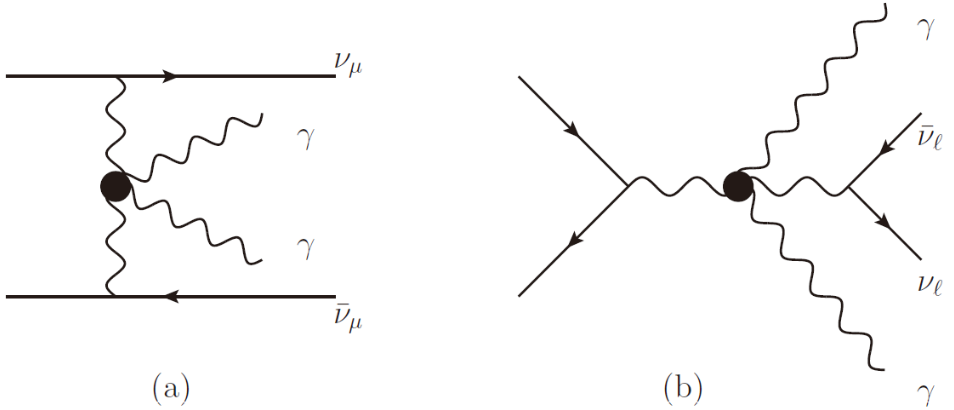
$$\frac{\sigma_{\rm VBF}^{\rm BSM}}{\sigma_{\rm ann}^{\rm BSM}} \propto \alpha_W^2 \frac{s}{m_X^2} \log^2\left(\frac{s}{m_V^2}\right) \log\left(\frac{s}{m_X^2}\right)\,,$$

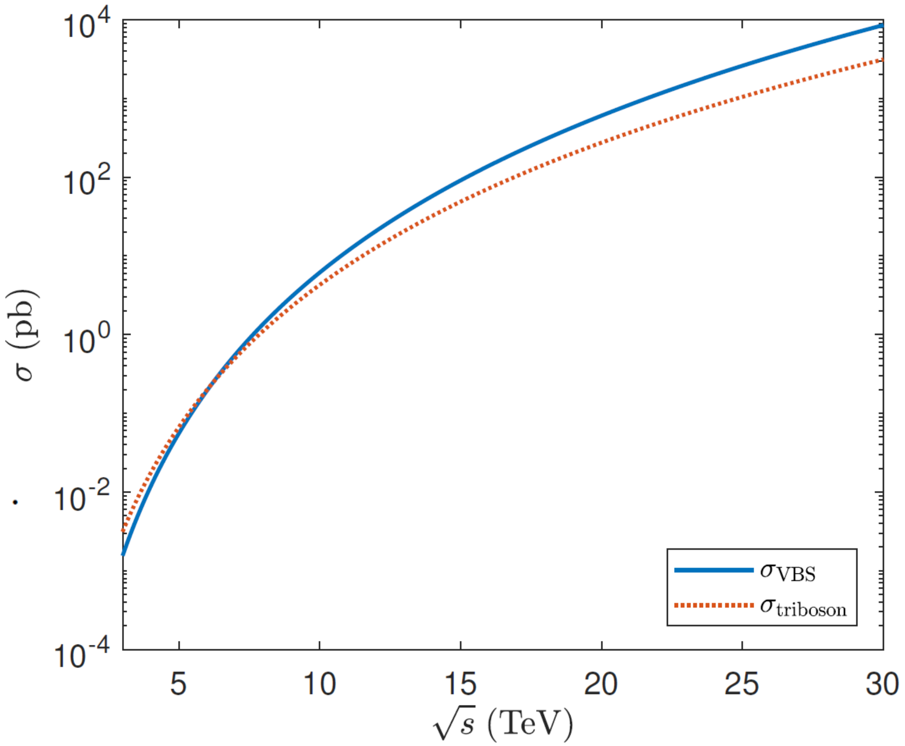
[H. Al Ali et al., Rept.Prog.Phys. 85 (2022) 8, 084201]

$\mu^+\mu^- \to \gamma\gamma\nu\bar{\nu}$ for illustration and take O_{T_5} as an example

$$\sigma_{\rm VBS} = \frac{e^4 f_{T_5}^2 s^3 \left(1-s_W^2\right)^2 \left[20\log\left(\frac{s}{16M_W^2}\right)\left(30\log\left(\frac{s}{16M_W^2}\right)-67\right)+943\right]}{110592000\pi^5\Lambda^8 s_W^4}\,,$$

$$\sigma_{\rm triboson} = \frac{e^2 f_{T_5}^2 s^3 \left(48s_W^8 - 64s_W^6 + 40s_W^4 - 12s_W^2 + 3\right)}{138240\pi^3\Lambda^8 s_W^2\left(s_W^2-1\right)} \times {\rm Br}(Z\to\nu\bar{\nu})\,.$$





The Contribution of aQGC to Tri-photon Process

$$\sigma_{\text{aQGC}}(f_{T_i}) = \sigma_{\text{SM}} + \sigma_{O_{T_i}}(f_{T_i}) + \sigma_{\text{int}}(f_{T_i})$$

	3 TeV	10 TeV	14 TeV	30 TeV
σ_{SM} (fb)	5.96	0.707	0.383	0.0953

$$\sigma_{\text{int}} = \frac{e^4 s(384\log(2) - 215)\left((1-4s_W^2)(4\alpha_1+3\alpha_2)+16c_W s_W(4\alpha_3+3\alpha_4)\right)}{110592\pi^3\Lambda^4 c_W s_W},$$

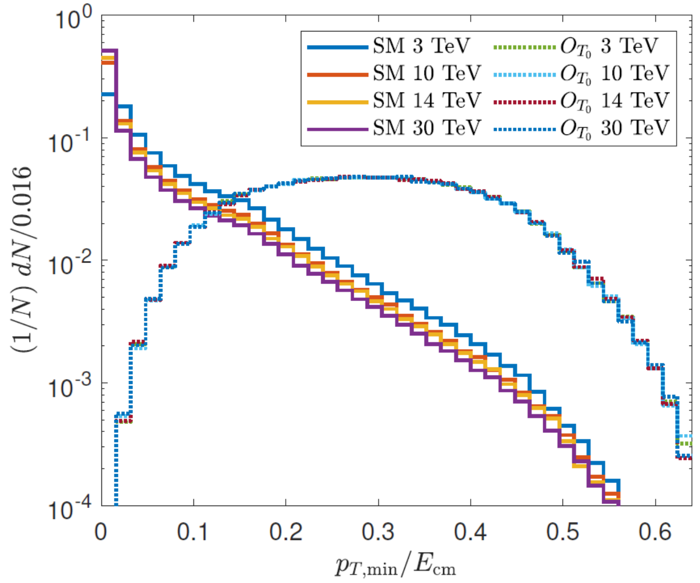
$$\sigma_{O_{T_i}} = \frac{e^2 s^3}{276480\pi^3\Lambda^8 c_W^2 s_W^2}\Big(8c_W s_W(1-4s_W^2)(16\alpha_1\alpha_3+7\alpha_1\alpha_4+7\alpha_2\alpha_3+4\alpha_2\alpha_4) + (1-4s_W^2+8s_W^4)\left(8\alpha_1^2+7\alpha_1\alpha_2+2\alpha_2^2\right)+128c_W^2 s_W^2\left(8\alpha_3^2+7\alpha_3\alpha_4+2\alpha_4^2\right)\Big),$$

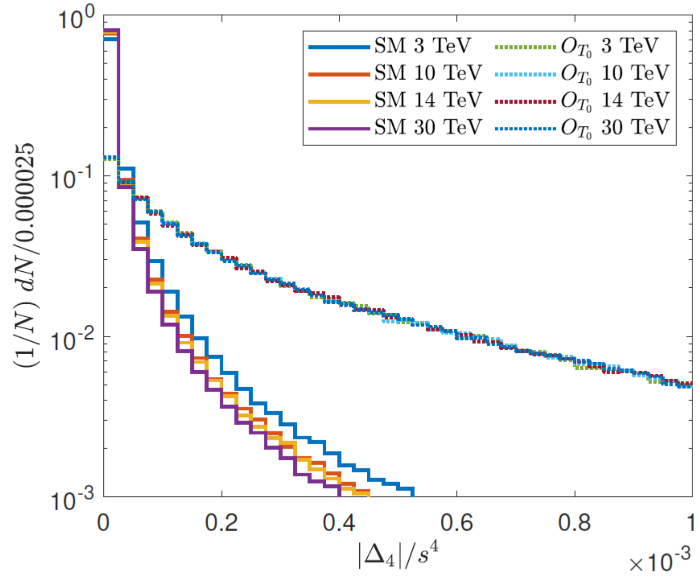
$$\alpha_1 = c_W^3 s_W(f_{T_5}+f_{T_6}-4f_{T_8})+c_W s_W^3(f_{T_0}+f_{T_1}-f_{T_5}-f_{T_6}),$$

$$\alpha_2 = c_W^3 s_W(f_{T_7}-4f_{T_9})+c_W s_W^3(f_{T_2}-f_{T_7}),$$

$$\alpha_3 = c_W^4 f_{T_8}+\frac{1}{2}c_W^2 s_W^2(f_{T_5}+f_{T_6})+\frac{1}{4}s_W^4(f_{T_0}+f_{T_1}),$$

$$\alpha_4 = c_W^4 f_{T_9}+\frac{1}{2}c_W^2 f_{T_7} s_W^2+\frac{f_{T_2}s_W^4}{4}.$$





Constraints on Dimension-8 Operators

	$\mathcal{S}_{\text{stat}}$	3 TeV 1 ab^{-1} (10^{-2} TeV^{-4})	10 TeV 10 ab^{-1} (10^{-4} TeV^{-4})	14 TeV 10 ab^{-1} (10^{-4} TeV^{-4})	30 TeV 10 ab^{-1} (10^{-5} TeV^{-4})
$\frac{f_{T_0}(f_{T_1})}{\Lambda^4}$	2	$[-43.49, 14.47]$t	$[-35.72, 12.19]$	$[-10.14, 4.09]$	$[-6.19, 3.30]$
	3	$[-48, 57, 19.55]$	$[-39.98, 16.45]$	$[-11.50, 5.46]$	$[-7.21, 4.32]$
	5	$[-57.08, 28.06]$	$[-47.10, 23.57]$	$[-13.77, 7.73]$	$[-8.92, 6.03]$
$\frac{f_{T_2}}{\Lambda^4}$	2	$[-108.0, 22.66]$	$[-87.71, 19.31]$	$[-24.37, 6.62]$	$[-14.06, 5.65]$
	3	$[-116.9, 31.59]$	$[-95.25, 26.85]$	$[-26.85, 9.09]$	$[-16.00, 7.58]$
	5	$[-132.4, 47.06]$	$[-108.3, 39.87]$	$[-31.08, 13.32]$	$[-19.27, 10.86]$
$\frac{f_{T_5}(f_{T_6})}{\Lambda^4}$	2	$[-10.78, 2.61]$	$[-8.81, 2.22]$	$[-2.45, 0.758]$	$[-1.44, 0.638]$
	3	$[-11.78, 3.61]$	$[-9.65, 3.05]$	$[-2.72, 1.03]$	$[-1.65, 0.846]$
	5	$[-13.49, 5.32]$	$[-11.08, 4.49]$	$[-3.19, 1.50]$	$[-2.01, 1.20]$
$\frac{f_{T_7}}{\Lambda^4}$	2	$[-27.54, 3.98]$	$[-22.47, 3.38]$	$[-6.17, 1.17]$	$[-3.41, 1.04]$
	3	$[-29.22, 5.66]$	$[-23.89, 4.80]$	$[-6.64, 1.65]$	$[-3.79, 1.43]$
	5	$[-32.22, 8.66]$	$[-26.42, 7.32]$	$[-7.48, 2.48]$	$[-4.46, 2.10]$
$\frac{f_{T_8}}{\Lambda^4}$	2	$[-1.74, 0.42]$	$[-1.42, 0.355]$	$[-0.399, 0.121]$	$[-0.233, 0.102]$
	3	$[-1.90, 0.58]$	$[-1.56, 0.490]$	$[-0.443, 0.165]$	$[-0.267, 0.136]$
	5	$[-2.17, 0.86]$	$[-1.79, 0.721]$	$[-0.518, 0.239]$	$[-0.325, 0.193]$
$\frac{f_{T_9}}{\Lambda^4}$	2	$[-4.50, 0.63]$	$[-3.66, 0.538]$	$[-1.00, 0.188]$	$[-0.553, 0.167]$
	3	$[-4.77, 0.90]$	$[-3.89, 0.765]$	$[-1.07, 0.264]$	$[-0.615, 0.229]$
	5	$[-5.25, 1.38]$	$[-4.29, 1.17]$	$[-1.21, 0.399]$	$[-0.723, 0.337]$

Summary

- Search for new physics indirectly as well as directly
- SMEFT is an effective, model-independent tool for probing indirectly possible BSM physics
- Physics at Dimension-8 provide windows of opportunity
- The unitarity bound is important when applying SMEFT
- Polarization and machine learning technology are powerful tools in the search for new physics

… the *direct* method may be used for joining battle, but *indirect* methods will be needed in order to secure victory.

The *direct* and the *indirect* lead on to each other in turn. It is like moving in a circle — you never come to an end. Who can exhaust the possibilities of their combination?

Sun Tzu, *The Art of War*

Thank you !