Construction of Operator Bases in Effective Field Theory

from On-shell Amplitudes to High-dimensional Operators

Zhe Ren

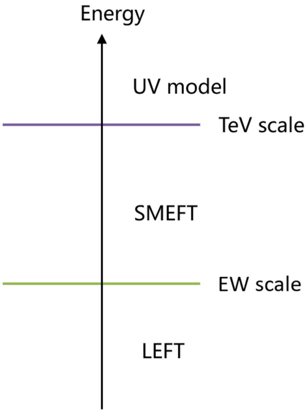
Hao-lin Li, Zhe Ren, Ming-Lei Xiao, Jiang-Hao Yu, Yu-Hui Zheng

ArXiv:2005.00008, 2007.07899, 2201.04639

Institute of Theoretical Physics, Chinese Academy of Science

August 10th,2022

EFT Framework



physical effect of heavy particles in UV model

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}}^{(4)} + \sum_{d>4} \left(\frac{1}{\Lambda_{NP}}\right)^{d-4} \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$$

physical effect of heavy particles in UV model and the standard model

$$\mathcal{L}_{\mathrm{LEFT}} = \mathcal{L}_{\mathrm{QCD+QED}}^{(4)} + \sum_{d>4} \left(\frac{1}{v}\right)^{d-4} \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$$

High-dimensional Operators

The dimension-6 operators in the SMEFT have been enumerated in 1986[1], yet the complete and independent basis (barring flavor structure) has not been presented until 2010, known as the Warsaw basis[2].

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r\varphi)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r\widetilde\varphi)$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^\star(\varphi^\dagger D_\mu\varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r\varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\,\widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\,\widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\,\widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\,\widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar q_s^k\sigma^{\mu\nu}u_t)$		

[1] W. Buchmuller and D. Wyler. "Effective Lagrangian Analysis of New Interactions and Flavor Conservation". In: *Nucl. Phys. B* 268 (1986), pp. 621–653. DOI: 10.1016/0550-3213(86)90262-2.

[2] B. Grzadkowski et al. "Dimension-Six Terms in the Standard Model Lagrangian". In: *JHEP* 10 (2010), p. 085. DOI: 10.1007/JHEP10(2010)085. arXiv: 1008.4884 [hep-ph].

High-dimensional Operators

Traditional Method:
One need to enumerate all possible operators, and remove redundant ones among them. The redundancies include equation of motion (EOM), covariant derivative commutator (CDC), integration by parts (IBP) and algebraic identities.

- EOM.

$$D^\mu D_\mu \phi + J_\phi = 0, \quad i\not{D}\psi + J_\psi = 0, \quad D_\mu F^{\mu\nu} + J_A^\nu = 0.$$

- CDC.

$$[D_\mu, D_\nu] = -iF_{\mu\nu}.$$

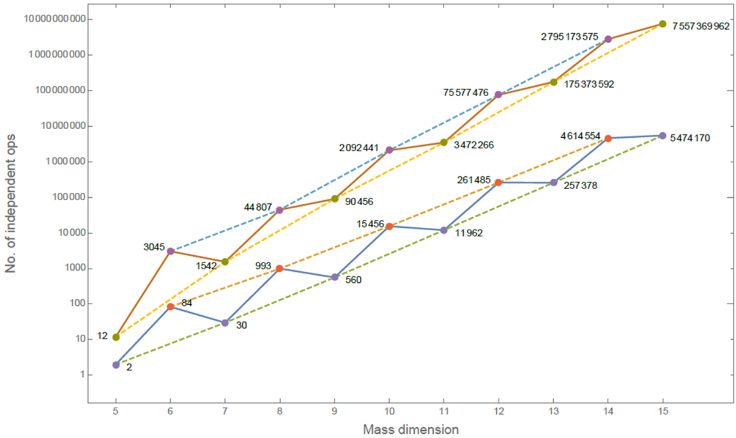
- IBP.

$$X\, D_\mu Y \sim -D_\mu X\, Y.$$

- Algebraic identities. Schouten identity, Bianchi identity, Fierz identity.

High-dimensional Operators

Hilbert Series in the SMEFT[3]:



[3] Brian Henning et al. "2, 84, 30, 993, 560, 15456, 11962, 261485, ...: Higher dimension operators in the SM EFT". In: *JHEP* 08 (2017). [Erratum: JHEP 09, 019 (2019)], p. 016. DOI: 10.1007/JHEP08(2017)016. arXiv: 1512.03433 [hep-ph].

High-dimensional Operators

Young tensor method[4,5,6]:
A systematical method to obtain the complete and independent operator basis of EFT that can be applied to generic models. The method was implemented in a Mathematica package: ABC4EFT.

- Pros:
  1. The operator bases are complete and independent, with flavor structure taken into account.
  2. The operator bases are on-shell, so operators in which are in one-to-one correspondence with on-shell amplitudes.
  3. The method can relate two different bases by giving the conversion matrix between them modulo the EOM and the CDC.

[4] Hao-Lin Li et al. "Complete set of dimension-eight operators in the standard model effective field theory". In: *Phys. Rev. D* 104.1 (2021), p. 015026. DOI: 10.1103/PhysRevD.104.015026. arXiv: 2005.00008 [hep-ph].

[5] Hao-Lin Li et al. "Complete set of dimension-nine operators in the standard model effective field theory". In: *Phys. Rev. D* 104.1 (2021), p. 015025. DOI: 10.1103/PhysRevD.104.015025. arXiv: 2007.07899 [hep-ph].

[6] Hao-Lin Li et al. "Operators for generic effective field theory at any dimension: on-shell amplitude basis construction". In: *JHEP* 04 (2022), p. 140. DOI: 10.1007/JHEP04(2022)140. arXiv: 2201.04639 [hep-ph].

On-shell Spinors as Building Blocks

Eg. Decompose $D^2\phi$ into irreducible representations of Lorentz group:

$$\underbrace{(D^2\phi)_{\alpha\beta\dot\alpha\dot\beta}}_{(\frac{1}{2},\frac{1}{2})\times(\frac{1}{2},\frac{1}{2})\times(0,0)} = \underbrace{\frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}D^\mu D_\mu\phi}_{(0,0)}$$

$$\underbrace{-\frac{i}{4}\epsilon_{\dot\alpha\dot\beta}\sigma^{\mu\nu}_{\alpha\beta}[D_\mu, D_\nu]\phi}_{(1,0)} \quad \underbrace{-\frac{i}{4}\epsilon_{\alpha\beta}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}[D_\mu, D_\nu]\phi}_{(0,1)}$$

$$\underbrace{+\frac{1}{4}(D^2\phi)_{(\alpha\beta)(\dot\alpha\dot\beta)}}_{\color{red}{(1,1)}}$$

EOM: $D^\mu D_\mu\phi$. CDC: $[D_\mu, D_\nu]\phi$. Highest weight: $(1,1)$.

On-shell Spinors as Building Blocks

- The highest weight of a field with derivatives on it corresponds to a set of non-vanishing on-shell spinors.

$$(D^2\phi_i)_{(\alpha\beta)(\dot\alpha\dot\beta)} \sim \lambda_{i\alpha}\tilde\lambda_{i\dot\alpha}\lambda_{i\beta}\tilde\lambda_{i\dot\beta}.$$
$$(D\psi_i)_{(\alpha\beta)\dot\alpha} \sim \lambda_{i\alpha}\tilde\lambda_{i\dot\alpha}\lambda_{i\beta}.$$
$$(DF_{\mathrm{L}i})_{(\alpha\beta\gamma)\dot\alpha} \sim \lambda_{i\alpha}\tilde\lambda_{i\dot\alpha}\lambda_{i\beta}\lambda_{i\gamma}.$$

- The lower weights of a field with derivatives on it, which involve the EOM or the CDC, automatically vanish when using on-shell spinors as building blocks.

$$D_\mu D^\mu\phi_i \sim \lambda_i^\alpha\tilde\lambda_{i\dot\alpha}\lambda_{i\alpha}\tilde\lambda_i^{\dot\alpha} = 0.$$
$$D^\alpha{}_{\dot\alpha}\psi_{i\alpha} \sim \lambda_i^\alpha\tilde\lambda_{i\dot\alpha}\lambda_{i\alpha} = 0.$$
$$D^\alpha{}_{\dot\alpha}F_{\mathrm{L}i\alpha\beta} \sim \lambda_i^\alpha\tilde\lambda_{i\dot\alpha}\lambda_{i\alpha}\lambda_{i\beta} = 0.$$

Semi-standard Young tableaux as the Lorentz Y-basis

Eg. The one-to-one correspondence between Young tableaux and on-shell amplitudes[7,8].

1	1	1	3
2	2	2	4

$$\sim \langle 12\rangle\langle 12\rangle\langle 34\rangle[34]$$

$$\sim F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}.$$

1	1	1	2
2	2	3	4

$$\sim \langle 12\rangle\langle 13\rangle\langle 24\rangle[34]$$

$$\sim F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}.$$

[7] Brian Henning and Tom Melia. "Constructing effective field theories via their harmonics". In: *Phys. Rev. D* 100.1 (2019), p. 016015. DOI: 10.1103/PhysRevD.100.016015. arXiv: 1902.06754 [hep-ph].

[8] Hao-Lin Li et al. "Complete set of dimension-eight operators in the standard model effective field theory". In: *Phys. Rev. D* 104.1 (2021), p. 015026. DOI: 10.1103/PhysRevD.104.015026. arXiv: 2005.00008 [hep-ph].

Semi-standard Young tableaux as the Lorentz Y-basis

In group theory, the semi-standard Young tableaux form a complete and independent basis for certain kind of Young tableaux, and the algebraic relations between them are Fock's condition.

- Fock's condition as the IBP.

$$\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 3 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline 3 & 1 & 1 & 1 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} + \begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 2 \\ \hline 3 & 2 & 2 & 4 \\ \hline \end{array},$$

$$\langle 12\rangle\langle 12\rangle\langle 34\rangle[34] = -\langle 12\rangle\langle 12\rangle\langle 14\rangle[14] - \langle 12\rangle\langle 12\rangle\langle 24\rangle[24].$$

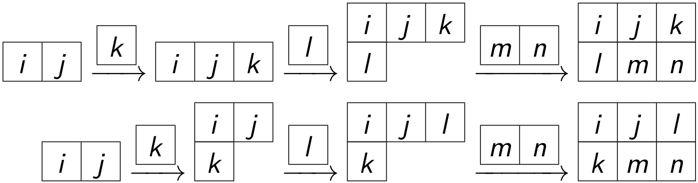
- Fock's condition as the Schouten identity.

$$\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 3 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} = \begin{array}{|c|c|c|c|}\hline 1 & 1 & 3 & 1 \\ \hline 2 & 2 & 2 & 4 \\ \hline \end{array} + \begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 2 \\ \hline 2 & 2 & 3 & 4 \\ \hline \end{array},$$

$$\langle 12\rangle\langle 12\rangle\langle 34\rangle[34] = \langle 12\rangle\langle 32\rangle\langle 14\rangle[34] + \langle 12\rangle\langle 13\rangle\langle 24\rangle[34].$$

Littlewood-Richardson Rule and the Gauge Y-basis

Eg. The $SU(2)$ representation of fields: ${\tau^I}_{ij} W^I \sim \boxed{i}\boxed{j}$, $L_k \sim \boxed{k}$, $H_l \sim \boxed{l}$, $H^\dagger_m H^\dagger_n \sim \boxed{m}\boxed{n}$.



$$\epsilon^{il}\epsilon^{jm}\epsilon^{kn}{\tau^I}_{ij} W^I L_k H_l H^\dagger_m H^\dagger_n, \quad \epsilon^{ik}\epsilon^{jm}\epsilon^{ln}{\tau^I}_{ij} W^I L_k H_l H^\dagger_m H^\dagger_n$$

From Y-basis to Flavor tensors

Eg. Y-basis of $Q^3 L$

$$O_1 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$
$$O_2 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{sbk})\,(Q_{raj}Q_{tcl})$$
$$O_3 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}\,(L_{pi}Q_{sbk})\,(Q_{raj}Q_{tcl})$$
$$O_4 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$

Each operator in y-basis is indeed a flavor tensor, $p, r, s, t = 1, \ldots, n_f$. Suppose that $n_f = 1$, how many independent operators are there for one generation of fermions?

Permutation Group and the P-basis

Totally symmetric Young Symmetrizer of the S_3 group:

$$\mathcal{Y}^{[3]} = \frac{1}{3!}\left(E + (12) + (13) + (23) + (123) + (132)\right).$$

$$\mathcal{Y}^{[3]}\begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \end{pmatrix} = \begin{pmatrix} -1/6 & 1/3 & 1/3 & -1/6 \\ -1/3 & 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 & -1/3 \\ -1/6 & 1/3 & 1/3 & -1/6 \end{pmatrix}\begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \end{pmatrix}.$$

The matrix is rank 1. So there is only 1 independent operator in Q^3L. For three generation of fermions, it is more complicated but the main idea is the same[9][10].

[9] Hao-Lin Li et al. "Complete set of dimension-nine operators in the standard model effective field theory". In: *Phys. Rev. D* 104.1 (2021), p. 015025. DOI: 10.1103/PhysRevD.104.015025. arXiv: 2007.07899 [hep-ph].

[10] Hao-Lin Li et al. "Operators for generic effective field theory at any dimension: on-shell amplitude basis construction". In: *JHEP* 04 (2022), p. 140. DOI: 10.1007/JHEP04(2022)140. arXiv: 2201.04639 [hep-ph].

Conversion among different bases

The Young tensor method include a systematical treatment for the IBP and the Schouten identity, which allow one to expand any operator onto the generated basis. Here are what one can do with the method:

- The conversion matrix of two different bases can be obtained throughout the y-basis.
- A basis can be expanded on the y-basis and check its completeness and independence. If the basis is over-complete, it can be reduced.
- For operators which involve repeated fields, a basis can be expanded on the p-basis, which automatically symmetrizes the operators in the basis, and the flavor relations of the basis can be easily found.

Conversion among different bases

$d^{\dagger}_{\mathbb{C}} e_{\mathbb{C}} L^{\dagger} Q u^2_{\mathbb{C}}$	
$\mathcal{O}^{(1)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})(u_{\mathbb{C}s}{}^{a} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(2)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})(u_{\mathbb{C}s}{}^{b} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(3)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{a})(Q_{rai} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(4)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{b})(Q_{rai} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(5)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}t}{}^{b})(Q_{rai} u_{\mathbb{C}s}{}^{a})$
$\mathcal{O}^{(6)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}t}{}^{a})(Q_{rai} u_{\mathbb{C}s}{}^{b})$
$\mathcal{O}^{(7)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(Q_{rai} \sigma^{\mu} L_v^{\dagger i})(u_{\mathbb{C}s}{}^{a} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(8)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(Q_{rai} \sigma^{\mu} L_v^{\dagger i})(u_{\mathbb{C}s}{}^{b} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(9)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}s}{}^{a} \sigma^{\mu} L_v^{\dagger i})(Q_{rai} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(10)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}s}{}^{b} \sigma^{\mu} L_v^{\dagger i})(Q_{rai} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(11)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{b} \sigma^{\mu} L_v^{\dagger i})(u_{\mathbb{C}s}{}^{a} Q_{rai})$
$\mathcal{O}^{(12)}$	$(e_{\mathbb{C}p} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{a} \sigma^{\mu} L_v^{\dagger i})(u_{\mathbb{C}s}{}^{b} Q_{rai})$
$\mathcal{O}^{(13)}$	$(Q_{rai} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}s}{}^{a} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(14)}$	$(Q_{rai} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}s}{}^{b} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(15)}$	$(Q_{rai} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{b} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{a})$
$\mathcal{O}^{(16)}$	$(Q_{rai} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{a} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{b})$
$\mathcal{O}^{(17)}$	$(u_{\mathbb{C}s}{}^{a} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{b} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})$
$\mathcal{O}^{(18)}$	$(u_{\mathbb{C}s}{}^{b} \sigma_{\mu} d^{\dagger}_{\mathbb{C}ub})(u_{\mathbb{C}t}{}^{a} \sigma^{\mu} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})$
$\mathcal{O}^{(19)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} Q_{rai})(u_{\mathbb{C}s}{}^{a} \sigma^{\mu\nu} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(20)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} Q_{rai})(u_{\mathbb{C}s}{}^{b} \sigma^{\mu\nu} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(21)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} u_{\mathbb{C}s}{}^{a})(Q_{rai} \sigma^{\mu\nu} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}^{(22)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} u_{\mathbb{C}s}{}^{b})(Q_{rai} \sigma^{\mu\nu} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}^{(23)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} u_{\mathbb{C}t}{}^{b})(Q_{rai} \sigma^{\mu\nu} u_{\mathbb{C}s}{}^{a})$
$\mathcal{O}^{(24)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} \sigma_{\mu\nu} u_{\mathbb{C}t}{}^{a})(Q_{rai} \sigma^{\mu\nu} u_{\mathbb{C}s}{}^{b})$

$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
-1 & -1 & 0 & 0 \\
0 & 0 & -1 & -1 \\
2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 \\
-2 & -2 & 0 & 0 \\
0 & 0 & -2 & -2 \\
-2 & -2 & 0 & 0 \\
0 & 0 & -2 & -2 \\
0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 \\
2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 \\
4 & 8 & 0 & 0 \\
0 & 0 & 4 & 8 \\
8 & 4 & 0 & 0 \\
0 & 0 & 8 & 4 \\
4 & -4 & 0 & 0 \\
0 & 0 & 4 & -4
\end{pmatrix} .
$$

M-basis	
$\mathcal{O}_1^{(m)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})(u_{\mathbb{C}s}{}^{a} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}_2^{(m)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{a})(Q_{rai} u_{\mathbb{C}t}{}^{b})$
$\mathcal{O}_3^{(m)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} Q_{rai})(u_{\mathbb{C}s}{}^{b} u_{\mathbb{C}t}{}^{a})$
$\mathcal{O}_4^{(m)}$	$(d^{\dagger}_{\mathbb{C}ub} L_v^{\dagger i})(e_{\mathbb{C}p} u_{\mathbb{C}s}{}^{b})(Q_{rai} u_{\mathbb{C}t}{}^{a})$

Conclusion

1. We present a general procedure to construct the independent and complete operator bases for generic Lorentz invariant EFTs, which is implemented into a publicly available Mathematica package: ABC4EFT (Amplitude Basis Construction for Effective Field Theories).
2. The Young tensor basis provides a modern view of the EFT operators from the on-shell perspective.
3. The method provide conversions between different operator bases, making it possible to compare results from various literature. Although the conversions can only be done inside a certain operator type modulo the EOM and the CDC now, the full conversions including all types at a certain dimension will be supported in the near future.

ABC4EFT

abc4eft is hosted by Hepforge, IPPP Durham

- Home
- Repo
- Downloads
- Contact

Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Theories (ABC4EFT).

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic Lorentz invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dimension.
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis conversion can be easily done.

Authors

The collaboration group at Institute of Theoretical Physics, CAS Beijing (ITP-CAS)

- Hao-Lin Li (previously postdoc at ITP-CAS, now postdoc at UC Louvain)
- Zhe Ren (4th-year graduate student at ITP-CAS)
- Ming-Lei Xiao (previously postdoc at ITP-CAS, now postdoc at Northwestern and Argonne)
- Jiang-Hao Yu (professor at ITP-CAS)
- Yu-Hui Zheng (5th-year graduate student at ITP-CAS)

Thanks!