

Axion Quality from Superconformal Dynamics

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Based on YN and M. Suzuki (TDLI), PLB 2021.

Strong CP Problem

QCD Lagrangian for strong interactions allows

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

explicitly violating **CP** symmetry.

The physical strong CP phase : $\bar{\theta} \equiv \theta - \arg \det (M_u M_d)$

The current upper bound on the neutron electric dipole moment

$$\rightarrow |\bar{\theta}| < 10^{-11}$$

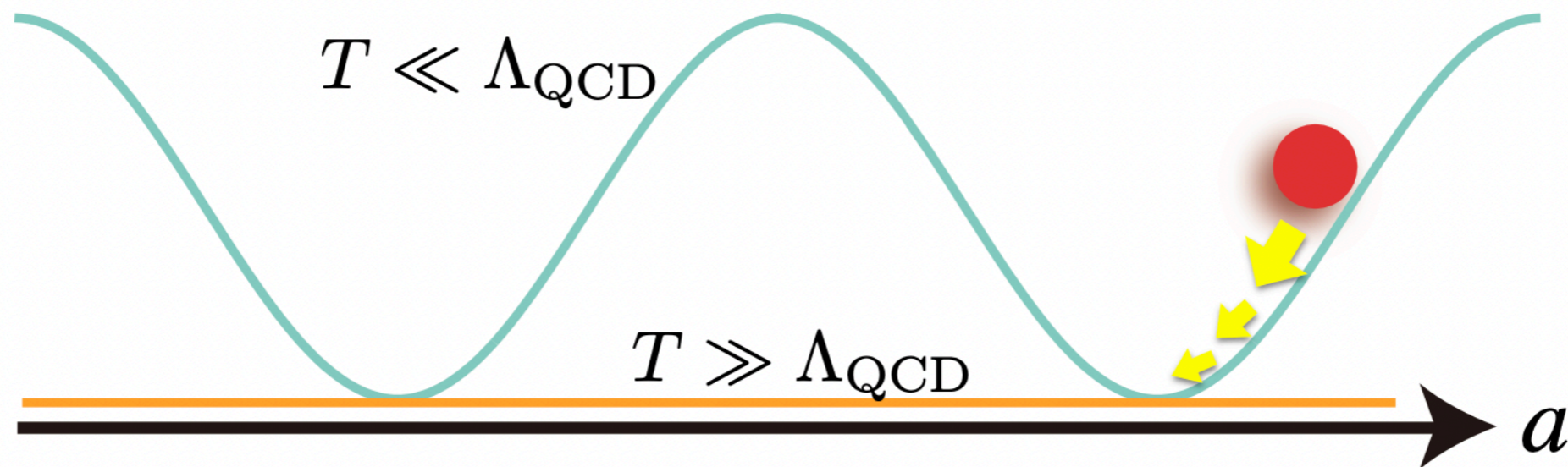
Why is $\bar{\theta}$ so small ??

Some shifts of $\bar{\theta}$ would not provide a visible change in our world.

Axion Solution

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



Fuminobu Takahashi slide

The **axion a** dynamically cancels the strong CP phase !

Axion Solution

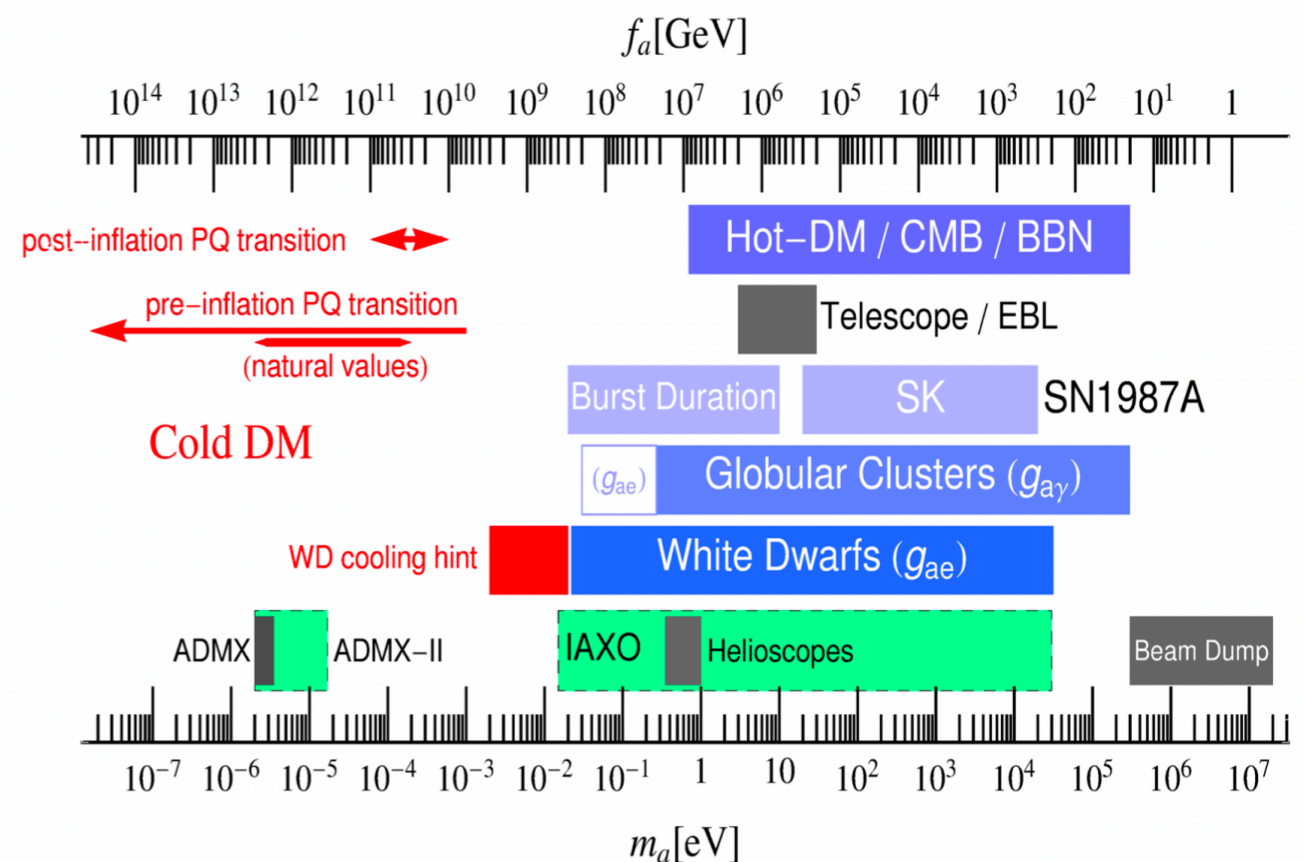
Axion is a pseudo-Nambu-Goldstone boson associated with spontaneous breaking of a **global U(1)_{PQ} symmetry**.

Non-perturbative QCD effects break the U(1)_{PQ} explicitly and generate the axion potential :

$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

Astrophysical observations put a lower limit :

$$f_a \gtrsim 10^8 \text{ GeV}$$



Axion Quality Problem

The small strong CP phase requires the $U(1)_{PQ}$ to be realized to an extraordinary high degree.



Quantum gravity effects do not respect such a global symmetry.

Planck suppressed $U(1)_{PQ}$ -violating operators are expected.

$$\Delta V(\phi) \sim \frac{|\phi|^{k+3}}{M_{\text{Pl}}^k} \phi + \text{h.c.}$$

➔ Destroy the Peccei-Quinn mechanism.

$$\Delta V(a) \sim f_a^4 \left(\frac{f_a}{M_{\text{Pl}}} \right)^k \cos \left(\frac{a}{f_a} - \varphi \right) \quad \langle \phi \rangle \equiv \frac{f_a}{\sqrt{2}}$$

Axion Quality Problem

The small strong CP phase requires the $U(1)_{PQ}$ to be realized to an extraordinary high degree.

Quantum gravity eff

Planck

metry.

**Superconformal
dynamics!**

$$M_{\text{Pl}}^k \phi + \text{h.c.}$$

➔ Destroy the Peccei-Quinn mechanism.

$$\Delta V(a) \sim f_a^4 \left(\frac{f_a}{M_{\text{Pl}}} \right)^k \cos \left(\frac{a}{f_a} - \varphi \right) \quad \langle \phi \rangle \equiv \frac{f_a}{\sqrt{2}}$$

Conformal Dynamics

The PQ breaking field marginally couples to **CFT sector fields**.

$$W_{\text{int}} = \lambda \phi \mathcal{O}_{\text{CFT}}$$

The PQ breaking field holds a large anomalous dimension.

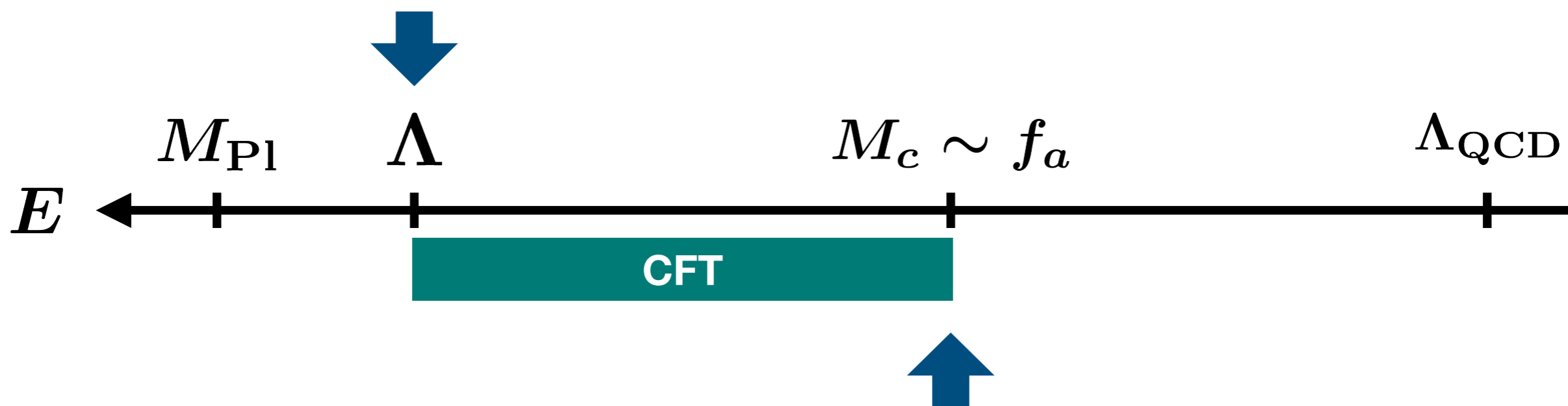
$$\rightarrow \epsilon_{\phi} \equiv Z_{\phi}^{-1/2}(\mu) = \left(\frac{\mu}{\Lambda} \right)^{\frac{\gamma_{\phi}}{2}} \ll 1$$

The $U(1)_{\text{PQ}}$ -violating operators are significantly suppressed at low-energies :

$$\Delta W \sim \frac{\phi^k}{M_{\text{Pl}}^{k-3}} \rightarrow \epsilon_{\phi}^k \frac{\phi^k}{M_{\text{Pl}}^{k-3}}$$

Conformal Dynamics

The theory flows into a conformal fixed point.



PQ breaking drives conformal breaking.

All the CFT sector fields become massive.

Integrating out the CFT sector fields generates

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

cf. The KSVZ axion model

The Model

A SUSY SU(N) gauge theory with N_f vector-like quarks :

$$Q_I, \bar{Q}_I \quad (I = 1, \dots, N_f) \quad N_f : \text{even}$$

The theory is in **conformal window** : $\frac{3}{2}N < N_f < 3N$

PQ singlet chiral superfields : $\Phi, \bar{\Phi}$

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$

$$m = 1, \dots, N_f/2 \quad k = N_f/2 + 1, \dots, N_f$$

The ordinary color is embedded in flavor symmetries :

$$\begin{aligned} & \underline{SU(N_f/2)}_1 \times SU(N_f/2)_2 \\ & \supset SU(3)_C \end{aligned}$$

The Model

	Q_m	\bar{Q}_m	Q_k	\bar{Q}_k	Φ	$\bar{\Phi}$
$SU(N)$	N	\bar{N}	N	\bar{N}	1	1
$U(1)_{PQ} (Z_N)$	$+1$	0	-1	0	-1	$+1$
$U(1)_R$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{2N}{N_f}$	$\frac{2N}{N_f}$

- The $U(1)_{PQ}$ symmetry is not anomalous under the $SU(N)$.



Axion does not couple to the $SU(N)$ gauge field so that no new axion potential is generated.

- Anomaly coefficient : $A_{U(1)_{PQ}-SU(3)_C-SU(3)_C} = N$

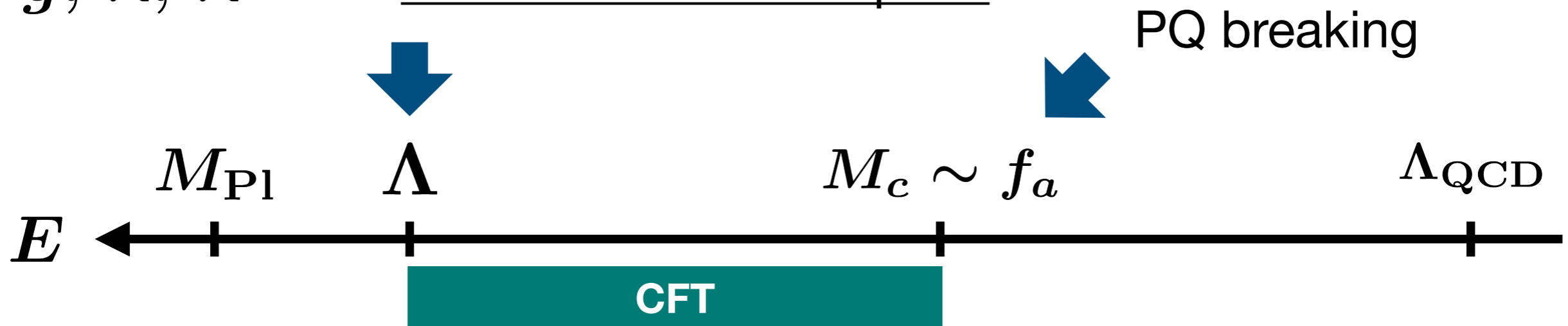


$Z_N \subset U(1)_{PQ}$ is **an anomaly-free discrete symmetry**.

It ensures the $U(1)_{PQ}$ at the renormalizable level.

Anomalous Dimension

$g, \lambda, \bar{\lambda}$ enter a non-trivial IR fixed point.

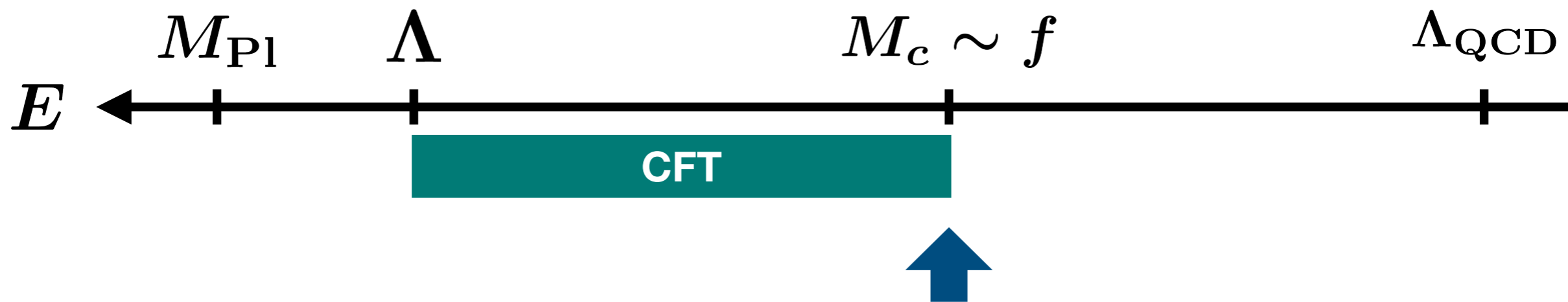


Anomalous dimension is determined by **the $U(1)_R$ charge.**

$$Z_{\Phi} = \left(\frac{M_c}{\Lambda} \right)^{-\gamma_{\Phi}} \quad \gamma_{\Phi} = 6 \frac{N}{N_f} - 2$$

Canonical normalization : $\Phi = \left(\frac{M_c}{\Lambda} \right)^{\gamma_{\Phi}/2} \hat{\Phi}$

Axion Potential



$\Phi, \bar{\Phi}$ obtain nonzero VEVs.

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$

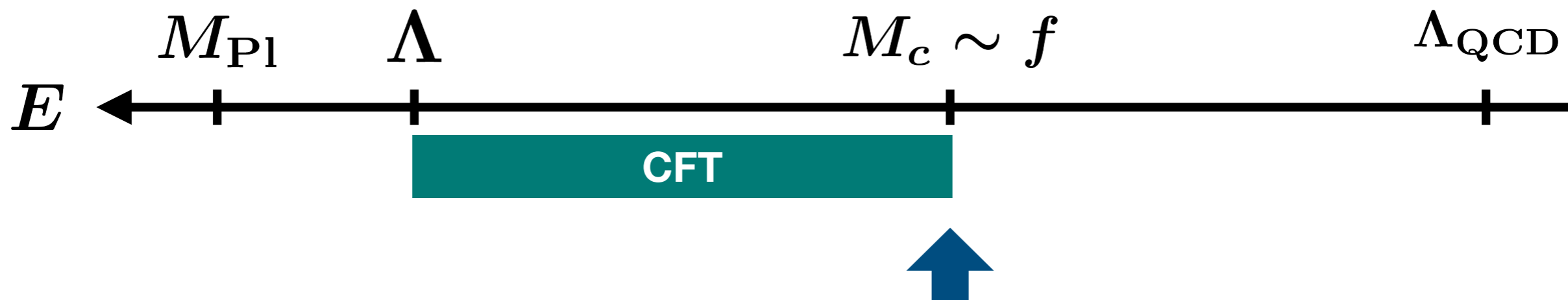
➔ All the new quarks become massive.

Integrating out the quarks ➔ $\mathcal{L}_{\text{eff}} \supset N \frac{a}{F_a} \frac{g_c^2}{32\pi^2} G \tilde{G}$

$$F_a/N = \sqrt{2} f/N$$

Axion potential : $V \sim m_\pi^2 f_\pi^2 \cos\left(N \frac{a}{F_a}\right)$ $m_\pi^2 f_\pi^2 = (0.1 \text{ GeV})^4$

Hidden Glueballs



$\Phi, \bar{\Phi}$ obtain nonzero VEVs.

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$

➔ All the new quarks become massive.

The model becomes a **SU(N) pure Yang-Mills theory.**

It confines just below the conformal breaking scale.

➔ Heavy **SU(N) glueballs** and their superpartners.

Emergent PQ

The most dangerous operator respecting the \mathbf{Z}_N symmetry :

$$W_{\mathbf{PQ}} \sim \frac{\Phi^N}{M_{\text{Pl}}^{N-3}} \sim \left(\frac{M_c}{\Lambda} \right)^{\frac{N\gamma_\Phi}{2}} \frac{\hat{\Phi}^N}{M_{\text{Pl}}^{N-3}}$$

The scalar potential in supergravity $V \supset -3WW^*/M_{\text{Pl}}^2$

$$W = m_{3/2} M_{\text{Pl}}^2$$

➔
$$V_{\mathbf{PQ}} = \left(\frac{M_c}{\Lambda} \right)^{\frac{N\gamma_\Phi}{2}} \frac{\kappa_{\mathbf{PQ}} m_{3/2} \hat{\Phi}^N}{M_{\text{Pl}}^{N-3}} \quad \kappa_{\mathbf{PQ}} : \text{model dependent coefficient}$$

The $U(1)_{\mathbf{PQ}}$ -violating axion potential :

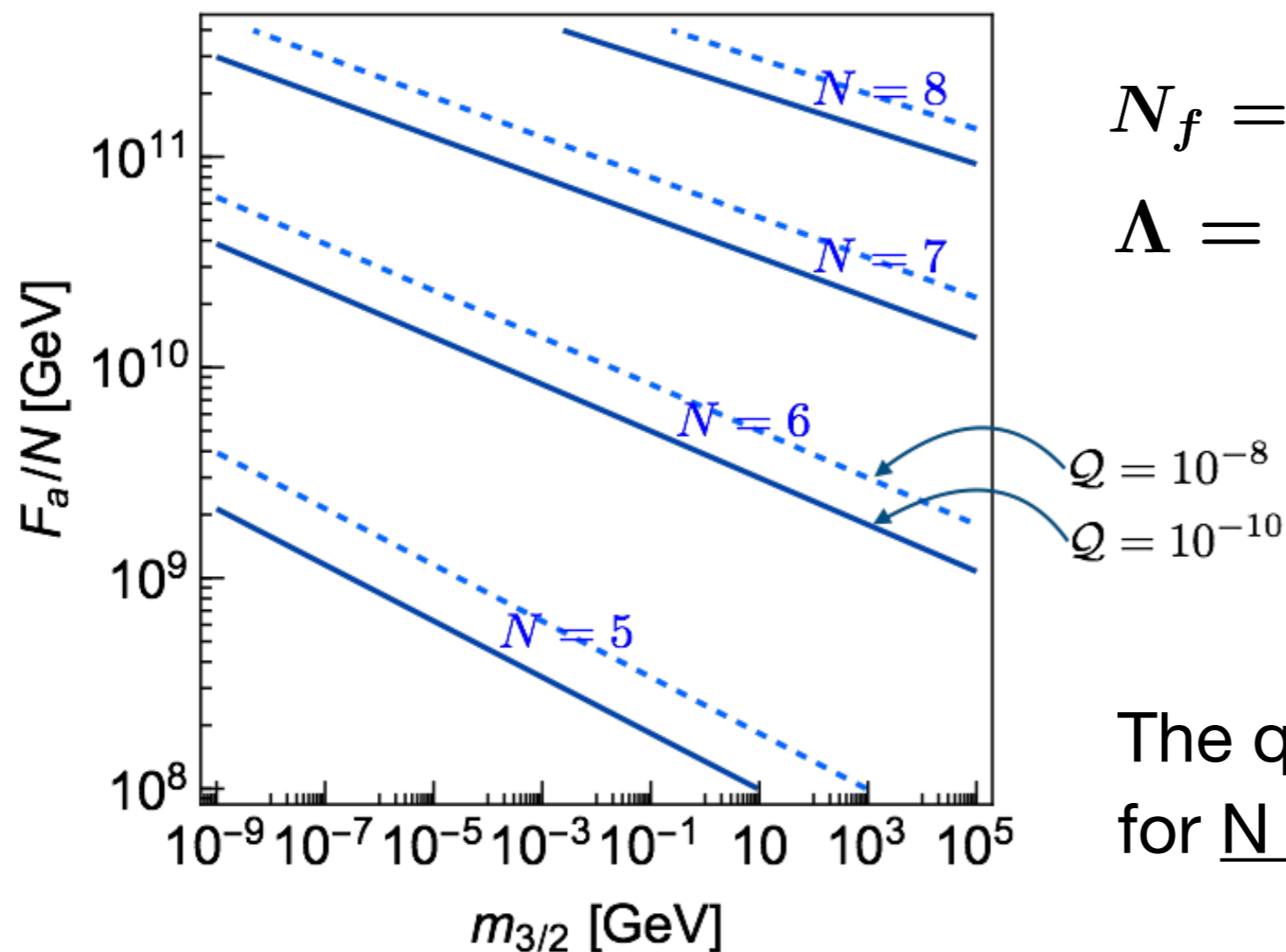
Arbitrary !

$$V_{\mathbf{PQ}} \supset \left(\frac{M_c}{\Lambda} \right)^{N(3N/N_f-1)} \frac{\kappa_{\mathbf{PQ}} m_{3/2} F_a^N}{M_{\text{Pl}}^{N-3}} \cos \left(N \frac{a}{F_a} + \varphi \right)$$

Emergent PQ

Axion quality factor : $V_{PQ} \equiv \mathcal{Q} m_\pi^2 f_\pi^2 \cos \left(N \frac{a}{F_a} + \varphi \right)$

Experimental upper bound requires $\mathcal{Q} \lesssim 10^{-10}$



$$N_f = 2N, M_c = F_a, \kappa_{PQ} = 1$$
$$\Lambda = 0.1 M_{Pl}$$

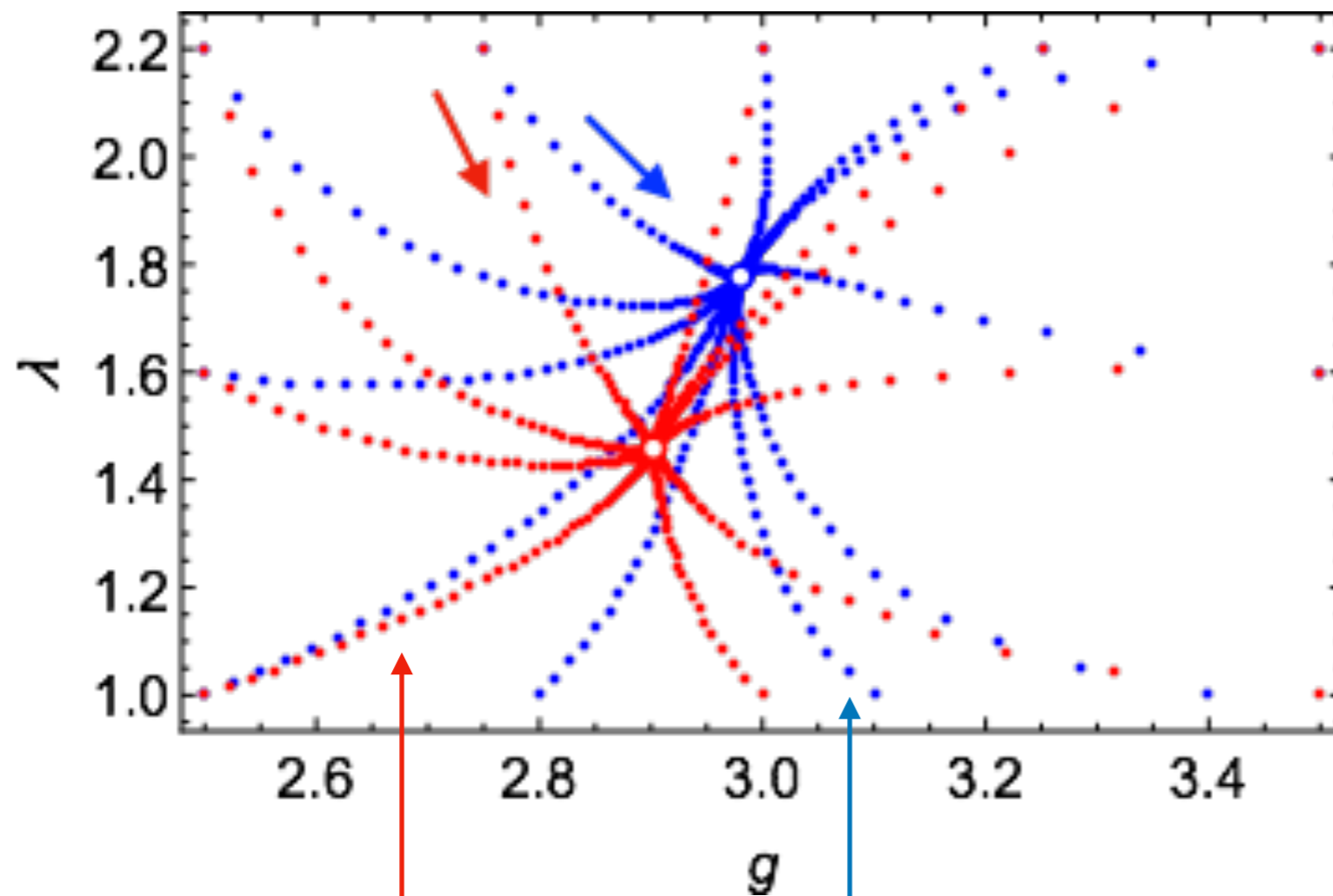
$$\mathcal{Q} = 10^{-8}$$
$$\mathcal{Q} = 10^{-10}$$

The quality problem is solved for $N = 5$ or larger !

The IR fixed Point

Check the existence of the IR fixed point for g , λ , $\bar{\lambda}$

RG flows from Λ_0 to $\mu = 10^{-9} \Lambda_0$



2-loop RGEs

1-loop RGEs

$$N = 5 \quad N_f = 10$$
$$\bar{\lambda} = 2 \text{ at } \Lambda_0$$

The effect of SU(3)_c gauge coupling is ignored.

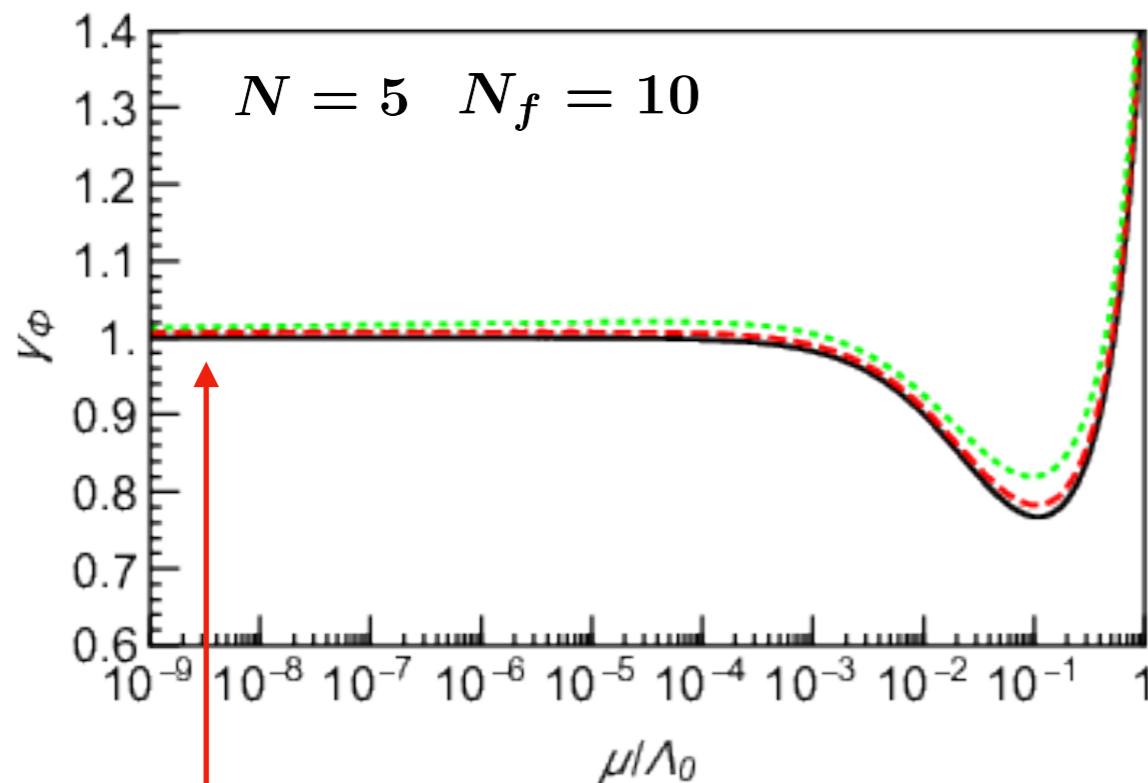
Both couplings flow into a non-trivial IR fixed point.

The IR fixed Point

Include the effect of the $SU(3)_c$ gauge coupling.

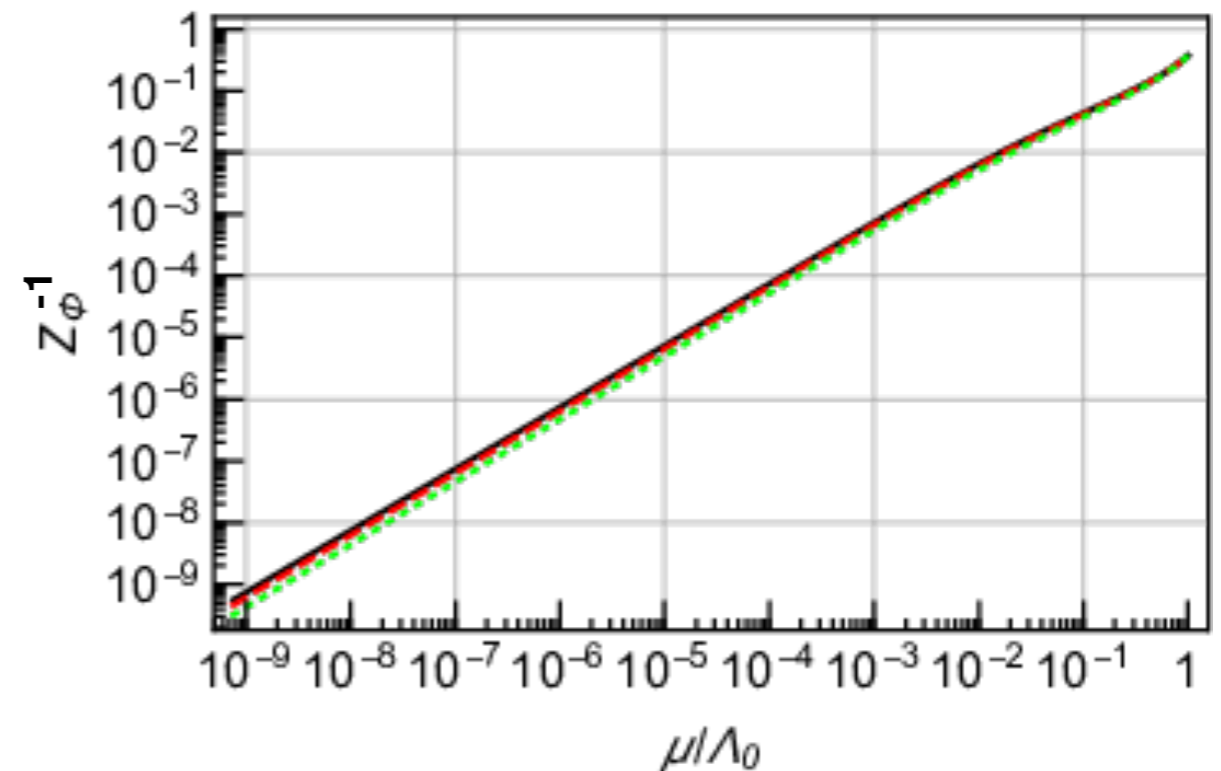
$$g = \lambda = \bar{\lambda} = 2 \quad g_c = 0, 1, 2 \quad \text{at } \Lambda_0$$

Anomalous dimension at 2-loop



The value without the QCD effect

Wave function renormalization factor



The smallness enables to solve the axion quality problem.

Summary



- **Superconformal dynamics** can address the axion quality problem.
- PQ breaking fields marginally couple to new quarks charged under the $SU(3)_c$ and a new $SU(N)$.
- A large anomalous dimension of PQ breaking fields leads to a strong suppression of explicit $U(1)_{\text{PQ}}$ -violating operators.
- PQ breaking drives conformal breaking and integrating out the new heavy quarks generates the desired axion coupling to gluons.

Thank you.

Backup Material

PQ Breaking

PQ breaking : $W'_X = \kappa' X (2\Phi\bar{\Phi} - f'^2)$

Canonical normalization : $\Phi = \left(\frac{M_c}{\Lambda}\right)^{\gamma_\Phi/2} \hat{\Phi}$

➔ $W_X = \kappa \left(\frac{M_c}{\Lambda}\right)^{\gamma_\Phi} X (2\hat{\Phi}\hat{\bar{\Phi}} - f^2)$

$$\kappa \sim \kappa'$$

$$f \sim \left(\frac{M_c}{\Lambda}\right)^{-\gamma_\Phi/2} f'$$

PQ (and conformal) breaking scale $M_c \sim f$