A formal notion of genericity and its application to supersymmetric Wess-Zumino models

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CHEP 2022, 11 August 2022

(J.Brister, Z.Sun, G.Yang, JHEP12(2021)199, [2111.09570])

Outline

A formal and analytical notion of genericity

Application to SUSY vacua in generic R-symmetric Wess-Zumino models

Summary and generalization

Notion of genericity

Genericity in high energy physics

- Generic: descriptive of an entire class, lacking specificity.
- Genericity in high energy physics: no parameter tuning.
- Small change of parameters from particular values (measured by experiments) does not affect the key property of the model, so no fine-tuning is needed.
- Related to naturalness and hierarchy problems.
- Genericity of the form of model is not considered here.

Building a generic model

- Criteria (1): fitting and predicting experiments.
- Criteria (2): symmetries, gauge invariance, renormalization...
- Criteria of genericity: parameters $\{c_\alpha\}$ satisfying (2) should take generic values and give consistent results for (1).

An analytic formalism of genericity

Genericiy represented by a function

- A key property of the model represented as $F(c_\alpha)$.
- The model possesses the property at $c_\alpha^{(0)} \Leftrightarrow F(c_\alpha^{(0)}) = C$.
- The property is generic: $F(c_\alpha) = C$ for c_α near $c_\alpha^{(0)}$.

An analytic formalism

- The model is described by a a set of parameters $\{\phi_i\}$, whose expectation values $\langle\phi_i\rangle = \phi_i(c_\alpha)$ depend on c_α.
- The notion of genericity is then expressed as:
  $F(\phi_i(c_\alpha^{(0)}), c_\alpha^{(0)}) = F(\phi_i(c_\alpha^{(0)} + \delta c_\alpha), c_\alpha^{(0)} + \delta c_\alpha) = C$.
- Assuming $F(\phi_i, c_\alpha)$ and $\phi_i(c_\alpha)$ are differentiable at $c_\alpha^{(0)}$,
  $\Rightarrow \dfrac{\mathrm{d}F}{\mathrm{d}c_\alpha} = \dfrac{\partial F}{\partial c_\alpha} + \dfrac{\partial F}{\partial \phi_i}\dfrac{\partial \phi_i}{\partial c_\alpha} = 0$ at $c_\alpha^{(0)}$.
- We will apply this formalism to SUSY Wess-Zumino models.

Supersymmetry (SUSY)

SUSY and SUSY breaking

- SUSY algebra: supercharges connecting fermions and bosons.
- Supermultiplets: chiral (ϕ, ψ, F), vector $(A_\mu, \lambda, \bar{\lambda}, D)$.
- Physics described by a SUSY invariant action.
- Why SUSY: phenomenology and math implications.
- Spontaneous SUSY breaking in a hidden sector, mediated to the Standard Model sector $\Rightarrow$ soft SUSY breaking terms.

Wess-Zumino models for F-term SUSY breaking

- A superpotential $W(\phi_i)$, a Kähler potential $K(\phi_i^*, \phi_j)$, $\Rightarrow V = K^{\bar{i}j}(\partial_{\phi_i} W)^* \partial_{\phi_j} W$, with $K_{\bar{i}j} = \partial_{\phi_i^*}\partial_{\phi_j} K$, $K_{\bar{i}j}K^{\bar{i}k} = \delta_j^k$.
- SUSY breaking $\Leftrightarrow \langle V \rangle > 0 \Leftrightarrow \langle -F_i^* \rangle = \langle \partial_{\phi_i} W \rangle \neq 0$.
- In SUGRA, $V = e^{K/M_{\mathrm{P}}^2}(K^{\bar{i}j}(D_{\phi_i} W)^* D_{\phi_j} W - 3W^*W/M_{\mathrm{P}}^2)$, SUSY breaking $\Leftrightarrow \langle D_{\phi_i} W \rangle = \langle \partial_{\phi_i} W + W \partial_{\phi_i} K / M_{\mathrm{P}}^2 \rangle \neq 0$.

The Nelson-Seiberg theorem and its extensions

The Nelson-Seiberg theorem

- An $U(1)$ R-symmetry transforms supercharges.
- $\hat{R}(\alpha)\phi_i = e^{ir_i\alpha}\phi_i$, R-charges r_i of ϕ_i.
- R-invariance of the action $\Rightarrow r_W = 2$.
- F-term SUSY breaking at the true vacuum $\Rightarrow$ R-symmetries, $\Leftarrow$ spotaneous R-symmetry breaking at the vacuum.
  (A.E.Nelson, N.Seiberg, [hep-ph/9309299])
- Requiring genericness in both parameters and R-charges.

The revised and generalized theorem

- SUSY breaking $\Leftrightarrow$ R-symmetries and one of the following:
  - W is singular at the origin;
  - $N_2 > N_0$ (R-charge 2 fields are more than R-charge 0 fields).

  (Z.Kang, T.Li, Z.Sun, [1209.1059]; Z.Li, Z.Sun, [2006.00538])
- Requiring genericness in both parameters and R-charges.

SUSY vacua in R-symmetric models

R-symmetric and R-symmetry breaking SUSY vacua

- R-symmetries and $N_2 \leq N_0 \Rightarrow$ R-symmetric SUSY vacua.
  (Z.Sun, [1109.6421])
- Counterexamples with $N_2 > N_0$ and non-generic R-charges $\Rightarrow$ R-symmetry breaking SUSY vacua.
  (Z.Sun, Z.Tan, L.Yang, [1904.09589]; A.Amariti, D.Sauro, [2005.02076]; Z.Sun, Z.Tan, L.Yang [2106.08879]; Z.Li, Z.Sun, [2107.09943])
- Both cases give SUSY vacua with $\langle W \rangle = 0$.

Applications

- $\langle \partial_i W \rangle = 0$ and $\langle W \rangle = 0$ gives $\langle D_i W \rangle = 0$ and $\langle V \rangle = 0$ in SUGRA extension of Wess-Zumino models.
- Model building, e.g. in string theory: SUSY breaking F-terms and the cosmological constant are perturbatively set to zero, and dynamically generated to archive an exponentially small scale (compared to the Planck scale).

SUSY vacua in generic R-symmetric models

Vanishing W at SUSY vacua

- Under an R-symmetry, $\hat{R}(\alpha)\phi_i = e^{ir_i\alpha}\phi_i$, $\hat{R}(\alpha)W = e^{2i\alpha}W$,
  $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\alpha}\hat{R}(\alpha)W = 2iW = ir_i\phi_i\partial_i W \Rightarrow W = \frac{1}{2}r_i\phi_i\partial_i W$.
- SUSY vacua: $\langle \partial_i W \rangle = 0 \Rightarrow \langle W \rangle = 0$, no need for genericity.

Term-by-term vanishing W in generic models

- Generic R-symmetric Wess-Zumino models: $W = c_\alpha p_\alpha(\phi_i)$.
- Assuming SUSY vacua generically exist $\Rightarrow$ solutions to $\partial_i W = 0$ and $W = 0$ generically exist at $\langle \phi_i \rangle = \phi_i(c_\alpha)$.
- Taking $\langle W \rangle$ as the F in the previous notion of genericity,
  $$\Rightarrow \frac{\mathrm{d}\langle W \rangle}{\mathrm{d}c_\alpha} = \frac{\partial \langle W \rangle}{\partial c_\alpha} + \langle \partial_i W \rangle \frac{\partial \phi_i}{\partial c_\alpha} = 0.$$
- $\langle \partial_i W \rangle = 0 \Rightarrow \frac{\partial \langle W \rangle}{\partial c_\alpha} = p_\alpha(\langle \phi_i \rangle) = 0$, W vanishes term-by term at SUSY vacua.

  (J.Brister, Z.Sun, G.Yang, [2111.09570])

Summary and generalization

Genericity and SUSY vacua in R-symmetric models

- A generic property: $F(c_\alpha) = C$ for c_α near $c_\alpha^{(0)}$,
  $\Rightarrow \dfrac{\mathrm{d}F}{\mathrm{d}c_\alpha} = \dfrac{\partial F}{\partial c_\alpha} + \dfrac{\partial F}{\partial \phi_i}\dfrac{\partial \phi_i}{\partial c_\alpha} = 0$ at $c_\alpha^{(0)}$.
- $\langle \partial_i W \rangle = 0$ in generic R-symmetric Wess-Zumino models
  $\Rightarrow \langle W \rangle = 0$ term-by-term.
- Constraint on the form of W may contribute to the classification of R-symmetric Wess-Zumino models.

Generalization to scalar potentials with scaling symmetries

- A scaling symmetry: $\hat{S}(\lambda) x^\mu = \lambda x^\mu$, $\hat{S}(\lambda)\phi_i = \lambda^{s_i}\phi_i$.
- Scaling invariance of the action $-\int \mathrm{d}^4 x V \Rightarrow s_V = -4$.
- Stationary points satisfying $\langle \partial_i V \rangle = 0 \Rightarrow \langle W \rangle = 0$, in generic models $\Rightarrow \langle V \rangle = 0$ term-by-term.
- Applications in scale-invariant systems?