



河南省科学院  
HENAN ACADEMY OF SCIENCES

# Explaining The New CDF II W-Boson Mass Data In The Georgi-Machacek Extension Models

**Xiaokang Du(都小康)**

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**Based on Xiaokang Du, Zhuang Li, Fei Wang, Yingkai Zhang. 2204.05760**

全国高能物理大会@辽宁师范大学

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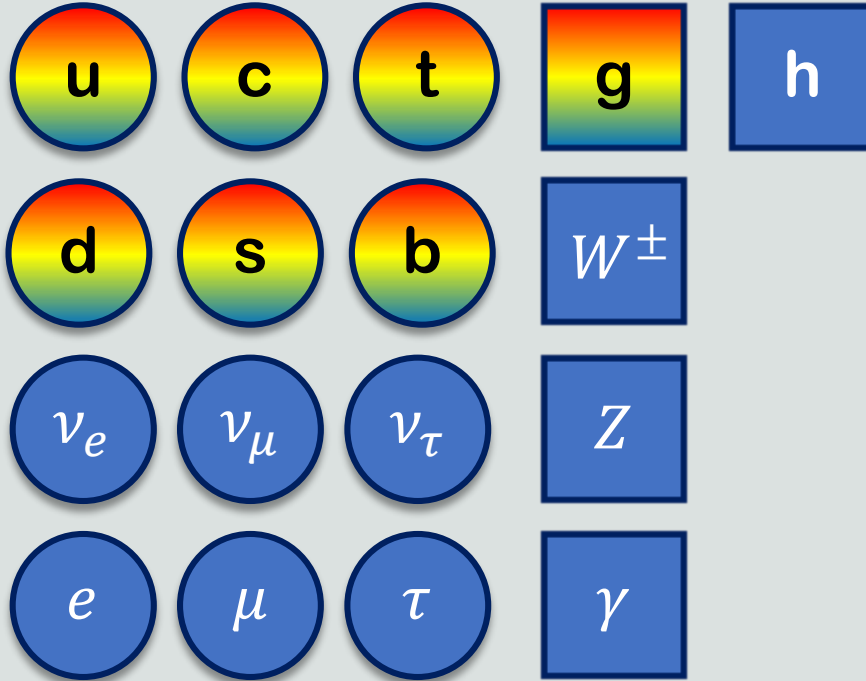
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# 1、CDF-II Results and the Georgi-Machacek Model

## the standard Model



Fields	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
$H$	1/2	2	1
$G^a$	0	1	8
$W^a$	0	3	1
$B$	0	1	1
$Q_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix}$	1/6	2	3
$u_{R_i}$	2/3	1	3
$d_{R_i}$	-1/3	1	3
$L_{L_i} = \begin{pmatrix} \nu_{L_i} \\ e_{L_i} \end{pmatrix}$	-1/2	2	1
$E_{R_i}$	-1	1	1

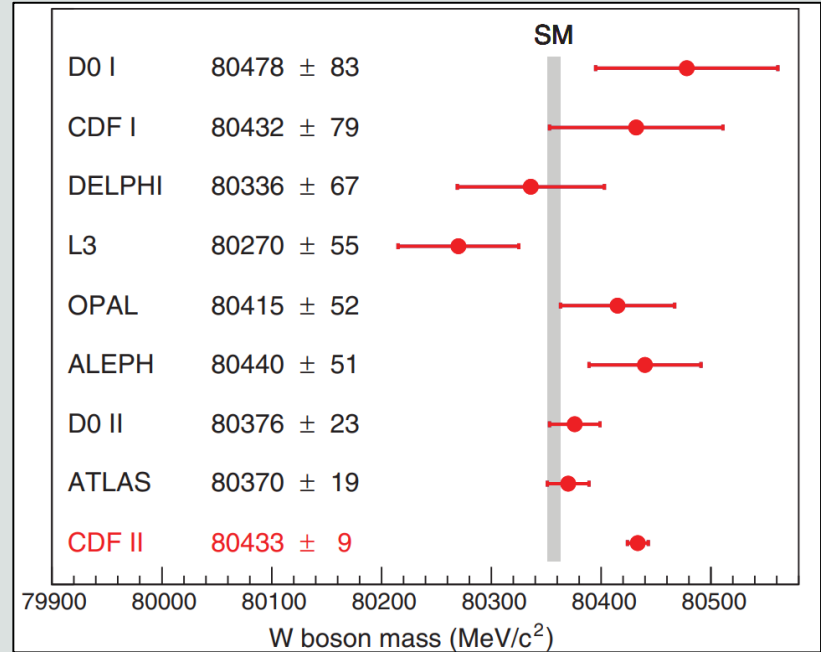
*Nucl.Phys.* 22 (1961) 579-588, *Phys.Lett.* 13 (1964) 168-171, *Phys. Rev. Lett.* 19 (1967) 1264-1266 ...

# 1、CDF-II Results and the Georgi-Machacek Model

## CDF-II Results on W Boson Mass



Science 376, 170-176 (2022)



$$\text{SM: } M_W = 80357 \pm 6 \text{ MeV}$$

$$\text{CDF-II: } M_W = 80433.5 \pm 9.4 \text{ MeV}$$

# 1、CDF-II Results and the Georgi-Machacek Model

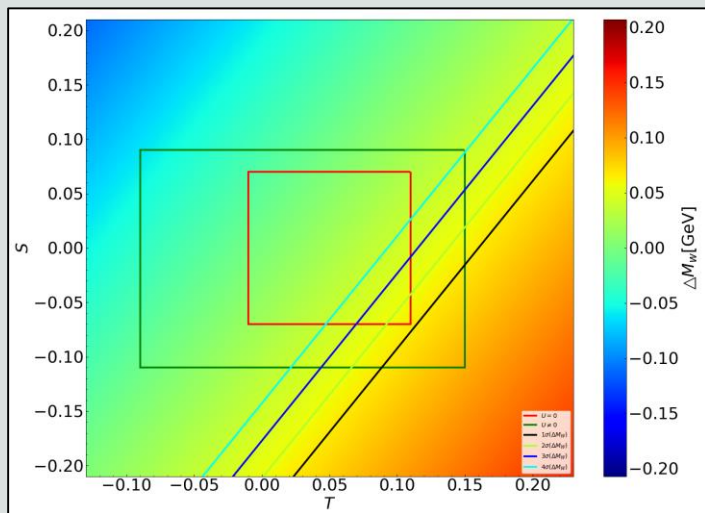
$\Delta m_W$  ?

$$\Delta m_W = \frac{\alpha M_W}{2(c_W^2 - s_W^2)} \left( -\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right),$$

$$\alpha S = 4s_W^2 c_W^2 \left[ \Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \Pi'_{ZY}(0) - \Pi'_{\gamma\gamma}(0) \right],$$

$$\alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},$$

$$\alpha U = 4s_W^2 \left[ \Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{ZY}(0) - s_W^2 \Pi'_{\gamma\gamma}(0) \right]$$



$$S = -0.01 \pm 0.10,$$

$$T = 0.03 \pm 0.012,$$

$$U = 0.02 \pm 0.11$$

$$S = 0.00 \pm 0.07,$$

$$T = 0.05 \pm 0.06,$$

$$U = 0$$

*Phys. Rev. D* 46 (1992) 381, *PTEP* 2020 (2020) 8, 083C01(PDG2020)

# 1、CDF-II Results and the Georgi-Machacek Model

The Georgi-Machacek Model

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

$$\mathcal{L}_{GM} = \mathcal{L}_{kin} + \mathcal{L}_Y + \mathcal{L}_\nu - V_H$$

$$\mathcal{L}_\nu \supset h_{ij} \overline{L_L^{ic}} i\tau_2 \chi L_L^j + h.c.$$

$$\begin{aligned} V(\Phi, \Delta) = & \frac{1}{2} m_\Phi^2 \text{tr}[\Phi^\dagger \Phi] + \frac{1}{2} m_\Delta^2 \text{tr}[\Delta^\dagger \Delta] + \lambda_1 (\text{tr}[\Phi^\dagger \Phi])^2 + \lambda_2 (\text{tr}[\Delta^\dagger \Delta])^2 \\ & + \lambda_3 \text{tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \text{tr}[\Phi^\dagger \Phi] \text{tr}[\Delta^\dagger \Delta] + \lambda_5 \text{tr} \left[ \Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] \text{tr}[\Delta^\dagger T^a \Delta T^b] \\ & + \mu_1 \text{tr} \left[ \Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] (P^\dagger \Delta P)_{ab} + \mu_2 \text{tr}[\Delta^\dagger T^a \Delta T^b] (P^\dagger \Delta P)_{ab} \end{aligned}$$

Fields	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
$\phi$	1/2	2	1
$\chi$	1	3	1
$\xi$	0	3	1

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$$

# 1、CDF-II Results and the Georgi-Machacek Model

$$\boxed{SU(2)_L \times SU(2)_R} \xrightarrow[\langle \chi^0 \rangle = \langle \xi^0 \rangle = v_\Delta]{\langle \phi^0 \rangle = v_\phi / \sqrt{2}} \boxed{SU(2)_c}$$

$$v_{EW}^2 = \sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i = v_\phi^2 + 4v_\chi^2 + 4v_\xi^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} \approx (246\text{GeV})^2$$

$$c_i = \begin{cases} 1, & (T, Y) \in \text{complex representation} \\ 1/2, & (T, Y = 0) \in \text{real representation} \end{cases}$$

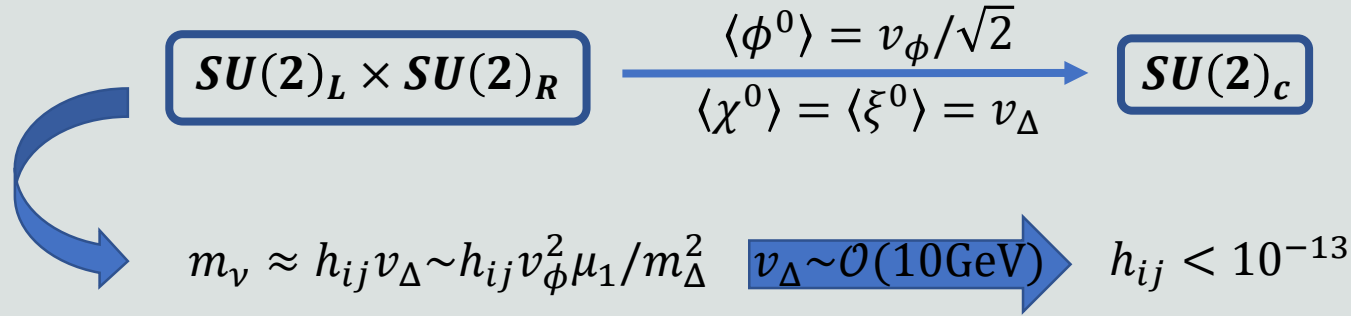
$$\rho_{tree} \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i / \sum_i 2Y_i |v_i|^2$$

$$\Delta\rho_{tree} \equiv \rho - 1 = \frac{v_\phi^2 + 4v_\chi^2 + 4v_\xi^2}{v_\phi^2 + 8v_\Delta^2} - 1 \approx \frac{4v_\chi^2 - 4v_\xi^2}{v_{EW}^2}$$

$$\tan \theta = 2\sqrt{2}v_\Delta / v_\phi < 0.2$$

# 1、CDF-II Results and the Georgi-Machacek Model

$$\mathcal{L}_{type-II} \supset h_{ij} \bar{L}_L^i c \tau_2 \chi L_L^j + \mu_1 \text{tr} \left[ \Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] (P^\dagger \Delta P)_{ab}$$



Vacuum stability

$$\begin{aligned} \lambda_1 > 0, \lambda_2 + \lambda_3 > 0, \lambda_2 + \frac{1}{2} \lambda_3 > 0, \\ -|\lambda_4| + 2\sqrt{\lambda_1(\lambda_2 + \lambda_3)} > 0, \\ \lambda_4 - \frac{1}{4} |\lambda_5| + \sqrt{2\lambda_1(2\lambda_2 + \lambda_3)} > 0. \end{aligned}$$

Perturbative unitarity

$$\begin{aligned} |\lambda_4 - \lambda_5| < 2\pi, \quad |2\lambda_3 + \lambda_2| < \pi, \\ |6\lambda_1 + 7\lambda_3 + 11\lambda_2| + \sqrt{(6\lambda_1 - 7\lambda_3 - 11\lambda_2)^2 + 36\lambda_4^2} < 4\pi, \\ |2\lambda_1 - \lambda_3 + 2\lambda_2| + \sqrt{(2\lambda_1 + \lambda_3 - 2\lambda_2)^2 + \lambda_5^2} < 4\pi. \end{aligned}$$

*Nucl.Phys.B* 262 (1985) 463-477, *JHEP* 01 (2013) 026, *JHEP* 01 (2016) 120



## 2、 Explaining W-Boson Mass In the GM and Extension Models

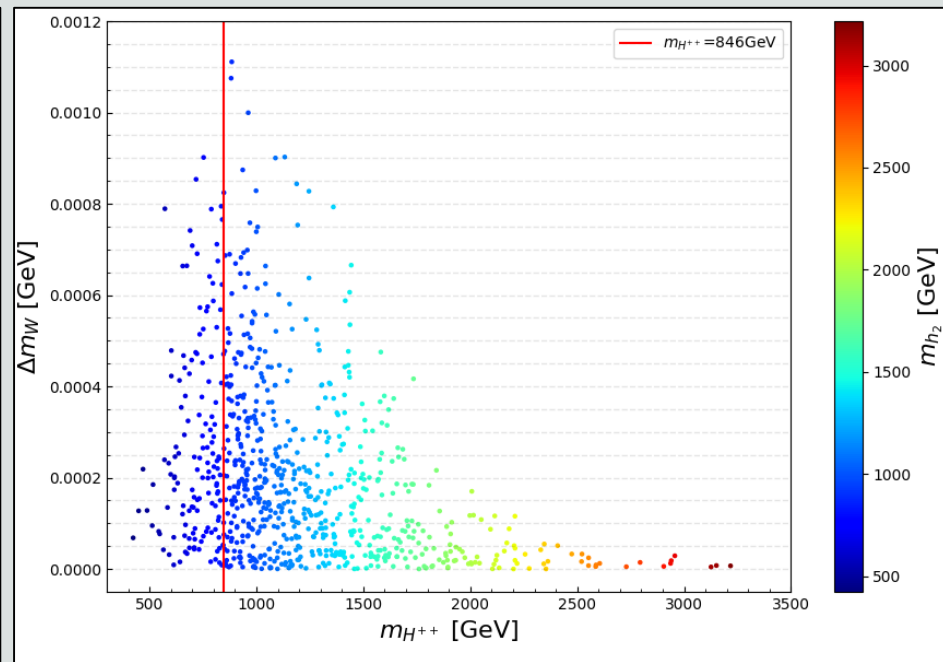
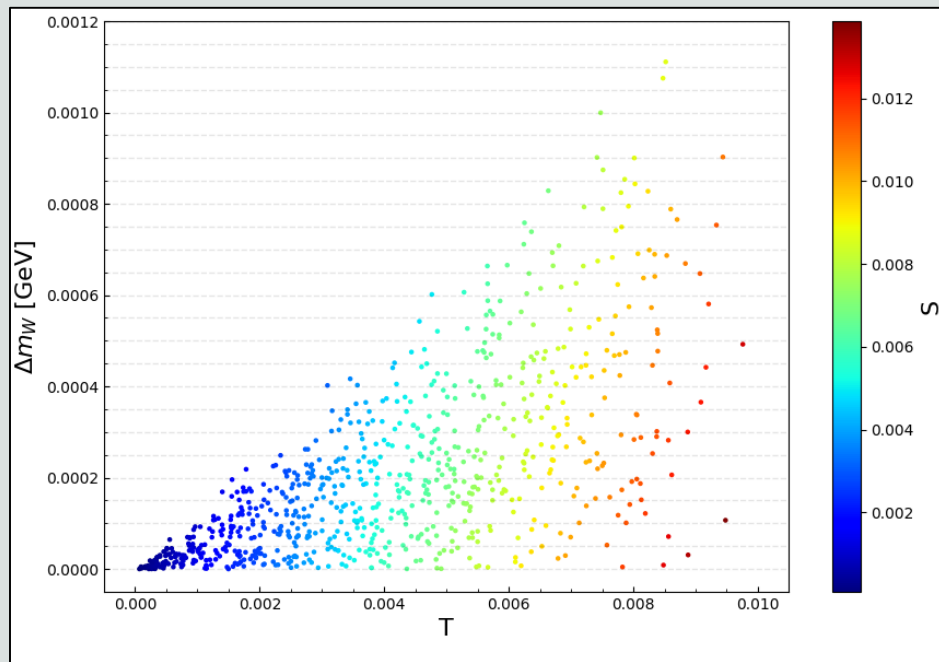
1

In the Original  
GM Model

$$SU(2)_L \times SU(2)_R$$

$$\begin{aligned} \langle \phi^0 \rangle &= v_\phi / \sqrt{2} \\ \langle \chi^0 \rangle &= \langle \xi^0 \rangle = v_\Delta \end{aligned}$$

$$SU(2)_c$$



## 2、 Explaining W-Boson Mass In the GM and Extension Models

2

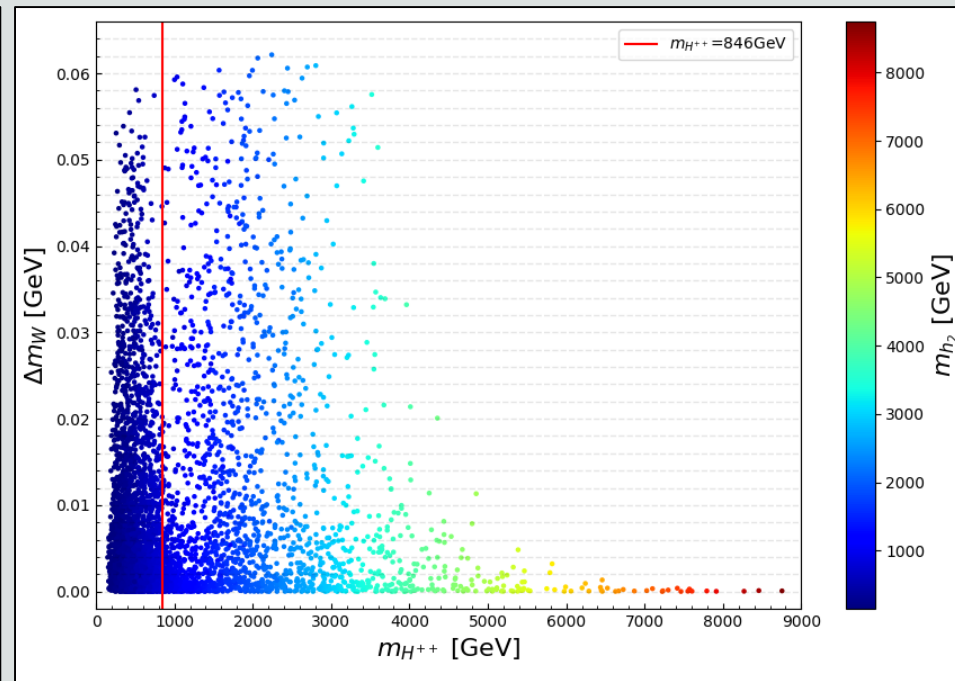
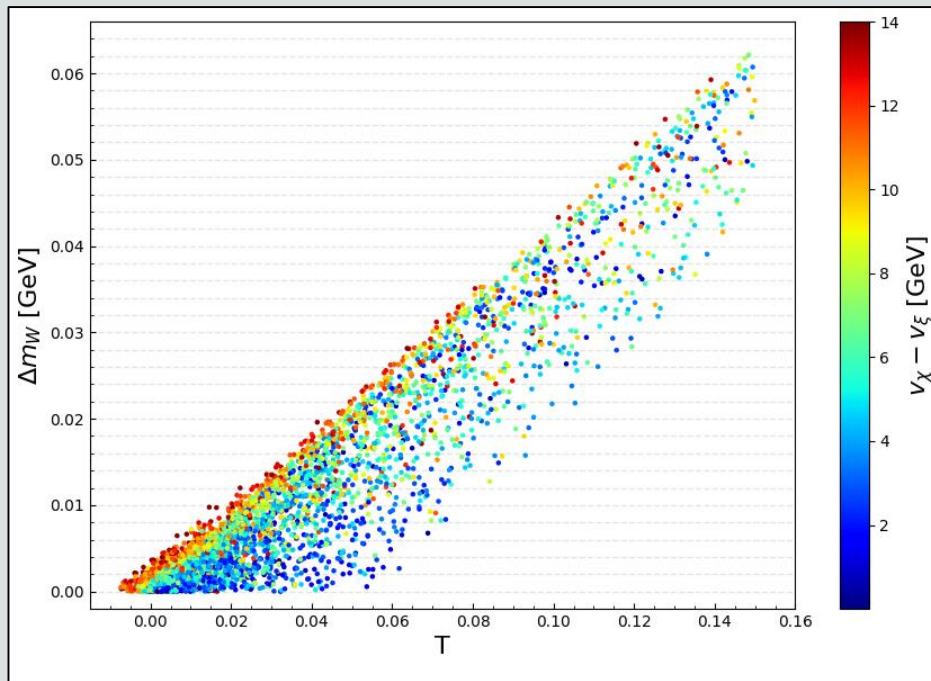
$$\Delta v = v_\chi - v_\xi > 0$$



~~$SU(2)_c$~~



$\Delta m_W$



## 2、 Explaining W-Boson Mass In the GM and Extension Models



$$-\mathcal{L}_\nu \supset y_{ij}^N \bar{L}_{L,i} \phi N_{R,j} + \frac{1}{2} (M_R)_{ij} N_{R,i}^T C N_{R,j} + h_{ij} \bar{L}_L^i c i \tau_2 \chi L_L^j + h.c.$$

$$M_\nu = \begin{pmatrix} h_{ij} v_\Delta & (y_{ij}^N)^T v_\phi \\ y_{ij}^N v_\phi & (M_R)_{ij} \end{pmatrix}$$

$\Downarrow$   $M_R \gg v_\phi \gg v_\Delta$

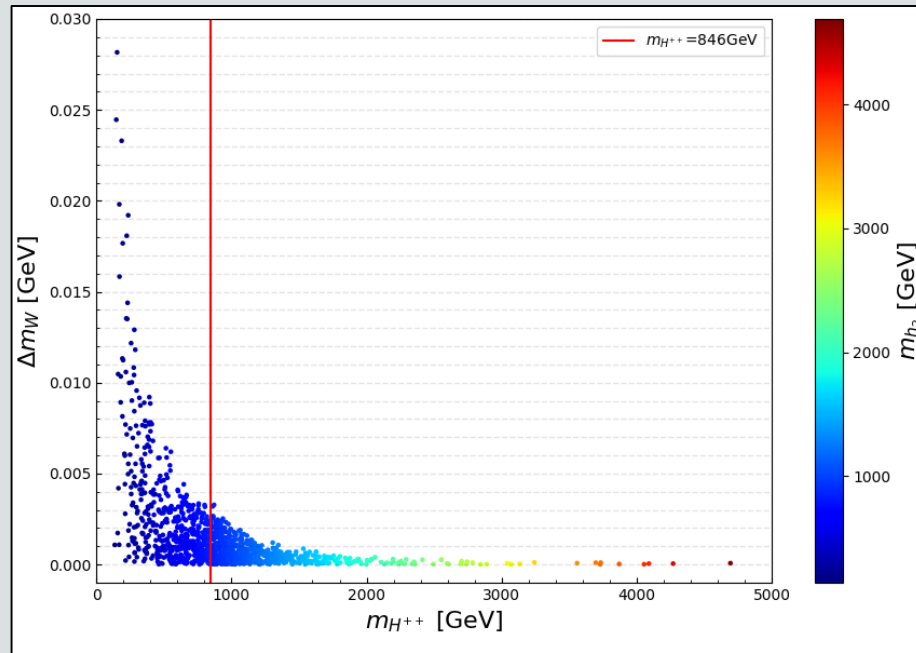
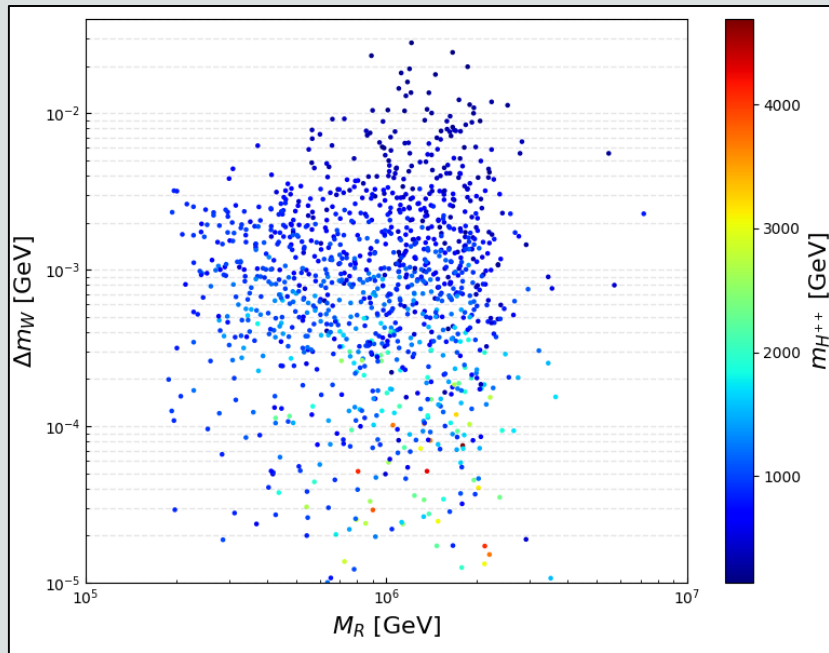
$$m_\nu \approx h_{ij} v_\Delta - v_\phi^2 (y_{ij}^N)^T M_{R,j}^{-1} (y_{ij}^N), \quad h_{ij} v_\Delta \approx (y^N v_\phi)^2 / M_{R,j}^{-1}$$

$$h_{ij} = 2\sqrt{2} (V_{PMNS}^T)^{-1} \left( \frac{v(1-s_H^2)}{s_H M_{R,i}} \right) \delta_{ij} (V_{PMNS})^{-1}, \quad y_{ij}^N = (V_{PMNS})^{-1}$$

## 2、 Explaining W-Boson Mass In the GM and Extension Models

$$BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}, BR(\mu \rightarrow 3e) < 10^{-12}$$

$$BR(\mu \rightarrow e\gamma) \sim \frac{\alpha_{EM}}{192\pi} |h_{ij}|^4 \left(\frac{m_W}{M_{H^{++}}}\right)^4 \quad \longrightarrow \quad h_{ij} < 10^{-2}, \quad M_R > 50\text{TeV}$$



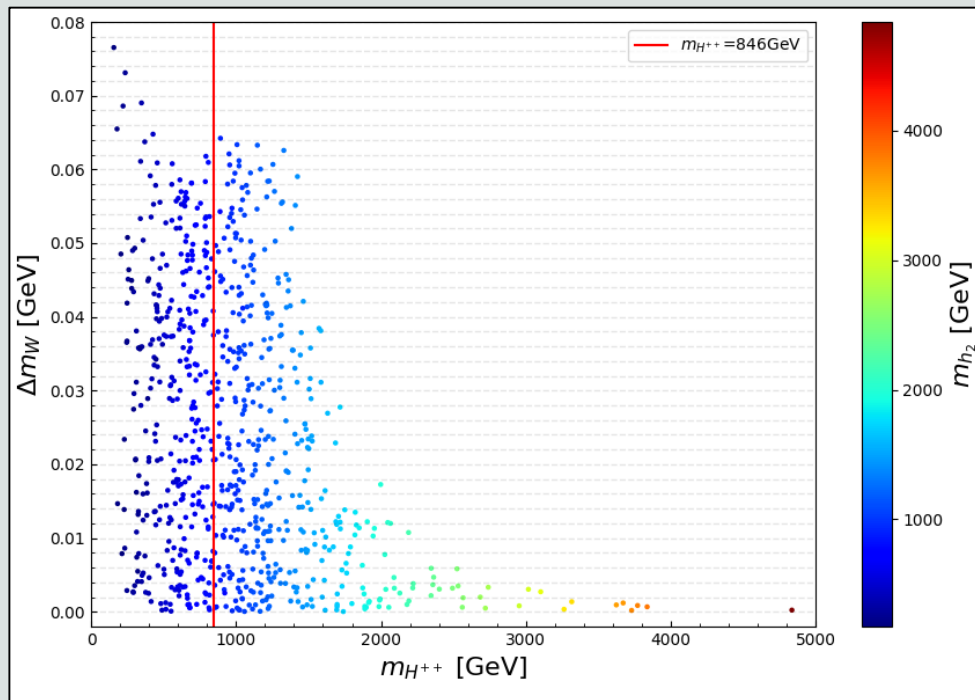
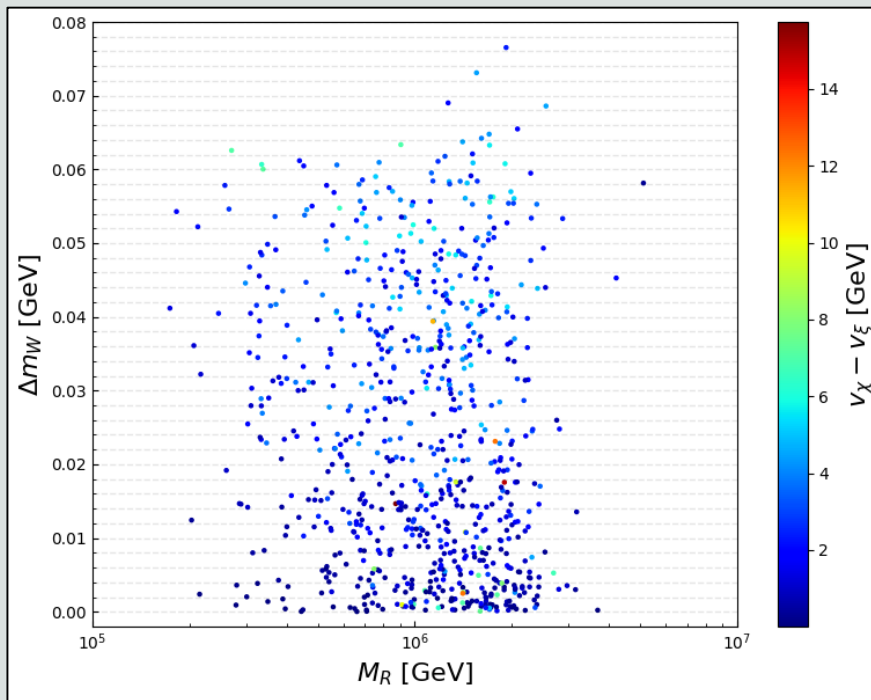
## 2、 Explaining W-Boson Mass In the GM and Extension Models

4

$$\Delta v = v_\chi - v_\xi > 0$$



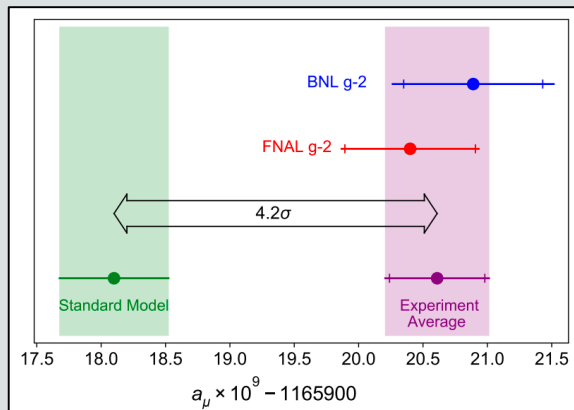
Type-I + Type-II



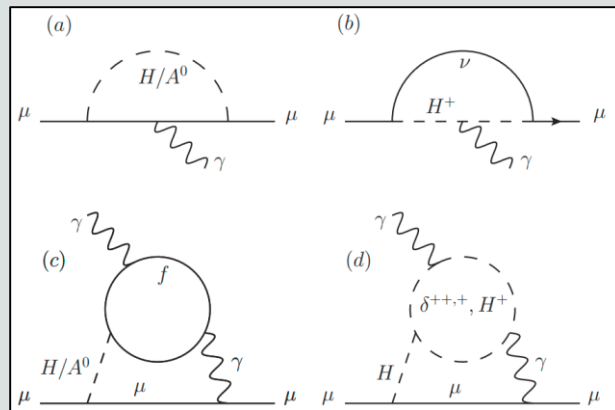
### 3、What to do Next ?

01

Muon  $g - 2$  in the GM Model



Phys.Rev.Lett. 126 (2021) 14, 141801



Phys.Rev.D 104 (2021) 5, 055011

02

Leptogenesis in the GM Model

To resolve BAU by the Leptogenesis

Heavy singlet neutrinos with hierarchical mass spectrum

lower bound of about  $10^8 \sim 10^9 \text{ GeV}$

Leptonic CP Asymmetry

Mass differences of heavy majorana neutrinos comparable to their decay width

lower bound of about  $\sim \text{TeV}$

Resonant Effect

Phys.Rev.D 56 (1997) 5431-5451

## 4、 Summary and Q&A

1. Taken a general discussion about the contribution to W boson mass in the original GM Model;
2. Explaining CDF-II results in the GM Extension Models.  
Misalignment among the triplet VEVs and large  $h_{ij}$  couplings allowed with RH neutrino sector;
3. Works what to do right now. Try to explain  $g_\mu - 2$  and CDF-II results simultaneously, try to understand BAU by leptogenesis in GM model with right hand neutrino extension.

**THANKS**

**Q&A**