中国物理学会高能物理分会第十一届全国会员代表大会暨学术年会

How arbitrary are perturbative calculations of the electroweak phase transition?

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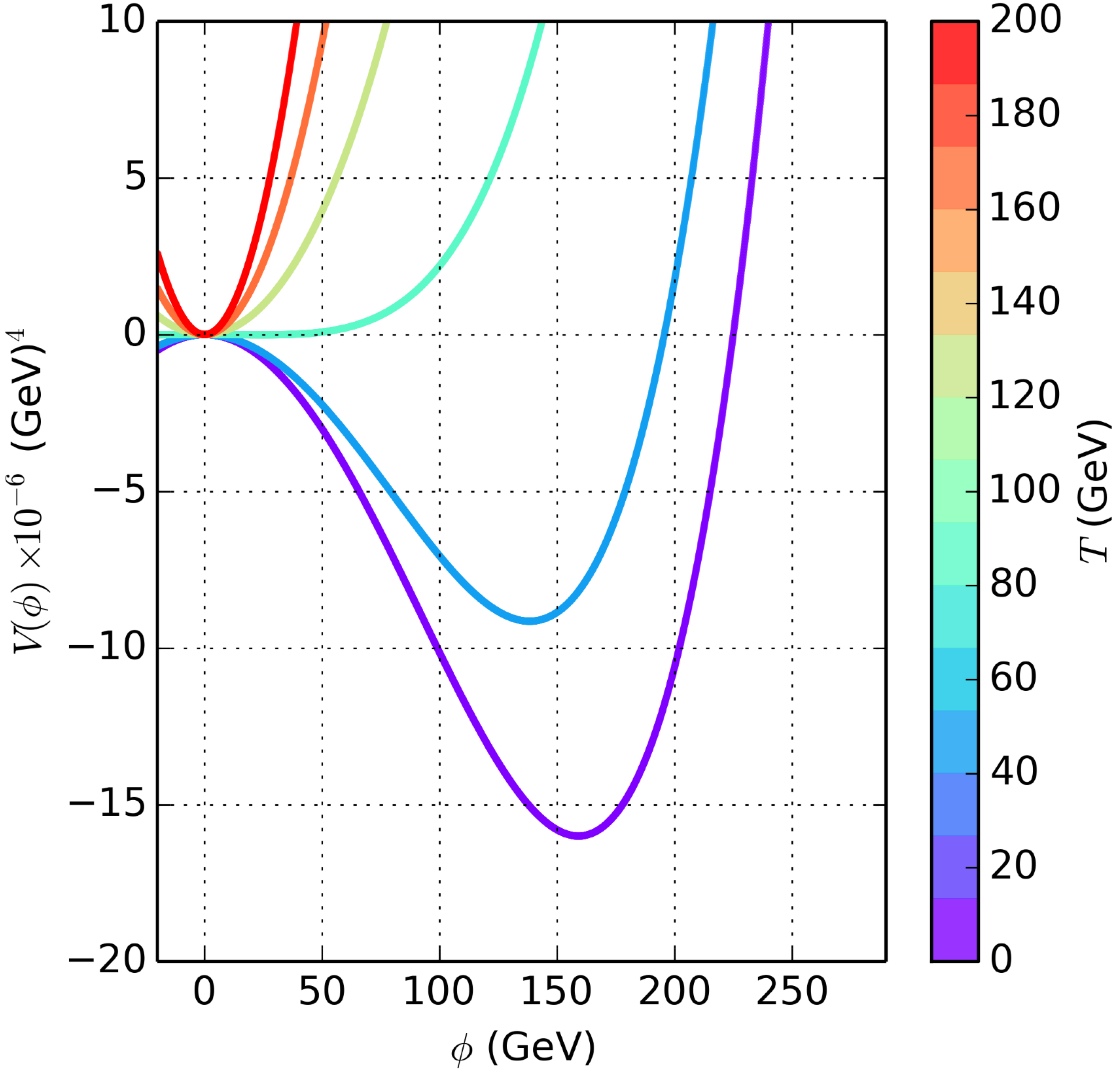
In collaboration with Peter Athron, Csaba Balazs, Andrew Fowlie, Lachlan Morris and Graham White

Based on arXiv:2208.01319

2022.08.09

Motivation



- Electroweak symmetry is broken in today's Universe, but was restored at very high temperatures.
- The transition between the symmetric and broken phases, the Electroweak Phase Transition (EWPT), is relevant to
  - Explanation of the baryon asymmetry of the Universe.
  - Generation of detectable gravitational wave.
  - Dark Matter relic density.
  - Vacuum stability.

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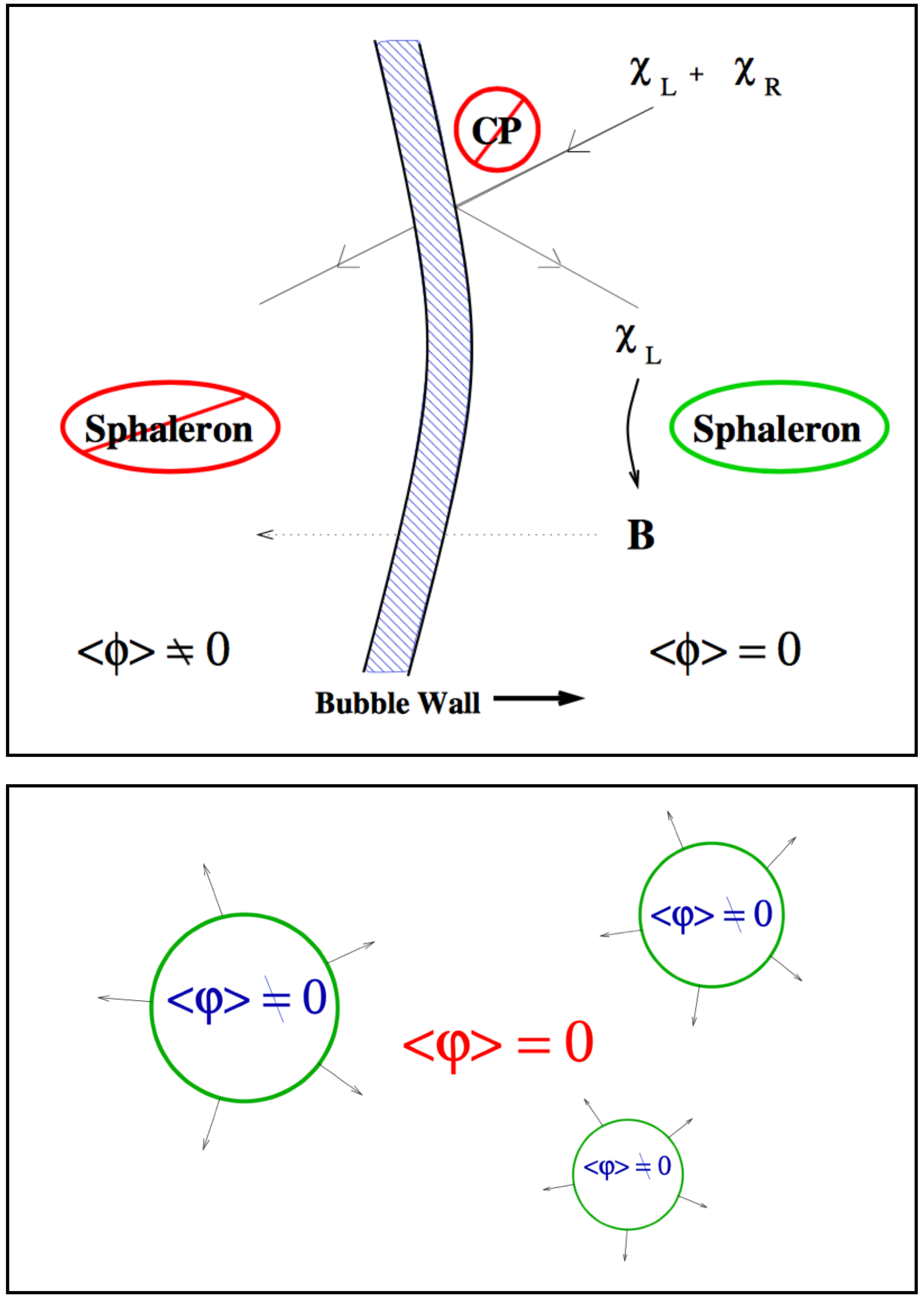

arXiv:1206.2942, David E. Morrissey, Michael J. Ramsey-Musolf

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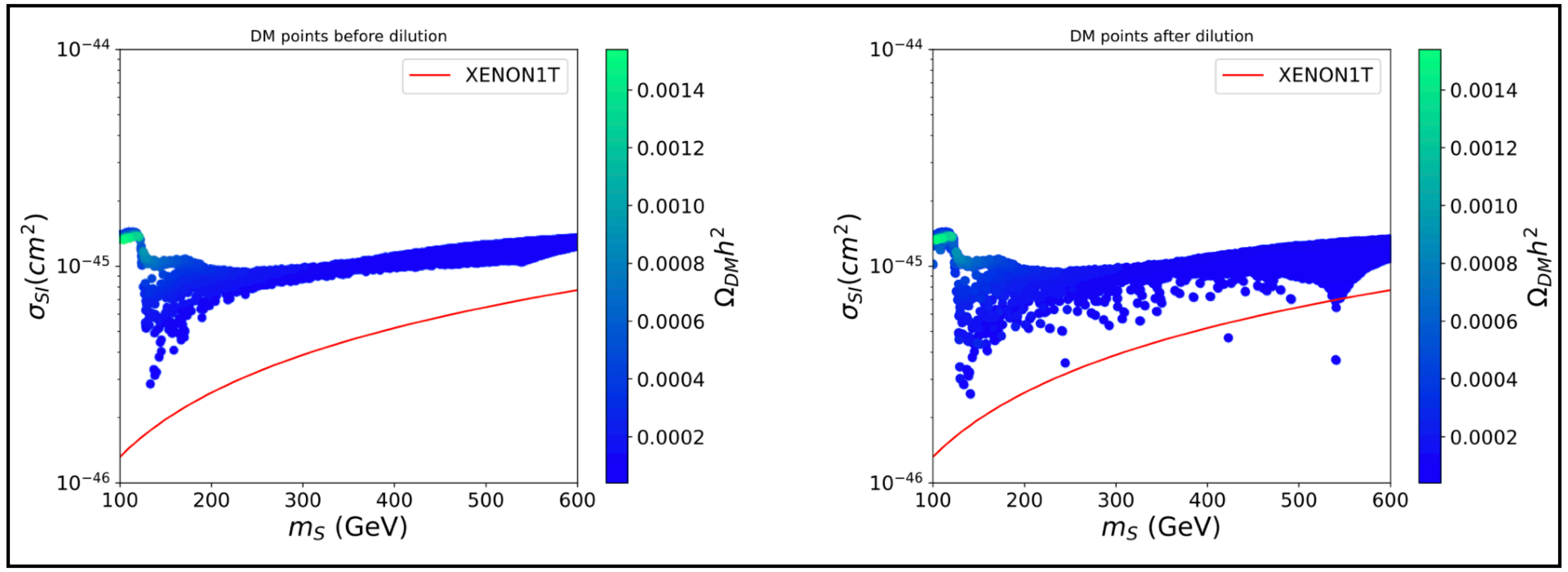
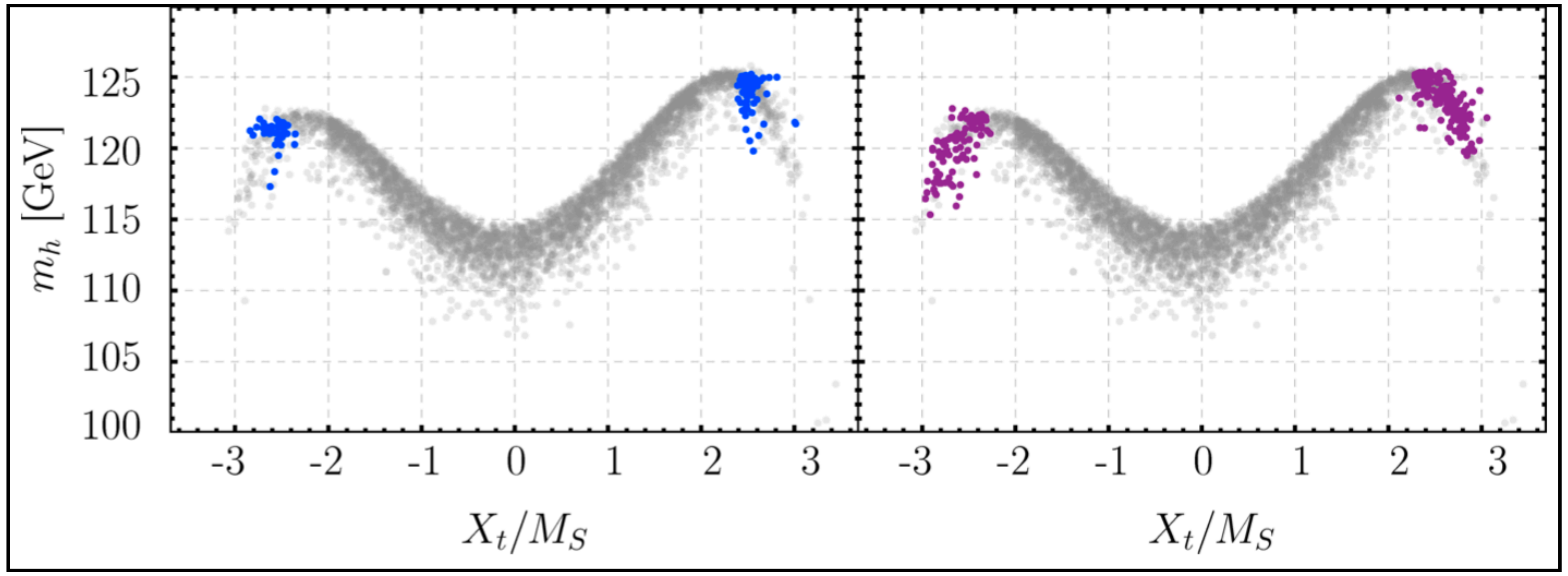
Motivation

arXiv:2207.14519, Yang Xiao, Jin Min Yang, Yang Zhang



arXiv:2207.14519, J. E. Camargo-Molina, B. Garbrecht, B. O'Leary, W. Porod, F. Staub



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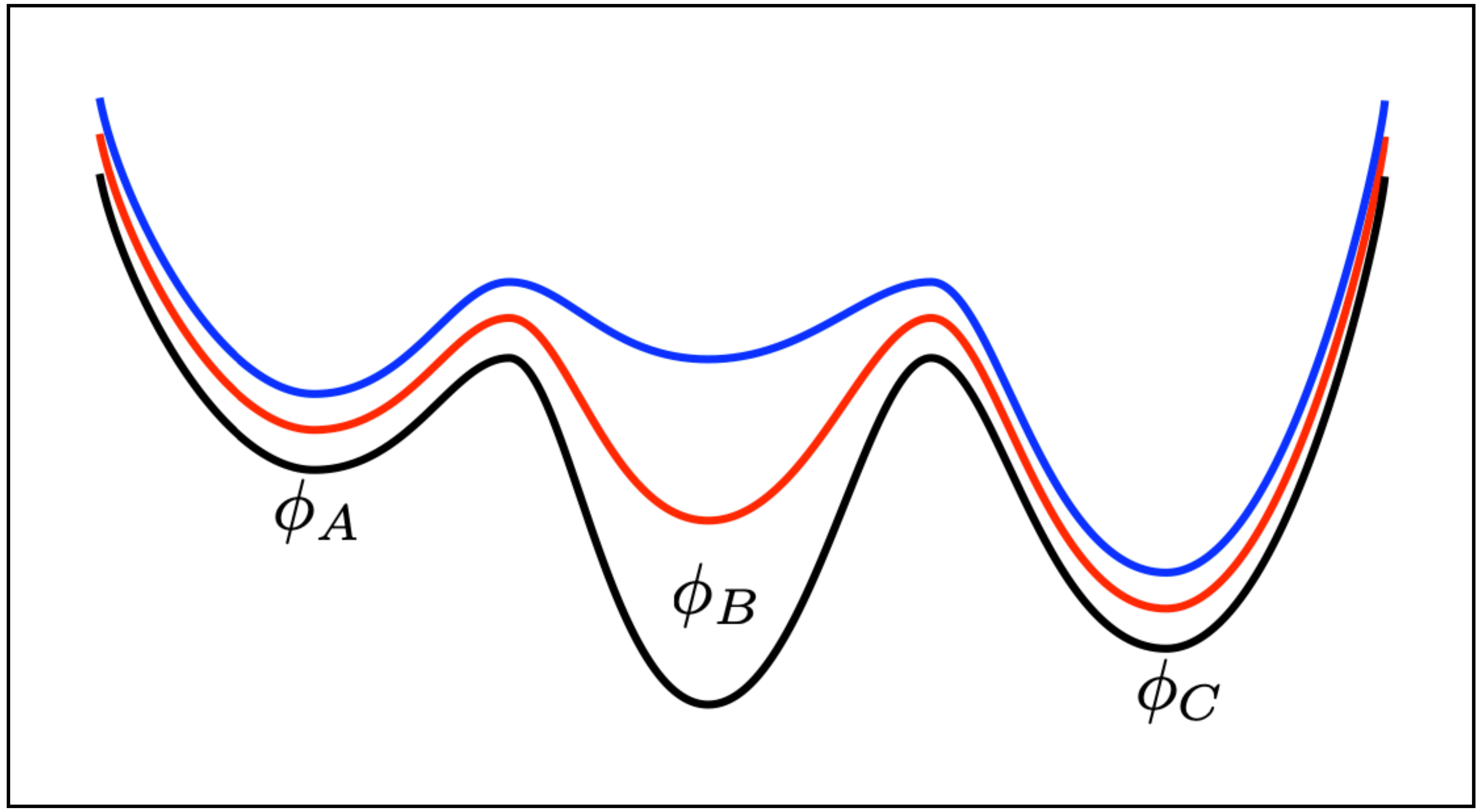
Motivation

- All these calculations need information about the minimum of the Higgs potential, such as their positions, first and second derivatives, and barriers among them at any temperature.
- The perturbative effective potential is usually used in new physics studies.
- At one-loop level, it is given by

$$V_{\mathrm{eff}}(\phi, T) = V_0(\phi) + V_1(\phi) + V_{1T}(\phi, T) + V_{\mathrm{ring}}(\phi, T)$$

arXiv:1109.4189, Carroll L. Wainwright



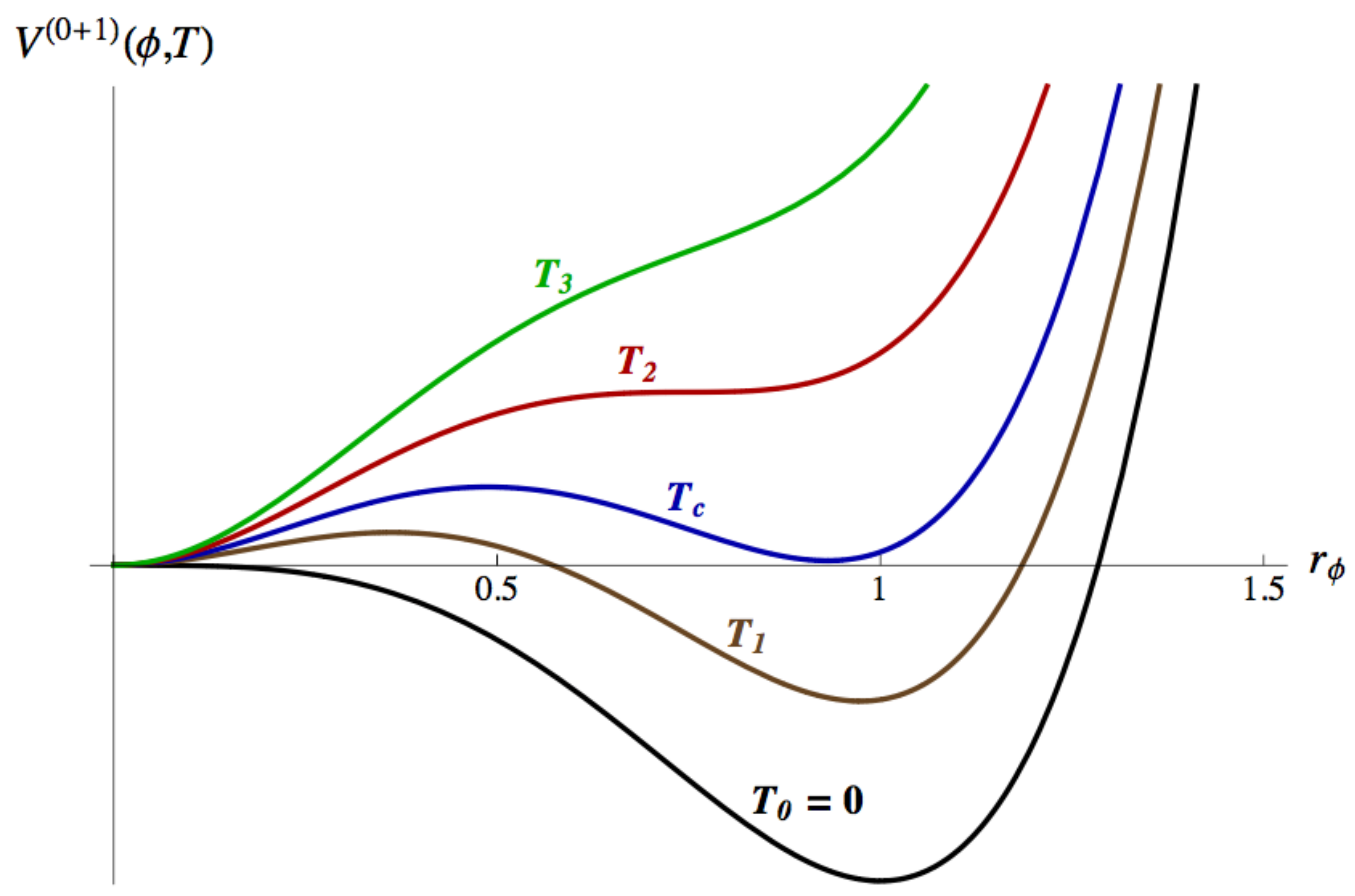
Motivation

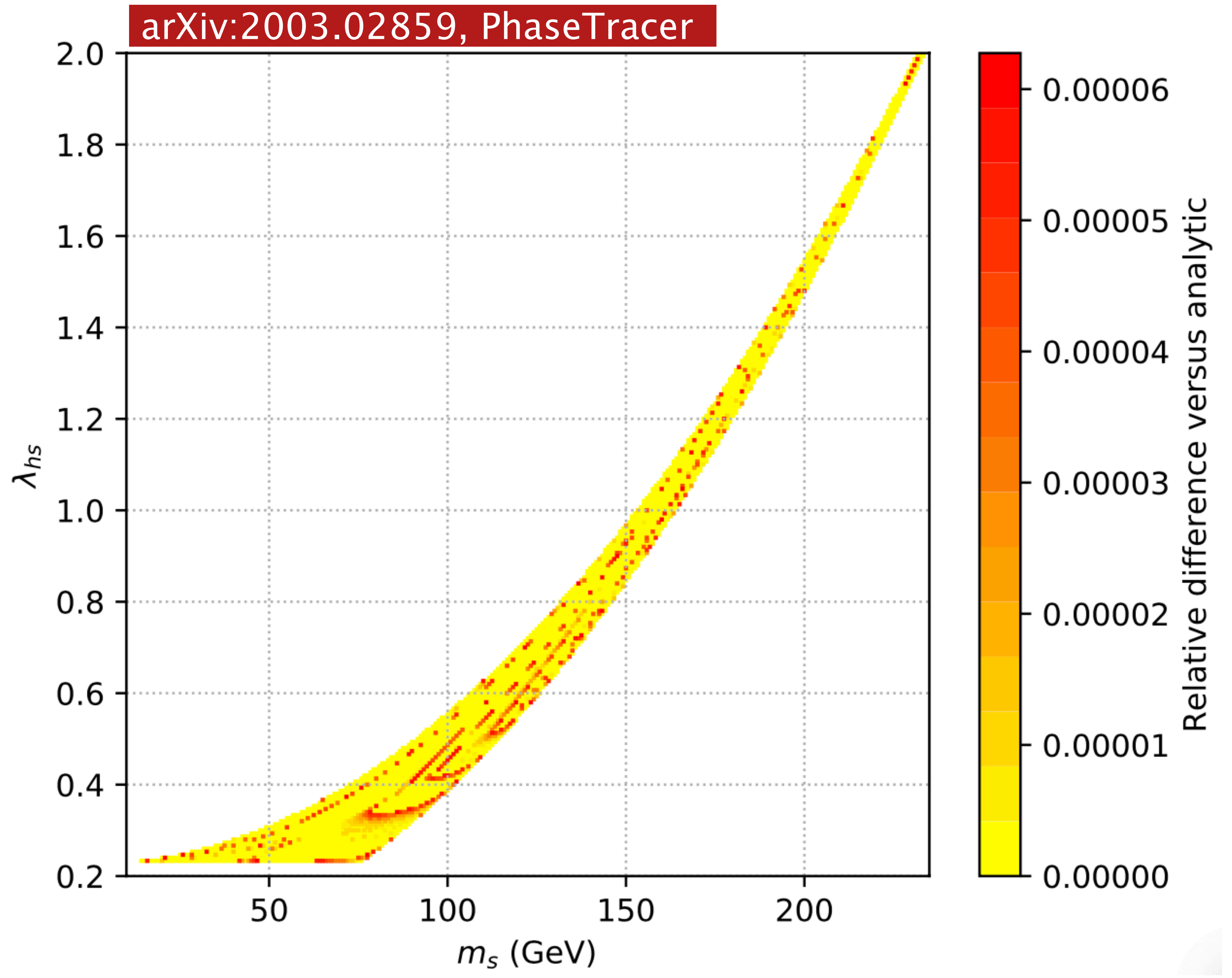
- As has long been recognized, the one-loop effective potential depends on
  - gauge and gauge fixing parameter ξ,
  - renormalisation scheme and renormalisation scale,
  - remedy of infrared divergences caused by Goldstone boson loops,
  - and treatment of daisy diagrams.
- Besides, H. H. Patel and M. J. Ramsey-Musolf (PRM) proposed a gauge-independent method, and high-temperature approximation is also widely used.

Name	Order	ξ dependence	μ dependence	Concern
PRM	Tree minima; 1-loop potential	Tadpoles	Explicit	No daisies
High-T	1-loop leading terms	Tadpoles	Implicit	Accuracy
$\overline{\text{MS}}$	1-loop potential	Explicit	Explicit	ξ & μ dependence
$\overline{\text{MS}}$ + RGE	1-loop potential; 2-loop RGE	Explicit	Reduced by RGE	ξ dependence
OS	1-loop potential	Explicit	No	ξ dependence
Covariant gauge	1-loop potential	Explicit	Explicit	ξ & μ dependence

Transition property

Here we investigate the extent to which perturbative calculations of the electroweak phase transition are arbitrary and uncertain, owing to above choices, by showing the impacts on the critical temperature T_c and the transition strength $\gamma_{\rm EW}$ in xSM.





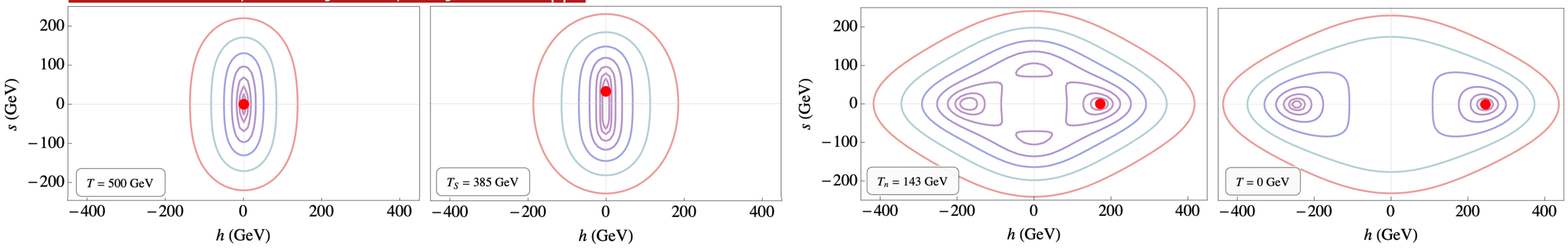
Model

We use the SM extended by a real scalar singlet (SSM) with $\mathbb{Z}_2$ symmetry as our benchmark model,

$$V_0(H,S) = -\mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2 - \frac{\mu_s^2}{2} S^2 + \frac{\lambda_s}{4} S^4 + \frac{\lambda_{hs}}{2} H^\dagger H S^2$$

because there are only three free independent parameters, m_s, λ_s, λ_{hs}, and there is only one first order phase transition in the evolution history of the Universe.

arXiv:1608.07578, Michael J. Baker, and Joachim Kopp

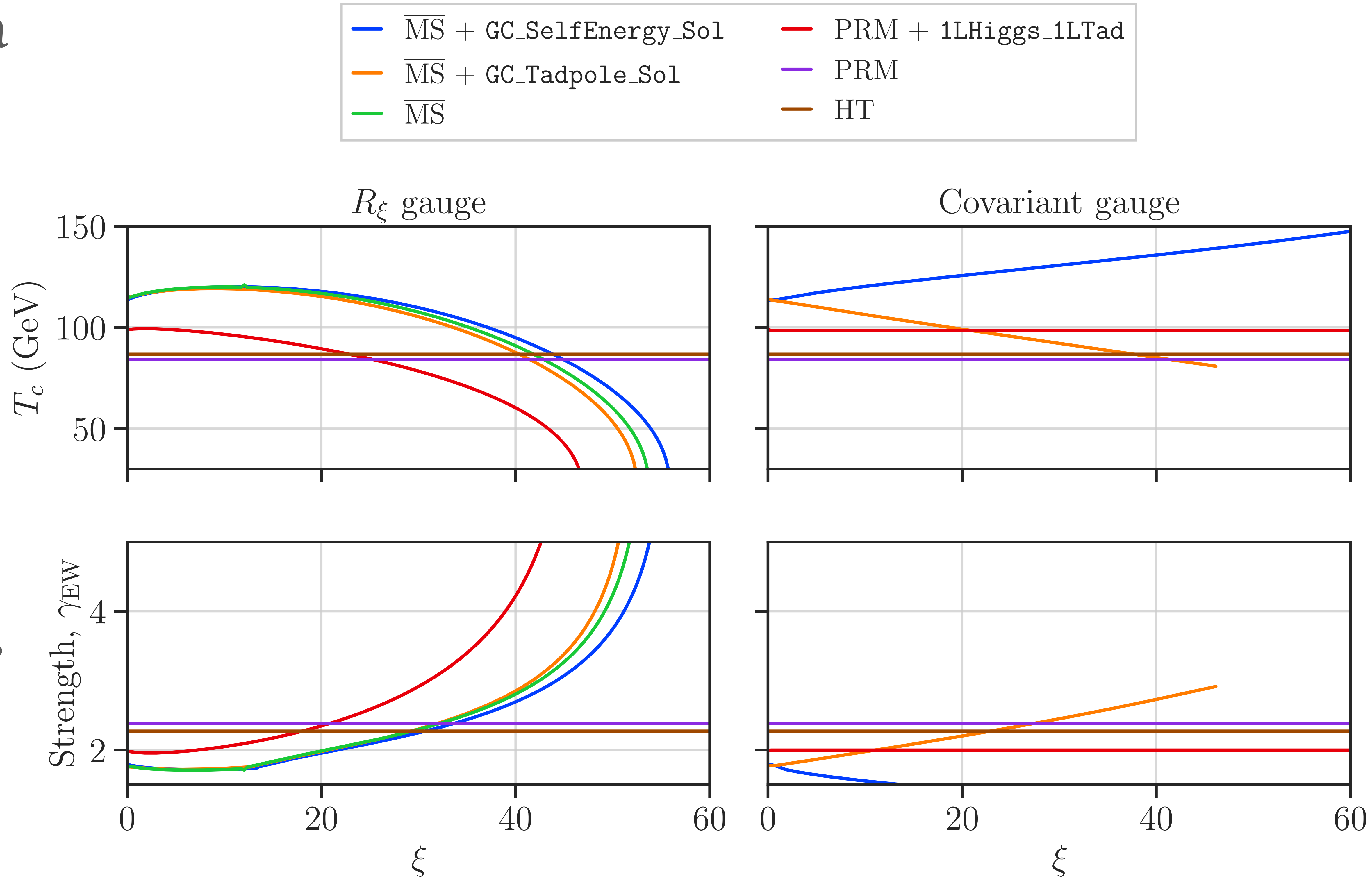


Gauge dependence

➤ We show the ξ dependence of the T_c and γ_{EW} in the R_ξ and covariant gauges for different methods, using a benchmark point of

$$\lambda_s = 0.1,\ \lambda_{hs} = 0.3 \text{ and } m_s = 65\ \mathrm{GeV}\,.$$

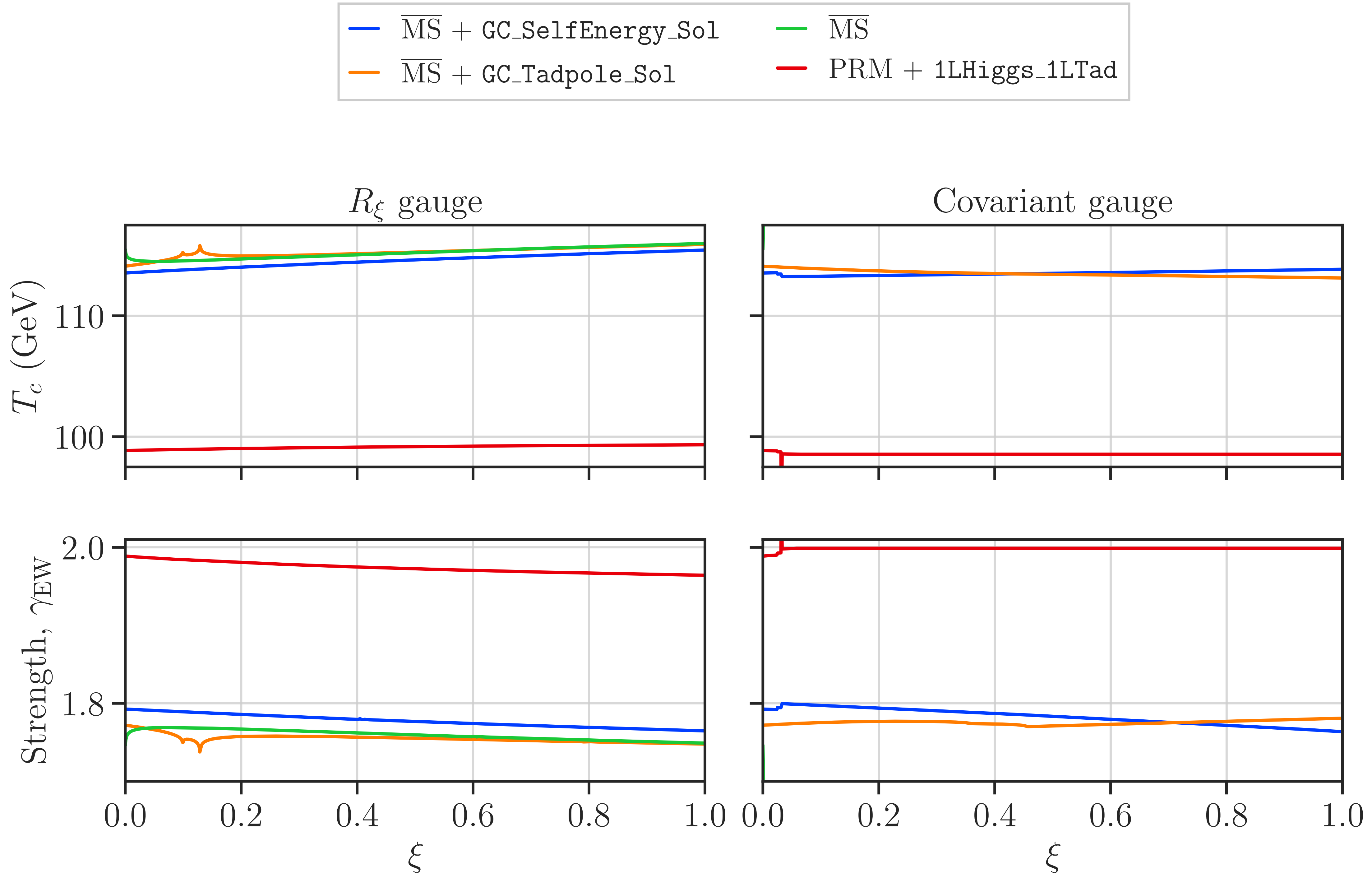
- The PRM and HT methods which, of course, are independent of the gauge parameter ξ.
- In the $\overline{\mathrm{MS}}$ scheme, with ξ increasing, the critical temperature T_c increases slightly for small values of ξ, and then decreases significantly for large values of ξ.
- The choice of treatment of the GC has limited impact.
- Results from the covariant and R_ξ gauges are identical at $\xi = 0$.



Gauge dependence

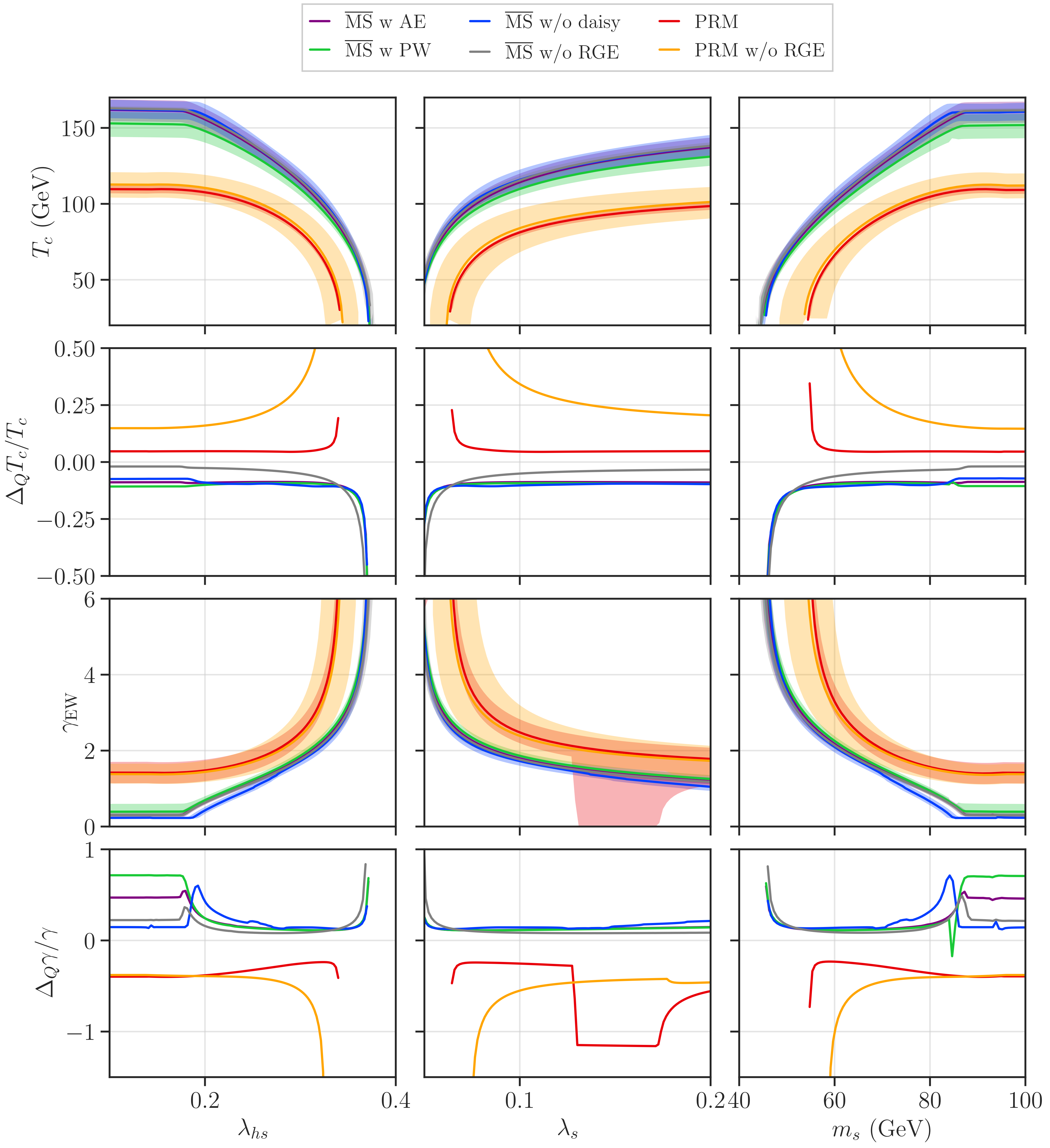
➤ However, there may be constraints on perturbativity that impose $\xi \lesssim 1$.

- Varying the gauge between $\xi = 0$ and 1 results in only slight changes in the properties of the FOPT.
- Some methods do not depend smoothly on ξ.
- Although using the modified μ from one-loop tadpole cures the GC at $\xi = 0$, the Goldstone may be massless at tree-level in the one-loop vacuum for other values of ξ.
- In the covariant gauge the T_c increases or decreases almost linearly with ξ increasing. This is because we take the real part of the square root in masses and for sufficiently large ξ, the ξ-dependent parts are imaginary.



Scale dependence

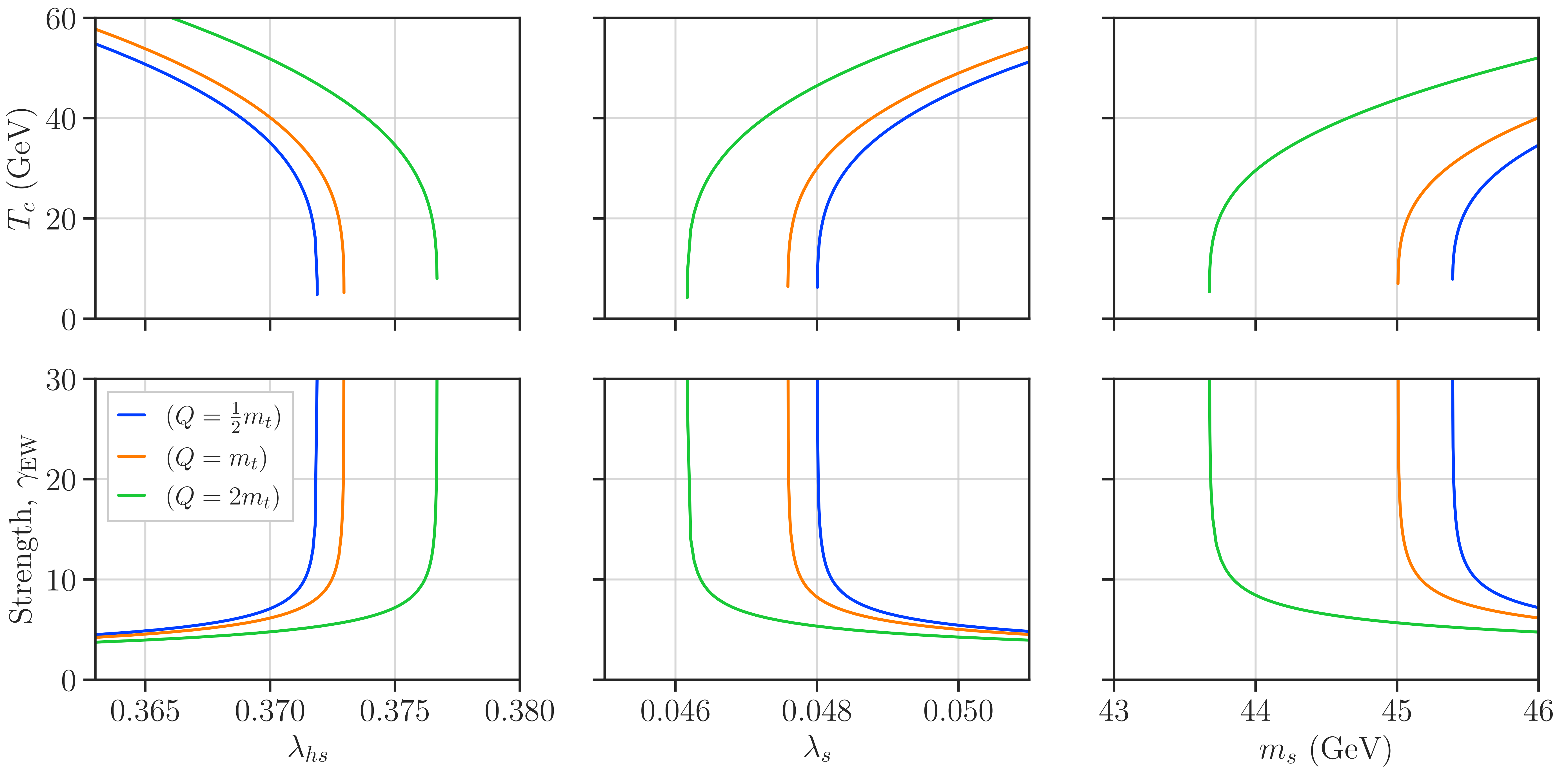
- The renormalisation scale dependence of the PT properties originates from the scale-dependence of the one-loop corrections to the effective potential.
- We show the renormalisation scale dependence in the $\overline{\text{MS}}$ scheme and PRM methods, by varying the renormalisation scale from $Q = \frac{1}{2}m_t$ to $Q = 2m_t$.
- In each of the panels, we vary one input parameter around the benchmark point
- We see almost no parameter dependence in the results at $\lambda_{hs} \lesssim 0.2$ and $m_s \gtrsim 90$ gev.
- A common and important feature in all the results is that the uncertainties rise sharply when the critical temperature is sensitive to the Lagrangian parameters.



Scale dependence

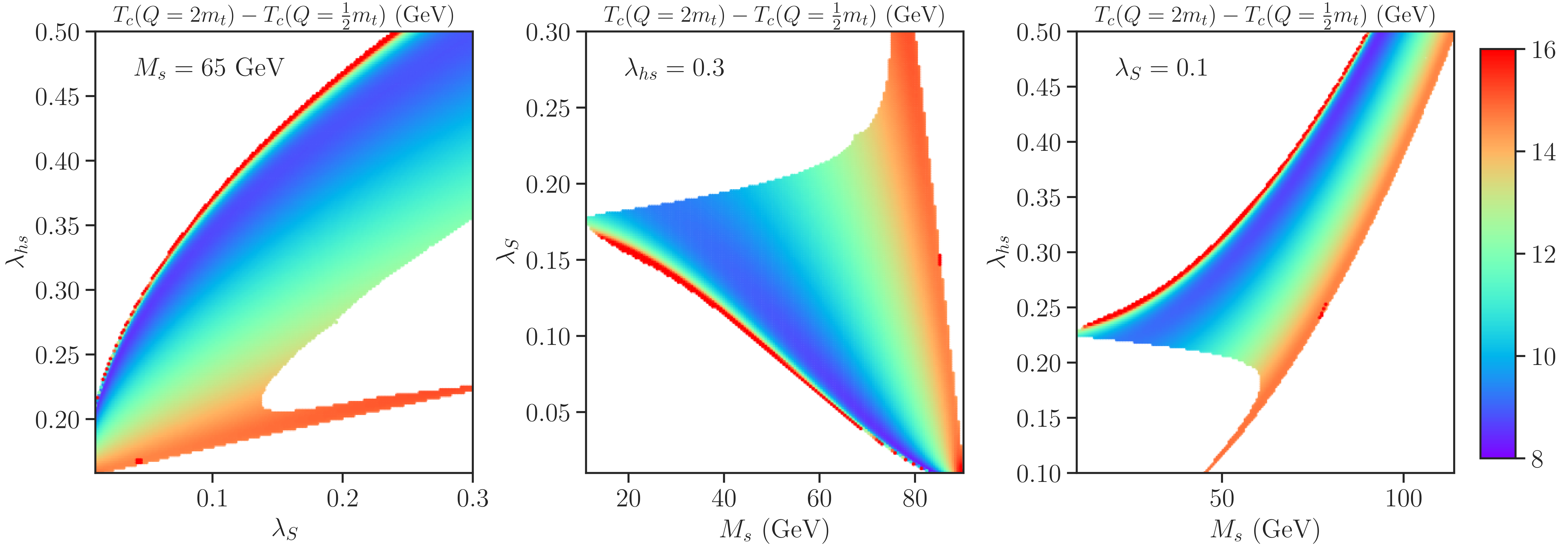
- To obtain $T_c < 5$ GeV, the model parameters have to be tuned to one part in 10^7.

$$\Delta V(T=0) = -\frac{1}{4}\frac{\mu_s^4}{\lambda_s} + \frac{1}{4}\frac{\mu_h^4}{\lambda_h} \approx -\frac{1}{4}\frac{(\frac{1}{2}\lambda_{hs}v_h^2 - m_s^2)^2}{\lambda_s} + \frac{1}{8}m_h^2 v_h^2$$



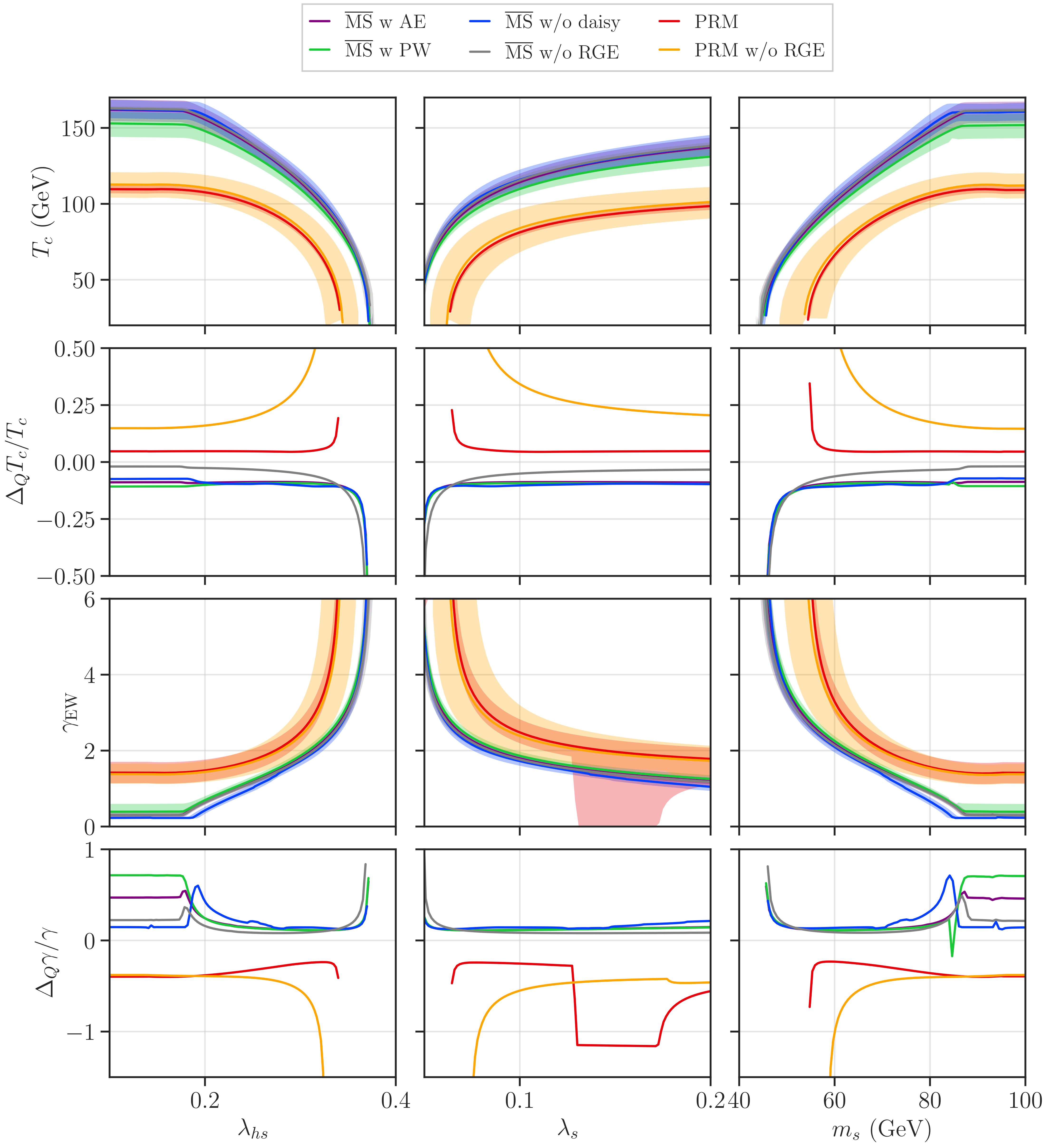
Scale dependence

- This can also been seen in the two-dimensional planes, which is consistent with that in the one-dimensional results.



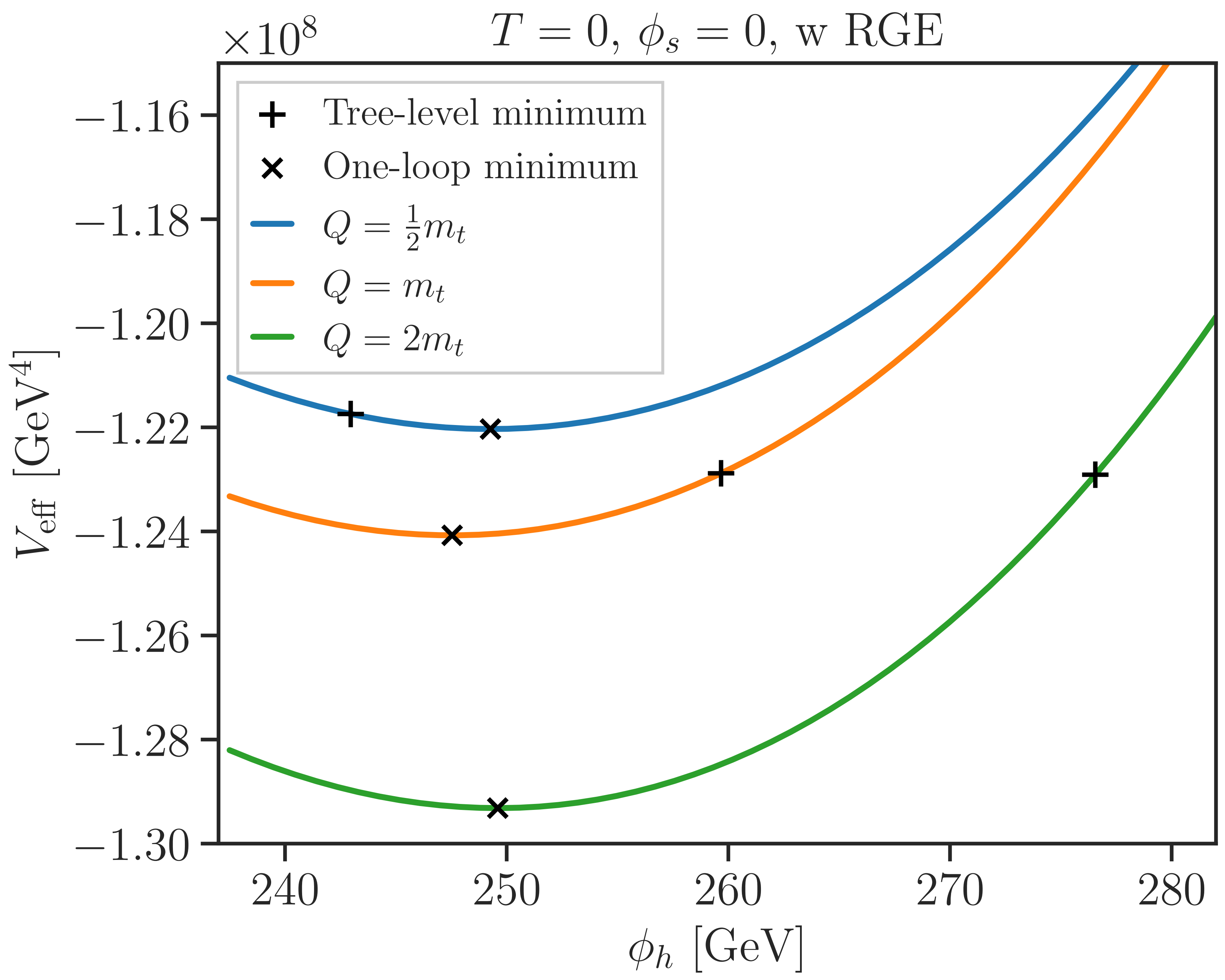
Scale dependence

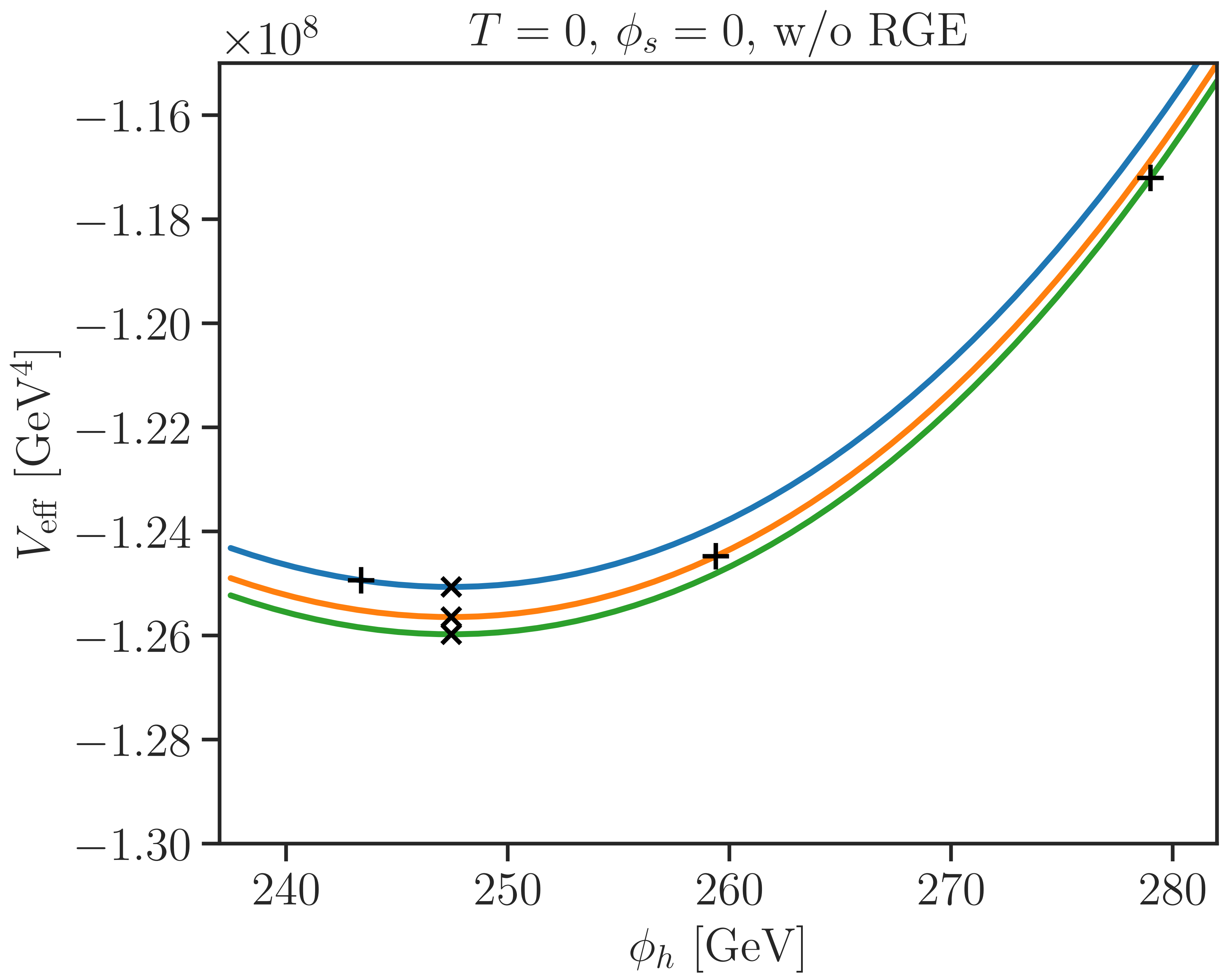
- With RGE running and the Parwani method, $\Delta_Q T_c$ in the $\overline{\text{MS}}$ result are about 16 GeV for large T_c.
- $\Delta_Q T_c$ of the $\overline{\text{MS}}$ scheme without RGE running is smaller than the one with RGE running, which is counter-intuitive.
- It is because the cancellation is spoilt by next-to-leading order terms, since we apply two-loop RGEs and omit the anomalous dimension.
- $\Delta_Q T_c$ of the PRM method with RGE running is rather small, typically about 5%. Without RGE running, $\Delta_Q T_c$ can be larger than 15%, and increases rapidly with T_c decreasing.



Scale dependence

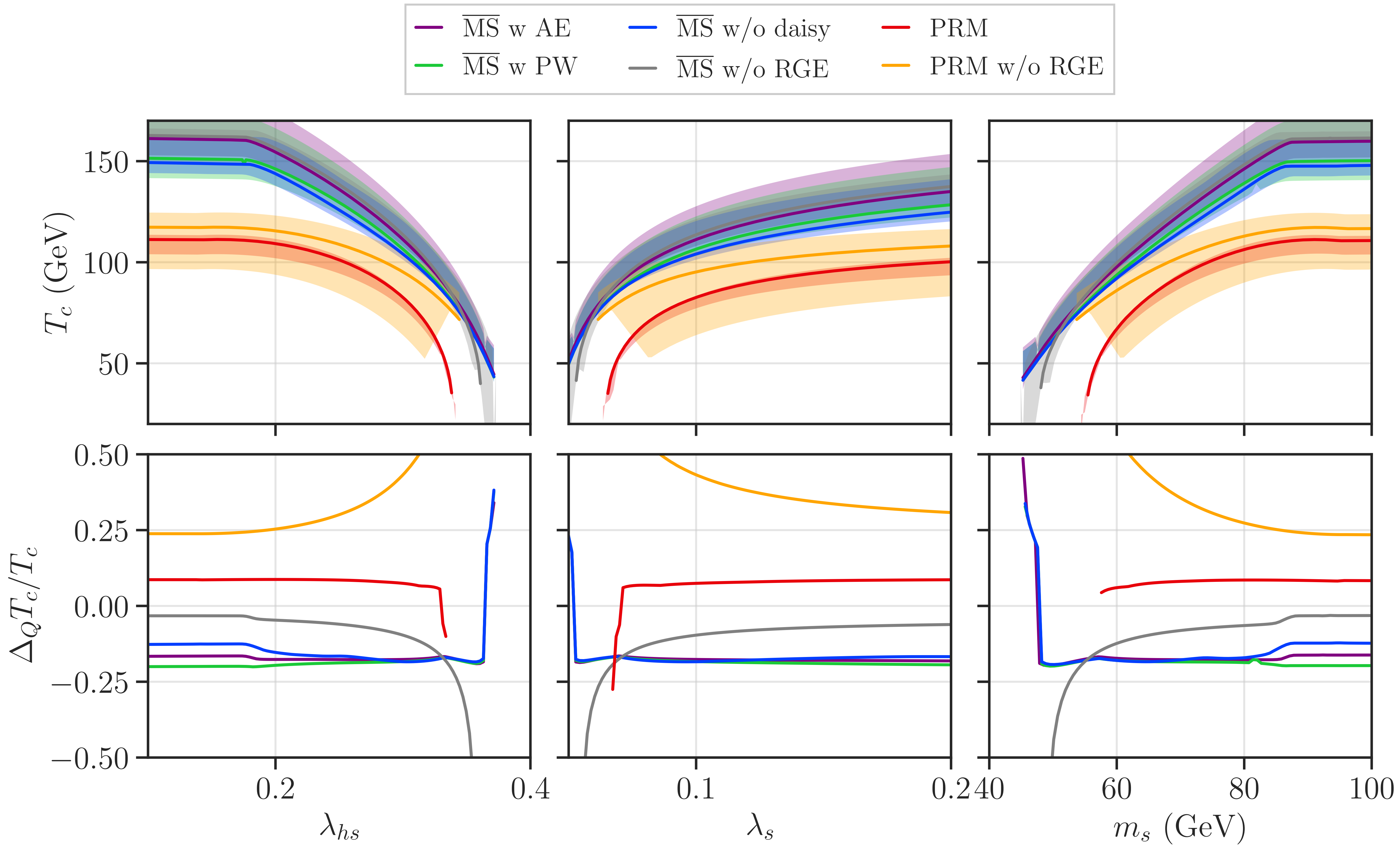
- This implies that proper RGE running is quite essential in the PRM method to reduce the renormalisation scale dependence.
- The zero-temperature potentials:





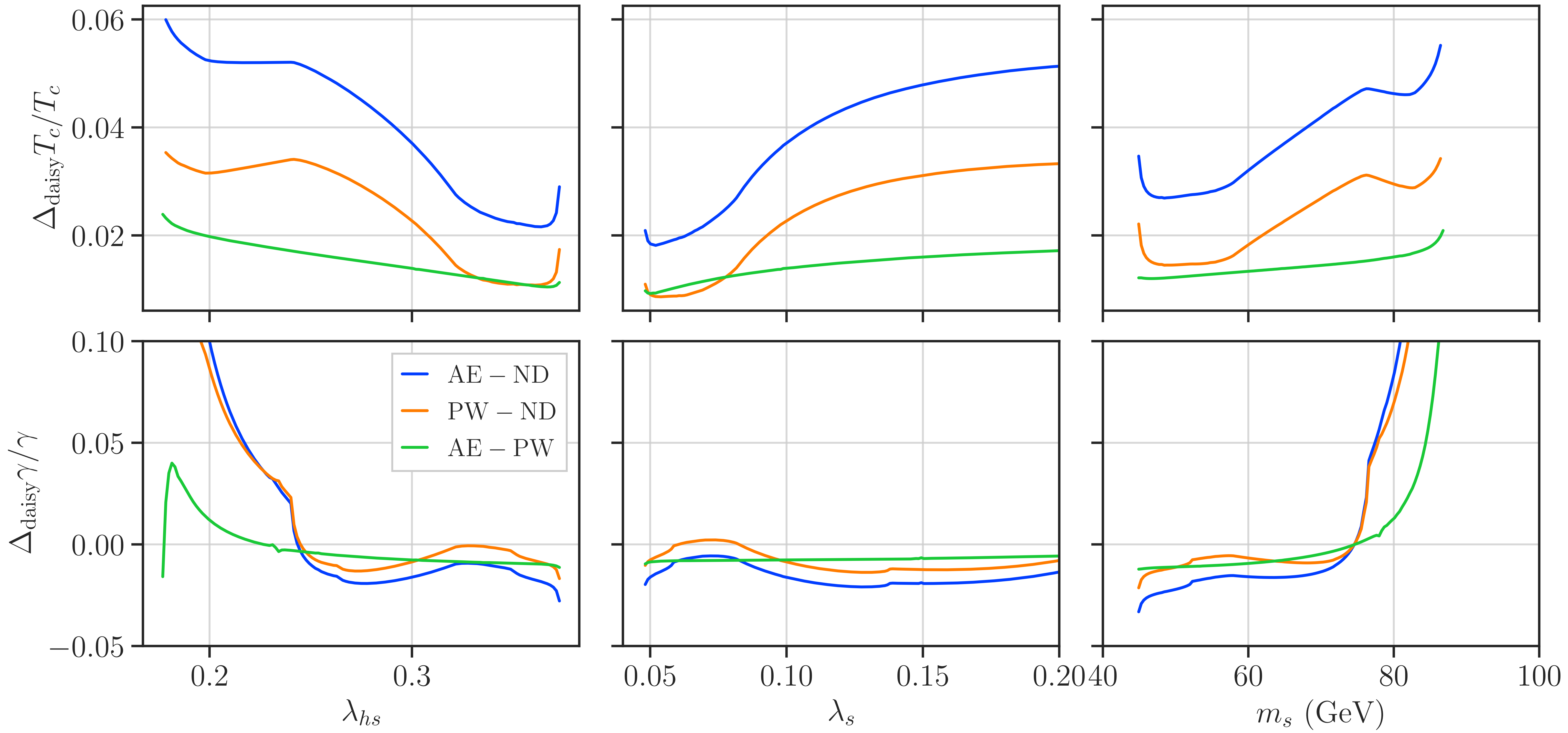
Scale dependence

- We also show the renormalisation scale dependence by varying the renormalisation scale from $Q = \frac{1}{2}T$ to $Q = 2\pi T$.



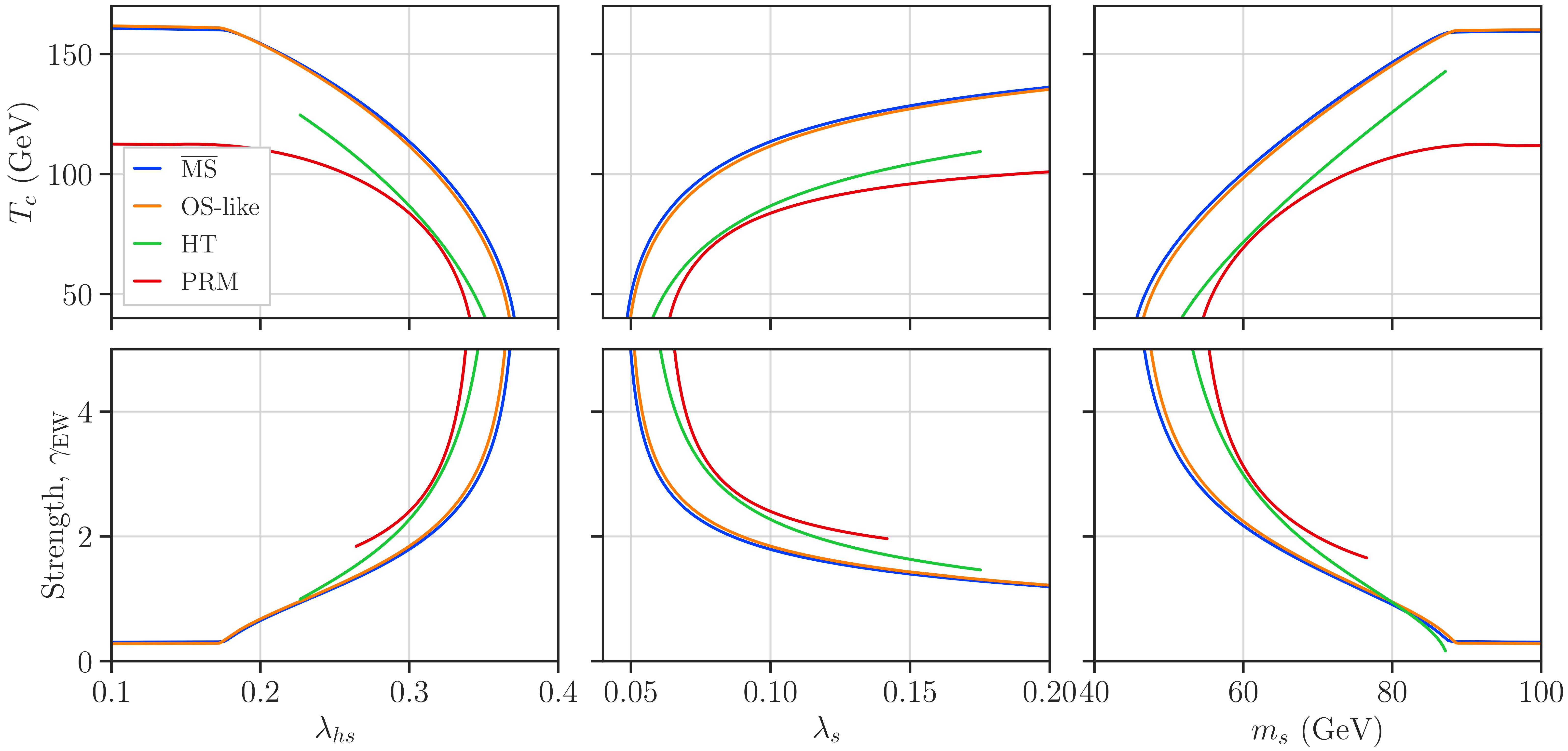
Daisy resummation dependence

- The changes in T_c caused by switching between the Parwani and the Arnold-Espinosa methods are less than 3% and the changes caused by switching off daisy resummation altogether were less than 6%.



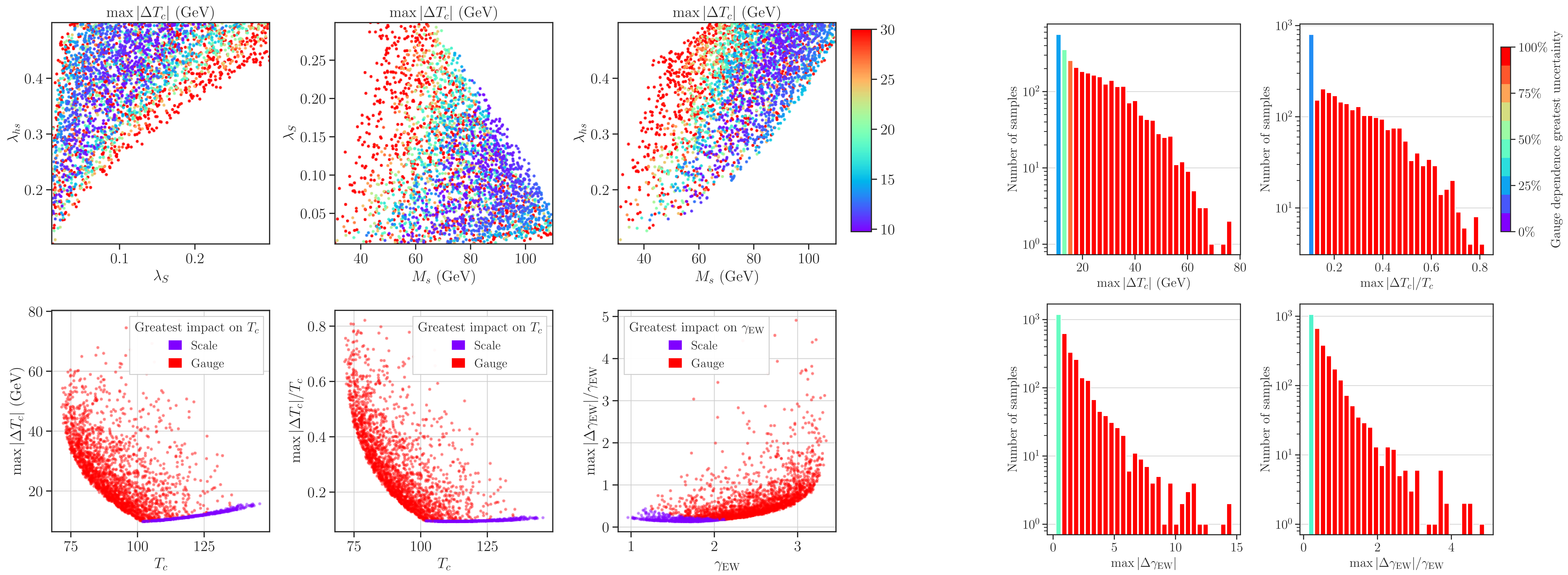
Comparison of renormalisation schemes and other methods

- The differences are typically mild with the OS and $\overline{\text{MS}}$ schemes, if we choose $Q = m_t$.
- The T_c of the high-temperature approximation and the PRM method are far lower than that of the OS-like scheme and the $\overline{\text{MS}}$ scheme.



Comparison in whole parameter space

- The greatest uncertainty of each point is the renormalisation scale uncertainty or gauge uncertainty.
- They are subject to the choice of scales and gauge parameters that we compare.



Conclusions

- ➤ We investigated the impact of choices in the perturbative effective potential on the properties of first-order phase transition.
- ➤ We refrain from making recommendations about the best choices, and instead wish to emphasize the sizes of the uncertainties and which choices influence the results the most.

- The scale dependence in the $\overline{\mathrm{MS}}$ scheme and gauge dependence were the most significant sources of arbitrariness.
- Scale dependence was particularly severe in the PRM scheme, and only somewhat alleviated by using RGE running.
- Whenever the PT properties strongly depend on the choices of Lagrangian parameters, they inevitably strongly depend on the choice of gauge and scale.
- The gauge-independent PRM and HT predictions for PT properties were similar to each other and quite different from those from the $\overline{\mathrm{MS}}$ method.
- The impact of the different treatments of daisy diagrams and the GC were relatively small, always changing the critical temperature by less than about 10 GeV.
- In the R_ξ gauge, we can avoid the GC at $\xi = 0$ by using the Higgs mass squared parameter obtained from one-loop tadpole conditions in the calculation of the field-dependent Goldstone masses. However, even with this approach, Goldstone catastrophes still occurred for other choices of ξ for which Goldstones were massless in the EW vacuum.

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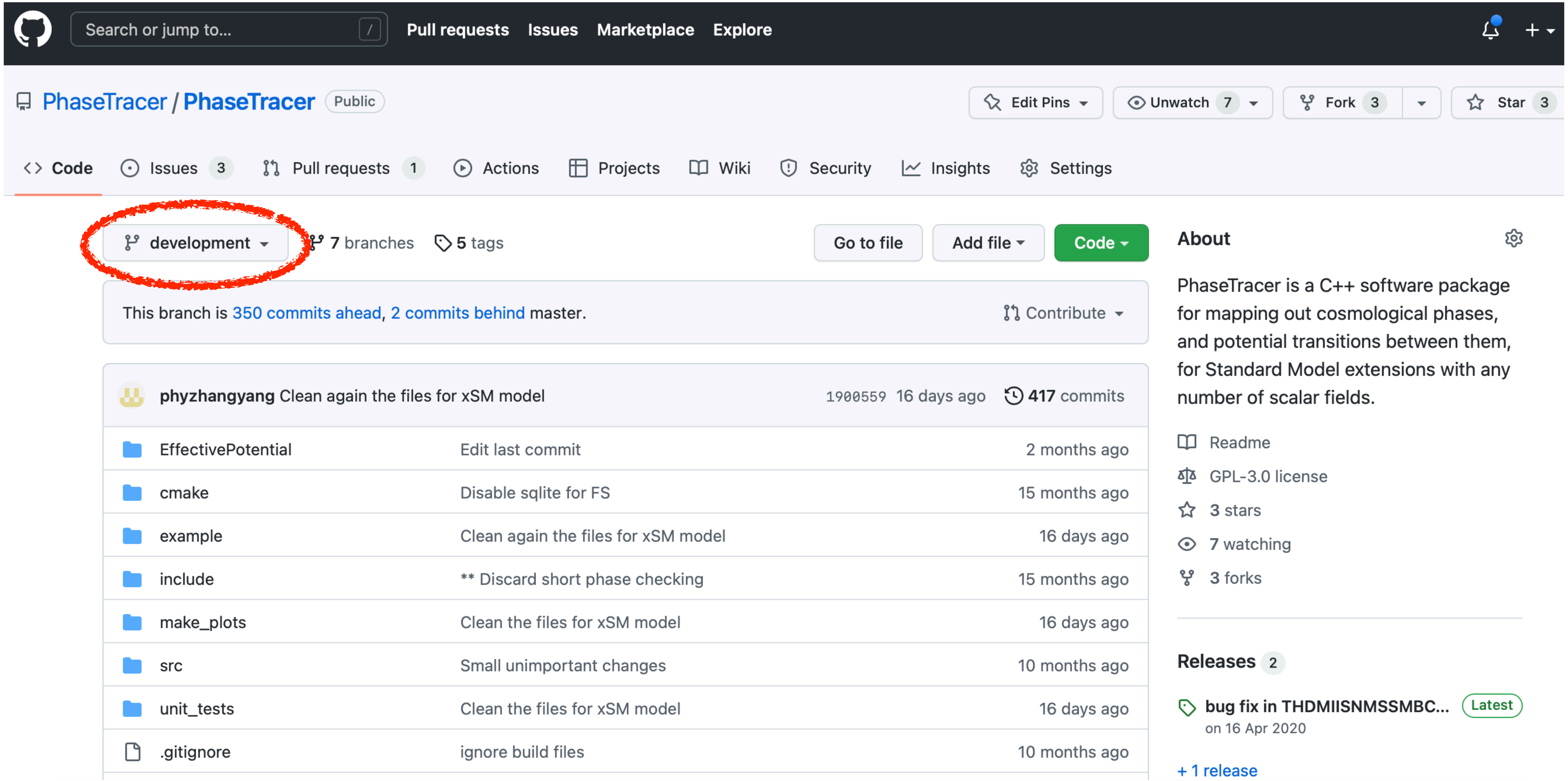

Thank you!

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