Cross Terms in the Fit Function for the Precession Frequency Analysis in the Fermilab Muon g-2 Experiment

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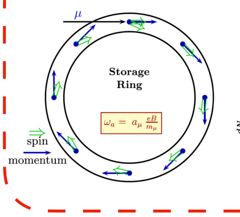
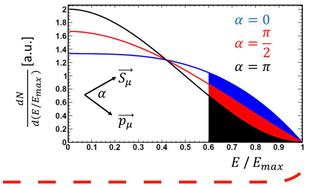
I. Muon g-2 experiment and ω_a fit function

1. Anomalous spin precession



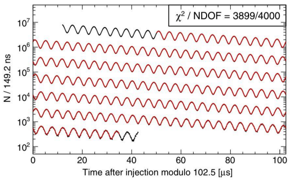


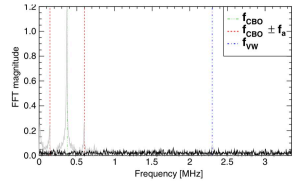
24 Parameters[1]

$$N = N_0 * N_{cbo}(t) * N_{vw}(t) * N_{dcbo}(t) * N_{vo}(t) * \Lambda(t)$$
$$* e^{-t/\tau}\left[1 - A_0 * A_{cbo}(t) * cos\left(\omega_a t + \phi_0 * \phi_{cbo}(t)\right)\right]$$

$$N_{cbo}(t) = 1 - A_{cbo} e^{-t/\tau_{cbo}} cos(\omega_{cbo} t + \phi_{cbo})$$

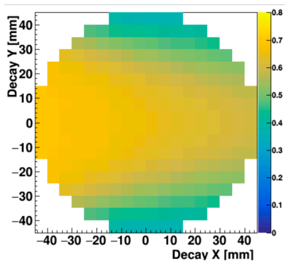
$$N_{vw}(t) = 1 - A_{vw} e^{-t/\tau_{vw}} cos(\omega_{vw} t + \phi_{vw})$$





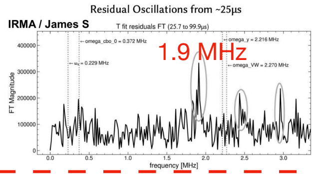
2. Beam dynamics[1] and detector acceptance[2]

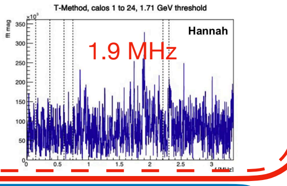
- Cyclotron frequency ~ 6.70 MHz
- Horizontal betatron frequency ~ 6.33 MHz
- Vertical betatron frequency ~ 2.20 MHz
- Coherent Betatron Oscillation frequency (CBO) ~ 0.37 MHz
- Vertical Waist frequency (WV) ~ 2.30 MHz



3. 1.9 MHz Puzzle

- No significant peak at 1.9 MHz in Run-1, but not the case in Run-2 [3,4].





II. ToyMC studies to understand the origin of 1.9 MHz

1. Beam dynamics

- Muon beam harmonically oscillates in both x and y directions.

$$x(t) = x_0 + A_x \cos(2\pi f_x t + \phi_x)$$
$$y(t) = y_0 + A_y \cos(2\pi f_y t + \phi_y)$$

3. Three modes

- Only consider detector's acceptance effeciency to generate muons.
- Study three oscillation modes separately.

$$N_{\text{gen}}(t) = N_0 * \epsilon_x(x) * \epsilon_y(y)$$
$$= N_0 * \epsilon_x(x(t)) * \epsilon_y(y(t))$$

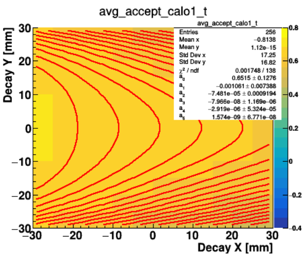
Initial parameters

Parameters	x-only oscillation	y-only oscillation	x and y oscillation
N_0	10000	10000	10000
x_0 [mm]	0	-	0
A_x [mm]	15.0	-	15.0
f_x [MHz]	6.332	-	6.332
ϕ_x [rad]	0	-	0
y_0 [mm]	-	0	0
A_y [mm]	-	15.0	15.0
f_y [MHz]	-	2.2	2.2
ϕ_y [rad]	-	0	0

2. Acceptance map

- Fit acceptance map with polynominal function.

$$\epsilon_{\text{fit}}(x, y) = a_0 + a_1 x + a_2 y^2$$
$$+ a_3 y^4 + a_4 x y^2 + a_5 x y^4$$



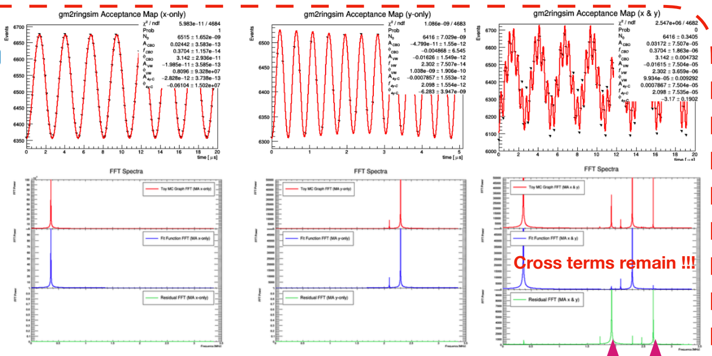
4. Fitting and FFT analysis

- Fit with traditional form.

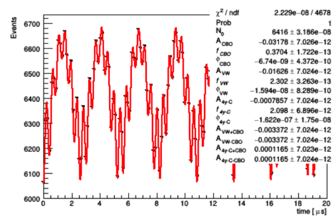
$$N_{\text{fit}}(t) = N_0 * N_x(t) * N_y(t)$$
$$N_x(t) = 1 + A_{CBO} \cos(2\pi f_{CBO} t + \phi_{CBO})$$
$$N_y(t) = 1 + A_{VW} \cos(2\pi f_{VW} t + \phi_{VW})$$
$$+ A_{4y-c} \cos(2\pi f_{4y-c} t + \phi_{4y-c}) \ ,$$

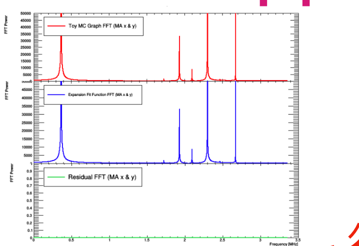
- Expand the original function and float the cross terms' amplitude !!!

$$N(t) = N_0 * N_x * N_y$$
$$N_x * N_y = 1 + A_{CBO} \cos(2\pi f_{CBO} t + \phi_{CBO})$$
$$+ A_{VW} \cos(2\pi f_{VW} t + \phi_{VW})$$
$$+ A_{4y-C} \cos(2\pi f_{4y-C} t + \phi_{4y-C})$$
$$+ A_{VW+CBO} \cos\left[2\pi(f_{VW} + f_{CBO})t + \phi_{VW} + \phi_{CBO}\right]$$
$$+ A_{VW-CBO} \cos\left[2\pi(f_{VW} - f_{CBO})t + \phi_{VW} - \phi_{CBO}\right]$$
$$+ A_{4y-c+CBO} \cos\left[2\pi(f_{4y-c} + f_{CBO})t + \phi_{4y-c} + \phi_{CBO}\right]$$
$$+ A_{4y-c-CBO} \cos\left[2\pi(f_{4y-c} - f_{CBO})t + \phi_{4y-c} - \phi_{CBO}\right]$$



(a) x-only mode (b) y-only mode (c) x and y mode





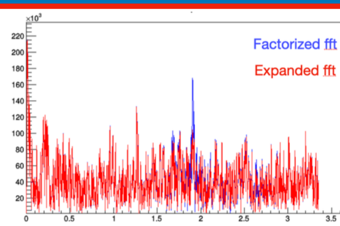
III. Conclusion and discussion

- The cross terms come from the correlation between x and y in position acceptance.
- The amplitudes of cross terms depend on the strength of the correlation.
- The puzzle can be resolved by floating cross terms' amplitude in ω_a fitting.

28 Parameters

$$N = N_0 * N_{cbo}(t) * N_{vw}(t) * N_{dcbo}(t) * N_{vo}(t) * \Lambda(t)$$
$$* e^{-t/\tau}\left[1 - A_0 * A_{cbo}(t) * cos\left(\omega_a t + \phi_0 * \phi_{cbo}(t)\right)\right]$$

$$N_{cbo}(t) * N_{vw}(t) = 1 - A_{cbo} e^{-t/\tau_{cbo}} cos(\omega_{cbo} t + \phi_{cbo}) - A_{vw} e^{-t/\tau_{vw}} cos(\omega_{vw} t + \phi_{vw})$$
$$+ e^{-t/\tau_{cbo} - t/\tau_{vw}}\left(A_{vw+cbo} cos\left[(\omega_{vw} + \omega_{cbo})t + \phi_{vw+cbo}\right] + A_{vw-cbo} cos\left[(\omega_{vw} - \omega_{cbo})t + \phi_{vw-cbo}\right]\right)$$



Reference:
[1] Muon g-2 collaboration, PHYSICAL REVIEW D 103, 072002 (2021)
[2] K.S. Khaw, Phase-acceptance update: final maps, Muon g-2 docdb 23766 (2020)
[3] H. Binney, UW ω_a update: t' clustering pileup, 2C+2D fitting, Muon g-2 docdb 25545 (2021)
[4] J. Stapleton, Run 2 Lessons from IRMA, Muon g-2 docdb 25688 (2021)