山东大学
SHANDONG UNIVERSITY

Triangle Singularity in the Production of T_{cc}^{+} and a Soft Pion

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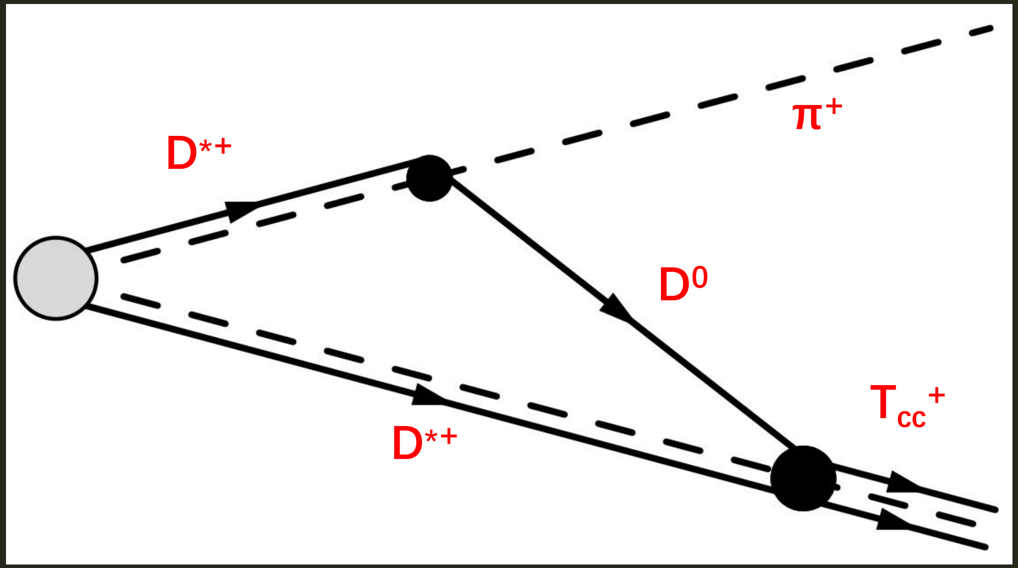
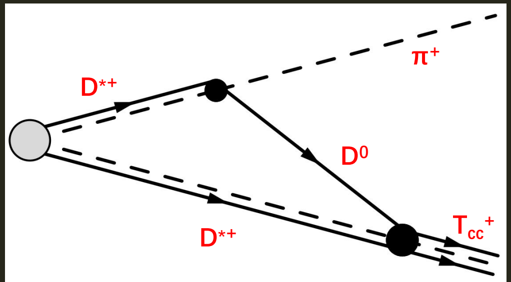
Triangle Singularity

- Kinematic singularity if three charm mesons in the loop are on their mass shells simultaneously.



1 Introduction

Triangle Singularity



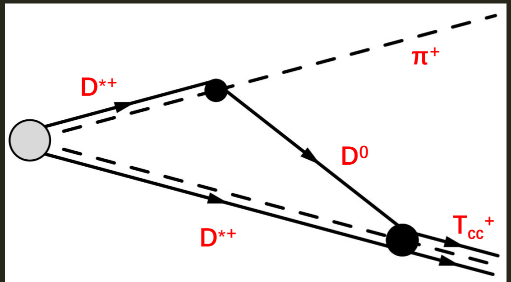
- $$T_+(q^2,\gamma^2) = \left(1+\frac{mb}{2M_T c}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_T c}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

  $$E_{\triangle+} = \frac{M_*}{4\mu^2}\left(\sqrt{2\mu E_+ - \gamma^2} - i\sqrt{m/M_T}\,\gamma\right)^2$$ where $E_+ = \delta_{0+} - \varepsilon_T - i\,\Gamma_{*+}$

- Log divergence in the limit of both binding energy and decay width go to zero.

  $$E_{\triangle+} \longrightarrow (M_T/2M)\delta_{0+} = 6.1\ \text{MeV}$$

Triangle Singularity



- $$T_+(q^2,\gamma^2) = \left(1+\frac{mb}{2M_Tc}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_Tc}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

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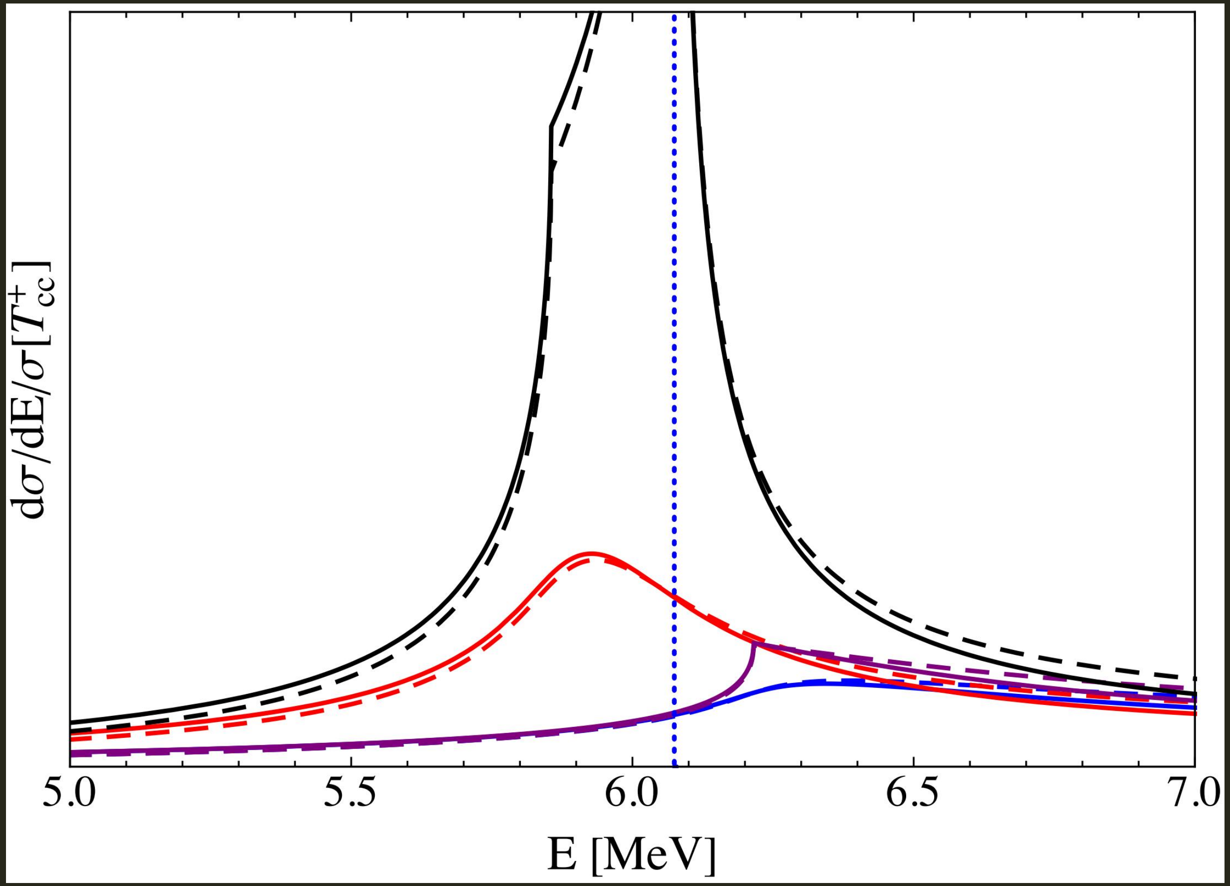
- Log divergence in the limit of both binding energy and decay width go to zero.

  $$E_{\triangle+} \longrightarrow (M_T/2M)\delta_{0+} = 6.1\ \text{MeV}$$

- Square-root branch point at $E = E_+$ from the $\sqrt{a}$ term.
- Limiting behaviour of $T_+(q^2,\gamma^2)$ determined by the interplay of above two items.

Triangle Singularity

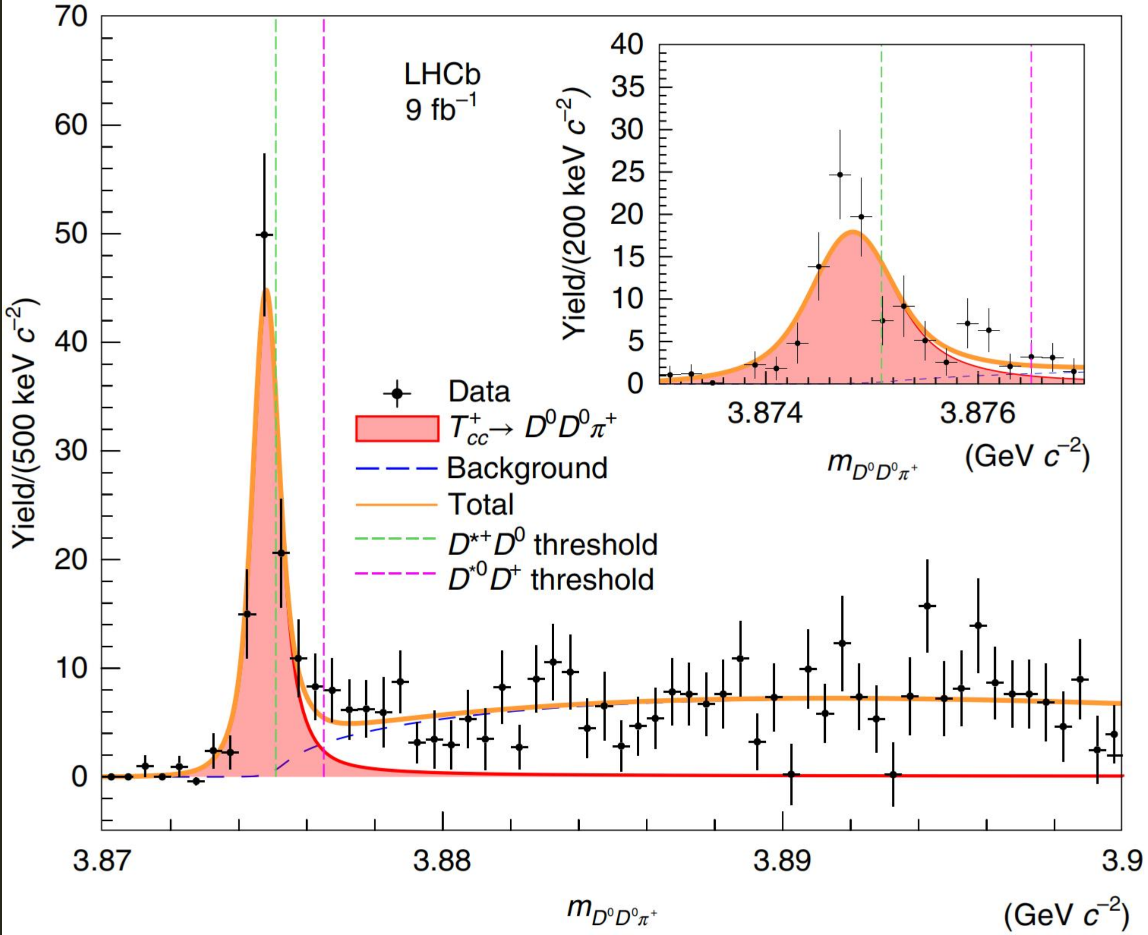
- *Black* $(|\varepsilon_T|, \Gamma_{*+}) = (0, 0)$
- *Red* $(|\varepsilon_T|, \Gamma_{*+}) = (0, 83\ keV)$
- *Purple* $(|\varepsilon_T|, \Gamma_{*+}) = (360\ keV, 0)$
- *Blue* $(|\varepsilon_T|, \Gamma_{*+}) = (360\ keV, 83keV)$



Solid: complete amplitude

Dashed: logarithmic approximation

T_{cc}^{+}(3875) or $T_{cc1}^{f}(3875)^{+}$



LHCb, Nature Physics 18, 751 (2022)

LHCb, Nature Commun. 13, 3351 (2022)

Binding Enegy & Width

$$\delta m_{\mathrm{BW}} = -273 \pm 61 \pm 5\,^{+11}_{-14}\,\mathrm{keV}\,c^{-2},$$

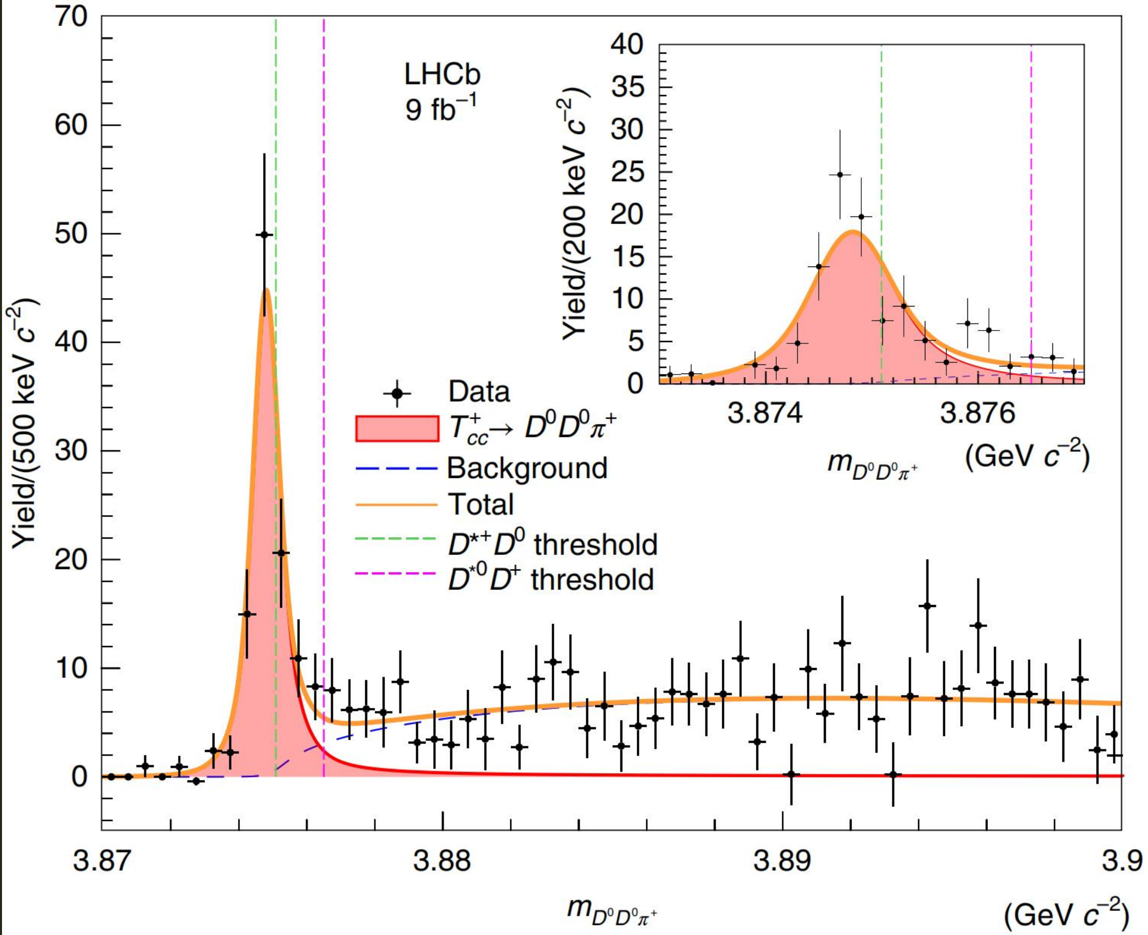
$$\Gamma_{\mathrm{BW}} = 410 \pm 165 \pm 43\,^{+18}_{-38}\,\mathrm{keV},$$

$$\delta m_{\mathrm{pole}} = -360 \pm 40^{+4}_{-0}\,\mathrm{keV}/c^2,$$

$$\Gamma_{\mathrm{pole}} = 48 \pm 2^{+0}_{-14}\,\mathrm{keV},$$

Resonant-like coupling!

T_{cc}^{+}(3875) or $T^{f}_{cc1}(3875)^{+}$



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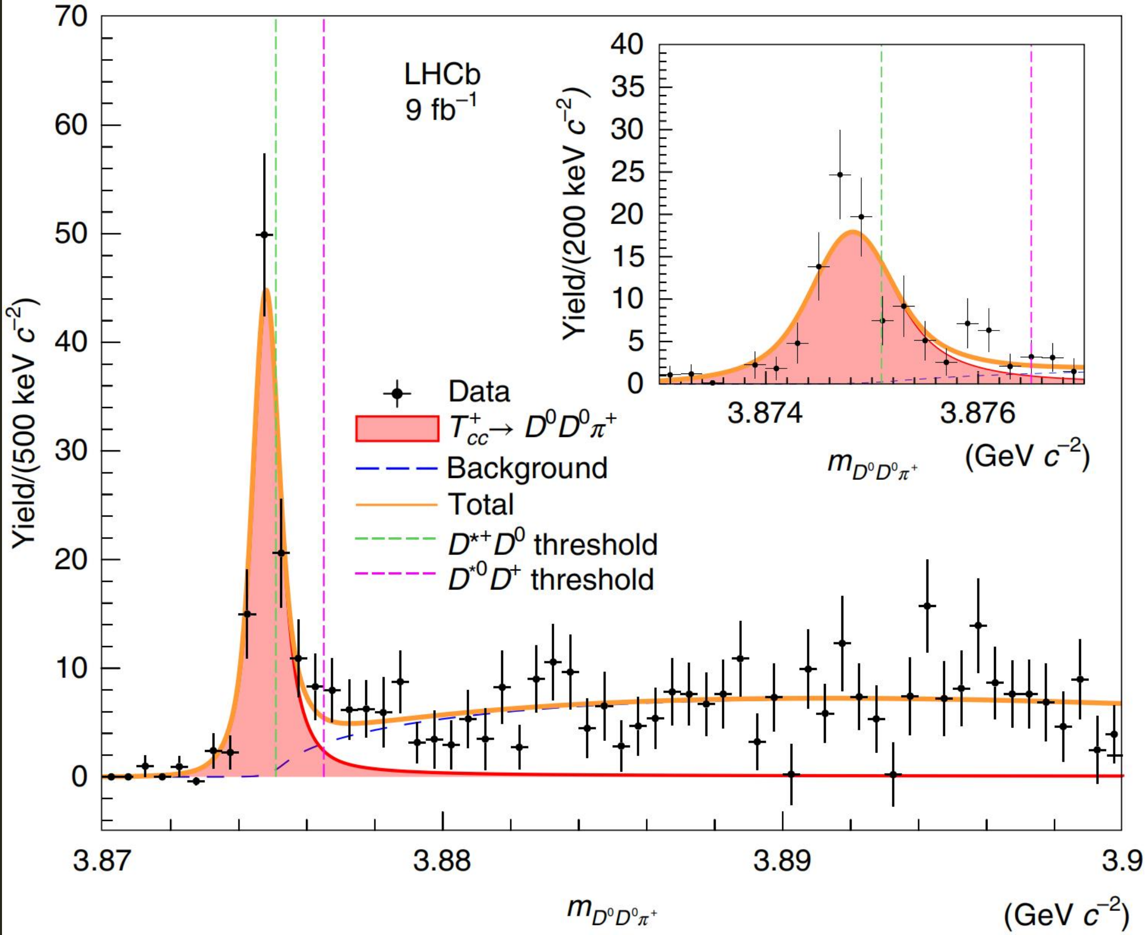
Characteristic Size

$$R_a \equiv -\Re\, a = 7.16 \pm 0.51\ \mathrm{fm}$$

$$R_{\Delta E} \equiv \frac{1}{\gamma} = 7.5 \pm 0.4\ \mathrm{fm}$$

Large!

T_{cc}^{+}(3875) or $T^{f}_{cc1}(3875)^{+}$



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Large!

J^P & I

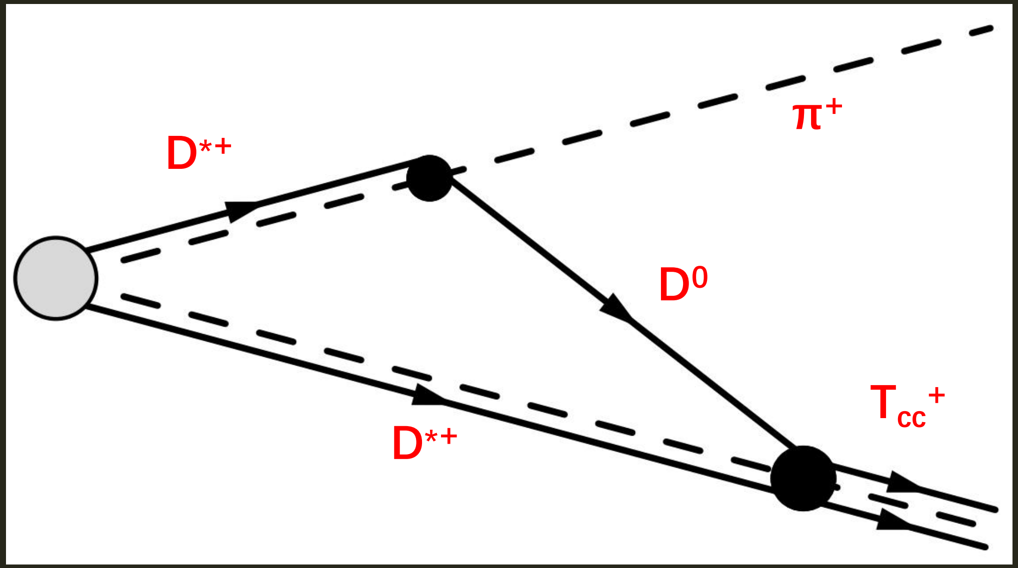
$$J^P = 1^+$$

$$I = 0$$

S-wave coupling to $D^{*+}D^0$!

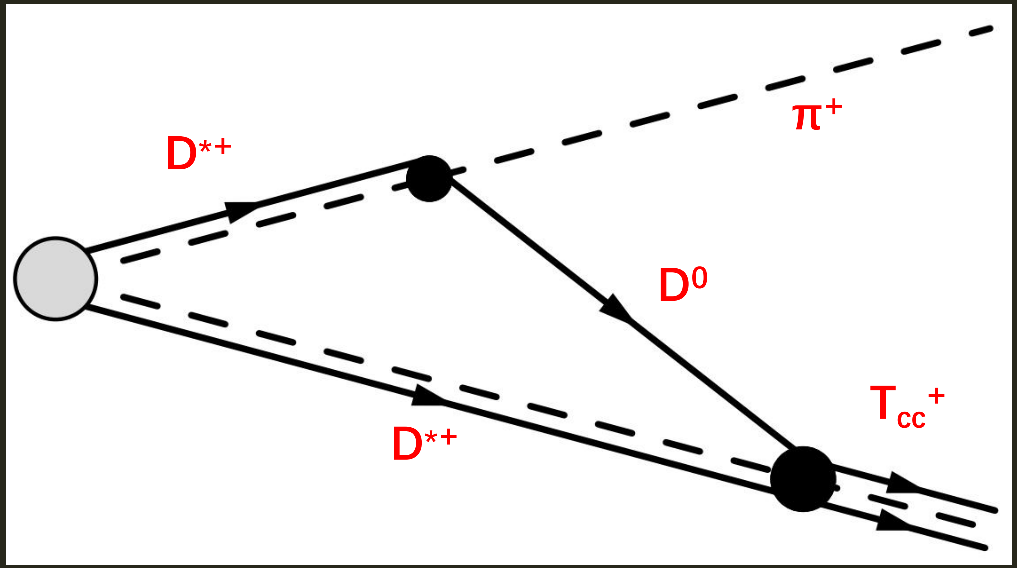
Motivation

- Better candidate as a loosely bound S-wave molecule than *X(3872)*



Motivation

- Better candidate as a loosely bound S-wave molecule than *X(3872)*
- Along with above mesearments, observation of the narrow peak from triangle singularity would support $D^{*}D$ molecule picture of T_{cc}^{+}.



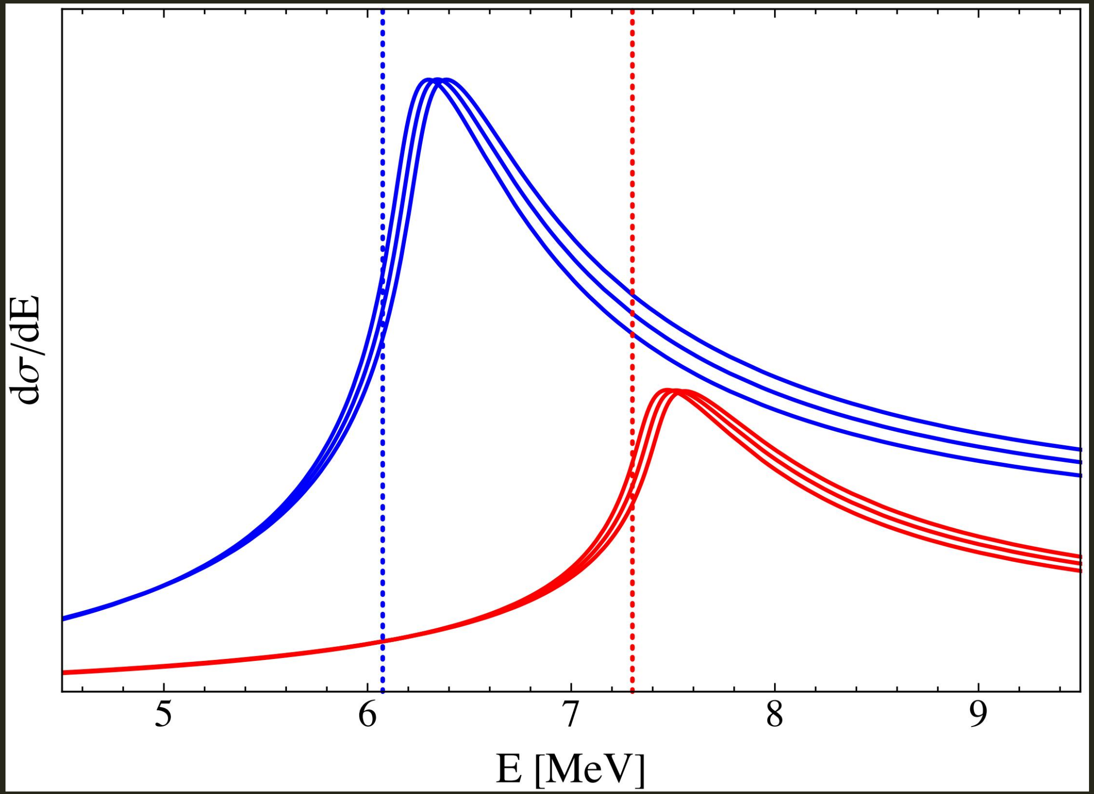
$T_{cc}^{+}: D^{*+}D^{0}$ channel only

Triangle Singularity

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{+}] = \left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_+(2\mu_{\pi T}E,\gamma^2)\right|^2,$$

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{0}] = \left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^* \right\rangle \frac{3G_\pi^2 M_T m \gamma_T}{32\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_0(2\mu_{\pi T}E,\gamma^2)\right|^2,$$

where $$\left\langle \mathcal{A}_{D^+D^0}\left(\mathcal{A}_{D^+D^0}\right)^* \right\rangle \equiv \frac{1}{\text{flux}}\sum_y \int d\Phi_{(DD)+y}\mathcal{A}_{D^+D^0+y}\left(\mathcal{A}_{D^+D^0+y}\right)^*$$



Binding energies are 320, 360, and 400 keV in order of increasing energy at the peak

2 Production of T_{cc}^{+} & Soft Pion

T_{cc}^{+}: $D^{*+}D^{0}$ channel only

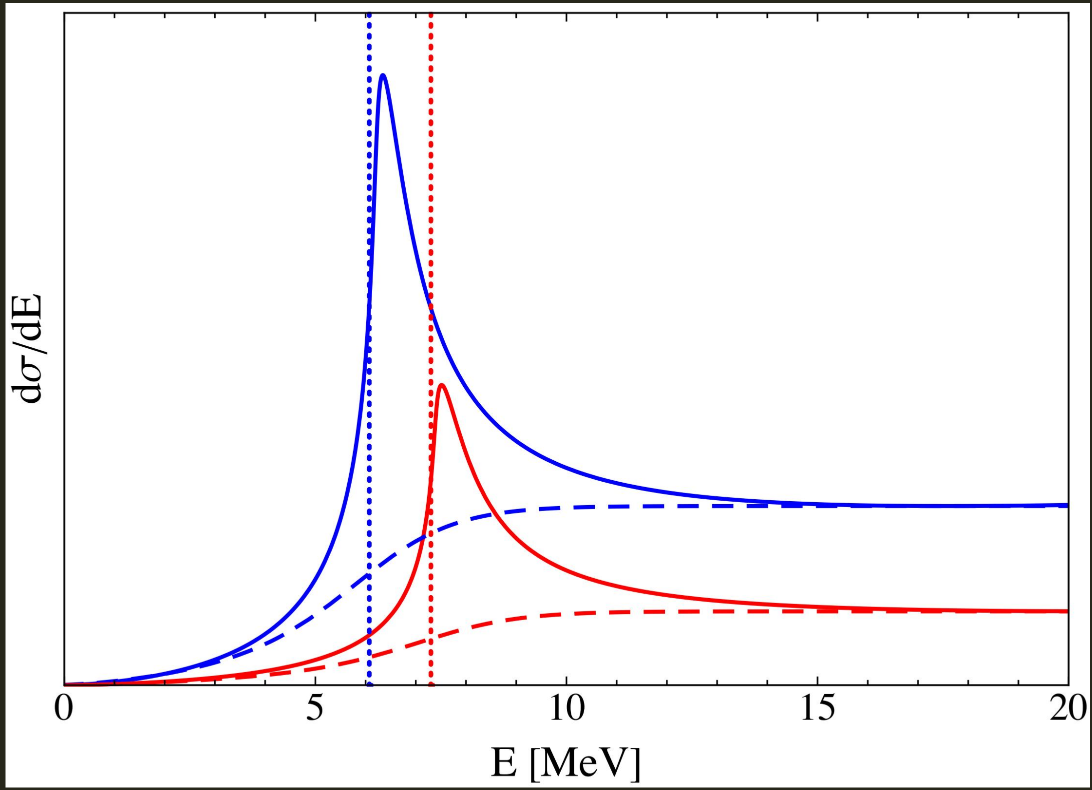
Peaks in the cross sections above background

$$\sigma\left[(T_{cc}^{+}\,\pi^{+})_{\triangle}\right] \approx (8.6 \pm 0.5)\,10^{-3}\,\left(\frac{m_\pi}{\Lambda}\right)^2 \sigma[T_{cc}^{+}, \mathrm{no}\,\pi]$$

$$\sigma\left[(T_{cc}^{+}\,\pi^{0})_{\triangle}\right] \approx (4.8 \pm 0.2)\,10^{-3}\,\left(\frac{m_\pi}{\Lambda}\right)^2 \sigma[T_{cc}^{+}, \mathrm{no}\,\pi]$$

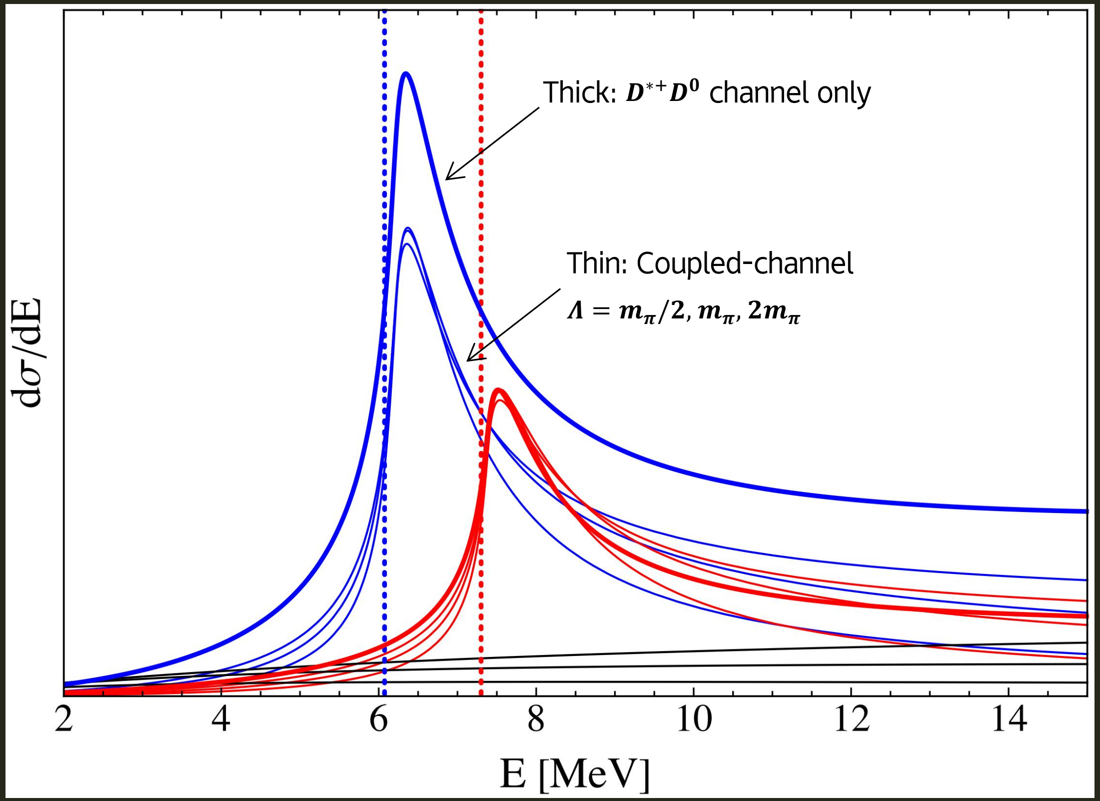
Where $\sigma[T_{cc}^{+}, \mathrm{no}\,\pi] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{3}{2\mu} \left|\psi_T(r=0)\right|^2$

is cross section for T_{cc}^{+} without any pion with relative momentum smaller than a ultraviolet cutoff.



2 Production of T_{cc}^+ & Soft Pion

T_{cc}^+: $(D^{*+}D^0 - D^{*0}D^+)/\sqrt{2}$ coupled-channel



Blue, red and black for $T_{cc}^+\pi^+$, $T_{cc}^+\pi^0$ and $T_{cc}^+\pi^-$, respectively.

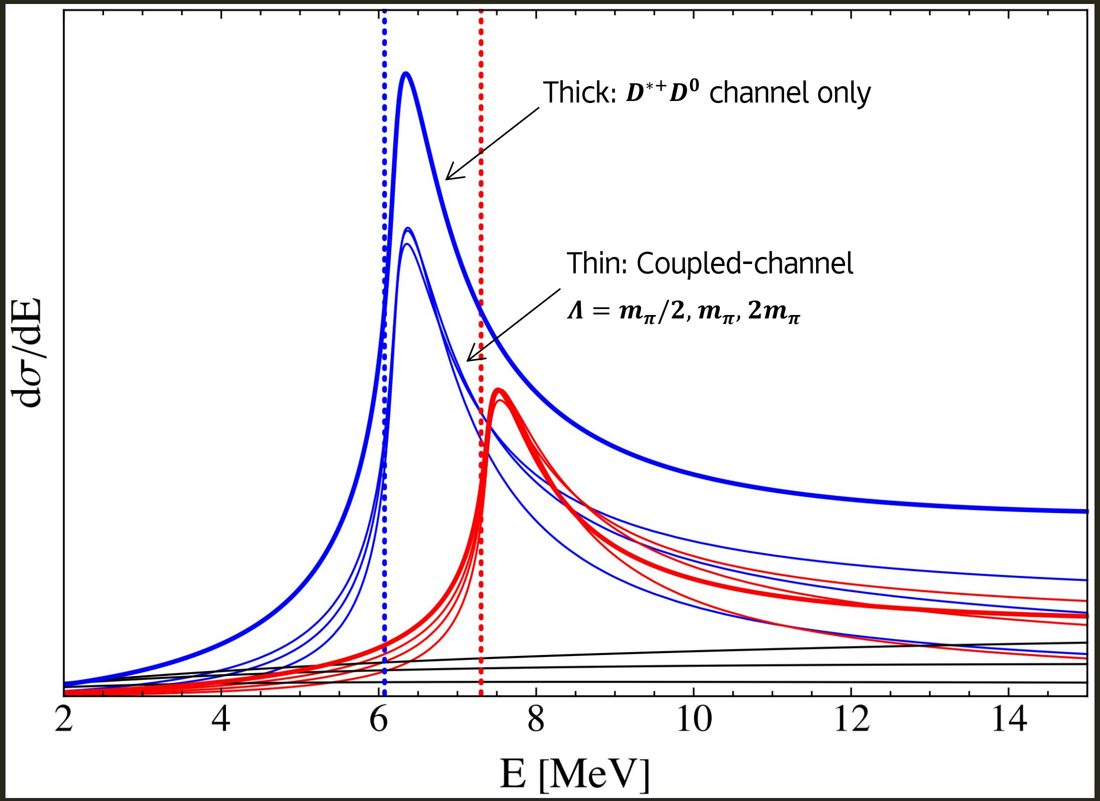
Triangle Singularity

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_+^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2,$$

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^0] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{3G_\pi^2 M_T m \gamma_T}{32\pi^2}(2\mu_{\pi T}E)^{3/2} \times \left(\left|T_0^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2 + \left|T_0'^{(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2\right),$$

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^-] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_-^{(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2.$$

T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$ coupled-channel



Blue, red and black for $T_{cc}^{+}\pi^{+}$, $T_{cc}^{+}\pi^{0}$ and $T_{cc}^{+}\pi^{-}$, respectively.

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$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{0}] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{3G_\pi^2 M_T m \gamma_T}{32\pi^2}(2\mu_{\pi T}E)^{3/2} \times \left(\left|T_{0}^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2 + \left|T_{0}^{\prime(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2\right),$$

$$\frac{d\sigma}{dE}[T_{cc}^{+}\pi^{-}] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_{-}^{(\Lambda)}(2\mu_{\pi T}E,\gamma_{0+}^2)\right|^2.$$

√ No triangle singularity in the production of $T_{cc}^{+}\pi^{-}$ channel, because the mass of D^{*0} is 2.4 MeV below $D^{+}\pi^{-}$ threshold which prevents the D^{*0} and D^{+} from being simultaneously on shell.

2 Production of T_{cc}^+ & Soft Pion

T_{cc}^+: $(D^{*+}D^0 - D^{*0}D^+)/\sqrt{2}$ coupled-channel

Asymptotic behaviors at large E

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \text{no}\,\pi]\,\frac{8G_\pi^2\mu_{\pi T}^2\mu_\pi}{3\pi}(2\mu_{\pi T}E)^{-1/2},$$
$$\frac{d\sigma}{dE}[T_{cc}^+\pi^0] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \text{no}\,\pi]\,\frac{2G_\pi^2\mu_{\pi T}^2\mu_\pi}{\pi}(2\mu_{\pi T}E)^{-1/2},$$
$$\frac{d\sigma}{dE}[T_{cc}^+\pi^-] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+, \text{no}\,\pi]\,\frac{8G_\pi^2\mu_{\pi T}^2\mu_\pi}{3\pi}(2\mu_{\pi T}E)^{-1/2}.$$

2 Production of T_{cc}^{+} & Soft Pion

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✓ Cross sections integrated up to E_{max} much larger than triangle-singularity energy

$$\sigma\left[T_{cc}^{+}\,\pi^{+}\right] \approx \left(3.2\sqrt{\frac{E_{\max}}{m_{\pi}}} - 0.0^{+1.8}_{-1.3}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^{+}, \text{no}\,\pi\right],$$
$$\sigma\left[T_{cc}^{+}\,\pi^{0}\right] \approx \left(2.4\sqrt{\frac{E_{\max}}{m_{\pi}}} - 0.0^{+1.3}_{-1.0}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^{+}, \text{no}\,\pi\right],$$
$$\sigma\left[T_{cc}^{+}\,\pi^{-}\right] \approx \left(3.2\sqrt{\frac{E_{\max}}{m_{\pi}}} - 1.3^{+0.3}_{-0.5}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^{+}, \text{no}\,\pi\right].$$

Errors from $\Lambda = 2^{0\pm1}m_{\pi}$

2 Production of T_{cc}^{+} & Soft Pion

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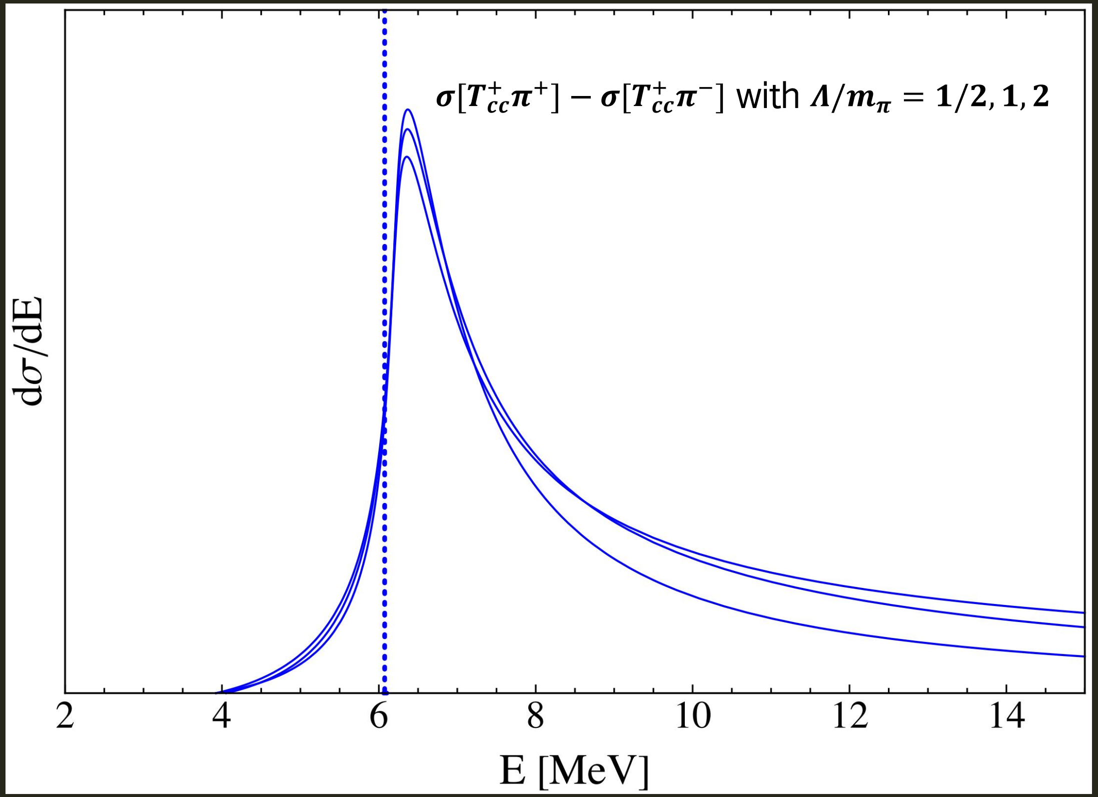

Peaks in the cross sections above background

$$\sigma\left[T_{cc}^{+}\,\pi^{+}\right] - \sigma\left[T_{cc}^{+}\,\pi^{-}\right] \approx \left(1.3^{+1.5}_{-0.8}\right) \times 10^{-2}\,\sigma^{(\Lambda)}\left[T_{cc}^{+}, \text{no}\,\pi\right]$$

2 Production of T_{cc}^+ & Soft Pion

T_{cc}^+: $(D^{*+}D^0 - D^{*0}D^+)/\sqrt{2}$ coupled-channel

LHCb data

- $\mathbf{117 \pm 16}$ events
- Inclusive production: $\boldsymbol{\sigma}^{\Lambda}[T_{cc}^+, no\ \pi] + \boldsymbol{\sigma}[T_{cc}^+\pi^+] + \boldsymbol{\sigma}[T_{cc}^+\pi^0] + \boldsymbol{\sigma}[T_{cc}^+\pi^-]$ with $E < E_{max} = q_{max}^2/(2\mu_{\pi T})$
- Fractions of events for $T_{cc}^+\pi^+$ and $T_{cc}^+\pi^-$:$(3.0^{+1.5}_{-1.2})$% and $(1.8^{+0.2}_{-0.4})$%, respectively

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- Fractions of events for $T_{cc}^{+}\pi^{+}$ and $T_{cc}^{+}\pi^{-}$: $(3.0_{-1.2}^{+1.5})\%$ and $(1.8_{-0.4}^{+0.2})\%$, respectively
- Fractions of events in the peak from triangle singularity for $T_{cc}^{+}\pi^{+}$: $(1.2_{-0.7}^{+1.3})\%$

  Small but all with energy within 1 MeV of the peak (6.1 MeV above threshold)

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Summary

- We used the coupled-channel model to calculate the cross sections for $T_{cc}^{+}\pi^{+}$, $T_{cc}^{+}\pi^{0}$ and $T_{cc}^{+}\pi^{-}$ at energies near the triangle-singularity peaks and at higher energies.

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- Backgrounds can be determined experimentally by measuring $T_{cc}^{+}\pi^{-}$ events.
- Fraction of $T_{cc}^{+}\pi^{+}$ events in the narrow peak is $(1.2^{+1.3}_{-0.7})\%$, all within $1\ MeV$ of the peak at $6.1\ MeV$.

Summary

- We used the coupled-channel model to calculate the cross sections for $T_{cc}^{+}\pi^{+}$, $T_{cc}^{+}\pi^{0}$ and $T_{cc}^{+}\pi^{-}$ at energies near the triangle-singularity peaks and at higher energies.
- Backgrounds can be determined experimentally by measuring $T_{cc}^{+}\pi^{-}$ events.
- Fraction of $T_{cc}^{+}\pi^{+}$ events in the narrow peak is $(1.2^{+1.3}_{-0.7})\%$, all within $1\ MeV$ of the peak at $6.1\ MeV$.
- Obervation of the narrow peak of triangle singularity supports molecule picture of $T_{cc}{}^{+}$.

Discussion

Molecule *vs* Commpact tetraquark

- Well above the triangle singularity energy E_{Δ}, $d\sigma/dE$ for $\boldsymbol{T_{cc}^{+}\pi^{+}}$ decreases as $E^{-1/2}$.

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- A compact tetraquark T_{cc}^{+} would have to have a suppressed coupling to $D^{*+}D^{0}$. Goldstone nature of the pion requires the production amplitude of $T_{cc}^{+}\pi$ to be proportional to the relative momentum of the pion. Therefore $d\sigma/dE$ should increase like $E^{3/2}$.

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Molecule *vs* Commpact tetraquark

- Well above the triangle singularity energy E_{Δ}, $d\sigma/dE$ for $\boldsymbol{T_{cc}^{+}\pi^{+}}$ decreases as $E^{-1/2}$.
- A compact tetraquark $\boldsymbol{T_{cc}^{+}}$ would have to have a suppressed coupling to $D^{*広}D^{0}$. Goldstone nature of the pion requires the production amplitude of $\boldsymbol{T_{cc}^{+}}\pi$ to be proportional to the relative momentum of the pion. Therefore $d\sigma/dE$ should increase like $E^{3/2}$.
- Measurements on the differential cross sections above the triangle singularity energy E_{Δ} provide important clues to the nature of $\boldsymbol{T_{cc}^{+}}$.

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THANK YOU

Triangle Singularity

in the Production of T_{cc}^{+} and a Soft Pion

汇报人：蒋军

Triangle amplitudes

$$T_+(q^2,\gamma^2) = \left(1+\frac{mb}{2M_T c}\right)\frac{1}{\sqrt{c}}\log\frac{\sqrt{a}+\sqrt{c}+\sqrt{a+b+c}}{\sqrt{a}-\sqrt{c}+\sqrt{a+b+c}} + \frac{m}{M_T c}\left(\sqrt{a}-\sqrt{a+b+c}\right)$$

$$a = (\mu/\mu_\pi)q^2 - M_* E_+,$$
$$b = -2(\mu/\mu_\pi)(\mu/M)q^2 + M_* E_+ - \gamma^2,$$
$$c = (\mu/M)^2 q^2.$$

$T_0(q^2,\gamma^2)$ can be obtained by replacing E_+ by E_0: $E_0 = \delta_{00} - \varepsilon_T - i\,(\Gamma_{*0} + \Gamma_{*+})/2$

Logarithmic approximationcan

$$T_+^{(\log)}(q^2,\gamma^2) = \sqrt{\frac{M/M_T}{\mu_{\pi T}\delta_{0+}}}\left(\frac{2M}{M_*}\log\frac{\sqrt{a}+(\mu/M)q+i\,\gamma}{\sqrt{a}-(\mu/M)q+i\,\gamma} + \frac{m}{M_*}\right)$$

Triangle amplitudes in coupled-channel model

$$T_+^{(\Lambda)}(q^2,\gamma^2) = \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma)}\left[T_+(q^2,\gamma^2) - T_+(q^2,\Lambda^2)\right]$$

$$T_0^{(\Lambda)}(q^2,\gamma^2) = \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma)}\left[T_0(q^2,\gamma^2) - T_0(q^2,\Lambda^2)\right]$$

$$T_0'^{(\Lambda)}(q^2,\gamma_{0+}^2) = -\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma_{0+})}\left[T_0(q^2,\gamma_{0+}^2) - T_0(q^2,\Lambda^2)\right]$$

$$T_-^{(\Lambda)}(q^2,\gamma_{0+}^2) = -\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma_{0+})}\left[T_-(q^2,\gamma_{0+}^2) - T_-(q^2,\Lambda^2)\right]$$

where $T_-(q^2,\gamma_{0+}^2)$ can be obtained by the right side of $T_+(q^2,\gamma^2)$ with the coefficients

$$a = (\mu/\mu_\pi)q^2 - M_* E_-,$$
$$b = -2(\mu/\mu_\pi)(\mu/M)q^2 + M_* E_- - \gamma_{0+}^2,$$
$$c = (\mu/M)^2 q^2.$$

with $E_- = \delta + \delta_{+-} - \varepsilon_T - i\,\Gamma_{*0}$ and $\delta_{+-} = M_{*0} - M_+ - m_- = -2.38\ \text{MeV}$

Asymptotic behavior of triangle amplitudes at large E

$$T_+(q^2,\gamma^2) \longrightarrow \left(\frac{M}{M_*} \log \frac{\sqrt{M_T/m}+1}{\sqrt{M_T/m}-1} + \frac{\sqrt{M_T m}}{M_*} \right) \frac{1}{q} - \frac{2iM_T m\gamma}{M_*^2 q^2}$$

Subtraction cancels the terms that decrease as $1/q$, so the triangle amplitude decreases as $1/q^2$:

$$T_+^{(\Lambda)}(q^2,\gamma^2) \longrightarrow i\,\frac{2\sqrt{(\Lambda+\gamma)\Lambda}\,M_T m}{\sqrt{1+Z_{0+}}\,M_*^2\,q^2}.$$

or

$$T_+^{(\Lambda)}(q^2,\gamma^2) \longrightarrow i\,\frac{4\mu_{\pi T}}{M_*\sqrt{\gamma_T/2\pi}}\,\frac{\psi_T^{(\Lambda)}(r=0)}{\sqrt{1+Z_{0+}}}\,\frac{1}{q^2}.$$

We verify that this gives the large-q^2 limit for a general wavefunction with a finite wavefunction at the origin.

Why is smooth cutoff wavefunction more physical at large E?

- We would like to estimate the cross sections at large E, so we should make sure the asymptotic behavior of $d\sigma/dE$ is correct.
- What is correct asymptotic behavior?

low-energy scattering of the two particles $f(E) = \frac{1}{-\gamma + \sqrt{-2\mu E}}$ with $E = k^2/(2\mu) + i\epsilon.$

$$\mathrm{Im}[f(E+i\epsilon)] = \frac{\pi\gamma}{\mu}\delta(E+\gamma^2/2\mu) + \frac{\sqrt{2\mu E}}{\gamma^2 + 2\mu E}\theta(E)$$

Delta function: the production of the bound state.

Theta function: the production of the two particles above the threshold, $\propto E^{-1/2}$.

Why is smooth cutoff wavefunction more physical at large E?

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] = \left\langle \mathcal{A}_{D^+D^0}(\mathcal{A}_{D^+D^0})^* \right\rangle \frac{G_\pi^2 M_T m \gamma_T}{4\pi^2}(2\mu_{\pi T}E)^{3/2}\left|T_+^{(\Lambda)}(2\mu_{\pi T}E,\gamma^2)\right|^2$$

At large E, since $T_+^{(\Lambda)}(q^2,\gamma^2) \longrightarrow i\,\frac{2\sqrt{(\Lambda+\gamma)\Lambda}\,M_T m}{\sqrt{1+Z_{0+}}\,M_*^2\,q^2}.$

so we have the physical asymptotic behavior:

$$\frac{d\sigma}{dE}[T_{cc}^+\pi^+] \longrightarrow \sigma^{(\Lambda)}[T_{cc}^+,\text{no }\pi]\,\frac{8G_\pi^2\mu_{\pi T}^2\mu_\pi}{3\pi}(2\mu_{\pi T}E)^{-1/2}$$

Why is smooth cutoff wavefunction more physical at large E?

$$T_{+}^{(\Lambda)}(q^2,\gamma^2) = \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\sqrt{1+Z_{0+}}\,(\Lambda-\gamma)} \left[T_{+}(q^2,\gamma^2) - T_{+}(q^2,\Lambda^2)\right]$$

where $$T_{+}(q^2,\gamma^2) \longrightarrow \left(\frac{M}{M_*}\log\frac{\sqrt{M_T/m}+1}{\sqrt{M_T/m}-1} + \frac{\sqrt{M_T m}}{M_*}\right)\frac{1}{q} - \frac{2iM_T m\gamma}{M_*^2 q^2}$$

Subtraction cancels terms that decrease as $1/q$, so the triangle amplitude decreases as $1/q^2$

Recall the replacement that considers both the smooth cutoff and doubled channel:

For $D^{*+}D^0$ configuration,

D^0 propagator replacement $$\frac{1}{k^2+\gamma^2} \longrightarrow \frac{1}{\sqrt{1+Z_{0+}}}\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2}\right)$$

Why is smooth cutoff wavefunction more physical at large E?

Smooth cutoff wavefunction leads the correct asymptotic behavior!

The D propagator replacement

$$\frac{1}{k^2+\gamma^2} \longrightarrow \frac{1}{\sqrt{1+Z_{0+}}}\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2}-\frac{1}{k^2+\Lambda^2}\right)$$

is equivalent to the universal wavefunction replacement

$$\psi(k)=\frac{\sqrt{8\pi\gamma}}{k^2+\gamma^2} \longrightarrow \psi^{(\Lambda)}(k)=\frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2}-\frac{1}{k^2+\Lambda^2}\right)$$

XEFT

- Effective field theory for charm mesons and pions
- Validity: kinetic energy of pions $\sim m_\pi$, kinetic energy of charm mesons $\sim m_\pi^2/M \sim 10MeV$

Fleming, Kusunoki, Mehen and Van Kolck, PRD 76, 034006 (2007)

XEFT

- Effective field theory for charm mesons and pions
- Validity: kinetic energy of pions $\sim m_\pi$, kinetic energy of charm mesons $\sim m_\pi^2/M \sim 10MeV$
- Galilean-invariant formulation of XEFT: conservation of pion and (anti)charm numbers, conservation of kinetic masses

  Pion number: sum of the numbers of $\pi, D^*, \bar{D}^*$ mesons

Fleming, Kusunoki, Mehen and Van Kolck, PRD 76, 034006 (2007)
Braaten, PRD 91, 114007 (2015)
Braaten, He, Jiang, PRD 103, 036014 (2021)

Wavefunction for loosely bound S-wave molecule

Sharp UV cutoff

Spatial wavefunction:

$$\psi(r) = \frac{\sqrt{\gamma/2\pi}}{r}\exp(-\gamma r)$$

Momentum-space:

$$\psi(k) = \frac{\sqrt{8\pi\gamma}}{k^2+\gamma^2}$$

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$

Braaten, Hammer, Phys. Rept. 428, 259 (2006)

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UV divergent integral:

$$\psi(r=0) = \int d^3k\, \psi(k)/(2\pi)^3$$

Sharp momentum cutoff:

$$|\boldsymbol{k}| < (\pi/2)\Lambda \text{ with } \Lambda \gg \gamma$$

At the origin:

$$\psi(r=0) = (\Lambda - \gamma)\sqrt{\gamma/2\pi}.$$

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$.

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Wavefunction
for loosely bound S-wave molecule

Smooth UV cutoff

Smooth momentum cutoff:

$$\psi^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2}\right)$$

At the origin:

$$\psi^{(\Lambda)}(r=0) = \sqrt{(\Lambda+\gamma)\Lambda\gamma/2\pi}$$

At large k:

$$\psi^{(\Lambda)}(k) \longrightarrow \sqrt{8\pi(\Lambda+\gamma)^3\Lambda\gamma}/k^4$$

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$

Suzuki, PRD 72, 114013 (2005)

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At the origin:

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At large k:

$$\psi^{(\Lambda)}(k) \longrightarrow \sqrt{8\pi(\Lambda+\gamma)^3\Lambda\gamma}/k^4$$

Why?

Same momentum dependence at small k

More physical qualitative behavior at large k, so we can make preditions.

Binding momentum: $\gamma = \sqrt{2\mu|\varepsilon|}$.

Suzuki, PRD 72, 114013 (2005)

Isoscalar T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$

Coupled-channel wavefunction

$$\psi_{\mathrm{cc}}^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda-\gamma_{cc}}\left(\frac{1}{k^2+\gamma_{\mathrm{cc}}^2} - \frac{1}{k^2+\Lambda^2}\right)$$

$$\gamma_{\mathrm{cc}} = \sqrt{2\mu(\delta+|\varepsilon|)}$$

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  $$\gamma_{\rm cc} = \sqrt{2\mu(\delta + |\varepsilon|)}$$

- Coupled-channel model for a loosely bound molecule with two channels related by symmetry
  $$\psi_{\rm cc}^{(\Lambda)}(r=0) = \psi^{(\Lambda)}(r=0)$$

Isoscalar T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$

Coupled-channel wavefunction

$$\psi_{\rm cc}^{(\Lambda)}(k) = \frac{\sqrt{8\pi(\Lambda+\gamma)\Lambda\gamma}}{\Lambda - \gamma_{\rm cc}} \left(\frac{1}{k^2+\gamma_{\rm cc}^2} - \frac{1}{k^2+\Lambda^2} \right)$$

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Coupled-channel model for a loosely bound molecule with two channels related by symmetry

$$\psi_{\rm cc}^{(\Lambda)}(r=0) = \psi^{(\Lambda)}(r=0)$$

Relative probability for the coupled-channel wavefunction

$$Z_{\rm cc} \equiv \int \frac{d^3k}{(2\pi)^3} \left|\psi_{\rm cc}^{(\Lambda)}(k)\right|^2 = \frac{(\Lambda+\gamma)\gamma}{(\Lambda+\gamma_{\rm cc})\gamma_{\rm cc}}$$

$\Lambda = m_\pi/2, m_\pi, 2m_\pi$, $Z_{0+} = 0.34, 0.38, 0.41$, respectively.

Isoscalar T_{cc}^{+}: $(D^{*+}D^{0} - D^{*0}D^{+})/\sqrt{2}$

"Feynman rules" : smooth cutoff + coupled channel

- $D^{*+}D^{0}$ configuration, D^{0} propagator replacement

$$\frac{1}{k^2+\gamma^2} \longrightarrow \frac{1}{\sqrt{1+Z_{0+}}}\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma}\left(\frac{1}{k^2+\gamma^2}-\frac{1}{k^2+\Lambda^2}\right)$$

- $D^{*0}D^{+}$ configuration, D^{+} propagator replacement

$$\frac{1}{k^2+\gamma_{0+}^2} \longrightarrow -\frac{1}{\sqrt{1+Z_{0+}}}\frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma_{0+}}\left(\frac{1}{k^2+\gamma_{0+}^2}-\frac{1}{k^2+\Lambda^2}\right)$$

2 Coupled-channel Model

Isoscalar T_{cc}^+: $(D^{*+}D^0 - D^{*0}D^+)/\sqrt{2}$

"Feynman rules" : smooth cutoff + coupled channel

- $D^{*+}D^0$ configuration, D^0 propagator replacement

$$\frac{1}{k^2+\gamma^2} \longrightarrow \frac{1}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma} \left(\frac{1}{k^2+\gamma^2} - \frac{1}{k^2+\Lambda^2} \right)$$

- $D^{*0}D^+$ configuration, D^+ propagator replacement

$$\frac{1}{k^2+\gamma_{0+}^2} \longrightarrow -\frac{1}{\sqrt{1+Z_{0+}}} \frac{\sqrt{(\Lambda+\gamma)\Lambda}}{\Lambda-\gamma_{0+}} \left(\frac{1}{k^2+\gamma_{0+}^2} - \frac{1}{k^2+\Lambda^2} \right)$$

- Same vertices for $D^{*+}D^0 - T_{cc}^+$ and $D^{*0}D^+ - T_{cc}^+$

Discussion

SPS *vs* DPS

- Calculation based on assumption that charm mesons are created at short distances much smaller than the mean radius ($3.7 \pm 0.2\ fm$) of $\boldsymbol{T_{cc}^{+}}$.

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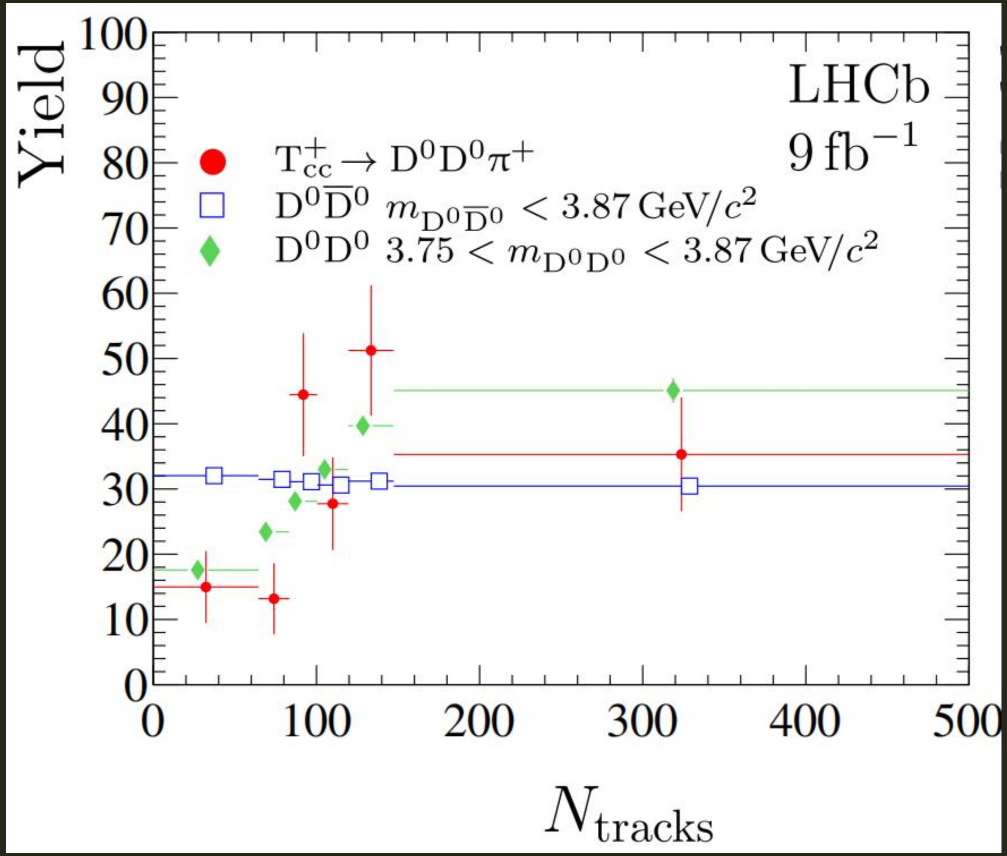
SPS *vs* DPS

- Calculation based on assumption that charm mesons are created at short distances much smaller than the mean radius ($3.7 \pm 0.2\ fm$) of T_{cc}^{+}.
- In double-parton scattering (DPS), the charm mesons maybe created at distances comparable to the radius of a proton ($\sim 1\ fm$).

Discussion

SPS *vs* DPS

- Calculation based on assumption that charm mesons are created at short distances much smaller than the mean radius ($3.7 \pm 0.2\ fm$) of T_{cc}^{+}.
- In double-parton scattering (DPS), the charm mesons maybe created at distances comparable to the radius of a proton ($\sim 1\ fm$).
- Single-parton scattering (SPS) makes the triangle-singularity peak stand out more clearly above the background.



LHCb, Nature Commun. 13, 3351 (2022)